ST. JOHN BAPTIST DE LA SALE CATHOLIC SCHOOL

Grade: 10 B

Physics group assignment on Kepler's laws

Group members

1. Bethelhem sisay
2. Bilen Habtom
3. Bitanya Endalew
4. Kenean Abebe
5. Simbo Dereje
6. Mahlet Hiruy
7. Mihret Zelalem
8. Lelise Tilahun
9. Selamawit Tolosa

Submitted to: Teacher Aaron GK
Submission date: January 12/2023

Tyco Brahe and Johannes Kepler

Tyco Brahe (1546–1601; from a wealthy Danish noble family) was fascinated by astronomy but dissatisfied with the time's planetary motion tables' accuracy. He made the decision to devote his entire life and a significant amount of resources to taking planetary positions ten times more precisely than the best earlier work. The king of Denmark bestowed upon Tyco a vast amount of resources, including an island with many families on it and money to construct an observatory, in exchange for his early successes and in appreciation for Tyco's uncle saving his life. Tyco constructed enormous instruments to set precise sights on the stars and employed numerous clocks and timekeepers (one estimate puts this at 10% of the GDP at the time).

The minute-of-arc measurement he set out to do was successful. Before the development of the telescope, this was a remarkable achievement. His goal was to support his own conception of the universe, which he believed to be a synthesis of Copernicus' and Ptolemy's ideas: the Earth was at rest, the sun revolved around it.

The Copernican theory was accepted by Johannes Kepler (1571–1630). Being raised in the Greek geometric tradition, he assumed that God must have had a geometrical reason for placing the six planets at the positions they did in relation to the sun. One inside the other, he imagined their orbits to be on spheres. One day, he suddenly realized there were only five perfect Platonic solids, which explained why there were six planets: the orbit spheres may have been designed so that a perfect solid could just fit between two successive ones. Given the limitations of observation at the time, he persuaded himself that this image might be the correct one. But that was before Tyco's findings were applied. Kepler went to work with Tyco in 1600 after realizing that Tyco's research might resolve the controversy in one direction or the other. The following year, Tyco passed away, and Kepler used the information for nine years.

He finally admitted that his geometric plans were incorrect and found his three laws of planetary motion which are:

- Planets move in elliptical orbits with the sun at a focus.
- Planets cover the same areas in the same amounts of time as they move around the sun.
- The cube of the distance from the sun is proportional to the square of the time.

Proofs for Kepler’s laws of planetary motion

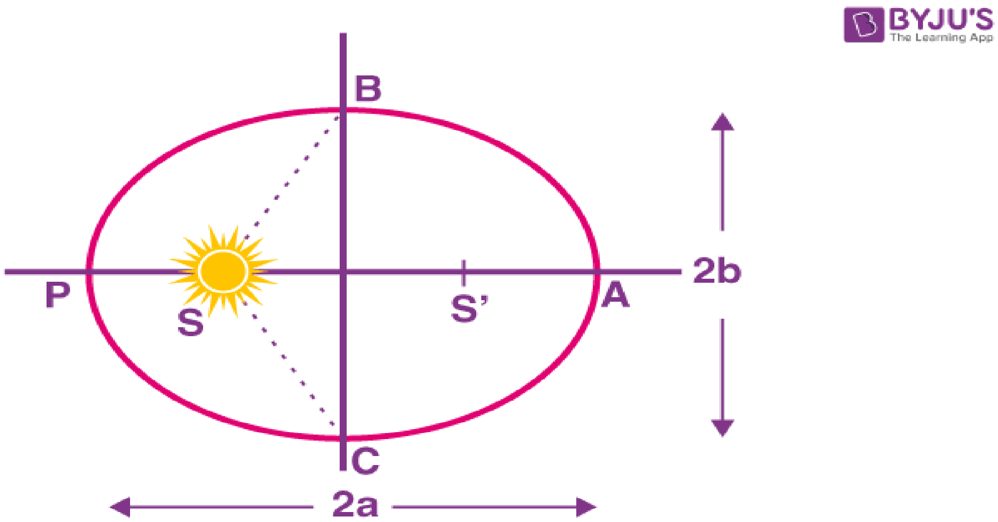
Kepler’s first law (the law of orbits)

According to Kepler's first law, "All planets orbit around the Sun in a path described by an ellipse such that the Sun at one of its two foci.

Semi-major axis (longest radius) is the average distance from the sun.

a = ½(aphelion + perihelion)

Eccentricity is the flatness of the orbit.



To proof this law from Newtonian dynamics,Take a mass m in a general elliptical orbit around a much more massive body M. When the separation of the masses is r, the total energy of the orbit is

$$E= \frac{1}{2}mv^2 - \frac{GMm}{r} \quad \text{.................. (1)}$$

Where v is the speed of the orbiting mass and G the constant of gravitation. The velocity of m has two components: a radial component equal to dr=dt (written $as\ \dot{r}$) and a component perpendicular to r which is the ‘circular’ component of the velocity, equal to rω where ω=$\dot{\theta}$ is the instantaneous angular velocity of the body, with θ as shown in the diagram. Because these components are orthogonal, the square of the total velocity equals the sum of the squares of these components. We can now write this energy equation in polar coordinates:

$E=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$…………… (2)

Similarly, we can write the angular momentum of m as

$L=mr^2\dot{\theta}$……………………….. (3)

As $r\dot{\theta}$ is the component of v perpendicular to r. Now make the substitution $\rho=\frac{1}{r}$, so that $\dot{\theta}$=Lρ^2 we can evaluate θ as

$\theta=\int\frac{L}{m}\rho 2$………………….. (4)

$=\int\frac{L}{m}\rho 2$…………………… (5)

But, $\dot{r}=-\frac{1}{\rho 2}\frac{d\rho}{dt}$……………. (6)

So that, $\theta=-\int\frac{L}{m\dot{r}}d\rho$………………………(7)

Rearranging equation 2 we can see that $\dot{r}$ is

$$\dot{r}^2=\frac{2E}{m}+2GM\rho-\frac{L^2}{m^2}\rho^2 \dots\dots\dots\dots\dots\dots(8)$$

Now for a further manipulation we make the following substitutions:

$$r_o=\frac{L^2}{GMm^2}\text{…………………………………………………. (9)}$$

$$e^2=1+\frac{2er_o}{GMm}\text{………………………………. ………… (10)}$$

Clearly, both r_o and e are constants. We choose them in this way so that our answer is immediately recognizable as an ellipse. After a little manipulation, equation 8 can be written as:

$$\dot{r}=\frac{L}{m}[e^2/r_o{}^2-(\rho-1/r\_o\ \ )\hat{}2\,]^{1/2}\text{………………………………… (11)}$$

Substituting this into equation 7 we get:

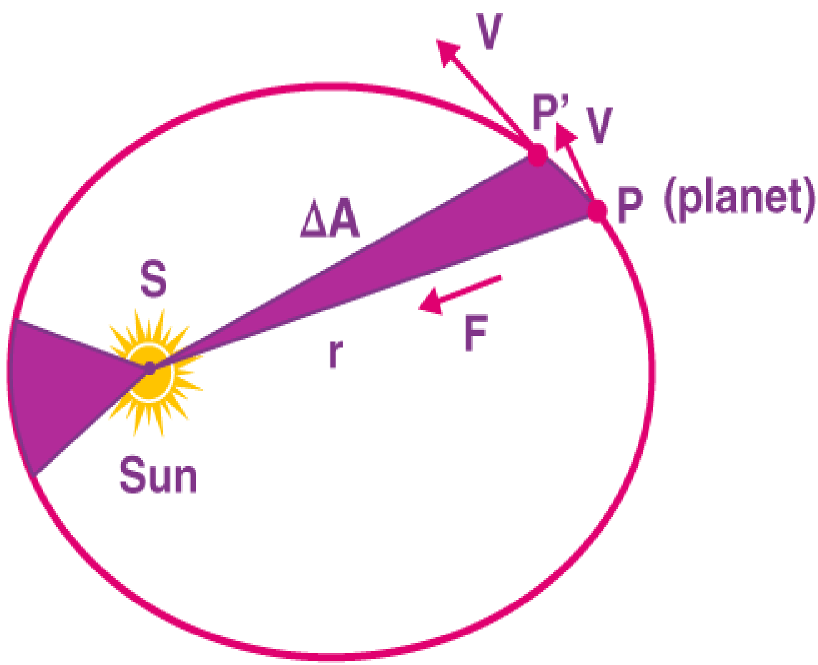
$$\theta=-\int\frac{1}{\sqrt{(e/r_o)^2-(\rho-1/r_0)^2}}d\rho\text{…………………………………… (12)}$$

$\cos^{-1}\left(\frac{\rho-\frac{1}{r_o}}{\frac{e}{r_o}}\right)$ ……………………………………………………………. **(13)**

r $=r_o/(1+e\cos\theta)$……………………………………………. **(14),** which is the equation of an ellipse in polar coordinates, with the origin at a focus. We can now identify r_o as the semi-latus rectum of the ellipse and e as its eccentricity.

Kepler's second law (Law of areas)

Kepler's second law of planetary motion describes the speed of a planet traveling in an elliptical orbit around the Sun. It states that a line between the Sun and the planet sweeps equal areas in equal times.



The planet's kinetic energy fluctuates along its path because its orbit is not circular. It has more kinetic energy near the perihelion and less kinetic energy near the aphelion, which suggests that it is moving faster at the perihelion and slower (v min) at the aphelion. If r is the distance between the planet and the sun at both its perihelion and aphelion, then:

r min + r max = 2a × (length of major axis of an ellipse) . . . . (1)

Using the law of conservation of angular momentum, the law can be verified. At any point of time, the angular momentum can be given as, **$L = mr^2\omega$.**

Now consider a small area ΔA described in a small time interval Δt and the covered angle is Δθ. Let the radius of curvature of the path be r, then the length of the arc covered = rΔθ.

$$\Delta A = \frac{1}{2}[r\,(r\Delta\theta)] = \frac{1}{2}r^2\Delta\theta$$

Therefore, $\Delta A/\Delta t = [\frac{1}{2}r^2]\, d\theta/dt$

Taking limits on both sides as, **Δt→0**, we get;

$$\lim_{\Delta t\to 0} \frac{dA}{dt}$$

$$\lim_{\Delta t\to 0} \frac{1}{2}r^2\frac{d\theta}{dt}$$

$$\frac{dA}{dt}=\frac{1}{2}r^2\omega$$

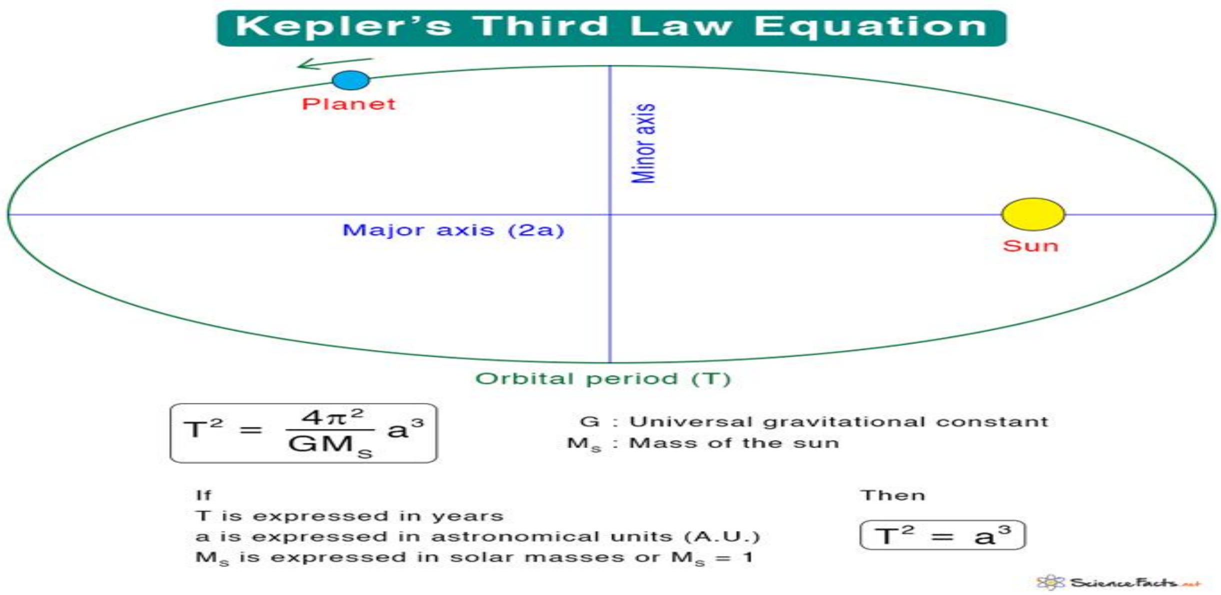
$$\frac{dA}{dt}=\frac{L}{2m}$$

Now, by conservation of angular momentum, L is a constant

Thus, $\frac{dA}{dt}$= constant.

Kepler’s 3rd law (law of period)

Kepler's Third Law states that the square of the time period of orbit is directly proportional to the cube of the semi-major axis of that respective orbit. (The semi-major axis for a circular orbit is of course the radius) Mathematically this can be represented as: $T^2 / r^3 = k$ where k is a constant. The value k is related to physical constants such that $k = 4\pi^2/GM$ where G is the gravitational constant and M the mass of the object at the center of the orbit (NOT the object doing the orbiting!)



To prove the 3rd law we will need the following four equations:

Circular Motion: $a = v^2/r$; $v = \omega r = 2\pi/T$

Gravitational attraction: $F = GMm/r^2$

Newton's Second Law: **F = ma**

Substituting circular motion and gravitational attraction into the above formula yields:

$$Mv^2/r = GMm/r^2$$

Cancelling the m's multiplying by r and by GM gives:

$$V^2/GM = 1/r$$

This is very close to the result we want; one more substitution should give us the desired equation. Notice that $v = \omega r = 2r\pi/T$ from circular motion equations.

And so we have:

$$\mathbf{4\pi^2/GMT^2 = 1/r^3}$$

Multiplying by T^2:

$\mathbf{T^2/r^3 = 4\pi^2/GM}$ as required!