

MAT102H5 - Introduction to Mathematical Proofs - Summer 2021 -UTM

Problem Set 3 - TO BE SUBMITTED SAT. MAY 29 , 11:59pm

Submit the following 7 problems in Crowdmarks. Late submission will be given 0, no excuse will be accepted. You are supposed to submit earlier than the deadline so in case of any issue you will have time to deal with it.

Only 2 or 3 problems will be graded. The problem set is 8 points. 6 points for problems, and 2 points for clean and neat writing. To get those 2 points, you must:

submit each question on its place in Crowdmark,
write the answer of each question in a separate sheet,
you will not get the point if your handwriting is hard to read, DO NOT write with script handwriting.

Question 1. Let $f : A \longrightarrow B$ be a function.

- (a) Prove that for any two sets $C, D \subseteq A$, we have $f(C \cap D) \subseteq f(C) \cap f(D)$.
- (b) Give an example of a function f , and sets C, D , for which $f(C \cap D) \neq f(C) \cap f(D)$.

Question 2. What is the image of the function $f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R}$, $f(x) = \frac{1+x^2}{x^2}$. Prove your answer. Do not use calculus.

Question 3. Let F be a field.

(a) **Cancellation rule for addition.** If $a, b, c \in F$ and $a + b = a + c$, prove that $b = c$.
(You might see the proof of this one in lectures)

(b) **Cancellation rule for multiplication.** If $a \neq 0, b, c \in F$ and $a.b = a.c$, prove that $b = c$.

Question 4. Construct a field with 5 elements. Draw the tables of addition and multiplication. (Hint: use question 3, you may use the axioms and claim/propositions about fields in the course notes).

Question 5. Show that in any field F , the equation $x^2 = 1$ can have at most two solutions. Can you think of a field in which the equation $x^2 = 1$ has exactly one solution?

Question 6. Express the following statement using the logic symbols, and decide, whether it is true or false. Explain your answer briefly.

The equation $x^2 + y^2 = 1$ has a solution (x, y) in which both x and y are natural numbers.

Question 7. For what value (or values) of $x \in \mathbb{R}$ the following statement is false? Why?

If $|x - 3| = 1$, then $|x - 2| = 2$.

The following problems from your course notes are only for extra practice. DO NOT submit them.

2.5.25 - 2.5.34 , 2.5.37 , 2.5.38 , 2.5.40 - 2.5.42 , 2.5.44 - 2.5.49 , 2.5.51
3.7.1 , 3.7.2, 3.7.4 -3.7.6