Assignment 1

(due on Fri Jan 29th at 11:59pm)

Rules:

- **You do the assignment YOURSELF**
- **You may consult classmates for hints, but do not copy others' solutions**
- **Do not share full solution ideas**
- **Answers on Piazza should follow the same spirit (just hints)**
- **Mention your references**

1. Let $M = (Q, \Gamma, \Sigma, \delta, q_0, q_{accept}, q_{reject}, \sqcup)$ be a Turing machine on the alphabet $\Sigma = \{0,1\}$, with the set of states Q, the tape alphabet Γ, and the transition function δ given in the table below:

	0	1	$\sqcup$
q_0	$q_1, \sqcup, R$	q_{reject}	q_{accept}
q_1	$q_1, 0, R$	$q_1, 1, R$	$q_2, \sqcup, L$
q_2	q_{reject}	$q_3, \sqcup, L$	q_{accept}
q_3	$q_3, 0, L$	$q_3, 1, L$	$q_0, \sqcup, R$

   (a) Does *M* accept or reject 0011? Does *M* accept or reject 0101? [1 mark]
   (b) Describe the language which *M* decides. [2 marks]

2. Prove that multiplication is primitive recursive [4 marks]

3. Let *A, B* be two computable sets of natural numbers. Show that $A \cap B$ is also computable. You may appeal to the Church-Turing thesis. [2 marks]

4. If A is computable, prove that $\bar{A}$ is also computable. [1 mark]