

$$C = Q/V$$

$$C = \frac{\epsilon A}{d}, \quad \epsilon = \kappa \epsilon_0$$

$$C = \frac{\kappa \epsilon_0 A}{d}$$

$\kappa$  - dielectric constant

$\epsilon$  - dielectric permittivity

$\epsilon_0$  - vacuum permittivity

$$QE = QV$$

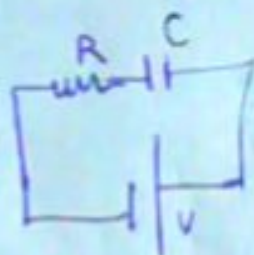
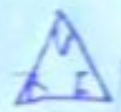
$$E = \frac{QV}{2}$$

$$Q = CV$$

$$E = \frac{CV^2}{2}$$

→ Capacitance

→ Potential difference



$$V = V_C + V_R$$

$$V = \frac{AQ}{AC} + RI$$

→ differential equation

$$V = \frac{AQ}{AC} + R \left( \frac{dQ}{dt} \right)$$

$$Q(t) = Q(1 - e^{-t/\tau})$$

$Q$  - full charge

$e$  - Euler number

$$e^{-t/\tau}, \quad \tau = RC \rightarrow \tau$$

time constant

Charging

$$Q(RC) = Q(1 - e^{-RC/RC})$$

$$= Q(1 - 1/e)$$

$$\approx 0.63Q$$

Discharging

$$Q(RC) = Q e^{-RC/RC}$$

$$= Q e^{-1}$$

$$\approx 0.37Q$$



$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{q_1 q_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

Electric field: the force acting on a charge per unit charge.

Source charge (Q)

test charge (q)

$$F = k \frac{Qq}{r^2}$$

$$F = \left( \frac{kQ}{r^2} \right) q$$

$$F = E q$$

Escape  $\rightarrow KE = PE$

Orbital  $\rightarrow F_g = F_c$   
velocities

$$v_e = \sqrt{\frac{2GM}{r}} = \sqrt{2} v_o$$

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{g r}$$

$\rightarrow$  Energy stored in a system due to masses in a gravitational field

$\Delta PE = -W$  (PE in a system is the work done by conservative forces)



$$U = (F_g)(\text{distance})$$

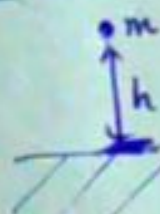
$$= \left( \frac{GMm}{r^2} \right) (r)$$

$$= \frac{GMm}{r}$$

PE at infinity  $\rightarrow 0$

PE = 0 at  $r \rightarrow \infty$

PE within a finite from a mass is negative



$\Delta PE = mgh$  (on a height h the ground is it is positive)

$\Delta PE \rightarrow$  positive

$$U = \frac{GMm}{r}$$

$$U(r) = \frac{GMm}{r}$$

$F_g$  initial & final position

$$F_g = \frac{GMm}{r^2}$$

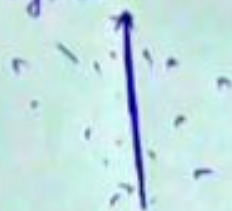
$$F_g = \left( \frac{GM}{r^2} \right) m$$

$$F_g = mg$$

$$g = \frac{GM}{r^2}$$



Moment of Inertia ( $I$ ): distribution of mass in a body/around axis of rotation  
 $I = \sum m_i r_i^2$  (point mass system)  
 → mass  
 → geometry



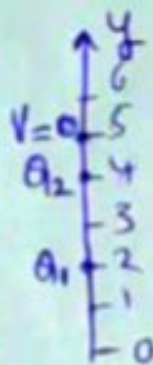
$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$= \sum_{i=1}^n m_i r_i^2$$

uniform object (Center of mass)

- rod  $= \frac{1}{12} m l^2$
- hollow sphere  $= \frac{2}{3} m r^2$
- solid sphere  $= \frac{2}{5} m r^2$

6



$$r_1 = 3$$

$$r_2 = 1$$

Potential at  $y=5$   
 ↓  
 Sum of the potential due to  $Q_1$  &  $Q_2$  (Scalar quantities)

$$V_{\square} = V_1 + V_2$$

$$0 = V_1 + V_2$$

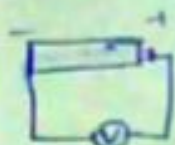
$$V_1 = -V_2$$

$$\frac{kQ_1}{r_1} = -\frac{kQ_2}{r_2}$$

$$\frac{Q_1}{3} = -\frac{Q_2}{1}$$

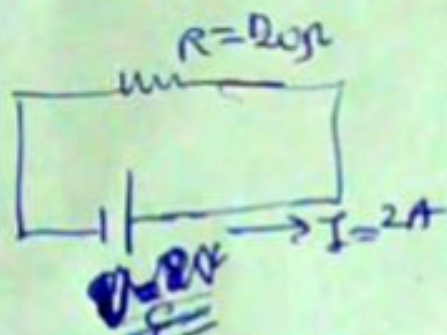
$$\boxed{Q_1 = -3Q_2}$$





terminal voltage

Emf: ideal voltage of a battery



What is the internal resistance

$$V = IR$$

$$= (2A)(20\Omega)$$

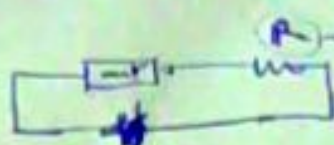
$$V = 40V$$

$$\boxed{E = V + Ir}$$

usually,  $r$  is very small

$$9V$$

$$(1.6 \times 10^{19}) (1.1 \times 10^{-19} C)$$



Circuit's total resistance before connecting to the battery

$$E = I(\text{total resistance})$$

$$\boxed{E = I(r + R)}$$

$$E = Ir + IR$$

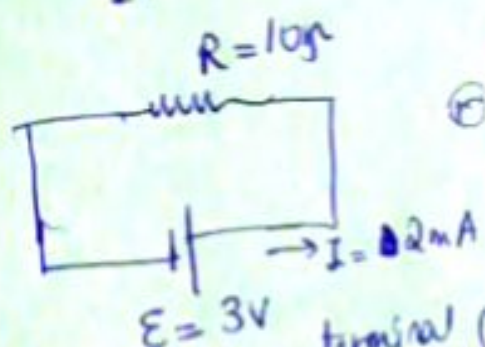
terminal voltage (V)

$$E = Ir + V$$

$\hookrightarrow$  voltage dissipated due to internal resistance

$$\boxed{Ir = E - V}$$

$$Ir = E - V$$



$$r = \frac{E - V}{I}$$

$$= \frac{3V - 2mV}{2mA}$$

terminal voltage  $(V) \Rightarrow V = IR$

$$V = 2mA(10\Omega)$$

$$V = 20mV$$

1eV

$$1 = 1$$

$$AV = 1V$$

$$V_e = \sqrt{\frac{2GM}{R}}$$

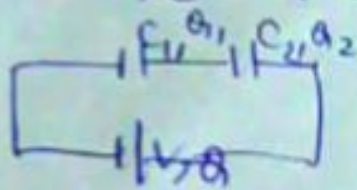
units of Energy

- Joule (SI unit)
- Calorie  $\rightarrow 1 \text{ Cal} = 4.2 \text{ J}$
- 1eV  $\rightarrow 1.6 \times 10^{-19} \text{ J}$
- kWh  $\rightarrow 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$
- 1eV  $\rightarrow 1.6 \times 10^{-19} \text{ J}$
- 1eV  $\rightarrow 1.6 \times 10^{-19} \text{ J}$



## Capacitors

Series  $\rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$   
 Parallel  $\rightarrow C = C_1 + C_2 + C_3$



$$C = Q/V$$

$$Q = Q_1 = Q_2$$

$$V = Q/C$$

$$V = V_1 + V_2$$

$$Q = CV$$

$$Q = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$Q/C = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

20 pF, 60 pF, 3 pF

$$E = \frac{1}{2} CV^2$$



$$Q = Q_1 + Q_2$$

$$\rightarrow V_2 = V_1 = V$$

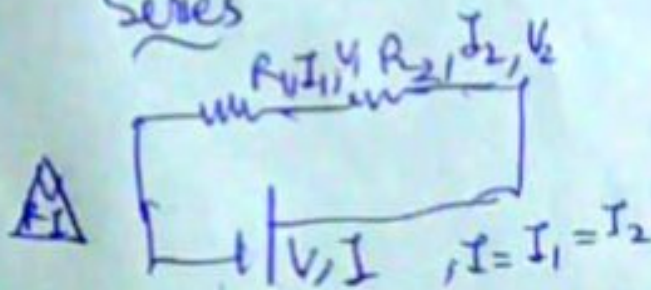
$$CV = C_1 V_1 + C_2 V_2$$

$$CV = C_1 V + C_2 V$$

$$C = C_1 + C_2$$

## Resistors

Series



$$V = RI$$

$$I = \frac{V}{R}$$

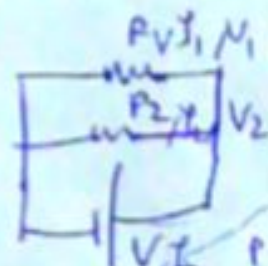
$$V = V_1 + V_2$$

$$RI = R_1 I_1 + R_2 I_2$$

$$RI = R_1 I + R_2 I$$

$$R = R_1 + R_2$$

Parallel



$$V = V_1 = V_2$$

$$I = I_1 + I_2$$

$$V = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$P = \frac{\text{Energy}}{\text{time}}$$

$$P = \frac{QV}{t}$$

$$P = IV$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

$$P = I(I_1 + I_2) = I^2 R$$



(Potential)(Absolute potential): the potential energy per unit charge

$$V = \frac{PE}{q} = \frac{kqQ}{r}$$

$$V = \frac{kQ}{r}$$

$$V_A = \frac{kQ}{r_A}, V_B = \frac{kQ}{r_B}$$

PE  $\rightarrow$  work done by conservative forces.

$$PE = W$$

Conservative  $\rightarrow$  PE Conserved

$$W = -\Delta PE$$

$\rightarrow$  path independent (initial & final states)

$$W = \int (q)(V_B - V_A)$$

disipative  $\rightarrow$  PE not conserved

$$= -\Delta PE$$

$\rightarrow$  path dependent

$$W = -(PE_B - PE_A)$$

$$W = -(q(V_B) - q(V_A))$$

$$= -(q(V_B - V_A))$$

potential difference (voltage)

$$W = -q(\Delta V)$$



equipotential surface

$$W = -q(\Delta V)$$

$$W = -q(V_B - V_A)$$

$V_B = V_A \rightarrow$  equipotential surface

$$W = -q(0)$$

$$= 0$$

equipotential surfaces  $\perp$  electric field lines

$$W = Fd \cos \theta$$

$$0 = Fd \cos \theta$$

$$\cos \theta = 0$$

$$\theta = 90^\circ / \pi/2$$



uniform



→ Resistivity ( $\rho$ )  
→ Conductivity ( $\sigma$ )

$$\boxed{\rho\sigma = 1}$$

$$\frac{1}{\sigma} = \rho$$

$$\frac{1}{\rho} = \sigma$$

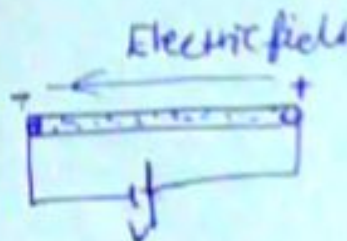
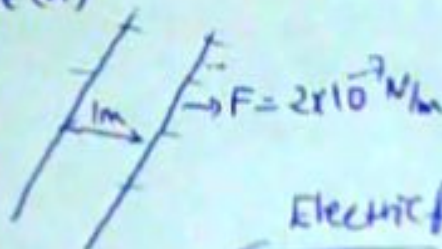
14.  $I = \frac{\Delta Q}{\Delta t}$  : the rate flow of Charges

SI unit: Ampere (A)

1 A:

$$n = \frac{ne}{V}$$

$$I = nAev$$



Conventional flow of positive charges  
→ the same direction as the electric field

Current  $\propto$  Electric field

$$\vec{J} \propto \vec{E}$$

microscopic  $\vec{J} = \sigma \vec{E}$

Ohm's law →

$$\vec{J} = \frac{\Delta Q}{\Delta t A} = \frac{I}{A}$$

$\vec{J}$  = Current density

$\vec{J}$ : the time rate of flow Charges in a given area

Electron Current

→ flow of electrons

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{\tau}_{net} = I\alpha$$

$$L = I\omega$$

$\tau_{net} = I\alpha$  - newton's second Law

$$\tau_{net} = I(\omega_f - \omega_i)$$

$$\tau_{net} = \frac{I\omega_f - I\omega_i}{t}$$

$$\tau_{net} = \frac{L_f - L_i}{t}$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\tau_{net} = \frac{\Delta L}{\Delta t} = \frac{\Delta L}{\Delta t} = \frac{\Delta L}{\Delta t}$$

angular momentum

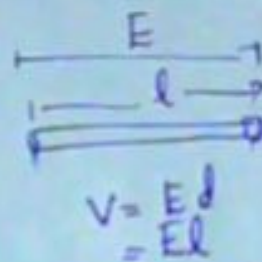


16. macroscopic form:  $\vec{J} \propto \vec{V}$ ,  $I \propto V$  at constant temperature  
 $I = \frac{1}{R} V$   
 $I = \frac{V}{R}$

microscopic form:  $\vec{J} = \sigma \vec{E}$   
 $\vec{J} = \frac{\vec{E}}{S}$

Ohm's law isn't necessarily always true

↓  
 Ohmic materials when it obeys Ohm's law.



$$\vec{J} = \frac{\vec{E}}{S}$$

$$\frac{I}{A} = \frac{E}{S} \Rightarrow$$

$$\frac{V}{RA} = \frac{E}{S}$$

$$\frac{El}{RA} = \frac{E}{S} \Rightarrow Sl = RA$$

$$R = \frac{sl}{A}$$

Temperature

$R_0$

$$\Delta R \propto \Delta T R_0$$

$$\Delta R = \alpha R_0 \Delta T$$

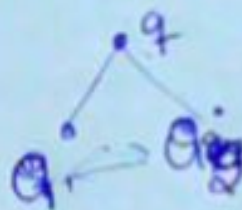
$$\Delta R = R - R_0$$

$$R = \Delta R + R_0$$

$$= \alpha R_0 \Delta T + R_0$$

$$= R_0 (1 + \alpha \Delta T)$$

Superposition principle



$$E_{\text{Earth}} = 6 \times 10^{24} \text{ kg}$$

$$r = \frac{2 \times G \times M \times E}{c^2}$$

18 Blackhole

$$V = \sqrt{\frac{2GM}{r}}$$

$$c = \sqrt{\frac{2GM}{r}}$$

$$c^2 = \frac{2GM}{r}$$

$$r = \frac{2GM}{c^2}$$

Schwarzschild radius

19.016



Salon

