CSC263 Tutorial #8

Graphs!

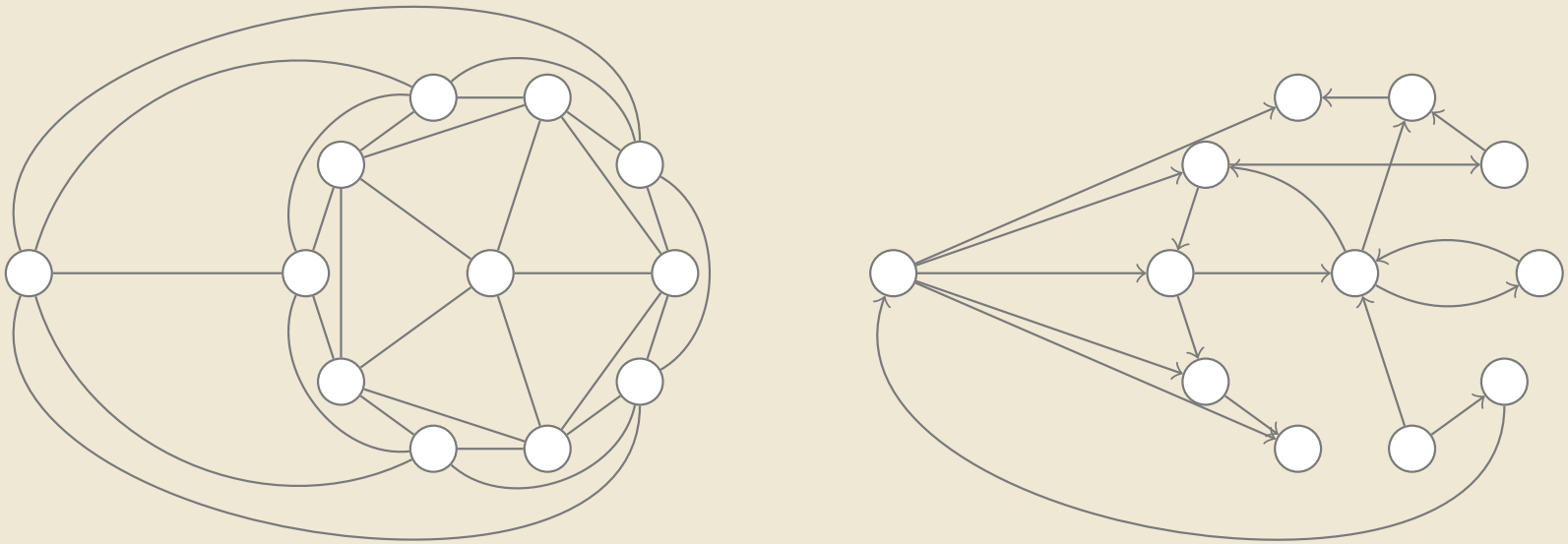
March 10, 2023

Things covered in this tutorial

- ⋆ What is a graph?
- ⋆ How can I perform BFS on a graph?
- ⋆ How can I find the shortest distance between two vertices in a graph?
- ⋆ How can I convert a real-life problem into a problem about graphs?
- ⋆ What's the tallest elevator in the world?

What is a graph?

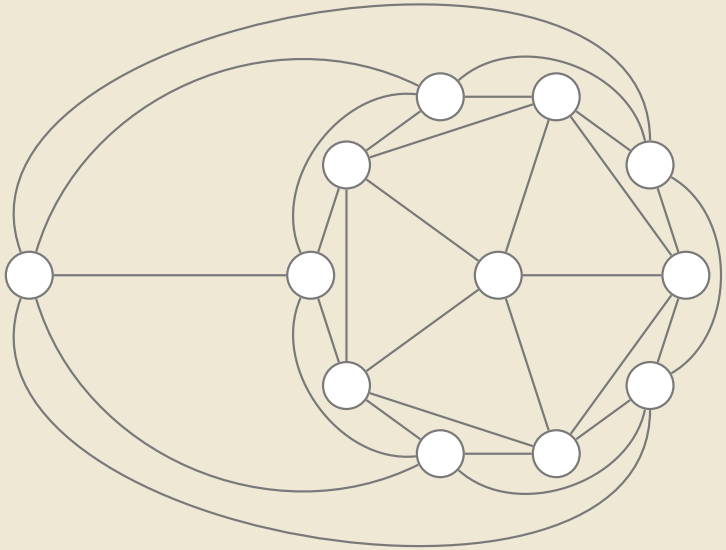
A graph is a funny-looking network of bubbles that are connected to each other.

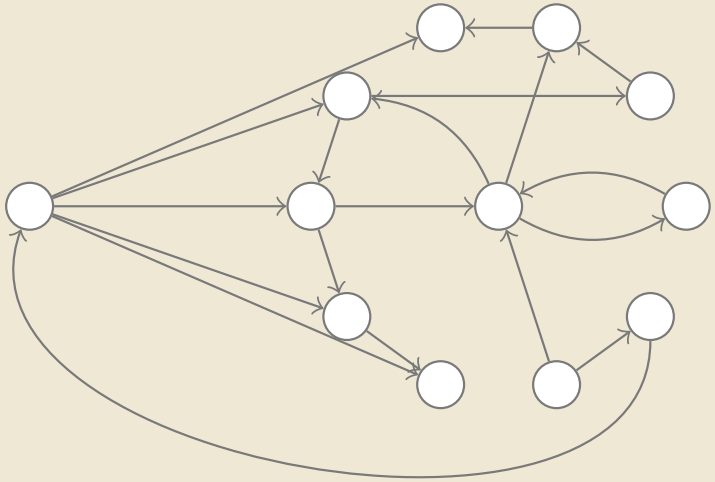


Left: An undirected graph. Right: A directed graph.

The bubbles are called the vertices (also called nodes). The connections between vertices are called edges. The edges may be directed or undirected.

What is a graph?





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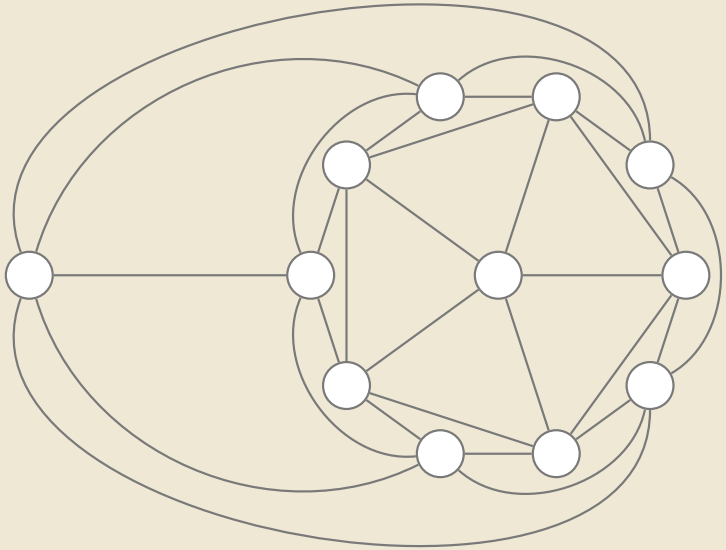

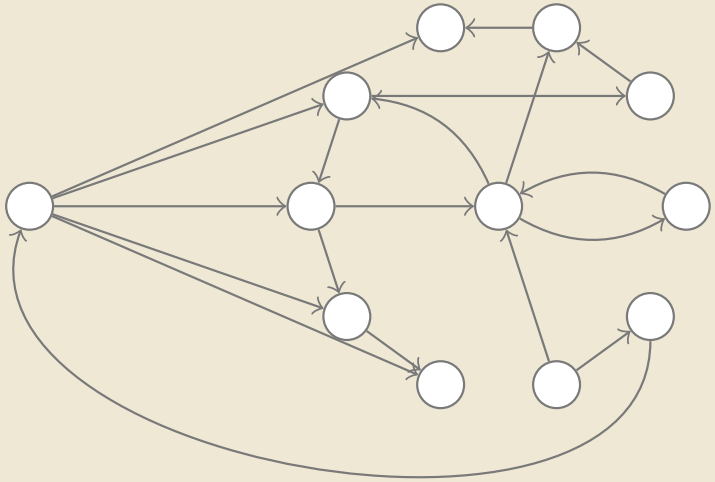

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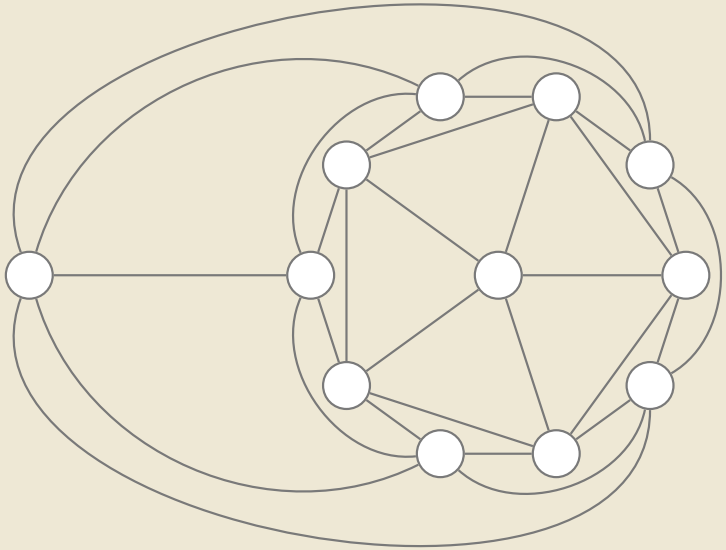

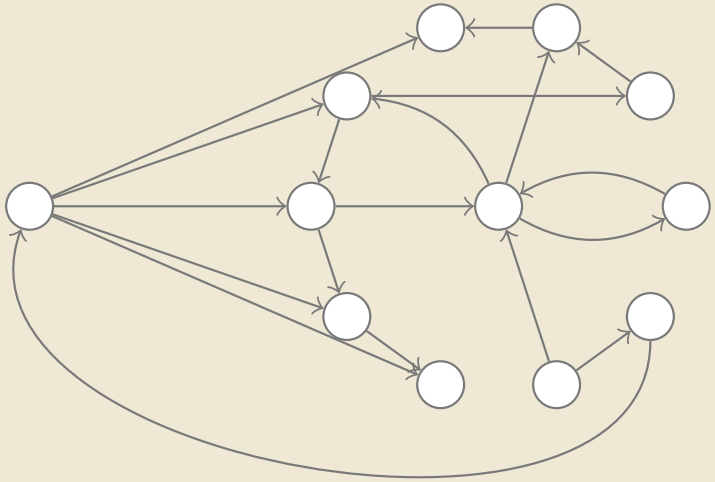

Formally, a graph $G = (V, E)$ consists of:

- ⋆ A set of vertices V. V can be any set, perhaps infinite.
- ⋆ A set of edges $E \subseteq V^2$. Given two vertices $u, v \in V$, we use $(u, v) \in E$ to denote that u and v are connected.

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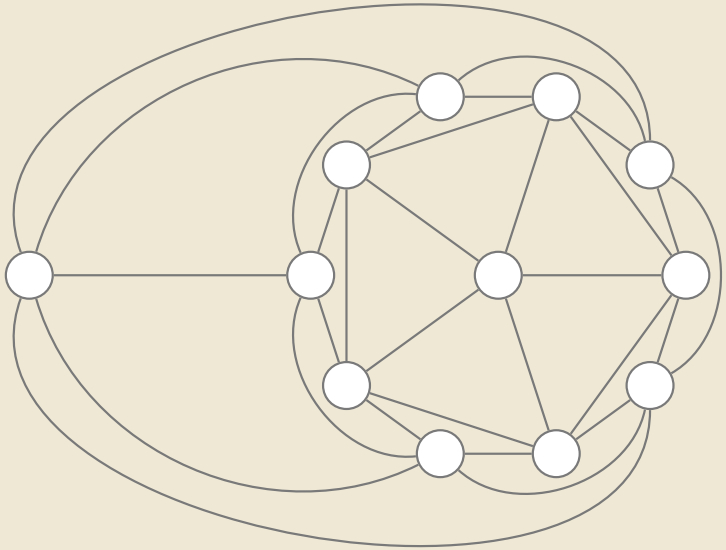

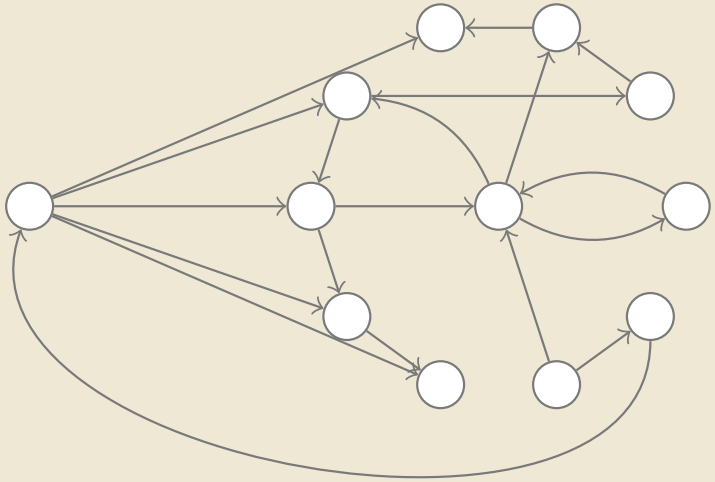

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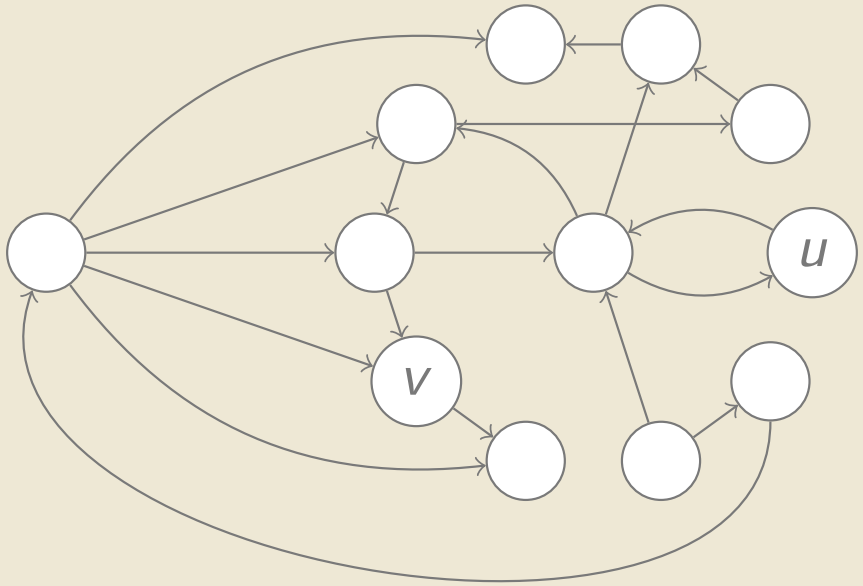
  For undirected graphs, E must be *symmetric*:

$$(u, v) \in E \Leftrightarrow (v, u) \in E.$$

Shortest Path

Task: Find the shortest path from u to v.



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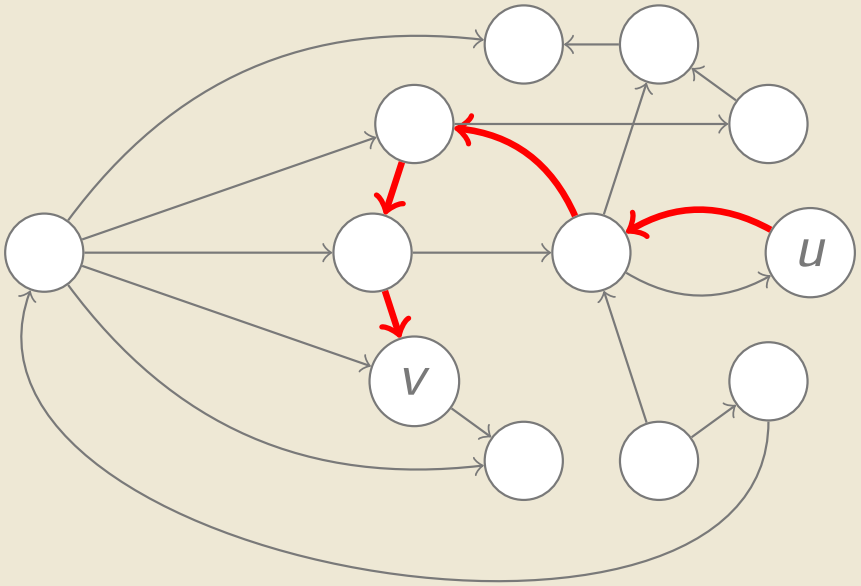


The shortest path from u to v has length 4.

BFS

BFS stands for

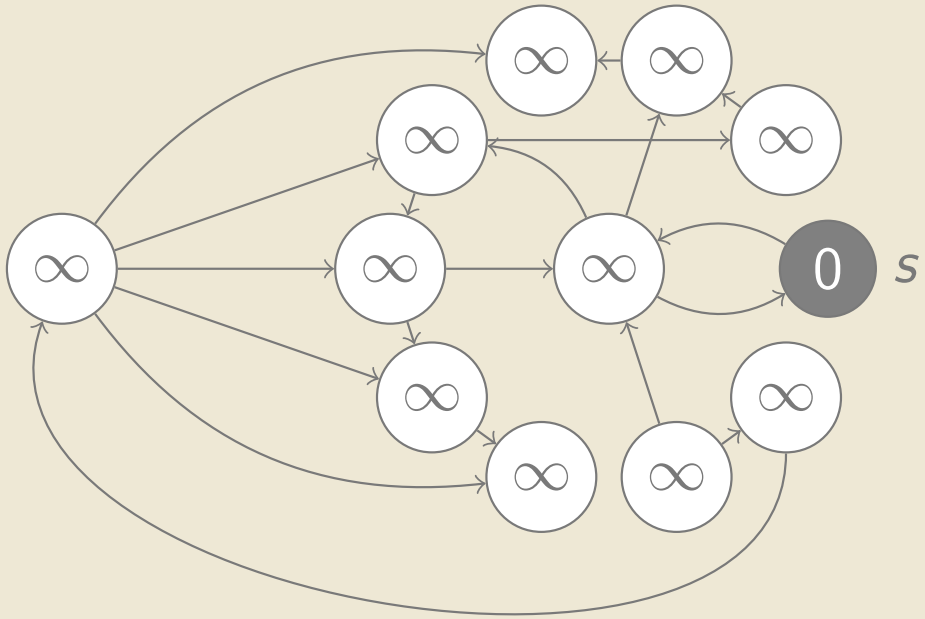
BFS

BFS stands for Breadth-First Search. BFS *traverses* every vertex in a graph, starting from a vertex $s \in V$.

```
BFS(G, s) :
  for each vertex v in V - {s}: # Initialize vertices
  v.colour = White
  v.d = oo
  v.p = Nil
  Q = {}
  s.colour = Gray
  s.d = 0
  Enqueue(Q, s)
  While Q not empty:
    u = Dequeue(Q)
    for each v in G.adj[u]:
      if v.colour == White # only visit unvisited vertices
        v.colour[v] = Gray
        v.d = u.d + 1
        v.p = u
        Enqueue(Q, v)
    colour[u] = Black
```

BFS

Task: Perform BFS starting from s, labelling the distances.



BFS

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Upon finishing, every node's `d` value is equal to the shortest path's distance from s to d.

Elevator problem

Tutorial Activity: Open Tutorial 8, and read the instructions.

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Here's an illustration of the sample test case `10 1 10 2 1`:

You start at X, and want to reach Y. You may only go up by 2 or down by 1 each move.

Fun facts about elevators

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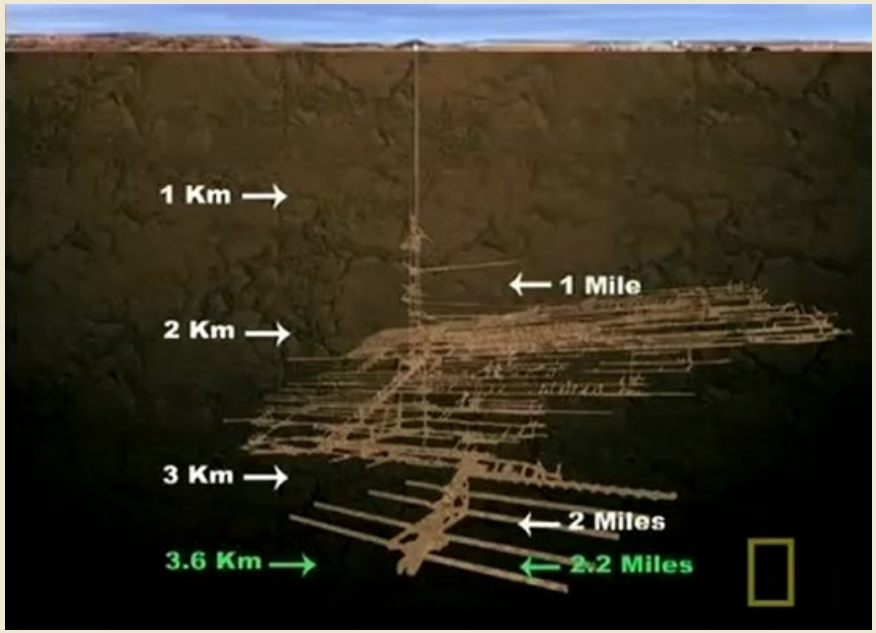
- ⋆ The tallest elevator in the world is found in the Mponeng Gold Mine in South Africa, owned by AngloGold Ashanti. The elevator is 2283 metres in height.



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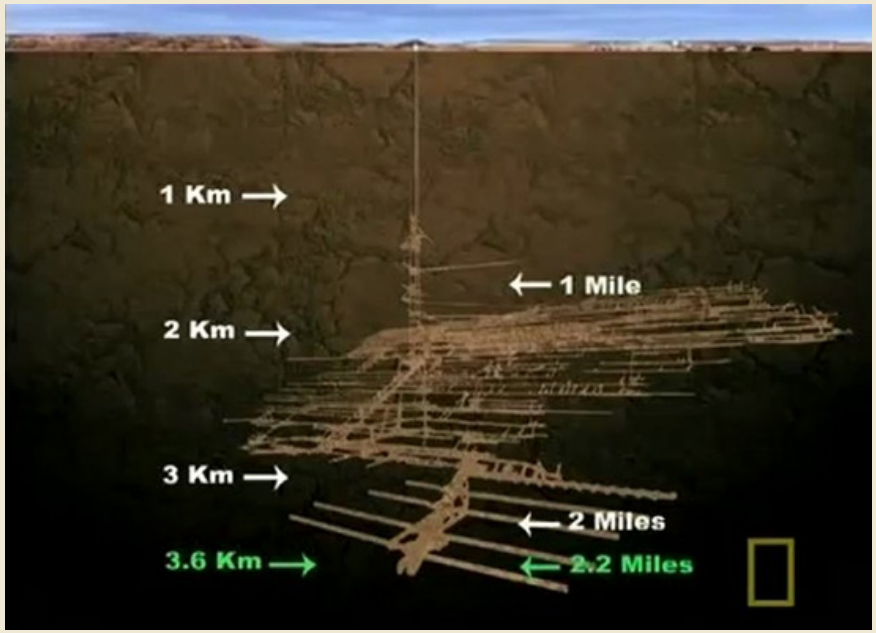

- ⋆ The deadliest elevator disaster in history happened in Vaal Reefs in South Africa, also owned by AngloGold Ashanti. A locomotive fell into the elevator shaft and caused the cage to detach and plunge 460 metres, killing 104 miners.