1: Deterministic Communication Complexity of Graph Isomorphism

The problem of Graph Isomorphism naturally seems very similar to the Equality function: a function we are very familiar with. Moreover, due to the inherent difficulty in accurately deciphering the pattern in the characteristic matrix for Graph Isomorphism, the approach taken was to reduce Equality of n^2 bits to the the problem of Isomorphism of Simple Undirected Graphs on n vertices.

Definitions:
Let $\mathbf{G_n}$ = the set of simple undirected graphs on n vertics
Let $IS_n : \mathbf{G_n} \times \mathbf{G_n} \to \{0,1\}$ be the graph isomorphism function i.e. 1 if the Graph x is an isomorphism of Graph y and 0 otherwise

Claim.
$D(IS_n) \geq n^2$

Proof. Reduction from equality of n^2 bit strings to isomorphism of simple undirected graphs on n vertices. (Note in the reduction can only use local work i.e. local work is free as Alice and Bob are computationally unbounded entities)
Show that the number of unique simple undirected graphs under isomorphism on n vertices are $\theta 2^{(n^2)}$ and therefore each can be represented using $\theta(n^2)$ bits. Therefore the n^2 bits of equality can be interpreted as graphs $\in \mathbf{G_n}$ and isomorphism $\leftrightarrow$ equality.

2: Deterministic Communication Complexity of Determining nth least significant bit of the product of two natural numbers

Product of n bits strings (do not need to consider entire number)

Definitions:
Let $Product_n : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ be the the function on n bits strings indicating the nth least significant bit of the product of the integers represented by the strings

Claim.
$D(Product_n) \geq n$ (maybe plus 1 look at the inequality)

Proof. The kernel has always dimension ¡ 2^{n-1} *vectors therefore by rank nullity*

proof for the above statement:

first show that the due to multiplication by 1, 2, 4 ... powers of 2 basis vectors must select from columns with sum = 0

then show that atleast 4 non zero elements are needed in a basis vector for the kernel are needed and that anymore than 4 can be reduced to exactly 4

Show that a + d = b + c = 2^n and that a, d correspond to 1 and b, c correspond to -1

rank $> 2^{n-1} \implies$ *log rank > n - 1* $\implies$ *ceiling log rank* $\geq n$ *and hence D(f)* $\geq$ *1*

3: Deterministic Communication Complexity of Computing a Boolean formula on n variables

some info

Definitions:
Let $Boolean_n : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$

Claim.
$D(boolean_n) = \omega(n)$

Proof. no idea as of now