MAT102H5 - Introduction to Mathematical Proofs - Summer 2021 -UTM

Problem Set 9 - TO BE SUBMITTED SAT. August 7, 11:59 pm

Submit the following 4 problems in Crowdmarks. Late submission will be given 0, no excuse will be accepted. You are supposed to submit earlier than the deadline so in case of any issue you will have time to deal with it.

Only 2 or 3 problems will be graded. The problem set is 8 points. 6 points for problems, and 2 points for clean and neat writing *and for submitting at least 3 of the problems.* **To get the 2 points, you must:**

submit at least 3 problems.
submit each question on its place in Crowdmark,
write the answer of each question in a separate sheet,
you will not get the point if your handwriting is hard to read, DO NOT write with script handwriting.

Question 1. Prove that the equation $2^x - 1 = y^2$ has no integer solution where $x > 1$.

Question 2. If $p_1, p_2, \cdots, p_k$ are prime numbers, prove that $\sqrt{p_1 p_2^{m_2} \cdots p_k^{m_k}}$ is irrational, $m_2, \cdots m_k$ are natural numbers. (The exponent of p_1 is 1). The prime numbers $p_1, p_2, \cdots, p_n$ are distinct.

Question 3. Let n be a natural number.

(a) What are all the possible value(s) of $gcd(n+1, 2-n)$? Explain.

(b) If $n \in \mathbb{N}$, what are the possible value(s) of $gcd(7^n, 7^n + 4)$?

Question 4. Consider the function $f : P(\mathbb{Z}) \to P(\mathbb{N})$, $f(A) = A \cap \mathbb{N}$.

(a) What are $f(\{-2, -1, 0, 1, 2\})$, $f(\{-1, -2, -3, \cdots\})$ and $f(\mathbb{N})$?

(b) Is f surjective? Explain.

(c) Is f injective? Explain.