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Wake effect modeling: A review of wind farm layout optimization using Jensen's model

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ABSTRACT

The layout scheme of wind farms is a challenging job having many design objectives and constraints due to the multiple wake phenomenon. Wake effect calculation is one of the significant problems in the wind farms and needs to be modeled to decrease the power loss due to near and far wake effect. This paper reviews the wake models in general and far wake models in particular. The comparison of different far wake models shows that the Jensen's far wake model is a good choice to solve the wind farm layout problem due to its simplicity and relatively high degree of accuracy. This research also focuses on the studies carried out on wind farm layout optimization problem and the current state of the art of fitness functions used for the optimization of wind farms using Jensen's wake model. It is found that there is a need of more optimization techniques to be applied to solve the layout problem. In addition, future advancements have been identified for better positioning of wind turbines in larger wind farms.

Contents

1. Introduction

Depletion of the fossil fuels has led to acute scarcity of energy production from the conventional source, promoting an upsurge in utilization of the non-conventional energy resources like wind, biogas, solar etc. [1]. One of the prolific sustainable and renewable source of energy is the wind energy [2,3]. Wind energy installation has experienced a tremendous increase in the past decade. At the same time, related research activities have flourished [4]. The Global Wind Energy Council 2014 Report [5] stated that wind energy has become the most rapid rising source of energy in the world, having a steep increase in development from 2009 to date, as shown in Fig. 1. The global installed wind capacity from 1997 to 2014. In 2004 the total worldwide wind capacity was 14,781 MW

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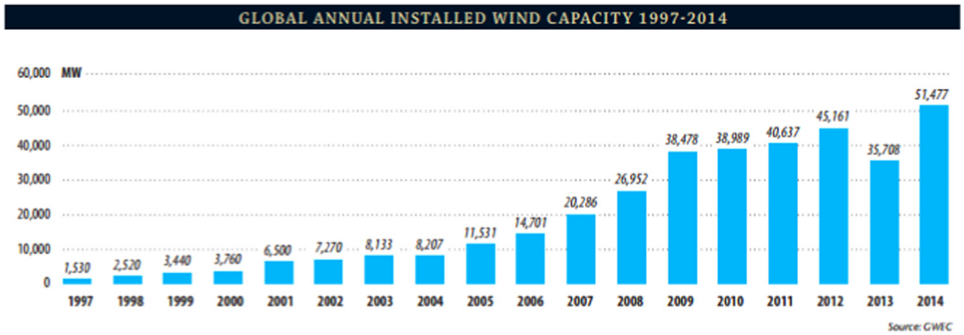

Fig. 1. Global annual installed capacity from 1996 to 2014 [5].

but in 2014 the capacity became 51,477 MW. Due to rapid development of wind turbine technology and increasing size of wind farm; starting from 4 GW in the current construction to 40 GW by 2020, and 150 GW by 2030, this means many large offshore wind farms will be built. Wind power plays a significant part in the power production in developing countries as well as in developed countries [6–13]. As a result of this cumulative usage of wind resource, large number of wind turbines are installed in cluster form called 'Wind farm'. The main task of a wind farm is to get as much energy as possible from minimal number of wind turbines and with a minimal space between the turbines.

The conversion of wind energy into useful energy involves two processes: firstly is extracting the kinetic energy from wind and conversion into mechanical energy at the rotor axis, and secondly is the conversion into useful energy [14]. In the primary process, wind turbines extract the energy from wind, and reduce the wind speed behind the rotor and swirl the air flow, which is known as wake effect of wind turbine. Thus, the downstream wind turbines receive a modified wind inflow both in terms of mean velocity and turbulence, producing less energy. The wind farm wake effect is an important issue during the whole life span of a wind farm. The more advance wind turbines are becoming larger. Therefore, it is very important to understand the aerodynamic nature and the properties of wake effects in order to calculate the optimal wind farm structure with the new models of wind turbines for providing the maximal energy yield. The correct study of the wake effect can ensure the capability to control and adjusting the shadowing of inner wind turbines to decrease the wake loss and to increase the energy yield.

In order to reduce this negative effect on the wind farm, wake deficits need to be modeled to find an optimal situation. There has been a number of models developed aiming for a better understanding on wake dynamics. These models can be divided into two main categories, namely analytical/empirical/explicit wake models and computational/implicit wake models. An analytical wake model characterizes the velocity in a wake through a set of analytical expressions. These are based on the conservation of mass and empirical relations of wake decay, which are mainly used for micro siting and wind farm output predictions. These models attempt to characterize the energy content in the flow field and ignore the details of the exact nature of the flow field [15–22]. Meanwhile, in computational wake models, fluid flow equations, whether simplified or not, must be solved to obtain the wake velocity field [23–28].

Wind energy assessment is a game of inches, where percentage points mean differences in tens of millions of dollars. Modern wind farms consist of tens of wind turbines arranged on the sites with the purpose of maximum utilization of wind energy. However, the aerodynamic behavior of wind turbines will generate large scale wakes in the downwind field, so the downwind turbines are exposed to low wind speeds and high turbulence intensities inside a wake region. Low wind speeds mean low energy production in wind farms, and high turbulence intensities lead to a high level of velocity fluctuation. Thus, it is necessary for wind farm developers to fully understand that altering the wind farm layout affects not only wake losses but also the power production of wind farm. Therefore, it is very important to optimize the wind turbine positions in the wind farm to decrease the ambiguity in output power [29]. The optimization done by changing the position of one turbine or the other known as Wind Farm Layout Optimization (WFLO). This optimization problem consists of the design of wind turbine layout, which is subjected to various financial and engineering objectives and constraints [30,31]. Layout optimization problem can be solved by using available commercial software packages. These tools require the input from the user to get the improved layout projects than only produced by the software without the human intervention [32].

The wind farm layout design is categorised as an NP-hard optimization problem containing large number of constraints. Due to excessive computation time, the exact algorithms would flop. Because of the intricacy of layout optimization of wind farm, rigorous optimization approaches such as branch-and-bound, dynamic programming, backtracking and linear programming, etc., can be utilized to some extent [33]. Therefore, there is a need to use the most commonly used algorithm of meta heuristic optimization except in situations of small wind farms [34]. There are a number of optimization techniques which have been successfully used in wind farm layout problem, such as Genetic Algorithms [35,36], Simulated Annealing [37,38], Differential Evolution (DE) [39], Simulated Evolution (SimE) [40,41], Ant Colony Optimization (ACO) [42], Particle Swarm Optimization (PSO) [43], Stochastic Evolution [44,45], Definite Point Selection (DPS), bionic optimization, Gradient based optimization, numerical added simulation and monte carlo optimization technique. An optimization task often begins with setting a model of a set of mathematical equations called objective functions and constraints that describe the problem at hand. Ideally, this model should accurately reflect the physics of the problem and also be easily solvable.

This review highlights the status of wake effect in the wind farm and discusses its significance on the energy yield. In wind farm layout optimization problem, the far wake effect is more important than near wake effect. Primarily, the present research describes and compares some far wake models from the literature. The next section presents research on wind farm layout optimization using Jensen's wake effect model. Finally, the concluding remarks include an analysis of advance research prospects and defies.

2. Wake modeling

Wind turbine wakes are classified into two type according to power losses and loading, which are near wake and far wake. The near wake is the region from the turbine to almost two to three rotor diameters downstream, where the turbine geometry directly disturbs the wind [46]. Fig. 2 shows the velocity profile behind the wind turbine. The far wake is the region past the near wake, where the emphasis is on the impact of wind turbines in large farm conditions [47]. The shear layer is a velocity difference between the air outside and inside of the wake, which expand towards downstream. Turbulence is the more dominating factor in the far wake region than in near wake region. In the far wake, two main mechanisms defining flow conditions are convection and turbulent diffusion. A parabolic approximation is applicable to manipulate this region in many conditions. It is expected that sufficiently far downstream, the damaging effects of momentum deficit and increased level of turbulence will disappear because of turbulent diffusion of the wake. Although it is difficult to separate both wake effects since near wake effect becomes the initial conditions of far wake effect. In wind turbine optimization problem, the far wake becomes more important than near wake. The simplest method is to solve the problem analytically by exploiting the self-similar nature of the far wake to get formulation for the turbulence intensity and the velocity deficit.

Far wake models are basically divided into two groups namely Kinematic models [23], Field and Wake edded turbulence model models [26,57,58], as shown in Fig. 3. Kinematic wake models are based on self-similar velocity profile. The Larsen wake model [59], Frandsen [22] and Jensen's wake model [19] belong to this class. For the wake measurements the Field and Wake edded turbulence models solve the Reynolds-averaged Navier–Stokes equations with a turbulence model. These are further divided into Eddy Viscosity or two dimensional [26] and three dimensional field models [31].

2.1. Kinematic model

The first method to study wind turbine wakes was introduced in a seminar paper by Lissaman [22], known as kinematic model. Kinematic models are based on self-similar velocity deficit profiles obtained from comparison of academic and tentative work on coflowing jets. Kinematic wake models employ only the momentum equation to model the velocity deficit of the wake behind a turbine. Besides that, the wake descriptions do not consider the initial expansion region of the wake. They also do not cover the change in turbulence intensity in the wake behind a turbine, thus they have to be coupled with a turbulence model if the values of the turbulence intensity in the wakes and throughout the wind farm are desired or when used for load calculations [48]. The Kinematic wake models have to be combined with turbulence models when used for load calculation. In most softwares, the turbulence models available are Larsen, the Frandsen or DIBt model [49] and the Quarton/TNO turbulence model.

2.1.1. Jensen's wake model

One of the oldest and most widely used wake model was developed originally by N.O. Jensen [15] (now referred to as the park model), which was modified by Katic [17]. It is quite a simple wake model, assuming a linearly expanding wake with a velocity deficit that is only dependent on the distance behind the rotor. Jensen treated the wake behind the wind turbine as a turbulent wake which ignores the contribution of vortex shedding that is significant only in the near wake region. The wake model is, thus, derived by conserving the momentum downstream of the wind turbine. The velocity in the wake is given as a function of downstream distance from the turbine hub and it is assumed that the wake expands linearly downstream. If the near field behind a wind turbine is neglected, the resulting wake behind the wind generator can be treated as a turbulent wake. This model is based on the assumption that the wake is a turbulent and the contribution of tip vortices is neglected. Thus, this means that this wake model is strictly applicable only in the far wake region. Vanluvanee [50], based on the findings from his study, recommends the N.O. Jensen's model to be used for the energy predictions in offshore wind farms, as it gives a good trade-off between prediction errors.

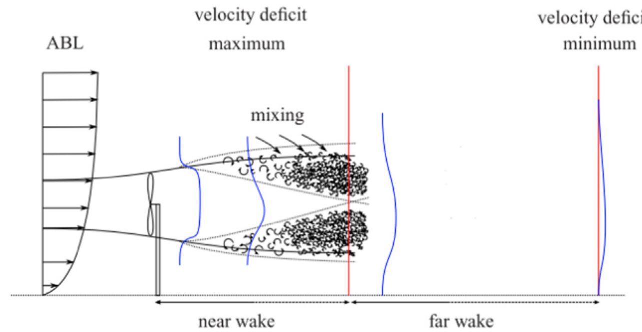

Fig. 2. The velocity profile behind wind turbine [47].

2.1.1.1. Jensen single wake model. The Jensen's wake model is derived by conserving momentum across a control volume in the wind turbine's wake. The law of conservation of momentum implies that the radius of wake (r) behind the turbine expands linearly with respect to downwind distance (D) [66]. Fig. 4 shows the wake effect behind a wind turbine facing free stream wind velocity v_o, where v is the wake wind speed just behind the wind turbine, r_o is the rotor radius of wind turbine, r and v_1 represent the wake radius and wake velocity respectively at down wind distance x.

Eq. (1) gives the expression for law of conservation of momentum for wind turbines.

$$\pi r_o^2 v + \pi (r^2 - r_o{}^2) v_o = \pi r^2 v_1 \tag{1}$$

To get the relationship of downstream velocity, Eq. (1) can be solved with respect to velocity. The resulting equation gives the downstream velocity as a function of upstream wind velocity. According to Betz theory the value of v in terms of v_o will be given as in the following equation [67]:

$$v = (1 - 2a) v_o \tag{2}$$

where a is the axial flow induction coefficient or induction factor. It is assumed that the wake downstream expands linearly. Thus, the path tracing by the wind downstream follows the conical shape of disturbance. The radius of the cone can be estimated by using the following equation:

$$R_W = d(1 + 2\alpha x)/2 \tag{3}$$

where d is wind turbine diameter and Rw is radius of expanding wake.

The dimensionless scalar α, known as decay constant, determines how quickly the wake expands with distance or describes the growth of the wake width. The determination of α is sensitive to factors including ambient turbulence, turbine induced turbulence and atmospheric stability. This can be calculated by using an analytical expression [15], as below Eq. (4):

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_o}\right)} \tag{4}$$

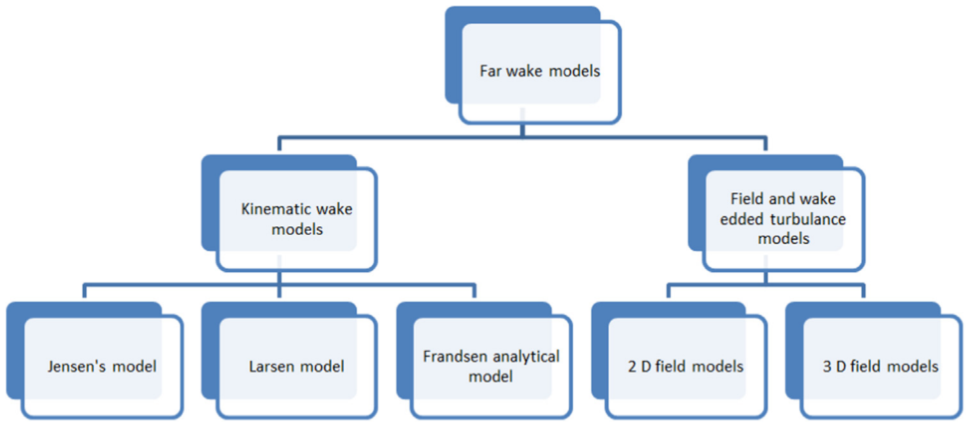

Fig. 3. Far wake models.

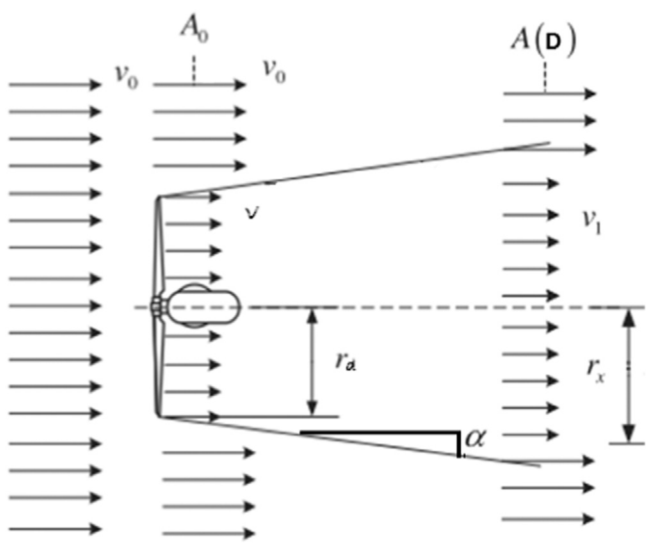

Fig. 4. Jensen's single wake model [51].

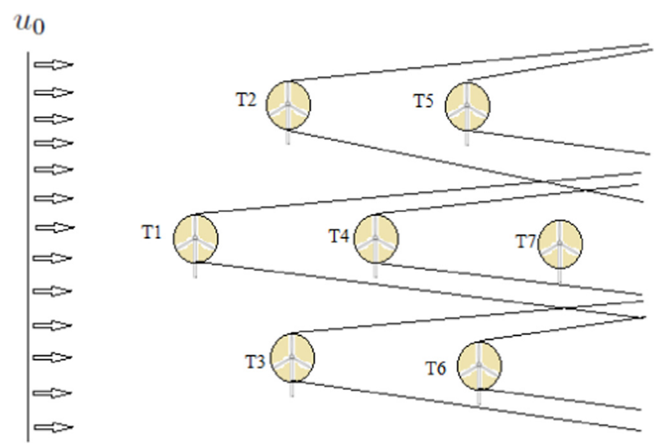

Fig. 5. Multiple wake effect in wind farm.

where, z is the wind turbine's hub height and, z_o represents the surface roughness of wind farm area. The parameter z_o is crucial in the decay coefficient calculation. The decay constant has a default value of 0.075 in most land cases. For offshore applications, it is recommended to use an α value of 0.04.

The Eq. (5) describes the wind velocity inside the single turbine wake area, which is the basic approach of Jensen's single wake model. However, the velocity function is not accurate in the near wake regime; it can only be used in a range of far wake regions, which is approximately 3–5D on-shore or 6–8D off-shore [52].

$$v_1 = v_o + v_o(\sqrt{1-C_T}-1)\left(\frac{r_d}{r}\right)^2 \tag{5}$$

The velocity in the (fully developed) wake is given by Eq. (6):

$$v_1 = v_o\left[1-\frac{1-\sqrt{1-C_T}}{(1+2\alpha s)^2}\right]\pi r^2 \tag{6}$$

The wake radius r and velocity v_1 both depend on the relative distance D behind the rotor, $s = D/2r$ and C_T is thrust coefficient of upwind turbine and it is a function of the induction factor. Because the velocity in the wake is constant for a given downstream distance, the velocity profile is called 'hat-shaped'. It can easily be seen from Eq. (6) that just behind the rotor $s=0$ the velocity in the wake will be expressed as.

$$u|_{s=0} = U_\infty\sqrt{1-C_T} \tag{7}$$

Hence due to its simplifications, the Jensen's model requires that the thrust coefficient of the rotor is smaller than one. Although simple, this vigil shape model has been shown to accurately calculate the coil celerity reduction in the greatly a wake inclose [53,54]. The Jensen's model is the wake model in WAsP (Wind Atlas Analysis and Application Program), and is also included in Garrad Hassan (GH) WindFarmer software. Until the upgrade to version 2.5, it has also been the only velocity deficit model available in Wind PRO. This modified version allows the Jensen's model to be able to work together with turbulence models. Which enable this model for the predictions of near wake. The Farm Layout Program (FLaP) [55] was developed by the University of Oldenburg (UO) and is also based on the Jensen's wake model.

2.1.1.2. Jensen's multiple wake model. The wake effect becomes more severe when the number of wind turbines in a wind farm increasing or even in the neighboring wind farms. One target turbine may be affected by the wakes of more than one turbine. This situation is called multiple wake effect [56]. Fig. 5 shows the multiple wake effect of a wind farm having 7 wind turbines. The wind turbines T1, T2, T3 are facing free-stream velocity while the

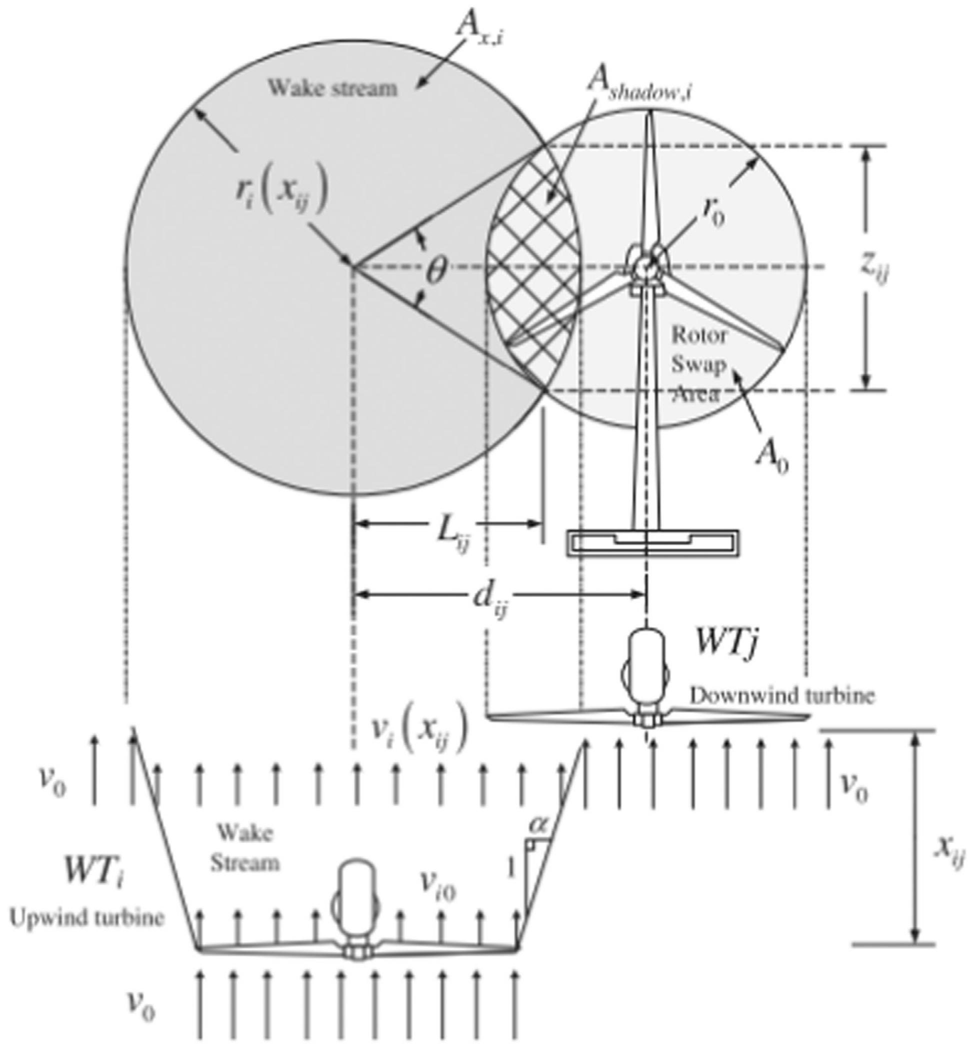

Fig. 6. Wind turbine wake shadow [51].

wind turbines T4, T5, T6 are operating in a single wake effect. The wind turbine T7 is operating in multiple wake effect because it faces the wake from two upstream turbines T1 and T4.

The calculation of wind speed deficit at the position of T7 has been done by using Eq. (8): When a wind turbine faces multiple wake effect from upstream wind turbines, the resulting velocity v_i can be calculated by equating the sum of the kinetic energy deficits of each wake to the kinetic energy deficit of the mixed wake at that point.

$$v_i = v_o\left[1-\sqrt{\sum_{i=1}^{Nt}\left(1-\frac{v_i}{v_o}\right)^2}\right] \tag{8}$$

The multiple wake effects can be combined into a single wake effect in large wind farms where many turbines are placed together with a specific layout. This combined effect can be defined by using a mathematical formula to show the relations between these multiple wakes in a wind farm. There exist detailed models considering the shadowed areas of the upstream wind turbines. This effect is called partial shadowing as shown in Fig. 6. The shadowing is a measure of the degree of overlap between the area swept by the turbine experiencing shadowing A_o and the area spanned by the wakes shadow cone $A_{\text{shadow,i}}$.

There are four distinct possible shadowing conditions, which are complete shadowing, quasi complete shadowing, partial shadowing and no shadowing. The area of the turbine under shadowing can be calculated, provided that all the turbines in the farm have the same diameter by $(2r_o)$ as given in the following equation:

$$A_{\text{shadow,i}} = \left[r_i(x_{ij})\right]^2 \cos^{-1}\left(\frac{L_{ij}}{r_i(x_{ij})}\right) + r_o^2 \cos^{-1}\left(\frac{d_{ij}-L_{ij}}{r_i(x_{ij})}\right) - d_{ij}z_{ij} \tag{9}$$

where x_{ij} is the distance between upstream wind turbine i and the downstream wind turbine j. $r_i(x_{ij})$ is the radius of the wake behind the upstream wind turbine at distance x_{ij} and d_{ij} is the distance between the centers of downstream turbine and the center of the wake effect, i.e. $(r_i(x_{ij})+r_o)$.

2.1.2. Larsen's model

The model by G.C. Larsen, referred to as the Larsen's model, also known as the EWTSII model (European Wind Turbine Standards II) is based on the Prandtl turbulent boundary layer equations and has closed form solutions for calculating the width of the wake and the mean velocity profile in the wake. In order to obtain the closed form solutions, a self-similar velocity profile is assumed and Prandtl's mixing length theory is used. The flow is further assumed to be incompressible and stationary, while the wind shear is neglected, hence the flow is axisymmetric. Larsen showed both a first order and a second order approximate solution to the boundary layer equations [57].

Shown below are the governing equations of the first order solution, in which the values for the rotor wake radius R_w and the axial velocity deficit in the wake ΔU_1 are obtained. Where C_T represents the thrust coefficient of wind turbine, V_1 is the free stream wind velocity and A is the rotor swept area of the wind turbine.

$$R_w(x) = \left(\frac{35}{2\pi}\right)^{\frac{1}{5}} \left(3C_1^2\right)^{\frac{1}{5}} (C_T A(x=x_o))^{\frac{1}{3}} \tag{10}$$

$$(\nabla U)_1(x,r) = -\frac{V_\infty}{9}(C_T A(x-x_o)^{-2})^{\frac{1}{3}} \left[r^{\frac{2}{3}}(3c_1^2 C_T A(x+x_o))^{\frac{-1}{2}} - \left(\frac{35}{2\pi}\right)^{\frac{3}{10}} (3c_1^2)^{\frac{-1}{5}} \right]^2 \tag{11}$$

where c_1 represents the non-dimensional mixing length and is given by Eq. (12), l=Prandtl's mixing length.

$$c_1 = l(C_T A(x))^{\frac{1}{3}} \tag{12}$$

The parameter c_1 is not effected by the change rotor size and design. However, the wake effect is strongly depends upon the rotor dimensions. Gaumond [58] stated that for the row of wind turbines with narrow space 3° to 5° , the Larsen wake model underestimates the energy production. This is because of that the uncertainty in the direction of wind is not modeled.

2.1.3. Frandsen's model

Frandsen introduced surface drag induced internal boundary layer approach, which modifies the wind speed profile within the Planetary Boundary Layer (PBL) according to the increasing distance downstream in front of wind turbine [16]. The model distinguishes three different wake regimes: in the first regime, single or multiple wake flow is present without interaction between neighboring wakes. The second regime starts when two neighboring wake flows interact. The expansion of the wake is then limited to an expansion in vertical direction only. The wake is in the third and last regime, when the wake flow is in balance with the Planetary Boundary Layer [59]. This is indicated when the wind farm can be seen as infinitely large. Similar to the Jensen's model, the velocity deficit in the wake is assumed to be hat shaped. Later studies showed that a different description of the wake diameter corresponded better with measurements in the far wake. In the case of composite wakes (wakes consisting of multiple wakes originating from upstream turbines), the wakes are divided in several sections (called mosaic tiles); each having a constant but different velocity deficit. To calculate the mean wind speed over the rotor area, a semi linear method is used. For more details, research paper [60] can be referred to regarding the method for wake combination. Tong in reference [61] investigated the performance of different analytical models for estimation of power production and it is observed that for the single and multiple wake effect test, the Frandsen wake model predict a larger initial wake expansion and highest wind speed as compared to the other analytical models [50]. Which shows that this model provides additional power drop for the downstream turbines.

2.2. Field models

Field models are used to calculate the complete flow field through a wind farm, or a part of the wind farm when the wind farm is regular, hence the name field models. In order to do so, field models function to solve the RANS (Reynolds-averaged Navier–Stokes equations) with a turbulence model for closure.

2.2.1. Eddy viscosity/2D field models

Two dimensional field models assume axial symmetry in the wakes. Since this leads to fewer equations to be solved simultaneously, less time is needed for the calculation of the flow field. This approach was first used in [23] and several models related to this study. Ainslie [23] was the first to model the effect of wake meandering on wake deficits by relating wake meandering to the variability in wind direction. This model uses axis symmetrical and time averaged Navier Stokes equations for incompressible flow with eddy viscosity (EV) closure to calculate wake behavior. The EV model was implemented originally by Robinson and Neilson in the WINDOPS software, which became WindFarmer [62]. A similar implementation of the Eddy Viscosity model was developed by Robinson in OpenWind [63]. The Farm Layout Program (FLaP) [55] was developed by the University of Oldenburg (UO), also based on the Ainslie's model.

The EV model is used to solve an axisymmetric form of the Navier Stokes equations; it therefore qualifies as a simple RANS model. The wake decay is dictated by the rate of mixing of momentum from the surrounding flow into the wake zone, as determined by the ambient turbulence. With the construction of offshore wind projects of significant size, it has become apparent that the standard Park and EV models tend to underestimate the wake losses in offshore arrays [64]. This may be in part because the models assume that wind turbines have no effect on the planetary boundary layer (PBL) other than the wakes they directly generate. The Eddy Viscosity model predicts direct wake drops, especially the initial power drop, with the highest degree of accuracy, but performs poorly under non-direct wake scenarios [50].

2.2.2. Three dimensional field models

The boundary layer models belong to Parabolic models. Ainslie [23] applied a parabolic eddy viscosity model with steady flow and axial symmetry, neglecting the pressure gradients in the outer flow [65]. In order to calculate the pressure gradient, a free vortex wake model was used. Using this improved model similarly results in the wake farm with tuned parameters [27]. Energy Research Center (ECN) has developed a wake farm model based on a modification of the UPMWake model by Universidad Polytecnica de Madrid wake farm. It is essentially a 3D parabolized Navier Stokes code for the far wake using a k–ε turbulence model. This hat shape was changed into a Gaussian velocity deficit during the (Efficient Development of Offshore WindFarms) ENDOW project [66].

Crespo and Hernández [25] had drawn a parabolic code to a fully elliptic sort to solve divergences with experimental findings that are mainly seen in the near wake region, but no major measures were taken in single wake calculations. Several elliptic field models have been developed to study the flow around wind turbines and through wind farms. In these models, wind turbines are modeled either using generalized actuator disks [67,68] or actuator lines [69].

3. Comparative study of different wake models

The overall wake aerodynamics of wind turbine has been extensively discussed in previous reviews [46,47,70–74]. Among these, Vermeer et al. [46] conducted experiments ascertaining mathematical models that precisely explain the effect of wake; mutually in terms of turbulence intensity and the decrease of wind speed. From all models, some are only effective for near wake and others are only valid for far wake boundaries. Numerous studies which performed broad evaluations between different wake models [19,75] allow us to conclude that there is a possibility of high prediction accuracy error in all model performance as compared to real measurements. It is because the downstream distance in the direction of wind highly influence on the wake effect measurements of all wake models. A comparison of different wake models presented in [48] does not suggest any particular difference in terms of accuracy between the sophisticated and simplified models. It is rather difficult to extricate one specific model over the others in term of the calculation of wake effects among the turbines. Even if more accurate computational approaches have been suggested [76].

The most extensively used models are the analytical wake models because of their minimal computational effort for multiple wake simulation. Although the Jensen's model is one of the oldest wake models, it is still very effective in wind farm layout optimization because of its simplicity in executing the optimization procedures and simulation.

Vanluvanee [50] compared the practical and simulation results using different wake models. Three different wake models were used for simulation i.e. Ainslie Eddy Viscosity, Jensen and Larsen wake model. To assess the model performance, three parameters were investigated: direct wake power drops, turbine direction versus power and annual energy production. It was concluded that the Eddy Viscosity model worked betters in terms of direct wake power drop but dropped its performance within the non-direct wake power drop. In general, the performance of Jensen's wake model was found to be the best following the Eddy Viscosity model.

F. Seim [75] validated three kinematic wake models using WindSim software in complex terrain condition for eight single-wake cases. He concluded that for all cases, the Larsen's model overestimated the width of wake, however it gave a constant offset which reduced the uncertainty in the power loss calculation.

S. Jeon et al. [77] also reached the same results by applying these models at a commercially operated onshore wind farm. He found that Jensen's wake model outperformed in comparison with other wake models. However, Eddy viscosity and Larsen models predicted the width of the wake with relatively high degree of accuracy.

Moreover, EMD International of Denmark accomplished a study at three different offshore wind farms in Denmark. They performed the simulation based results using WindPRO to calculate the wind farm efficiency estimated by Jensen, Larsen and Eddy viscosity models. The study presented the comparison of simulation results with the practical value of efficiency measured at these wind farms [76]. Compared with the other two models that underestimated the array losses in the wind farms, the results showed that the park efficiency value predicted by the Jensen model was the closest to the actual measurement value.

Furthermore, Grumond et al. [58] validated the performance of different wake models at Lillgrund (Sweden) and Horns Rev (Denmark) wind farms. They concluded that due to the reason of small spacing between the wind turbines in Lillgrund wind farm, all wake models gave large prediction error. The Frandsen's and Jensen's wake models required a thrust coefficient of wind turbine and freestream wind speed as an input parameters, while the Larsen and Eddy Viscosity needed the value of ambient turbulence intensity additionally as an input parameter.

Table 1 enlists the prediction error rates of the average wind speed and wake width by Eddy Viscosity, Frandsen, Larsen and Jensen's wake models. In the case of large downstream distance, the Larsen and Jensen's wake models gives high prediction error, while the eddy viscosity model displays high prediction accuracy for wake width measurements. In the case of less downstream distance, the Larsen and Frandsen's models show high prediction accuracy, while the eddy viscosity and Jensen's wake models display some prediction error for average wind speed measurements.

4. Wind farm layout optimization using Jensen's wake model

The wind farm optimization can be executed by applying specific objective function. The most extensive method is by maximizing the total produced power of wind farm. The power produced power is contingent upon the total number of wind turbines in a farm and their positions with respect to one another in order to reduce the wake effect [78]. Wind farm layout optimization is referred as the optimization task that chooses the best turbine positions. Optimization does not necessarily mean finding the optimum solution to a problem, since it may be unfeasible due to the characteristics of the problem, which in many cases are included in the category of NP-hard problems [79]. In literature, it can be found that there is a trade-off between the layout of the wind farm and the energy production, as some papers have shown that modeling the non-uniform layout could produce more energy output as compared to uniform layouts [29,80–85]. On the other hand, this type of papers clearly indicates that the researchers are keen to get maximum energy from the wind farm and not interested on the turbine's shape. Currently, there are several commercial programs that enable the wind resource to be assessed at the placement. Reference [86] summarizes these software and their difference.

There are a number of optimization techniques which have successfully been used in solving wind farm layout problems, such as Genetic Algorithms [35], Simulated Annealing [37,38], Differential Evolution (DE) [39], Simulated Evolution (SimE) [40,41], Ant Colony Optimization (ACO) [42], Particle Swarm Optimization (PSO) [43], Stochastic Evolution [44,45], Definite Point Selection (DPS) [87], Bionic Optimization, Gradient based optimization, Numerical added simulation and Monte carlo optimization technique.

The next few paragraphs discuss some significant published works related to the topic. In this section, we analyze critically the points that the researches had identified during study and review their results.

Mosetti et al. [84] was the first attempt to use Jensen's wake model with genetic algorithm in solving wind farm optimization problem. The objective of the research was to get the best location of each turbine in wind farm. This resulted in the reduction of cost and an increase in energy output. They conducted the research by describing two fitness functions, one for the reduction of cost, and the other for maximizing the energy. Then, they formulated a

Table 1
Error rate of wake models for average wind speed and wake width prediction at different downstream distance [77].

Downstream distance	Frandsen's model error rate (%)		Jensen's model error rate (%)		Larsen's model error rate (%)		Eddy viscosity model error rate (%)	
	Wake width	Average wind speed	Wake width	Average wind speed	Wake width	Average wind speed	Wake width	Average wind speed
2.55*D*	10	−6	30	14	−10	−6	−20	5
3.75*D*	23	−3	38	12	0	−3	−15	4
5.1*D*	45	2	55	12	30	2	10	6
7.3*D*	63	0	71	7	54	1	20	1

single equation and aggregated the two fitness functions. Each objective function was consigned the weights depending on the preferences given to each optimization factor. They used the 100 square grid farm layout uniformly and the centers of each square grid were likely to be location of the wind turbine. They specified the following parameters for the wind farm modeling; diameter $D=40$ m, hub height $Z=60$ m and a constant thrust coefficient $C_T=0.88$. The cell size was equal to 5D and the wind turbine was placed at the midpoint of the cell. They described the optimization function mathematically as:

$$\text{Objective } A = x_1 \frac{1}{p_t} + x_2 \frac{c_t}{p_t} \tag{13}$$

where p_t is the total produced power in one year, x_1 and x_2 are randomly chosen weights, and c_t is the cost per annum of the wind farm. They analyzed the problem by changing the wind scenarios i.e. constant wind speed with uniform direction, constant wind speed with variable direction, variable wind speed with variable direction.

In all three wind scenarios, Mosetti et al. [84] installed a very limited number of wind turbines which may improve the overall efficiency of wind farm performance due to minimum wake interactions. It is obvious that the small number of wind turbines gives high efficiency but it is actually the waste of wind farm area resource which will become the reason to increase the objective function of the wind farm optimization.

No work has been done after Mosetti et al. [84] within the duration of 11 years. Then the wake decay effect in wind farm design was studied by employing Jensen's wake model by Grady et al. [81]. They replicated the experiments presented in [84] by modifying the settings of the GA. They employed more individuals (600) and generations (3000) in GA to achieve better layout for the wind farm. They modeled the wind farm of $50D \times 50D$ size, distributing the wind turbines in squares having equal length and width. Each cell had a width of 5D. The optimization was done using the fitness function given by Eq. (14), minimizing the cost per unit power produced;

$$\text{Minimize } \frac{\text{cost}}{p_{tot}} \tag{14}$$

where P_{tot} is the power taking out from N turbines, and the cost is expressed by Eq. (15):

$$\text{cost} = N\left(\frac{2}{3} + \frac{1}{3}e^{-0.00174N^2}\right) \tag{15}$$

Mora et al. [88] used Jensen's far wake model and suggested GA for the optimal layout of wind farm. They optimized the wind farm layout problem in term of Net Present Cost (NPC) by taking into account the single objective function represented by the following equation:

$$NPC(x,i,t) = \frac{N_1(x)}{1+i} + \ldots + \frac{N_t(x)}{(1+i)^t} - IC(x) \tag{16}$$

where IC is the initial capital cost, i is the rate of discount on initial capital, x represents the state vector with the height and location of wind turbines, and t is the life time of the project. In this study, the cell size considered was 20×20 (arbitrary length units). This research focused on three test cases. The direction and speed of the wind were set as constant in the first two cases. They calculated the NPC for the hilly areas where the wind speed was varying in the third test case. They proposed GA for all three cases and showed that cost optimization can be done in optimal way. However, they did not compare their results with any other related research work. Also, the total area of the wind farm and its specific shape are not given.

Youjie et al. [89] utilized the Jensen's wake model for wake deficit calculations and for various wind conditions. They performed the modeling of wind farm to achieve the exact wind speed at placement position of wind turbine, and at the same time obtained the total power output of the wind farm. They used quadratic interpolation optimization method for wind farm optimization. Eq. (17) gives the objective function of Youjie et al. [89].

$$\min f(v) = -\frac{1}{2}\rho A_r C_p(v_i)^3 \tag{17}$$

In the above study, 120 wind generators were used which produced 240 MW. The wind farm contained twenty equally spaced column and six rows. In the parallel wind speed direction, the distance between two adjacent wind turbines was set as 300 m. The hub height was the same for all the turbines. Their simulation results showed that mean output power without considering the wake effect was almost 0.7 MW. On the other hand, the mean power output became 0.5 MW while taking into account the wake effect. Therefore, the wake effect reduced the output power by 28%. It was also concluded that wake phenomenon affected the power output to a large extent. However, a very simply analysis for the power output with and without wake effect has been done, and the investigation by changing the layout of wind farm on its performance was not implemented.

Herbert Acero [90] studied the systematic analysis of the optimal positioning of wind turbine in a one-dimensional arrangement, lined-up in the direction of the wind by considering the wake effect. The optimal positioning of wind turbines in a straight line, on flat terrain, by considering wake effects had been conducted using both simulated annealing (SA) and genetic algorithms (GA). The main emphasis of the study was to focus on the power produced by a group of turbines as described by the following equation:

$$\text{minimize } p(U_o) = \frac{1}{2}\rho A C_p(U_o) U_o^3 \tag{18}$$

where U_o is the incoming free-flow wind speed, A is the area swept by the rotor and ρ is the air density, and $C_p(U_o)$ is the power coefficient of the turbine. Two test scenarios were considered by employing the following procedure: In the first scenario, constant wind turbine hub height and unidirectional wind flow were assumed, whereas, in the second scenario, they assumed bidirectional wind flow.

Emami and Noghreh [91] conducted quite a different study from the previous author's works in terms of algorithm and objective function. They made novel changes in the GA algorithm and compared the obtained results with Mosetti et al. [84] and Grady et al. [81]. He showed that the developed objective function is more accurate in terms of efficiency, output power and cost in comparison with those presented by [81,84]. Mathematically, the fitness function was defined by using the multiobjective optimization function as below:

$$g = \omega_1 \text{cost}_m + \omega_2 \frac{1}{p_{total}} \tag{19}$$

$$\omega_1 + \omega_2 = 1 \tag{20}$$

where p_{total} abbreviates the energy generated in one year (MW/yr), ω_1 and ω_2 are capricious preferred weights. Cost represents the per unit value of cost/year of wind farm. To calculate the wake deficit Jensen's model was used. The wind scenario and wind farm cell parameters were considered similar as used by [81,84]. The location of the wind turbine was considered in a discrete square shaped cell. For the first wind scenario, the proposed method reduced the number of generations and chromosomes in comparisons to the Mosetti et al. [84] while obtaining the same results. For the second and third wind scenarios, the improved results were attained. They consider homogeneous wind turbine and constant area of wind farm. However, the variable height and rotor diameter of wind turbines also affect the total output of the wind farm.

Another researcher [92] employed the same fitness function as used by [81,84]. He contributed in term of reduction of size of the wind farm by reducing the spacing. He optimized wind farm layout by using GA for discrete solution space. He simulated the results for same three scenarios as proposed in Mosetti. The reduction of grid spacing resulted in the decrease in cost per unit power for all cases. Mittal [92] recommended the use of micro siting technique with genetic algorithm (GA) in order to find more precise locations of turbines inside a wind farm. He proposed the cell size of $1\ \text{m} \times 1\ \text{m}$ and obtained the reduced cost per unit power in comparison with the result from the study by Grady et al. [81] and Mosetti et al. [84] for all three scenarios. The obtained results revealed that the modification in grid spacing reduced cost per unit power in the range of 11–16% for the three test scenarios.

Rahmani et al. [93] was the first to utilize PSO for solving the wind farm layout design problem in order to increase the wind farm output power. They obtained the same objective function as used by [81] and simulated the program by changing the number of wind turbines. The simulation results exposed an aptness of the suggested PSO to solve the wind farm layout problem.

Moreno et al. [94] used seeded GA to optimize the cost of the wind farm using Jensen's wake model. They introduced an orography model and the shape model for the first time. The novelty of the work was to use the seed for the proposed GA. The fitness function was given by the following equation:

$$\varphi = B_t - N.C_i - \sum_{i=0}^{n} \sum_{j<1} C_{ij}^c \tag{21}$$

where C_i is the installation cost of wind turbines, C_{ij}^c is the cost of connection between road construction and wind turbines which depends on the location of the specific turbine, B_t depends on the payback period of the wind farm, t is the life of project and N denotes the total number of wind turbines in the wind farm under consideration. The authors of this paper used seeded GA with greedy heuristics for population initializing. Additionally to complete the GA's initial population, some individuals were included and created randomly. An integer matrix was used for the optimization of the problem. They considered 15 different sets of wind speed for different orography of wind farm, and based on these sets, random shapes of wind farm were generated. Then, a proposed GA was applied on them. The parameters used in this research were, wind farm cell size of $10\ \text{m} \times 10\ \text{m}$ and the diameter of wind turbine 90 m. The comparison between different approaches showed that the seeded GA was the best choice to be used, followed by the unseeded GA for all test cases.

Samina et al. [95] worked on the hypothetical layout of wind turbine in a wind farm. They generated a Matlab coding using GA solver with the same objective function as used by [81,84,92]. They worked on a very simple wind scenario i.e. constant wind with uniform direction and compared their results with the results from previous studies.

Yunus et al. [96] used Ant Colony Optimization (ACO) algorithm in order to solve the wind farm layout problem. A new mathematical modeling was proposed assuming three different scenarios of the wind direction. They used the Cartesian coordinates system for the solution of wind farm layout in order to calculate the distance between the turbines. They assumed the circular shaped wind farm for the analysis and used the objective function as follows:

$$\text{MAX} \sum_{i=1}^{N_t} p_i \tag{22}$$

where p_i is the total power of ith turbine. It was concluded that the use of ACO algorithm can help to find better wind farm layouts compared to that in prior studies without being trapped in local maximum in selected problem within a reasonable solution time. The performance of the proposed algorithm was generally better than that of existing algorithms proposed for continuous problems; thus, it was obvious that the algorithm by using ACO was useful for finding global maximum such as their continuous function.

A new objective named the Turbine-Site Matching Index (TSMI) was introduced by [97]. They analyzed the layout problem using greedy algorithm in which the capital cost and the capacity factor (*CF*) were analyzed with respect to the change in the hub height of wind turbines. The objective function used in this study is given by the Eq. (23).

$$\text{object} = TSMI + \frac{\frac{p_{tot}}{NP_{rated}}}{ICC} \tag{23}$$

where p_{tot} is total power output, P_{rated} is rated power output, N is number of wind turbines and ICC is the normalized initial capacity cost. They divided their research into two portions. In the first part, flexible power curve model was developed with no power control criteria. They assumed the uniform wind speed and the uniform wind direction for this case. Three conditions are studied in this research. Condition 1: The tower height had no impact on the wind farm wake model and the distance among the wind turbines. Condition 2: The relative spacing between the wind turbines are fixed and the wake model changes with the tower height. Condition 3: As the tower height changes, the wake model and the distance among the wind turbines varies. In second portion, they investigated the power curve model with power control mechanisms. For this, it is assumed that the turbine wake model and the distance amongst the wind turbines depend on the tower height. In this study, they used 40 m diameter wind turbines with hub height varies from 40 m to 600 m. The effectiveness and the applicability of proposed method is illustrated through the numerical studies. They concluded that the tower height had not much impact on the optimization of wind farm layout.

Reference [98] investigated the effects of varying hub heights of wind turbines within wind farm on the power output. The authors finally reached the result that the power output became better in case of changing the hub height of different wind turbines from 50 m to 78 m, even when using the same number of wind turbines. They also analyzed different cost models and gave concluding remark that one can reduce the cost per unit power produced by using different hub height wind turbines in the wind farm. On the other hand, this is generally valid only for the large wind farm and large hub height variation.

Shakoor et al. [99] introduced a new concept, the effect of wind farm area dimensions on the layout optimization. They used GA for the optimization of cost per unit power obtained from the wind farm. The outcome shows that area shape strongly affects the total free stream wind velocity entering in the boundaries of wind farm and wind turbine power output.

A binary particle swarm optimization (BPSO) was applied to locate the optimal position of the turbines within the wind farm by [100]. They introduced the variable 'time varying acceleration coefficient (TVAC)' in their study. The BPSO–TVAC algorithm was applied to a $10\ \text{m} \times 10\ \text{m}$ square shaped farm site, considering uniform wind and non-uniform wind speed with variable direction characteristics. The analysis of cost versus maximum output power of the wind farm was then performed. The parameters used were the same as in the previously discussed studies [81,84]. The objective function for the optimization layout was defined as:

$$\text{Fitness Function} = \frac{\text{cost}(N)}{p_T} = \frac{N\left(\frac{2}{3}+\frac{1}{3}e^{-0.00174N^2}\right)}{\sum_{K=0}^{360}\sum_{i=1}^{N} f_k p_i(u_i} \tag{24}$$

where N is equal to the number of wind turbines installed in a wind farm $(0<N\leq 100)$, $\overline{u_i}$ is mean wind velocity depending on wind distribution, f_k is wind frequent distribution $(\sum_{k=0}^{360} f_k = 1)$. It is established that algorithm efficiency was maximum for smaller wind farms. On the other hand, for larger wind farms, the algorithm might not work much or end up with the same layout as in previous studies.

Turner et al. [101] developed mixed integer linear and quadratic optimization formulations and applied them to several example layout cases in the literature. Compared to previous approaches, their models produced layouts that tended to be more symmetric and that generated slightly more power. They considered a 10×10 grid, varying the number of turbines placed and wind directions. The formulations were modeled in C++ and solved using CPLEX 12.1. The fitness function is given by following equation.

$$\text{minimize} \sum_{j=1}^{N} y_j\left(\sum d_{iji}^2 y_i\right) \tag{25}$$

where d_{ij} is the wind speed deficit generated at position j due to a turbine placed at position i, y_j is the distance (along the direction of the wind) between positions i and j, binary variables y_i, define y to be the feasible region of formulation. In the study, wind turbine with rotor diameter of 40 m and hub height of 60 m and thrust coefficient of 0.88 was used.

Table 2 presents the work done on wind farm layout optimization by using Jensen's wake model for wake deficit measurement and different optimization methods for wind turbine positioning. It is clear from the table that most of the research has been done by using heuristic methods. However, it can also be done by mathematical techniques. There are two main approaches for modeling turbine locations i.e. continuous and discrete. A continuous location model allows turbines to be placed anywhere within the farm, subject to boundary and proximity constraints. A discrete location model only allows turbines to be placed at a finite number of places.

5. Conclusion

This paper focuses on the existing literature on the wake effect modeling and explains the important distinction between far and near wake effect in large wind farms. The detailed comparison of wind turbine wake effect models in terms of wake effect prediction and wind turbine power calculation is presented. The results have shown that the prediction accuracy of all wake models is highly affected by the spacing and downstream distance between the wind turbines.

It is particularly difficult to predict the accuracy of the specific models or group of models in term of multiple wake. The capability of wake models to predict atmospheric and sea stability effects and losses due to nearby farm may highly appear to be lacking. Increased spacing can undeniably decrease wake losses, but wake models must be improved for the optimization of wind farm layout. It is found that most related studies on wind farms utilize the Jensen's wake model which is the simplest available method to date. This model gives an acceptable degree of prediction accuracy for the off-shore as well as for the flat terrain onshore wind farms.

The wind farm layout design is categorized as an NP-hard optimization problem containing large number of constraints. Majority of researchers are working to solve the problem by implementing the existing optimization techniques. On the other hand, only some researchers focus on the solution method itself. The main studies that have been carried out to date are certainly a good starting point for further research to obtain more effective solution methods, but they cannot be considered entirely satisfactory for several reasons. First, most of the studies consider a discrete domain dividing the total wind farm area space into small

Table 2
Different characteristics of wind farm optimization problems using Jensen's wake model, *HH*=hub height and *RD*=rotor diameter.

Ref, year	Optimization method	Equation no (Objective function)	Wind behavior	*HH*, *RD* (m)	Computational domain
[84], 1994	GA	13	Mean	60,40	Discrete
[81], 2005	GA	14	Mean	60,40	Discrete
[88], 2007	GA	16	Mean	Variable	Discrete
[89], 2009	QIO	17	–	Unspecified	Discrete
[90], 2009	SA,GA	18	Mean	Variable,77	Unspecified
[91],2010	GA	19	Weibull	60,40	Discrete
[92], 2010	GA	14	Mean	Unspecified	Discrete
[93], 2010	PSO	14	Mean	60,40	Continuous
[94], 2011	GA	21	Weibull	Unspecified	Discrete
[95], 2012	SS, GA	14	Mean	60,40	Discrete
[96], 2012	ACSA	22	Mean	Unspecified	Continuous
[97], 2013	GGA	23	Mean	Variable,40	Continuous
[100], 2013	BPSO	24	Mean	60,40	Continuous
[101], 2014	MILQO	25	Mean	60,40	Continuous
[99], 2015	GA	14	Mean	60,40	Discrete
[87], 2015	DPS, GA	14	Mean	60,40	Discrete

GA – Genetic Algorithm.
QI – Quadratic Interpolation Optimization.
SA – Simulated Annealing.
PSO – Particle Swarm Optimization.
SS – Spread Sheet.
ACSA – Ant Colony Search Algorithm.
GGA – Global Greedy Algorithm.
BPSO – Binary Particle Swarm Optimization.
MILQO – Mixed Integer Linear and Quadratic Optimization.
DSP – Definite Point Selection.

cells for wind turbine positioning. However, the continuous space search gives more optimal points for turbine installation. Second, for on-shore wind farm, the topography of the area must be described as one of the optimization objectives or constraints in layout optimization techniques. Third, except the relative positions of the wind turbines in the installation site, the boundaries of the installation area also affect the final electricity production, so it must be considered as an optimization step. Fourth, the review of previous work also shows that most of the research has been done using heuristic techniques and even the share of genetic algorithms is more than 75% for wind farm layout optimization. Hence, it is required to explore new optimization techniques. Further, different optimization approaches can be hybridized for solving the problem to get optimum results.

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