MAT102H5 - Introduction to Mathematical Proofs - Summer 2021 -UTM

Problem Set 7 - TO BE SUBMITTED SAT. JULY 24, 11:59pm

Submit the following 4 problems in Crowdmarks. Late submission will be given 0, no excuse will be accepted. You are supposed to submit earlier than the deadline so in case of any issue you will have time to deal with it.

Only 2 or 3 problems will be graded. The problem set is 8 points. 6 points for problems, and 2 points for clean and neat writing *and for submitting at least 3 of the problems*. **To get the 2 points, you must:**

submit at least 3 problems.
submit each question on its place in Crowdmark,
write the answer of each question in a separate sheet,
you will not get the point if your handwriting is hard to read, DO NOT write with script handwriting.

Question 1.
Find the mistake in the following “proof”:

Claim: The numbers $0, 1, 2, 3, \cdots$ are all even.
Proof: We use strong induction to prove the statement “n is even” for $n = 0, 1, 2, 3, \cdots$.
Base case: $n = 0$ is an even number, hence the statement is true for $n = 0$.
Assume that the statement is true for $n = 0, 1, 2, 3, \cdots, k$, and consider $n = k + 1$.
By assumption, both 1 and k are even numbers, and hence so is their sum $k + 1$. It thus follows that the statement holds for all $n = 0, 1, 2, 3, \cdots$.

Question 2.

Prove that every $n \in \mathbb{N}$ can be written as a product of an odd integer and a nonnegative integer power of 2.

Question 3.

Consider the function $f : [0, \infty) \to [0, \infty)$, $f(x) = \frac{x}{1+x}$. Prove, by induction, that for any $n \in \mathbb{N}$, $f^n(x) = \frac{x}{1+nx}$.

(f^n is the function obtained by composing n copies of f : $f^n = f \circ f \circ f \cdots \circ f$,the composition is repeated n times).

Question 4.

Show that the following pairs of sets have the same cardinality.

(a) Integers divisible by 3, and the even positive integers.

(b) $\mathbb{R}$ and the interval $(0, \infty)$.