

Mental Model Updating and Pupil Response

Sungjoon Park

Supervisor: Dr. Britt Anderson

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Department of Psychology

University of Waterloo

Abstract

To adapt to our dynamic environment, we detect unexpected changes in the form of surprises. However, not all surprises update our mental model about our environment. Mental model updating occurs selectively for informative surprises, known as Bayesian surprise. Surprises that are uninformative, or purely information theoretic, do not contribute to our model. My proposed study will investigate whether mental model updating is reflected in pupil response by comparing the pupil response to the two types of surprises. In addition, it will investigate whether the reliability of the stimulus affects the pupil response to surprises and mental model updating. This will be done by devising a probabilistic inference task where participants need to learn what the contingent is. I expect participants to show greater pupil response and greater change in reported belief after observing Bayesian surprises than after observing information theoretic surprises.

Introduction

We create mental models (internal representations of the world) to behave adaptively by predicting how our actions will interact with our environment (Johnson-Laird, 2013). These models are built on our experiences and it informs many of our behaviors, decision making process and expectations (Filipowicz, Anderson, & Danckert, 2016). Since the utility of a mental model depends on how accurate its predictions are, it must be able to update itself with new information about our dynamic world (Filipowicz et al., 2016). One of the key challenges of updating mental models is the identification of the relevant information from the constant flow of sensory input (Filipowicz et al., 2016). Reisenzein, Horstmann, and Schützwohl (2019) claim that prediction errors, or surprises, inform us of flawed mental models and that we attempt to explain the cause of the surprise and/or update our mental model to mend such errors. However, not all surprises are thought to initiate mental model updating (O'Reilly et al., 2013), and we often ignore surprising information when we think the source is unreliable. Schwartenbeck, FitzGerald, and Dolan, (2016) suggest that Bayesian surprises, but not information theoretic surprises, are associated with mental model updating. Is it possible to distinguish, behaviorally, the two surprises? And how might their reliability contribute to mental model updating?

I argue it is possible to observe mental model updating through pupillometry. My proposed study will investigate whether pupil size change can reflect neural signals, implicated in mental model updating, previously investigated with invasive or functional imaging tools. Past studies show that the noradrenergic locus coeruleus (LC) is a key brain structure implicated in regulating central arousal, pupil dilation (Urai, Braun, & Donner, 2017) and signaling uncertainties about the environment (Yu & Dayan, 2005). As the LC is intimately connected

with the dopaminergic midbrain area, Van Slooten, Jahfari, Knapen, and Theeuwes (2018) claim that the dopaminergic activities associated with belief updating, a form of mental model updating, can elicit pupil dilations. I will proceed by reviewing literature that suggest pupil response reflect belief states, explain the differences between information theoretic and Bayesian surprise, and review the behavioral tasks that have been used to test these associations. I will conclude by making specific predictions to how surprise, pupil response, belief, confidence, and updating all relate and present my adaptation of a task to test my predictions.

Colizoli, de Gee, Urai, and Donner (2018) suggest pupil response varies with belief state. In their study, participants were tasked to identify the overall movement direction of random dot kinematograms. During their task, “pupil responses during feedback anticipation and after reward feedback were modulated by decision-makers’ internal belief states” (Colizoli et al., 2018). In other words, depending on what the observers believe they have observed, their pupil responded differently before and after observing a confirmation or a refutation of it. In addition, the magnitude of pupil response varied depending on the ambiguity of the dot direction (difficulty condition of their study). As shown in Figure 1, correct identifications predicted smaller pupil response compared to incorrect identification, and greater difficulty predicted larger pupil response. However, for incorrect identifications, greater difficulty predicted smaller pupil response (Colizoli et al., 2018). Their result suggests that pupil respond differently depending on whether an expectation was met or not. It also suggests that pupil response is moderated by the confidence in a prediction. However, this experiment only suggests that the pupil respond to surprises (prediction errors) in general, and does not claim whether the increased pupil response from surprises can be associated with mental model updating. My

proposed study attempts to answer whether pupil responds differently to surprises that contribute to mental model updating.

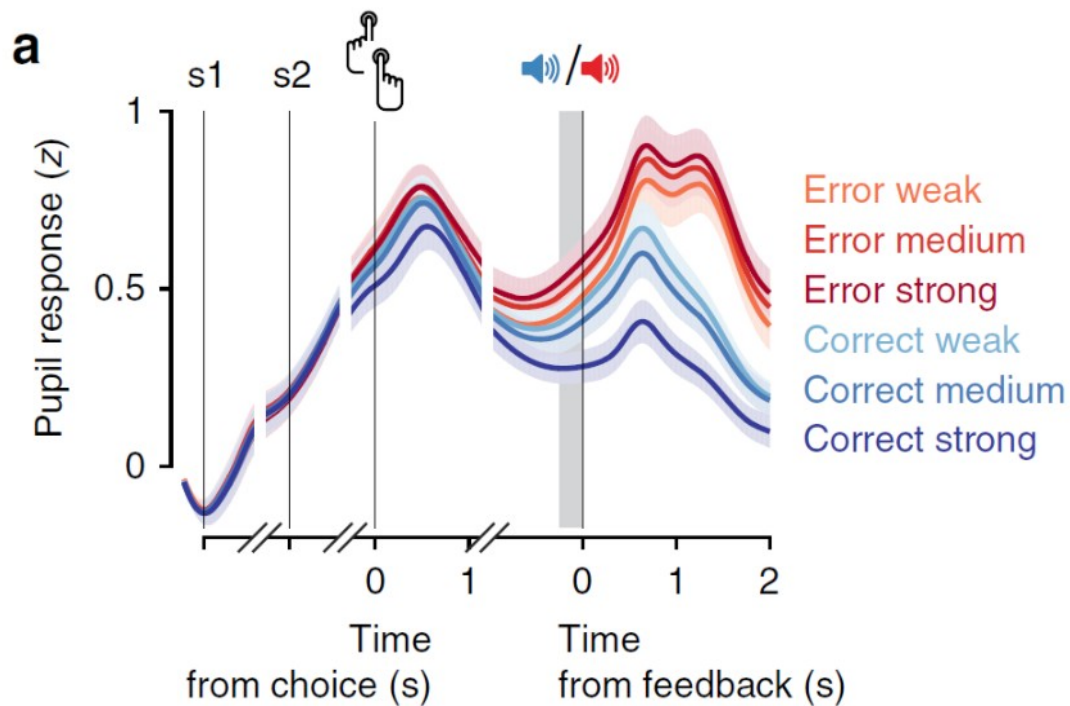


Figure 1: Graph illustrating the pupil response pre- and post-feedback. Pupil response is greater for prediction errors and it scaled positively with difficulty for correct predictions but negatively for prediction errors. It suggests that pupil response varies with the participant's expectations and task difficulty. (Urai, Braun, & Donner, 2017)

Both O'Reilly et al. (2013) and Schwartenbeck et al. (2016) differentiate surprise into two varieties; information theoretic surprise and Bayesian surprise. Information theoretic surprise is defined purely by how unexpected the observation was based on one's mental model and it does not indicate how useful the observation was to better represent the environment (Schwartenbeck et al., 2016). It corresponds to its Shannon information, the negative log probability of the observation based on the brain's representation before the observation

(O'Reilly et al., 2013). On the other hand, Bayesian surprise is defined by how meaningful it was or by its epistemic value. It corresponds to the Kullback–Leibler divergence between one's belief before and after the observation (Schwartenbeck et al., 2016). In other words, information theoretic surprise is evaluated by the probability of an observation based on one's expectations, and Bayesian surprise is evaluated by the probability of whether one's expectations will change after an observation (O'Reilly et al., 2013). Of interest to me is how these types of surprises might be reflected as pupil responses. Although, O'Reilly et al., (2013) claims pupil respond differently to Bayesian and information theoretic surprises, it is not clear what kind of relationship they have.

I hypothesize that Bayesian surprise will elicit greater pupil response compared to information theoretic surprise because of its association with greater activity in the dopaminergic midbrain area (e.g., Substantia Nigra Compacta, Ventral Tegmental Area) (Schwartenbeck et al., 2016). Schwartenbeck, FitzGerald, & Dolan (2016) claim that these dopaminergic activities play an important role in belief updating. Although it is not clear how dopamine modulates pupil response, several studies suggest that the close connection between the dopaminergic midbrain structures and the noradrenergic locus coeruleus (LC), a key brain structure associated with pupil responses (Urai et al., 2017) and uncertainty (O'Reilly et al., 2013), play a vital role in modulating the pupil during a reinforcement learning context (a form of mental model updating) (Van Slooten, Jahfari, Knapen, & Theeuwes, 2018). It may be possible to observe pupil responses associated with Bayesian surprises beyond what is observed with information theoretic surprises that do not contribute to mental model updating. However, we do not always accept

useful information at first sight. Sometimes, we doubt its legitimacy and question how reliable it is. Do we give Bayesian surprise different weight depending on how reliable we think it is?

I predict that more reliable Bayesian surprises will elicit greater pupil response and mental model updating than less reliable ones. In the context of computational learning theories, Meyniel, Sigman, and Mainen (2015) state that confidence is used to determine how much of the new information should be incorporated during learning. It is thought that the more confident we are about the new information, the more it changes our beliefs and the more confident we are about our previous beliefs, the less the new information changes them (Meyniel et al., 2015). Boldt, de Gardelle, and Yeung (2017) claim that our confidence in our beliefs are reflected by the evidence reliability that they are based on. In Urai et al. (2017) and Colizoli et al. (2018)'s studies, the investigators manipulated evidence reliability by manipulating the ambiguity (task difficulty) of random dot kinematograms; where some dot kinematogram directions were easier to identify than others (Beck et al., 2008). They observed that pupil response to prediction errors for more difficult dot kinematograms were smaller than for easier dot kinematograms. This suggests that the arousal to prediction error is weaker when one is less confident about their prediction. Since the stimulus was less reliable, one might be less confident about their prediction and be less surprised that they were wrong. I will investigate whether Bayesian surprise will also be less surprising/arousing when it is less reliable, and whether this relationship will predict less mental model updating.

I will implement a probabilistic inference task and manipulate the reliability of the cues that are used to make predictions; so some cues will be more predictive than others. In addition, this task will involve participant's explicit report of their belief state and confidence to

corroborate whether Bayesian surprises are associated with mental model updating and whether the reliability manipulation is changing participant's confidence. I expect to observe larger changes in reported confidence after observing Bayesian surprises, and I expect reported confidence to be smaller during the low reliability conditions. As for the confidence or belief change related to the reliability manipulation, it is possible that the confidence level before the surprise will determine this effect and we may observe an exploration-exploitation dilemma (Sutton & Barto, 2018). For instance, if one has high confidence before observing a reliable surprise, one's confidence might not change significantly. When we are deeply entrenched in an ideas, even the most respected and reliable source might not be able to change our mind. On the other hand, if we have low confidence in our beliefs, even an unreliable surprise might lead to a significant change in confidence or outright change in our beliefs. Because of these possibilities, I expect confidence change to be generally larger during the high reliability conditions while belief changes to be quicker during the low reliability conditions.

In summary, this study will attempt to i) replicate the findings by Urai et al. (2017) and Colizoli et al. (2018) showing prediction errors elicit greater pupil response than correct predictions, ii) investigate whether Bayesian surprises elicit greater pupil response than information theoretic surprises, iii) investigate whether lower signal reliability predicts less confidence, iv) investigate whether lower signal reliability will result in less pupil response to Bayesian surprise, and v) investigate whether Bayesian surprise predicts explicit change in mental model in the form of reported belief and confidence change and whether this change varies with signal reliability. All of my hypotheses will be investigated with an experimental task

inspired by Schwartenbeck, FitzGerald and Dolan (2016) which will be examined in the following paragraphs.

As depicted in Figure 2, Schwartenbeck, FitzGerald and Dolan (2016) designed a task involving Bayesian and information theoretic surprises where participants inferred which one of two contingents predicted the outcomes during an fMRI study. This study involved two contingent cues, an audio cue and a visual cue. Both cues had a “good” and a “bad” version. The “good” version predicted a 90% chance for a monetary reward and the “bad” version predicted a 90% chance for a monetary loss. Participants were exposed to both (audio and visual) cues simultaneously, with each being one of the two versions (“good” or “bad”). After observing the cues, participants observed the outcome (gain vs loss). Next, participants indicated their confidence in which contingent they believed predicted the outcome. Participants were shown a slider with the text “shape” and “tone” on either ends to indicated how confident they were about the two contingents. When the participants knew which type of cue was the contingent, they were surprised when they lost points.

Surprising outcomes were of two types; information theoretic and Bayesian surprises. Cues pertaining to Bayesian surprises predicted different outcomes and cues pertaining to information theoretic surprises predicted the same outcome. For instance, a Bayesian surprise involved a participant who believed that the visual cue was the contingent cue, observed a “good” audio and a “bad” visual cue and faced a loss outcome. This individual might have gained more confidence that the audio is the correct contingency and/or lose confidence in the visual cue. On the other hand, cues related to information theoretic surprise involved both visual and audio cues predicting a “win” while the outcome being a loss. While both scenarios are

prediction errors, only Bayesian surprise allows for the participant to be clearly informed to whether their confidence in the controlling contingency should be altered. Once the participants were confident that the alternative cue was the contingent, they would have updated their mental model.

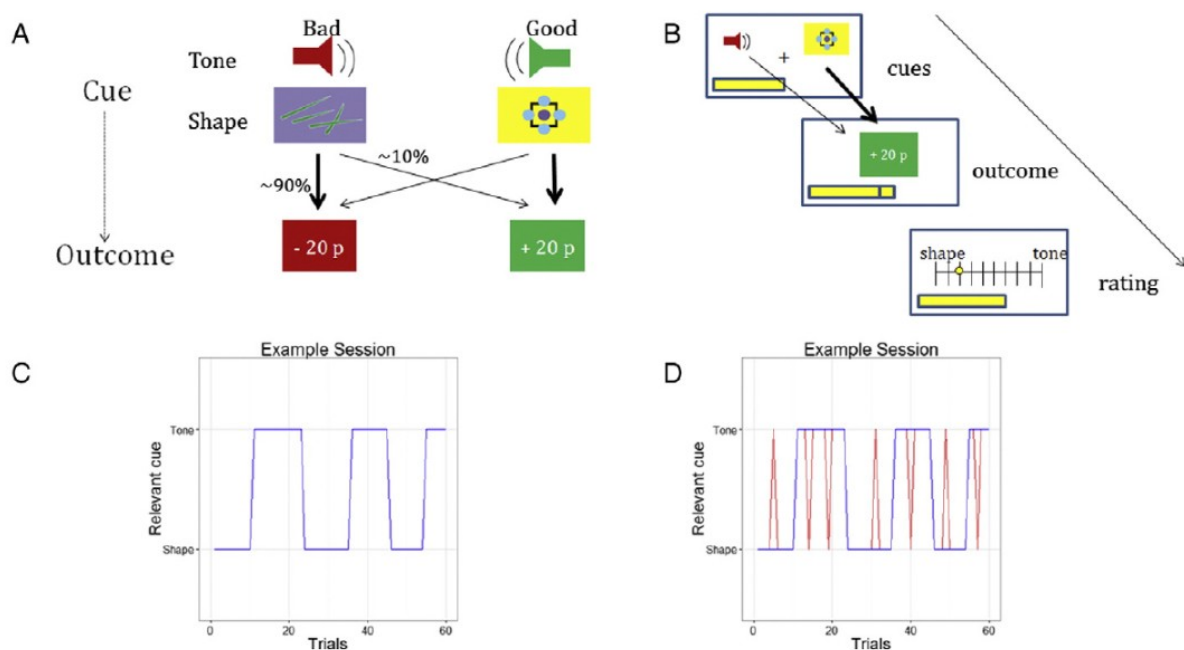


Figure 2: A task designed by Schwartenbeck, FitzGerald, & Dolan (2016) where either the visual or audio cue predicts a monetary gain with 90% reliability. Periodically the contingent switched between the visual and audio cue. Participants were tasked to indicate their confidence in which cue they thought was the contingent.

In the above study, the investigators observed greater activity in the “dopamine-rich mid-brain regions including the Substantia Nigra Compacta and [the] Ventral Tegmental Area” for Bayesian surprises compared to information theoretic surprises (Schwartenbeck et al., 2016). If there is a close interaction between these dopaminergic regions and the noradrenergic LC as several studies suggest (Van Slooten et al., 2018), I expect pupil response to Bayesian surprise to

be greater than those in response to information theoretic surprise. If this is the case, it could mean that the heightened dopaminergic activity from the midbrain area, associated with Bayesian surprise and mental model updating, can be observed through pupillometry and pupillometry may be a relatively accessible, non-invasive, method of monitor mental model related neural activities. Finer details regarding my version of this task will be explored in the following section.

Materials and Methods

Participants

Neurologically healthy students will be recruited from the University of Waterloo to participate in this study for course credit and/or monetary compensation. None of the participants will have a history of brain injury and all must have normal or corrected-to-normal vision. All participants will provide informed consent prior to participation. The research protocol will be approved by the office of research ethics at the University of Waterloo. The experiment is expected to take no more than three hours across three sessions. Participants recruited through SONA will be compensated with one course credit per hour. Participants recruited outside of SONA will be compensated with \$14 per hour. In addition, both types of participants will be further incentivized to place effort by providing additional monetary compensation between \$0 and \$10, linearly spaced between 50% and 100% of the total points accumulated on each session (50% = \$0, 75% = \$5, 100% = \$10), and an additional \$15 for completing all three sessions.

Stimuli

This probabilistic reversal learning task will be similar to that of Schwartenbeck, FitzGerald, & Dolan (2016); an example of a trial is illustrated in Figure 3. Participants will be tasked to imagine a game scenario where they must predict whether a fictitious character will be safe or face danger in its time sensitive quest to earn gold (points). If the participant correctly predicts safety or danger, the character will successfully complete its quest in time and earn 10 gold. If the participant makes the wrong prediction, the character will either underestimate or overestimate the danger and fail to complete the quest and earn no gold. Participants will make this prediction by observing a single visual cue with both a geometric and color property (contingents). There will be two colors and shapes. One color and shape will predict safety and the others will predict danger. On any individual trial it is either the shape or the color that determines the outcome, never both (equiluminant colors will be used to control for the effect of luminance on pupil diameter and the contingents will be counterbalanced between participants).

Participants will be exposed to high and low reliability conditions. In the high reliability condition, optimal predictions will be correct 90% of the trials. During the low reliability condition, optimal predictions will be correct 70% of the trials. Thus, when a participant makes a prediction, there is always a chance that their prediction will be wrong despite knowing the correct contingent.

When determining contingency changes (shape to color and vice versa) outcomes will be surprising because they no longer corresponds to the established rule. Participants will only be able to infer this change after seeing a Bayesian surprise where the shape and color predict the

opposite outcome. However, if both contingents predicted the same outcome, and the participant made a prediction error, they cannot determine whether the contingent has changed because both color and shape were incorrect; this is an example of an information theoretic surprise.

Behavioral Paradigm

Participants will be briefed about their task of guiding a fictitious character on its quest to earn gold and how they must predict whether it will face dangers that affect the probability of success. Participants will be seated in front of a table with a typical computer setup including a monitor, mouse and keyboard. An eye-tracker will be placed between the monitor and the keyboard without interfering the participant's view of the screen. In addition, a chin rest will be fixed between the participant and the table so they could rest their head and maintain stability. After calibrating the eye-tracker, participants will be instructed to complete ten practice trials. First, participants will be instructed to fixate on a central cross for 500ms. Next, a visual cue will replace the fixation cross for 2000ms. They will be instructed that observing a green circle will predict safety and a blue square will predict danger (counterbalanced across all participants). On the next screen, participants will be instructed to indicate their safe/danger prediction by clicking on their respective text elements and to indicate the confidence they have for which contingency is in effect, all within 5000ms. If the participant does not respond in time, they will be shown a screen informing them of their failure to respond for 1000ms and return to the start of the trial. Next, participants will see a fixation cross for 500ms and then an outcome screen for 5000ms, where they will see either a vertical or a horizontal Gabor patch that signifies whether Alex earned gold or not.

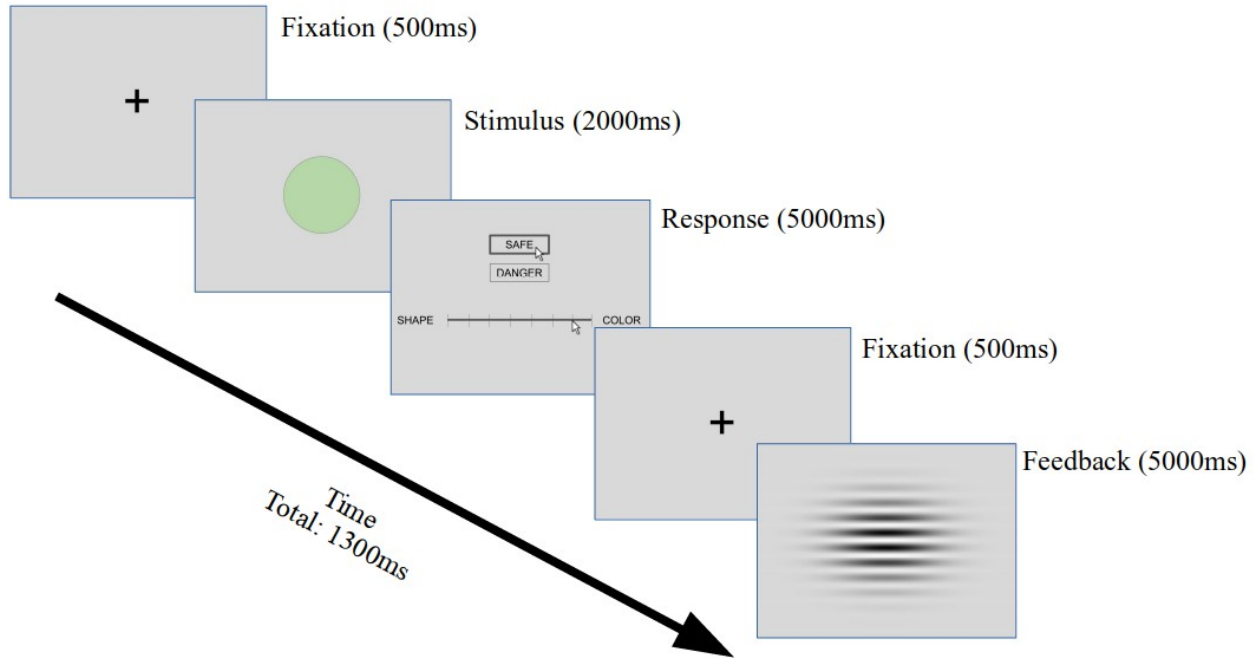


Figure 3: Diagram of a typical trial of the proposed experimental task.

Before initiating the experimental trials, the participants will be instructed that in the following trials only shape or color will predict the outcome. They will be explicitly instructed that when the shape predicts the outcome, a circle will predict safety and square will predict danger (counterbalanced). When the color predicts the outcome, green will predict safety and blue will predict danger. In addition, they will be instructed that throughout the trials they will notice that the contingency (shape or color) that predicts the outcome has changed, so that the contingent they thought predicted the outcome is no longer predictive. In addition, Participants will be instructed that the outcome is stochastic and sometimes the it will contradict their predictions despite their knowledge of the true contingent.

The experimental trials will span a total of 648 trials divided into 36 blocks across three sessions and each session of experimental trials will last approximately 47 minutes. The number

of trials on each block will be generated from a random number generator, and this number will be no greater than 26 and no lesser than 10. The predictive contingent will alternate throughout the blocks (shape → color → shape → color, or vice versa) and each pair of blocks will have a 50% chance to be either high reliability (90%) or low reliability (70%) blocks. Each participant will be randomly assigned to conditions where in the experimental trials the safe and dangerous versions of the shapes and colors will be counterbalanced between participants (square and blue predicting safe conditions instead of circle and green). Participants will be given a break half way through the experiment to rest and to re-calibrate the eye tracker.

Anticipated Data Analysis

As pupil size fluctuates with unintentional cognitive and luminance related factors, I will consider recommendations by Knapen et al. (2016) before analysis. As illustrated in Figure 4, raw pupil data will be preprocessed, which involves linear blink interpolation, accounting for pupil size change in response to blinks and saccades, and low-pass filtering to remove minor pupil changes unrelated to my stimulus (Knapen et al., 2016). Knapen et al. (2016) made their FIRDeconvoluiton python package openly available on GitHub. This package was designed to perform the aforementioned preprocessing and I will adopt and/or modify it for my use. After preprocessing, pupil responses will be computed. As pupil responses are typically delayed by 220ms after the onset of the manipulation (Mathôt, Fabius, Van Heusden, & Van der Stigchel, 2018), I will use the average pupil diameter of the first 100ms after feedback onset as the baseline pupil diameter and the area under the curve during the rest of the feedback duration as

the post-feedback pupil response (Swallow, Jiang, & Riley, 2019). However, as there is no established standard for how pupil response should be measured, other methods such as peak pupil dilation or proportional changes will be explored (Winn, Wendt, Koelewijn, & Kuchinsky, 2018).

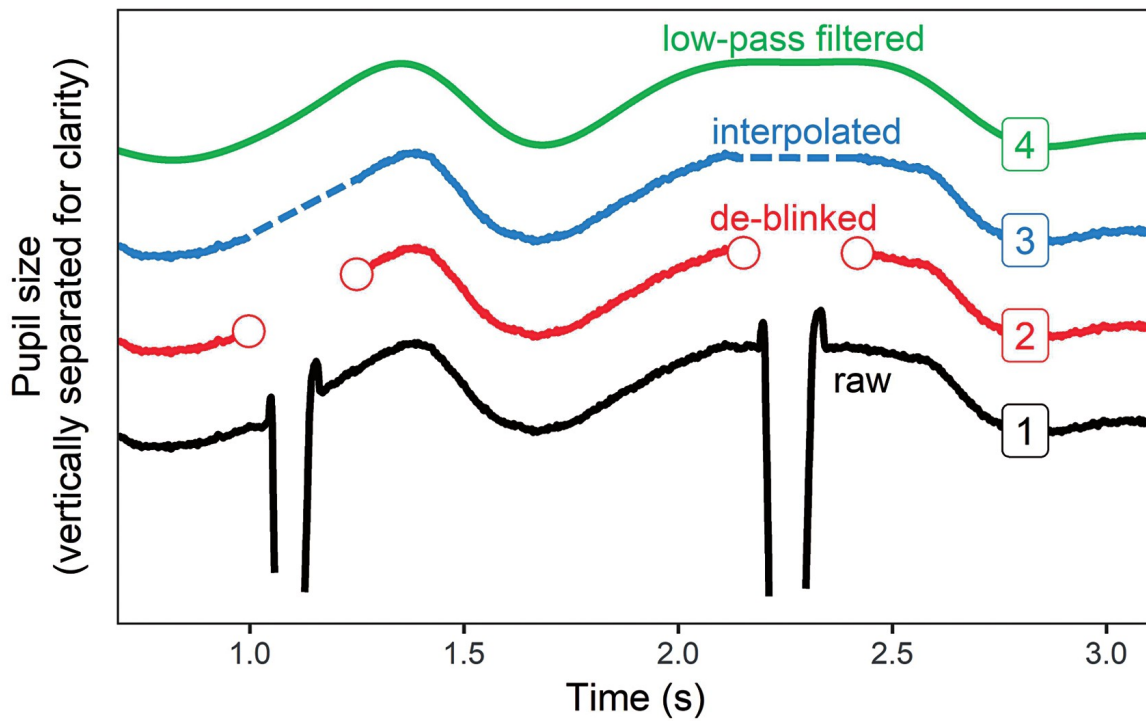


Figure 4: Illustration of preprocessing of raw pupil data. Blinks are detected and linearly interpolated. Lastly, a low-pass filter is applied to smoothen the signal, removing minor pupil changes. (Winn, Wendt, Koelewijn, & Kuchinsky, 2018)

Data analysis will be conducted using the R statistical software (R Core Team, 2019) and linear mixed effects models (LMEM) will be used to fit the data with the lme4 package (Bates, Mächler, Bolker, & Walker, 2015). The general form of the model will involve a dependent variable (e.g., post-feedback pupil response), fixed effect independent variable(s), and a random effect variable(s) (e.g., individual participant). Table 1. lists all the variables that will be

analyzed. A maximal model will involve all fixed effect variables and their interactions with random slopes. Although some argue this is the best way to model (Barr, Levy, Scheepers, & Tily, 2013), without sufficient data points, its complexity is prone to lead to convergence failures and I will likely fit multiple, more parsimonious, models for each of my hypothesis (Bates, Kliegl, Vasishth, & Baayen, 2015). For example, the simplest model to tests whether Bayesian surprise predicts greater post-feedback pupil response would follow the the formula: Pupil response \sim Surprise Type + Baseline Pupil Diameter + (1 | Participant).

Table 1. Variables of interest

Dependent Variable	Fixed Effects	Random Effects
Post-feedback pupil response	Prediction	Participant
Change in Contingent Confidence	Contingent Belief	Session
Change in Contingent Belief	Contingent Confidence	
	Stimulus Reliability	
	Surprise Type	
	Baseline Pupil Diameter	

Potential Pitfalls

Although this study is a within-subject design and participants will conduct numerous trials across three sessions, it is questionable whether my analysis will have sufficient power. The occasions in which participant face surprises may not be sufficient to make a confident inference. My trial number and parameter values provides a sample size of 89 information theoretic surprises per participant and likely less for Bayesian surprises. This may be sufficient to test the effect of prediction error and surprise types, however, it may not be enough to test the effect of signal reliability. It is also not clear whether my manipulation of signal reliability will

be meaningful and whether the participants can notice the difference; both implicitly and explicitly. The design and the narrative involved in my experiment may cause confusion for some participants and the time frame in which participants must report their prediction and confidence may be too short and too demanding for the participant to sustain for approximately 23 minutes before the break.

Next Step

An ethics application will be submitted by mid June. Coding of the protocol will begin early June and finished by the end of the Spring 2019 term. I anticipate data collection and analysis to begin during the start of the Fall 2019 term.

References

- Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language*, 68(3), 255–278. <https://doi.org/10.1016/j.jml.2012.11.001>
- Bates, D., Kliegl, R., Vasishth, S., & Baayen, H. (2015). Parsimonious Mixed Models. Retrieved from <http://arxiv.org/abs/1506.04967>
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting Linear Mixed-Effects Models Using {lme4}. *Journal of Statistical Software*, 67(1), 1–48. <https://doi.org/10.18637/jss.v067.i01>
- Beck, J. M., Ma, W. J., Kiani, R., Hanks, T., Churchland, A. K., Roitman, J., ... Pouget, A. (2008). Probabilistic population codes for Bayesian decision making. *Neuron*, 60(6), 1142–1152. <https://doi.org/10.1016/j.neuron.2008.09.021>
- Boldt, A., de Gardelle, V., & Yeung, N. (2017). The impact of evidence reliability on sensitivity and bias in decision confidence. *Journal of Experimental Psychology: Human Perception and Performance*, 43(8), 1520–1531. <https://doi.org/10.1037/xhp0000404>
- Colizoli, O., de Gee, J. W., Urai, A. E., & Donner, T. H. (2018). Task-evoked pupil responses reflect internal belief states. *Scientific Reports*, 8(1), 13702. <https://doi.org/10.1038/s41598-018-31985-3>
- Filipowicz, A., Anderson, B., & Danckert, J. (2016). Adapting to change: The role of the right hemisphere in mental model building and updating. *Canadian Journal of Experimental Psychology/Revue Canadienne de Psychologie Expérimentale*, 70(3), 201–218. <https://doi.org/10.1037/cep0000078>
- Johnson-Laird, P. N. (2013). Mental models and cognitive change. *Journal of Cognitive Psychology*, 25(2), 131–138. <https://doi.org/10.1080/20445911.2012.759935>
- Knapen, T., De Gee, J. W., Brascamp, J., Nuiten, S., Hoppenbrouwers, S., & Theeuwes, J. (2016). Cognitive and ocular factors jointly determine pupil responses under equiluminance. *PLoS ONE*, 11(5), 1–13. <https://doi.org/10.1371/journal.pone.0155574>
- Mathôt, S., Fabius, J., Van Heusden, E., & Van der Stigchel, S. (2018). Safe and sensible preprocessing and baseline correction of pupil-size data. *Behavior Research Methods*, 50(1), 94–106. <https://doi.org/10.3758/s13428-017-1007-2>

- Meyniel, F., Sigman, M., & Mainen, Z. F. (2015). Confidence as Bayesian Probability: From Neural Origins to Behavior. *Neuron*, 88(1), 78–92.
<https://doi.org/10.1016/j.neuron.2015.09.039>
- O'Reilly, J. X., Schuffelgen, U., Cuell, S. F., Behrens, T. E. J., Mars, R. B., & Rushworth, M. F. S. (2013). Dissociable effects of surprise and model update in parietal and anterior cingulate cortex. *Proceedings of the National Academy of Sciences*, 110(38), E3660–E3669.
<https://doi.org/10.1073/pnas.1305373110>
- R Core Team. (2019). R: A Language and Environment for Statistical Computing. Vienna, Austria. Retrieved from <https://www.r-project.org/>
- Reisenzein, R., Horstmann, G., & Schützwohl, A. (2019). The Cognitive-Evolutionary Model of Surprise: A Review of the Evidence. *Topics in Cognitive Science*, 11(1), 50–74.
<https://doi.org/10.1111/tops.12292>
- Schwartenbeck, P., FitzGerald, T. H. B., & Dolan, R. (2016). Neural signals encoding shifts in beliefs. *NeuroImage*, 125, 578–586. <https://doi.org/10.1016/j.neuroimage.2015.10.067>
- Sutton, R. S., & Barto, A. G. (2018). *Reinforcement learning: an introduction* (Second edi). Cambridge, Massachusetts: The MIT Press.
- Swallow, K. M., Jiang, Y. V., & Riley, E. B. (2019). Target detection increases pupil diameter and enhances memory for background scenes during multi-tasking. *Scientific Reports*, 9(1), 5255. <https://doi.org/10.1038/s41598-019-41658-4>
- Urai, A. E., Braun, A., & Donner, T. H. (2017). Pupil-linked arousal is driven by decision uncertainty and alters serial choice bias. *Nature Communications*, 8, 1–11.
<https://doi.org/10.1038/ncomms14637>
- Van Slooten, J. C., Jahfari, S., Knapen, T., & Theeuwes, J. (2018). How pupil responses track value-based decision-making during and after reinforcement learning. *PLoS Computational Biology*, 14(11), 1–24. <https://doi.org/10.1371/journal.pcbi.1006632>
- Winn, M. B., Wendt, D., Koelewijn, T., & Kuchinsky, S. E. (2018). Best Practices and Advice for Using Pupillometry to Measure Listening Effort: An Introduction for Those Who Want to Get Started. *Trends in Hearing*, 22, 233121651880086.
<https://doi.org/10.1177/2331216518800869>
- Yu, A. J., & Dayan, P. (2005). Uncertainty, Neuromodulation, and Attention. *Neuron*, 46(4), 681–692. <https://doi.org/10.1016/j.neuron.2005.04.026>