

**Problem 1**

Construct a truth table for each of the following predicates. Which of them are logically equivalent?

- (a)  $Q \Leftrightarrow (P \vee Q)$ .
- (b)  $P \vee \neg Q$ .
- (c)  $Q \Rightarrow P$ .
- (d)  $(P \vee \neg P) \wedge (Q \Leftrightarrow (Q \wedge \neg Q))$ .

**Problem 2**

Show that the polynomial

$$p(x) = 4x^5 - 3x^3 + x^2 + 1$$

has no rational roots. *Hint: See example 3.6.3 in the Course Notes.*

**Problem 3**

Let  $S = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$ .

1. List all subsets of  $S$ . (How many subsets are there?)
2. Determine which of the following statements are true.

- |                                 |                                     |
|---------------------------------|-------------------------------------|
| • $\emptyset \subseteq S$ .     | • $\{\emptyset\} \subseteq S$ .     |
| • $\emptyset \in S$ .           | • $\{\emptyset\} \in S$ .           |
| • $\emptyset \subseteq \{S\}$ . | • $\{\{\emptyset\}\} \subseteq S$ . |
| • $\emptyset \in \{S\}$ .       | • $\{\{\emptyset\}\} \in S$ .       |

**Problem 4**

Prove that  $3^{2^n} - 1$  is divisible by 10 for all  $n \in \mathbb{N}, n \geq 2$ .