Problem 1
Determine which of the following expressions are mathematical statements. Of those that are statements, determine whether they are true or false.

1. $(\forall x \in \mathbb{R})(x > y)$.
2. $(\forall x \in \mathbb{R})(x > \pi)$.
3. $(\exists x \in \emptyset)(x = x)$.
4. $(\forall x \in \emptyset)(x = x)$.
5. $\emptyset \Leftrightarrow (\forall x \in \mathbb{R})(x \notin S)$ (where S is some predetermined set).
6. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(x^2 > y)$.
7. $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 > y)$.
8. $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x^2 > y)$.
9. $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 > y)$.

Problem 2
For each valid mathematical statement in Problem 1, write out its negation.

Problem 3
Let P, Q, R be statements. Use a truth table to show "$P \Rightarrow ((Q \wedge R) \vee (\neg Q \wedge \neg R))$" is logically equivalent to "$\neg P \vee (Q \Leftrightarrow R)$".

Problem 4
Let $f : \mathbb{R} \to \mathbb{R}$ be a function, and $c, L \in \mathbb{R}$. We say $\lim_{x \to c} f(x) = L$ if

$$(\forall \epsilon > 0)(\exists \delta > 0)(0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon).$$

(ϵ and δ are the Greek letters Epsilon and Delta respectively.)

1. Let $f(x) = x$. Show that
$$\lim_{x \to 0} f(x) = 0.$$
2. Let $f(x) = ax$, where $a \in \mathbb{R}$ is some constant. Show that
$$\lim_{x \to 1} f(x) = a.$$
3. (Harder) Let $f(x) = x^2$. Show that
$$\lim_{x \to 2} f(x) = 4.$$