CSC363H5 Tutorial 5

warning: do not attempt to learn social skills from me

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Learning objectives this tutorial

By the end of this tutorial, you should...

- Be able to come up with terrible CSC363 flirtatious quotes that are almost as bad as mine.
- Be able to state what $A \leq_m B$[1] means.
- Understand why if $A \leq_m B$ and A is c.e., then so is B.
- Appreciate the fact that reading week is in 3 days, and then realize your assignment is also in 3 days ;-;[2]

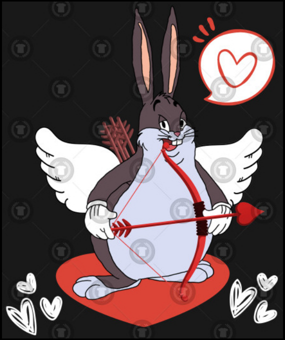
[1] This is read "A is m-reducible to B".
[2] so ask me any questions you have!

Some readings (again, certified by `helo_fish.jpg`)

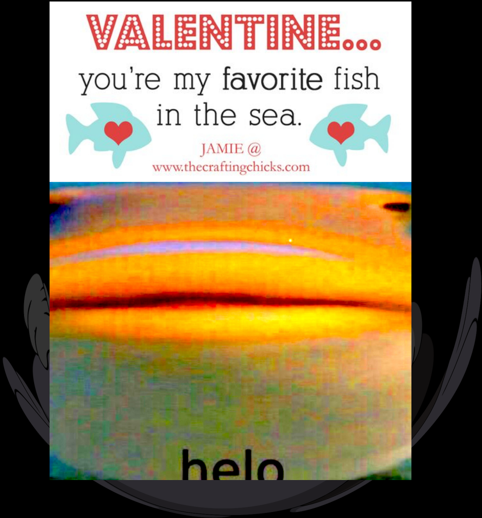
- Chapter 7.1, 7.2 (up to page 107)
- Chapter 10.1, 10.2, 10.3

Again, read those to cheat on the homework! honestly though, it would really help with the homework questions, and it contains a solution to at least one of the homework questions.

here's valentines day chungus <3



pls ignore watermarks.
because i'm low budget.



DISCLAIMER

DO NOT attempt to use any of the terrible pick-up lines you encounter in this tutorial, labelled in red. You have been warned.[3]

Using these pick-up lines may result in:

- Being called to the principal's office.
- Lovesickness, emotional pain, melancholy.
- Severe social withdrawal and repulsion.
- Prosecution via the *Copyright Act* (or whatever copyright policy your jurisdiction has).
- Forfeiture of privilege of eating sushi (or whatever your favourite food is).
- Music torture via the song "Big Chungus".

[3] I do not make any copyright claims to any of these awkward flirting lines.

Just a quick note for Q4 of the assignment!

hopefully you have started the assignment already! D:

in Q4, by "the set of partial computable functions is c.e.", we mean the set $\{e \in \mathbb{N} : \varphi_e \text{ is p.c.}\}$ is c.e.. By "the set of (total) computable functions is not c.e.", we mean the set $\{e \in \mathbb{N} : \varphi_e \text{ is total}\}$ is not c.e..

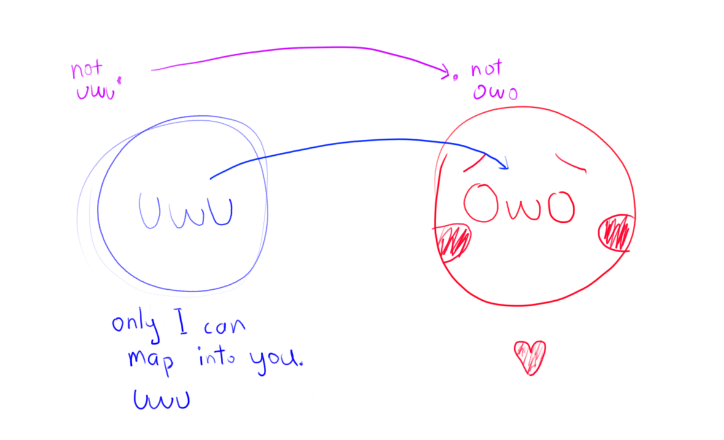
Task: prove that the set of good memories we will create is not computably enumerable.

m-reduction?

Let $A, B \subseteq \mathbb{N}$ (as always!). We say $A \leq_m B$ (read "A is *m*-reducible to B") if there exists a *computable* function f such that

$$x \in A \Leftrightarrow f(x) \in B.$$

Note: f doesn't have to be a bijection! it doesn't even have to be injective.



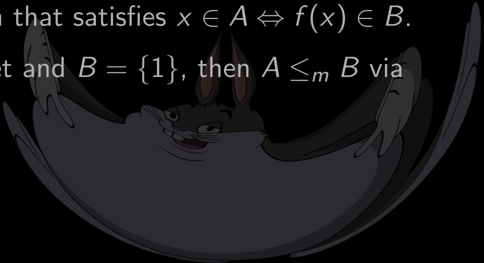
m-reduction?

Let $A, B \subseteq \mathbb{N}$ (as always!). We say $A \leq_m B$ (read "A is *m*-reducible to B") if there exists a *computable* function f such that

$$x \in A \Leftrightarrow f(x) \in B.$$

Example: if $A = \{0, 2, 4, \ldots\}$ and $B = \{0, 4, 8, \ldots\}$. Then $A \leq_m B$, since $f(x) = 2x$ is a computable function that satisfies $x \in A \Leftrightarrow f(x) \in B$.

Example: if A is any computable set and $B = \{1\}$, then $A \leq_m B$ via $f(x) = I_A(x)$.



m-reduction?

Are you a natural number? Cuz I am, and we can apply the Cantor pairing function

Again, do not attempt to use these lines. I take no responsibility for any potential injuries you may incur from using these quotes.

(either way, I hope you recall the Cantor pairing function!)

m-reduction? 

Task: Let $K_0 = \{\langle x, y\rangle : \varphi_x(y)\downarrow\}$. Show that $K \leq_m K_0$ by finding a computable function f such that $x \in K \Leftrightarrow f(x) \in K_0$.[4]

Task: Show that I am *m*-reducible to you. Conclude that there exists a computable function f that maps me to you exclusively. <3

Task: Show that my feelings for you are in $\overline{K}$.

cursed smiley.

[4] $K = \{x : \phi_x(x)\downarrow\}$.

m-reduction?

Task: Let $K_0 = \{\langle x, y\rangle : \varphi_x(y)\downarrow\}$. Show that $K \leq_m K_0$ by finding a computable function f such that $x \in K \Leftrightarrow f(x) \in K_0$.[5]

Answer: let $f(x) = \langle x, x\rangle$. Then

$$x \in K \Leftrightarrow \varphi_x(x)\downarrow \Leftrightarrow \langle x, x\rangle \in K_0.$$

[5] $K = \{x : \phi_x(x)\downarrow\}$.

m-reduction?

The following theorem is saying that if $A \leq_m B$, then B is "at least as hard to compute as A", in some sense.

Theorem

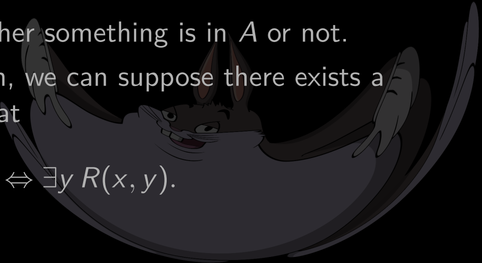
1. *If $A \leq_m B$ and B is computable, then A is computable.*
2. *If $A \leq_m B$ and B is c.e., then A is c.e..*

Task: Prove the above.
Hint:

1. Show that we can decide whether something is in A or not.
2. Using the normal form theorem, we can suppose there exists a computable relation R such that

$$x \in B \Leftrightarrow \exists y\, R(x, y).$$

Show that $A \in \Sigma^1_0$.



m-reduction?

Theorem

1. *If $A \leq_m B$ and B is computable, then A is computable.*
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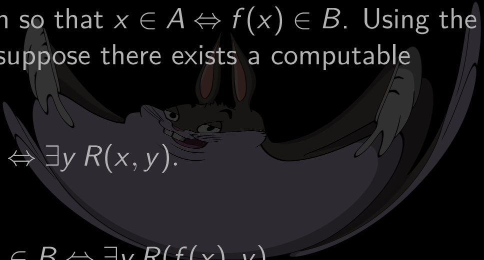
Proof:

1. Let f be a computable function so that $x \in A \Leftrightarrow f(x) \in B$. Then to check whether some arbitrary $x \in A$, we just check whether $f(x) \in B$ or not.
2. Let f be a computable function so that $x \in A \Leftrightarrow f(x) \in B$. Using the normal form theorem, we can suppose there exists a computable relation R such that

$$x \in B \Leftrightarrow \exists y\, R(x, y).$$

Then

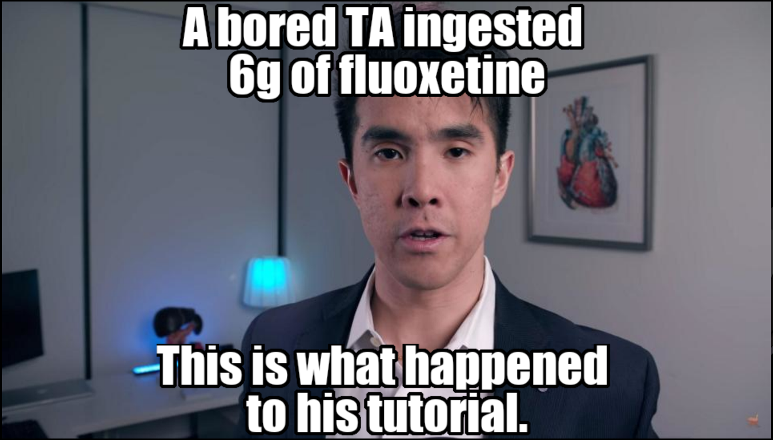
$$x \in A \Leftrightarrow f(x) \in B \Leftrightarrow \exists y\, R(f(x), y).$$



<3

Theorem

If your feelings are so much harder to compute than my feelings, then I <3 you.



Exercise 7.1.6

Again, $K_0 = \{\langle x, y\rangle : \varphi_x(y)\downarrow\}$.

Task: Convince yourself that K_0 is c.e..

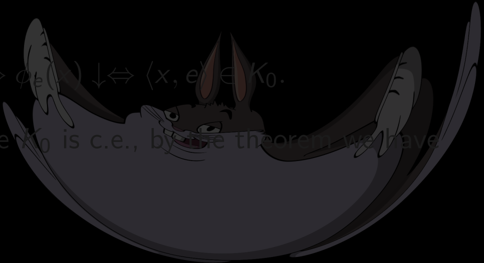
Task: Let $A \subseteq \mathbb{N}$. Show that A is c.e. if and only if $A \leq_m K_0$.

Hint: Normal form theorem! A is c.e. implies $A = W_e$ for some e.

Answer: Suppose A is c.e.. Then $A = W_e$ for some e. Consider the function $f(x) = \langle x, e\rangle$:

$$x \in A \Leftrightarrow x \in W_e \Leftrightarrow \phi_e(x)\downarrow \Leftrightarrow \langle x, e\rangle \in K_0.$$

Conversely suppose $A \leq_m K_0$. Since K_0 is c.e., by the theorem we have proven, A is also c.e..



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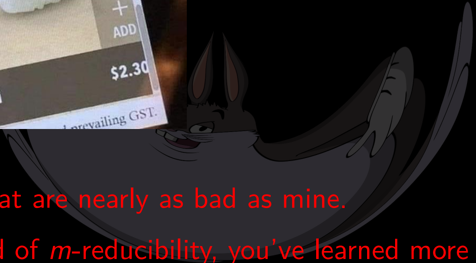
363 is hard ;-;

are you CSC363? because i don't want to fail you ;-;

are you a math course? i'm sorry, i'd prefer passing on you. :(

as you can see, you shouldn't ask me for relationship advice.

i planned the tutorial to end here, i don't have any more content prepared. sorry ;-; and have a nice day! here's some plain sushi



Task: come up with pickup lines that are nearly as bad as mine.

Task: convince yourself that instead of *m*-reducibility, you've learned more about how to convince people to stay away from you.