Assignment 2

Due on Feb 12

Q1 [2 marks]

You saw in the tutorials that an infinite set is c.e. iff it is the range of a 1:1 computable function. If we replace c.e. by computable, we can replace 1:1 by strictly increasing.

In other words, an infinite set is computable iff it is the range of a strictly increasing computable function. Prove that statement.

Recall: a function f is strictly increasing if $x < y$ implies that $f(x) < f(y)$

Q2 [1 mark]

Use Q1 to show that every infinite c.e. set contains and infinite computable subset.

Q3 [2 marks]

Prove the **Padding Lemma**: *Every partial computable function has infinitely many indices*. In other words, for any given partial computable function f, the set $\{e: f = \varphi_e\}$ is infinite.

Don't mix this with the fact that every program has a unique Gödel number. There could be more than one program to compute the same function.

Q4 [2 marks]

You saw that the set of partial computable functions is c.e. Now, show that the set of (total) computable functions is NOT c.e.

Hint: Assume it is c.e. This implies that for some computable f, $\{\varphi_{f(e)}: e \in \mathbb{N}\}$ is the set of computable functions (why?). This will imply that the function given by $g(x) = \varphi_{f(x)}(x) + 1$ is computable (why?). This gives a contradiction (explain it).

Q5 [3 marks]

(i) Prove that the set of c.e. sets is countable [2 marks].
(ii) Prove that $|P(\mathbb{N})|$ is uncountable [1 mark].