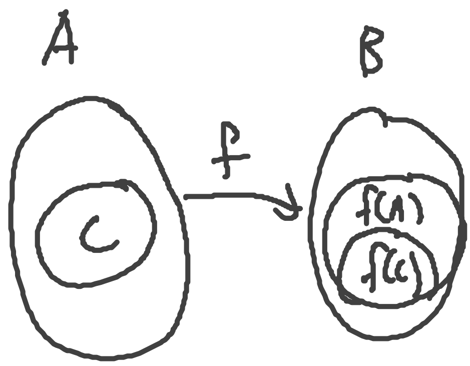
w3

Monday, May 24, 2021 6:37 PM



my computer died! D: i'll try to get it repaired within the next week.
in the mean time, you'll have to deal with this browser-based version of onenote. :(
also, sorry for the bad microphone quality today!
(i'll also be late uploading this week's notes)

things covered in this tutorial:
- more sets, of course!
- images of sets under functions
- there's also one question on fields
- hmm... if we're done early, i could go over some quiz questions on request?

1) (a) Let $f : A \longrightarrow B$ be a function, and $C, D \subseteq A$. Prove that $f(C) \setminus$

Recall: $f(C) = \{f(x) : x \in C\}$

Pf: Let $y \in f(C) \setminus f(D)$. $y \in f(C)$, so $y = f(x)$ for some x
$y \notin f(D)$, so $y \neq f(z)$ for <u>any</u>

(b) Prove or disprove: if $f : A \longrightarrow B$ be a function, and $C, D \subseteq A$
$f(C \setminus D)$. (false)

try some examples!

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$C = (-\infty, 0)$ $D = (0, \infty)$

$f(C) = (0, \infty)$ $f(D) = (0, \infty)$

$f(C) \setminus f(D) = \emptyset \neq f(C \setminus D) = f(C) = (0, \infty)$

$f : \mathbb{R} \to$

$C =$

$f(C) = \{$

$f(C$

2) Is the set $\mathbb{R}^2$, with addition and multiplication defined below a fi

$\{(x, y) : x, y \in \mathbb{R}\}$

$(a, b) + (c, d) = (a + c, b + d)$ $(a, b).(c, d) = (ac, bd)$ Hint:

bonus: is there a way to define '+' and '.' over R^2 so that it is a field? (not that this is relevant for MAT102, but it's a nice thing to know)

if this were a field, it must have an additive identity (x_0, y_0) s.t.

$(x_0, y_0) + (a, b) = (a, b)$ (for any $(a, b) \in \mathbb{R}^2$).

$(x_0 + a, y_0 + b) = (a, b)$

$x_0 + a = a \Rightarrow x_0 = 0$

$y_0 + b = b \Rightarrow y_0 = 0$

So the additive identity must be $(0,0)$.

it must also have a multiplicative identity $(x_1,$

$(x_1, y_1) \cdot (a,b) = (a,b)$ (for any $(a,b) \in \mathbb{R}^2$)

$x_1 \cdot a = a$ if $a=1$, $x_1 \cdot 1 = 1$

$y_1 \cdot b = b$ if $b=1$, $y_1 \cdot 1 = 1$

So the multiplicative identity is $(1,1)$.

Consider $(0,1)$. This is not equal to

hence it must have

(x,y) s.t. $(0,1) \cdot (x,$

$0 \cdot x$

this

Bonus: yes!

$(a,b) + (c,d) = (a+b, c+d)$ $(a+b$

$(a,b) \cdot (c,d) = (ac-bd, ad+bc)$

complex numbers!