LOGICAL DEDUCTION IN AI

PREDICATE LOGIC FUNDAMENTALS

Partha P Chakrabarti
Indian Institute of Technology Kharagpur

Predicate Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

New Additions in Proposition (First Order Logic)

Variables, Constants, Predicate Symbols and

New Connectors: ∃ (there exists), ∀(for all)

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: $\forall x(\text{goes}(\text{Mary}, x) \rightarrow \text{goes}(\text{Lamb}, x))$

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: $(F1 \wedge F2) \rightarrow G)$ is always true

Use of Quantifiers

EXAMPLES:

Someone likes everyone

Everyone likes someone

There is someone whom everyone likes

Everyone likes everyone

If everyone likes everyone then someone likes everyone

If there is a person whom everyone likes then that person likes himself

LAWS of NEGATION:

Likes(x, y)

1. $\exists x(\forall y \text{ likes}(x, y)) \to F_2$
2. $\forall x(\exists y \text{ likes}(x, y))$
3. $\exists y(\forall x \text{ likes}(x, y))$
4. $\forall x(\forall y \text{ likes}(x, y)) \to F_1$
5. $F_1 \to F_2$

$\forall x_1(\forall y_1 \text{ likes}(x_1, y_1)) \to \exists x(\forall y \text{ likes}(x, y))$

6. $\exists y(\forall x \text{ likes}(x, y) \to \text{likes}(y, y))$

$\neg \forall x(P(x) \wedge q(x))$

$\exists x($

Blocks World Problem

Define an appropriate language and formalize the following sentences in First Order Logic:

1. A is above C, D is on E and above F.
2. A is green while C is not.
3. Everything is on something.
4. Everything that is free has nothing on it.
5. Everything that is green is free.
6. There is something that is red and is not free.
7. Everything that is not green and is above B, is red.

Use of Function Symbols

If x is greater than y and y is greater than z then x is greater than z.

The age of a person is greater than the age of his child.

Therefore the age of a person is greater than the age of his grandchild.

The sum of ages of two children are never more than the sum of ages of their parents.

1. $\forall x \forall y \forall z ((g(x,y) \wedge g(y,z)) \rightarrow g(x,z))$ $g(x,y)$ $x > y$

2. $\forall x \forall y (Child(x,y) \rightarrow g(Age(y), Age(x)))$ $Age(x)$ $\rightarrow$

Variables and Predicate / Function Symbols

Variables, Free variables, Bound variables

Symbols – proposition symbols, constant symbols, function symbols, predicate symbols

Variables can be quantified in first order predicate logic

Symbols cannot be quantified in first order predicate logic

Interpretations are mappings of symbols to relevant aspects of a domain

Terminology for Predicate Logic

Domain: D

Constant Symbols: M, N, O, P, ….

Variable Symbols: x,y,z,….

Function Symbols: F(x), G(x,y), H(x,y,z)

Predicate Symbols: p(x), q(x,y), r(x,y,z),

Connectors: ~, Λ, V, →, ∃, ∀

Terms:

Well-formed Formula:

Free and Bound Variables:

Interpretation, Valid, Non-Valid, Satisfiable, Unsatisfiable

Validity, Satisfiability, Structure

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

Is the same as:

F1: ∀x(www(M, x) → www(L, x))

F2: www(M, S)

G: www(L, S)

To prove: (F1 ∧ F2) → G) is always true

Interpretations

What is an Interpretation? **Assign a domain set D, map constants, functions, predicates suitably.**
The formula will now have a truth value

Example:

F1: $\forall x(g(M, x) \rightarrow g(L, x))$

F2: $g(M, S)$

G: $g(L, S)$

Interpretation 1: D = {Akash, Baby, Home, Play, Ratan, Swim}, etc.,

Interpretation 2: D = Set of Integers, etc.,

How many interpretations can there be?

To prove Validity, means $((F1 \wedge F2) \rightarrow G)$ is true under all interpretations

To prove Satisfiability means $((F1 \wedge F2) \rightarrow G)$ is true under at least one interpretation

In Its Power Lies Its Limitations

Russell's Paradox (The barber shaves all those who do not shave themselves. Does the barber shave himself?)

- There is a single barber in town.
- Those and only those who do not shave themselves are shaved by the barber.
- Who shaves the barber?

Checking Validity of First order logic is undecidable but partially decidable (semi-decidable) {Robinson's Method of Resolution Refutation}

Higher order predicate logic - can quantify symbols in addition to quantifying variables.

$$\forall p((p(0) \wedge (\forall x(p(x) \rightarrow p(S(x))) \rightarrow \forall y(p(y))$$

Thank you