MAT102H5 - Introduction to Mathematical Proofs - Summer 2021 -UTM

Problem Set 5 - TO BE SUBMITTED SAT. JUNE 12 , 11:59pm

Submit the following 4 problems in Crowdmarks. Late submission will be given 0, no excuse will be accepted. You are supposed to submit earlier than the deadline so in case of any issue you will have time to deal with it.

Only 2 or 3 problems will be graded. The problem set is 8 points. 6 points for problems, and 2 points for clean and neat writing *and for submitting at least 3 of the problems.* **To get the 2 points, you must:**

submit at least 3 problems.
submit each question on its place in Crowdmark,
write the answer of each question in a separate sheet,
you will not get the point if your handwriting is hard to read, DO NOT write with script handwriting.

Question 1. Use **contrapositive** to prove the following statements.

a) Let m and n be two integers. Prove that if $m^2 + n^2$ is divisible by 4, then both m and n are even numbers.

b) Let $x, y \in \mathbb{R}$. If $x \neq y$, then $\frac{x}{\sqrt{x^2+1}} \neq \frac{y}{\sqrt{y^2+1}}$.

Question 2. Prove that the following equation has **no rational** solutions.

$$x^5 + 3x^3 + 7 = 0$$

Question 3. Prove that a natural number is divisible by 9 if and only if its sum of digits is divisible by 9.

Question 4. Prove that the equation $x^2+x+1 = y^2$ has no natural solutions. (Multiply by 4 and complete the square).