1.

a)

P	Q	$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

since $P \Rightarrow Q$ and $\neg Q \Rightarrow \neg P$ are True in exactly the same worlds they are equivalent

b)

P	Q	$P \Leftrightarrow \neg Q$	$(P \wedge \neg Q) \vee (\neg P \wedge Q)$
F	F	F	F
F	T	T	T
T	F	T	T
T	T	F	F

since $P \Leftrightarrow Q$ and $(P \wedge \neg Q) \vee (\neg P \wedge Q)$ are True in exactly the same worlds they are equivalent

2.

a)

Smoke	Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
F	F	T
F	T	F
T	F	T
T	T	T

Satisfiable since ∃ w where
the statement holds but Not a Tautology
the statement doesn't hold for ∀ w

b)

Smoke	Fire	Heat	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ heat) ⇒ Fire)
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	F	T

Satisfiable since $\exists w$ where
the statement holds but Not a Tautology
since the statement doesn't hold for $\forall w$

c)

Smoke	Fire	Heat	$((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Tautology as true for all worlds w

3. Symbols

a) & Variables:

- $My \Leftrightarrow$ unicorn is mythical
- $I \Leftrightarrow$ unicorn is immortal
- $Mam \Leftrightarrow$ unicorn is a mammal
- $H \Leftrightarrow$ unicorn is horned
- $Mag \Leftrightarrow$ unicorn is magical

KB

1. $My \Leftrightarrow I$
2. $\neg My \Rightarrow Mam$
3. $Mam \vee I \Rightarrow H$
4. $H \Rightarrow Mag$

6) Step 1: Eliminate $\Rightarrow$ and $\Leftrightarrow$

1 $\quad$ 1 $\neg My \lor I$, 2 $\neg I \lor My$

2 $\quad$ 3 $My \lor Mam$

3 $\quad$ 4 $\neg(Mam \lor I) \lor H$

4 $\quad$ 5 $\neg H \lor Mag$

Numbers in white - correspondence to old clauses

Numbers in yellow = new clauses

Step 2: Use Demorgans to push $\neg$ inwards

1, 2, 3, 5 remain same

4 $\quad (\neg Mam \land \neg I) \lor H$

Step 3: Distribute $\lor$ over $\land$

again rest stay same

4 $\quad (\neg Mam \lor H) \land (\neg I \lor H)$

Resulting CNF:

1. $(\neg My \vee I)$
2. $\wedge (\neg I \vee My)$
3. $\wedge (My \vee Mam)$
4. $\wedge (\neg Mam \vee H)$
5. $\wedge (\neg I \vee H)$
6. $\wedge (\neg H \vee Mag)$

c)

- can't prove that the unicorn is mythical since the following assignment

$\{My: F, I: F, Mam: T, H: T, Mag: T\}$

satisfies the KB but not $\alpha = My$

- proving by resolution that the unicorn is horned

Resolve (3) and (4)

(7) = $(My \vee H)$

Resolve (7) and (1)

(8) $(I \vee H)$

Resolve (8) and (5)

(9) (H)

$\therefore$ H must be true, hence the unicorn must be horned

Resolve 9 and 6

(Mag)

∴ Mag must be True

Unicorn must be magical.

4. <u>Figure 1:</u>

- Decomposable - can check that no AND gate has overlapping amongst its two inputs

- Not smooth - as 2nd OR gate from left i.e. OR(C, AND(¬D, ¬C)) violates the condition

- Deterministic - since all ORs have at most one high input

* OR(AND(¬A, B), AND(¬B, A))

if this is true then
A = False, B = True ∴
AND(¬B, A) is False

* OR(C, AND(¬D, ¬C))

if C is True
∴ AND(¬D, ¬C)

* $OR(AND(\neg A, \neg B), A)$

if this is true, $A = False$

$\therefore$

must be false

* $OR(AND(C, \neg D), AND(D, \neg C))$

if this is true, C is true

$\therefore$ the second and is false

as $\neg C = False$

Figure 2

- Decomposable - can check that no AND gate has overlapping amongst its two inputs

- Smooth - can check that subcircuits that are inputs to OR gates have same atoms

- Non-deterministic - can check that the first or gate can have both inputs true

$$OR(\underbrace{AND(\neg A, B)}, AND(\neg A, B))$$

if this is true, so is this

$\therefore$ non deterministic

5.

a) Models are: Let F represent false, T represent True

- $A=F, B=T, C=F, D=F$ weight $= 0.32 \times 0.08$
- $A=F, B=T, C=F, D=T$ weight $= 0.32 \times 0.32$
- $A=F, B=T, C=T, D=F$ weight $= 0.32 \times 0.12$
- $A=F, B=T, C=T, D=T$ weight $= 0.32 \times 0.48$

- $A=T, B=F, C=F, D=F$ weight $= 0.12 \times 0.08$
- $A=T, B=F, C=F, D=T$ weight $= 0.12 \times 0.32$
- $A=T, B=F, C=T, D=F$ weight $= 0.12 \times 0.12$
- $A=T, B=F, C=T, D=T$ weight $= 0.12 \times 0.48$

Adding them up we get $= 0.44$

Hence WMC for the formula is 0.44

b) OR(AND($\neg A$, B), AND($\neg B$, A))

$$= \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} \left(AND(\neg A, B) \right) + \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} \left(AND(\neg B, A) \right)$$

$$= \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} (\neg A) * \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} (B) + \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} (\neg B) * \underset{\text{ROOT}}{\overset{\text{COUNT}}{\text{ON}}} (A)$$

$$= 0.8 \times 0.4 + 0.6 \times 0.2$$

$$= 0.44$$

This method is equivalent to the WMC for the formula

c) Let C be the function that represents count on root

using the properties of count on root, we can simplify as follows:

Let OR be represented by O
AND " A

1. $A(\neg A, B) = 0.8 \times 0.4 = 0.32$
2. $A(\neg A, \neg B) = 0.8 \times 0.6 = 0.48$
3. $A(B, A) = 0.2 \times 0.4 = 0.08$
4. $A(\neg B, A) = 0.6 \times 0.2 = 0.12$

5. $A(C, \neg D) = 0.6 \times 0.2 = 0.12$
6. $A(C, D) = 0.6 \times 0.8 = 0.48$
7. $A(\neg D, \neg C) = 0.2 \times 0.4 = 0.08$
8. $A(D, \neg C) = 0.8 \times 0.4 = 0.32$

9 OR(1, 4) = 0.32 + 0.12 = 0.44

10 OR(6, 7) = 0.48 + 0.08 = 0.56

11 OR(2, 3) = 0.48 + 0.08 = 0.56

12 OR(5, 8) = 0.12 + 0.32 = 0.44

13 AND(9, 10) = 0.44 × 0.56 = 0.2464

14 AND(11, 12) = 0.44 × 0.56 = 0.2464

15 OR(13, 14) = $\boxed{0.4928}$