LOGICAL DEDUCTION IN AI

PROPOSITIONAL LOGIC TO PREDICATE LOGIC

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Deduction Using Propositional Logic: Steps

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

Boolean Formulae developed using well defined connectors ~, ∧, ∨, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of Sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a Combined Formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Deduction Using Propositional Logic: Example 1

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \land F2) \rightarrow G$, that is:

$((a \rightarrow b) \land a) \rightarrow b$

Deduction Using Propositional Logic: Example 1

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Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is: $((a \rightarrow b) \wedge a) \rightarrow b$

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Deduction Using Propositional Logic: Example 2

Boolean variables a, b, c, d, … which can take values true or false.

Boolean formulae developed using well defined connectors ~, ∧, ∨, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If I am the President then I am well-known. I am not the President. So I am not well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \to b$

F2: $\sim a$

G: $\sim b$

The final formula for deduction: $(F1 \wedge F2) \to G$, that is: $((a \to b) \wedge \sim a) \to \sim b$

a	b	$a \to b$	$(a \to b) \wedge \sim a$	$((a \to b) \wedge \sim a) \to \sim b$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Model and Solve Using Propositional Logic

Suppose we know that: “if Arjun is thin, then Mohit is not bearded or Julia is not tall” and “if Julia is tall then Devika is graceful” and “if Devika is graceful and Mohit is bearded then Arjun is thin” and “Mohit is bearded”. Can we deduce that “Julia is not tall” ?

Solve Using Propositional Logic

Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

- **Box 1 "The gold is not here"**
- **Box 2 "The gold is not here"**
- **Box 3 "The gold is in Box 2"**

Only one message is true; the other two are false. Which box has the gold?

Insufficiency of Propositional Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

Predicate Logic

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New Additions in Proposition (First Order Logic)

Variables, Constants, Predicate Symbols and

New Connectors: ∃ (there exists), ∀(for all)

Formulating Predicate Logic Statements

New Additions in Proposition (First Order Logic)

Variables, Constants, Predicate Symbols and

New Connectors: ∃ (there exists), ∀(for all)

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

Predicates: contractor(x), dependable(x), engineer(x)

F1: ∀x(contractor(x) → ~dependable(x))

[Alternative: ~∃x (contractor(x) ∧ dependable(x))]

F2: ∃x(engineer(x) ∧ contractor(x))

G: ∃x(engineer(x) ∧ ~dependable(x))

To prove: (F1 ∧ F2) → G) is always true

More Examples

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

$dancer(x),\ student(x),\ g(x)$

$\forall x\,(dancer(x) \rightarrow g(x))$

$student(Ayesha)$

$dancer(Ayesha)$

$\exists x\,(student(x) \rightarrow g(x))$

$\exists x\,(student(x) \wedge g(x))$

$st(x) \rightarrow g(x)$

$\neg st(x) \vee g(x)$

$s(x) \quad g(x) \quad \rightarrow$

$0 \quad 0 \quad 0$

Every passenger is either in first class or second class. Each passenger is in second class if and only if the passenger is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

$p(x),\ f(x),\ s(x),\ w(x)$

$\forall x\,(p(x) \rightarrow ((f(x) \wedge \neg s(x)) \vee (\neg f(x) \wedge s(x))))$

$\forall x\,(p(x) \rightarrow ((s(x) \rightarrow \neg w(x)) \wedge (\neg w(x) \rightarrow s(x))))$

$\exists x\,(p(x) \wedge w(x))$

$\neg \forall x\,(p(x) \rightarrow w(x))$

$\neg \forall x\,(p(x) \wedge w(x))$

$\exists x\,(\neg p(x) \vee \neg w(x))$

$\neg f(x) \vee w(x)$

$\exists x\,(p(x) \wedge \neg w(x))$

$\exists x\,(p(x) \wedge s(x))$

Thank you