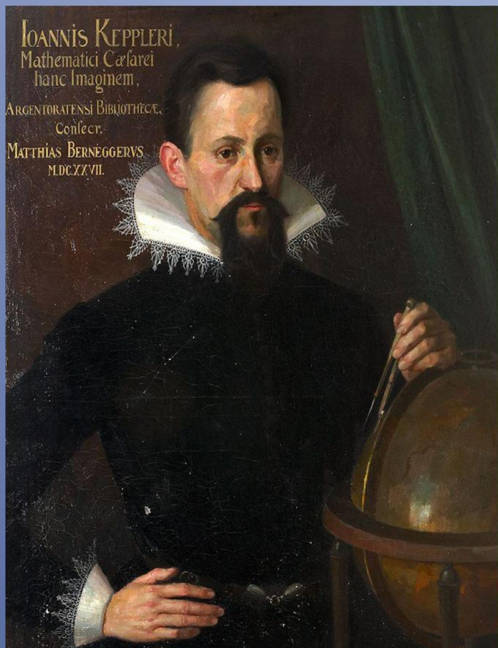

Johannes kepler

Kepler's laws and their proof

Group members

- Esubalew Tizazu Israel kassahun
- Mihiretab Abebe Elias Gerbaw
- Robsan Taye Yabsira Tilahun
- Mikiyas Hayilu Natnael Wendesen
- Daniel Getachew yonnatan Adisu

Submitted to :Teacher Aaron (our favorite)

Submission date: Jan 9th 2023

Table of content

Topic **page**

Acknowledgement

- We want to thank our teacher for giving us these assignment to work on ,to be honest the topic wasn’t interesting but we tried to make it more attractive despite that it gave us knowledge about the motion of celestial bodies and other staffs .

- Secondly we want to thank Esubalew’s family for providing the materials and ideas for the success of the assignment.

- At last but not the least we want to thank Johannes kepler for changing the phenomena of the world specially in Astronomy and Optics ,we wish for him to rest in peace.

Introduction

Kepler's Laws of Planetary Motion

Kepler's three laws describe how planetary bodies orbit the Sun. They describe how (1) planets move in elliptical orbits with the Sun as a focus, (2) a planet covers the same area of space in the same amount of time no matter where it is in its orbit, and (3) a planet's orbital period is proportional to the size of its orbit (its semi-major axis).

Explore the process that Johannes Kepler undertook when he formulated his three laws of planetary motion.

The planets orbit the Sun in a counterclockwise direction as it is known the Sun's north pole, and the planets' orbits all are aligned to what astronomers call the ecliptic plane.

The story of our greater understanding of planetary motion could not be told if it were not for the work of a German mathematician named Johannes Kepler. Kepler lived in Graz, Austria during the tumultuous early 17th century. Due to religious and political difficulties common during that era, Kepler was banished from Graz on August 2nd, 1600

In this paper we will try to see about kepler laws and there proof as well in detail. We hope you will find something new and important in the paper.

-To make things more interesting we add some facts about

Johannes kepler

1. Kepler's childhood passion

Johannes was introduced to astronomy at an early age and developed a strong passion for it that would span his entire life.

He was introduced to astronomy by his mother who took him out at night to show him interesting things in the skies, including the Great Comet of 1577, writing that he "was taken by [his] mother to a high place to look at it."

In 1580, at age nine, he observed another astronomical event, a lunar eclipse, recording that he remembered being "called outdoors" to see it and that the moon "appeared quite red".

2. Kepler's humble career beginnings

Kepler's remarkable scientific career had a humble start, as he started out as a math teacher in a seminary in the Austrian city of Graz.

Despite his desire to become a minister, near the end of his studies at Tübingen, Kepler accepted an offer to teach mathematics and astronomy, in April 1594, as a replacement to Georg Stadius at the Protestant school.

While teaching there, he developed an instrumental friendship with Prince Hans Ulrich von Eggenberg, who financed most of his later experiments and inventions.

Through this friendish, Keppler became the chief mathematician of Emperor Rudolf II. At that time, he also met Tycho Brahe in Prague and began a long and fruitful cooperation with him.

3. The strange partnership that revolutionized science

Kepler had a tumultuous relationship with Danish astronomer Tycho Brahe; but it was the accurate planetary data of Brahe that enabled Kepler to make his great discoveries in astronomy.

Tycho Brahe was a Danish astronomer, known for his accurate and comprehensive astronomical observations. Kepler become Tycho's assistant in about 1600.

The two had different personalities, with Tycho being arrogant and merciless towards his opponents and Kepler being self-effacing and self-disparaging, referring to himself as a "housedog" and a "worm".

Upon Tycho Brahe's untimely death in 1601, his data fell into the hands of his mathematically gifted assistant. Through the use of this data, he was able to rise to the ranch of Imperial Mathematician in Prague.

Kepler incorporated religious arguments and reasoning into his work, motivated by the religious conviction and belief that God had created the world according to an intelligible plan that is accessible through the natural light of reason.

He regarded his three laws of planetary motion as celestial harmonies that reflected God's design for the universe. In his famous work Harmonices Mundi, he found harmonies in nature to claim that the Earth has a soul because it is subjected to astrological harmony.

It was while finding these harmonies that Kepler discovered what came to be known as the third law of planetary motion: that the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

4. Kepler laid the foundation of modern optics

Apart from his laws of planetary motion, Kepler made important contributions to optics, including formulation of the inverse-square law governing the intensity of light.

Kepler was able to invented his own version of the refracting telescope, which helped him understand the structure and working of the eye.

Understanding the path

-**Fortunately,** an opportunity to work as an assistant for the famous astronomer Tycho Brahe presented itself and the young Kepler moved his family from Graz 300 miles across the Danube River to Brahe's home in Prague. Tycho Brahe is credited with the most accurate astronomical observations of his time and was impressed with the studies of Kepler during an earlier meeting. However, Brahe mistrusted Kepler, fearing that his bright young intern might eclipse him as the premier astronomer of his day. He, therefore, led Kepler to see only part of his voluminous planetary data.

He set Kepler, the task of understanding the orbit of the planet Mars, the movement of which fit problematically into the universe as described by Aristotle and Ptolemy. It is believed that part of the motivation for giving the Mars problem to Kepler was Brahe's hope that its difficulty would occupy Kepler while Brahe worked to perfect his own theory of the solar system, which was based on a geocentric model, where the earth is the center of the solar system. Based on this model, the planets Mercury, Venus, Mars, Jupiter, and Saturn all orbit the Sun, which in turn orbits the earth. As it turned out, Kepler, unlike Brahe, believed firmly in the Copernican model of the solar system known as heliocentric, which correctly placed the Sun at its center. But the reason Mars' orbit was problematic was because the Copernican system incorrectly assumed the orbits of the planets to be circular.

After much struggling, Kepler was forced to an eventual realization that the orbits of the planets are not circles, but were instead the elongated or flattened circles that geometers call ellipses, and the particular difficulties Brahe hand with the movement of Mars were due to the fact that its orbit was the most elliptical of the planets for which Brahe had extensive data. Thus, in a twist of irony, Brahe unwittingly gave Kepler the very part of his data that would enable Kepler to formulate the correct theory of the solar system, banishing Brahe's own theory.

Since the orbits of the planets are ellipses, let us review three basic properties of ellipses. The first property of an ellipse: an ellipse is defined by two points, each called a focus, and together called foci. The sum of the distances to the foci from any point on the ellipse is always a constant. The second property of an ellipse: the amount of flattening of the ellipse is called the eccentricity. The flatter the ellipse, the more eccentric it is. Each ellipse has an eccentricity with a value between zero, a circle, and one, essentially a flat line, technically called a parabola.

The third property of an ellipse: the longest axis of the ellipse is called the major axis, while the shortest axis is called the minor axis. Half of the major axis is termed a semi-major axis. Knowing then that the orbits of the planets are elliptical, johannes Kepler formulated three laws of planetary motion, which accurately described the motion of comets as well.

Laws of kepler

Kepler's First Law (The law of orbits): each planet's orbit about the Sun is an ellipse. The Sun's center is always located at one focus of the orbital ellipse. The Sun is at one focus. The planet follows the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit.

Kepler's Second Law (The law of equal areas): the imaginary line joining a planet and the Sun sweeps equal areas of space during equal time intervals as the planet orbits. Basically, that planets do not move with constant speed along their orbits. Rather, their speed varies so that the line joining the centers of the Sun and the planet sweeps out equal parts of an area in equal times. The point of nearest approach of the planet to the Sun is termed perihelion. The point of greatest separation is aphelion, hence by Kepler's Second Law, a planet is moving fastest when it is at perihelion and slowest at aphelion.

Kepler's Third Law(The law of periods): the squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axes of their orbits. Kepler's Third Law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus we find that Mercury, the innermost planet, takes only 88 days to orbit the Sun. The earth takes 365 days, while Saturn requires 10,759 days to do the same. Though Kepler hadn't known about gravitation when he came up with his three laws, they were instrumental in Isaac Newton deriving his theory of universal gravitation, which explains the unknown force behind Kepler's Third Law. Kepler and his theories were crucial in the better understanding of our solar system dynamics and as a springboard to newer theories that more accurately approximate our planetary orbits.

Proof of Kepler's laws from Newtonian dynamics

It would be a pity to have a course on dynamical astronomy and not at least see a proof of Kepler's laws from Newton's laws of motion and gravitation. Importantly, these proofs are not examinable! They are presented here purely to satisfy curiosity and for your entertainment.

KEPLER'S FIRST LAW Ki:

A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse. Take a mass $\underline{m}$ in a general

elliptical orbit around a much more massive body M. When the separation of the masses is r the total energy of the orbit is E = 1 2 mv2 – G , (1) where v is the speed of the orbiting mass and G the constant of gravitation. The velocity of m has two components: a radial component equal to dr/dt (written) and a component perpendicular to $\dot{r}$ r which is the 'circular' component of the velocity, equal to $r\omega$ where ω ($\equiv \dot{\theta}$) is the instantaneous angular velocity of the body, with θ as shown in Fig. 1. Because these components are orthogonal, the square of the total velocity equals the sum of the squares of these components. We can now write this energy equation in polar coordinates:

E = 1/2 ($\dot{r}$2 + r2$\dot{\theta}$ 2) – $GMm/\ r$. (2)

KEPLER'S SECOND LAW Let's now prove Kii:

The line joining a planet to the Sun sweeps out equal areas in equal intervals of time. The proof of Kii highlights the generality of the 'sweeping out area' rule for motion under any central force.

In a time dt the planet will move by a small amount d$\boldsymbol{r}$. The small triangle this vector makes to the Sun (Fig. 2) has an area

$$\mathbf{d}A = \mathbf{1\ /2}\ r\mathbf{d}r\ \mathbf{sin}\ \alpha, \qquad \mathbf{(a)}$$

where α is the angle between $\boldsymbol{r}$ and d$\boldsymbol{r}$ (remember the area of a triangle is $\frac{1}{2}\ ab$ sin C). This can be usefully written as a (pseudo)vector perpendicular to the plane of the triangle with magnitude dA using the vector cross-product:

$$\mathbf{d}\boldsymbol{A} = \mathbf{1/\ 2}\ \boldsymbol{r} \times \mathbf{d}\boldsymbol{r}. \qquad \text{(b)}$$

The rate of sweeping out area due to movement is therefore

$$\dot{\boldsymbol{A}} = \mathbf{d}\boldsymbol{A}/\ \mathbf{d}t = \mathbf{1/\ 2}\ \boldsymbol{r} \times \dot{\boldsymbol{r}}\ \mathbf{.} \qquad \text{(c)}$$

Kepler's second law states that this is a constant for the orbital motion, so $\boldsymbol{A}$should be zero if Kii holds. Differentiating with respect to time again gives

$$\ddot{\boldsymbol{A}} = \mathbf{1/\ 2}\ (\dot{\boldsymbol{r}} \times \dot{\boldsymbol{r}} + \boldsymbol{r} \times \ddot{\boldsymbol{r}}). \qquad \text{(d)}$$

The first term on the right-hand side certainly equals zero, as it is the cross-product of a vector with itself, but the second term is not zero for general motion. However, $\ddot{\boldsymbol{r}}$ is just the acceleration of the planet, and by Newton's second law that is in the direction of the applied (gravitational) force, so is also directed along $\boldsymbol{r}$. Therefore the second term must also be zero in this case. We can therefore say that $\ddot{\boldsymbol{A}} = 0$ and so $\dot{\boldsymbol{A}}$ is a constant. It's clear this would be true for any 'central force', where the force is directed along the line connecting the centres of mass.

Similarly, we can write the angular momentum of m as

$$L = mr\mathbf{2}\dot{\theta}, \qquad \mathbf{(3)}$$

as $r\dot{\theta}$ is the component of v perpendicular to r. Now make the substitution ρ = 1/r, so that $\dot{\theta}$ = $L\rho^2/m$. We can evaluate θ as

$$\theta = \int L/(\rho 2)\,\mathrm{d}t \qquad (4)$$

$$= \int L/m\,(\rho 2)\,\mathrm{d}t/\mathrm{d}\rho\,(\mathrm{d}\rho) \qquad (5)$$

$$\text{but } \dot{r} = -(1/\rho 2)\,(\mathrm{d}\rho/\mathrm{d}t) \qquad (6)$$

$$\text{so } \theta = -\int L/m\,\dot{r}\,(\mathrm{d}\rho). \qquad (7)$$

Rearranging Eqn. (2) we can see that $\dot{r}$ is

$$r/\,2 = 2E\,/m + 2GM\rho - L2/\,m2(\,\rho 2)\,. \qquad (8)$$

Now a further manipulation. We make the substitutions

$$r0 = L2/GMm2 \qquad (9)$$

$$e2 = 1 + 2Er0/\,GMm. \qquad (10)$$

Clearly, both $r0$ and e are constants. We choose them in this way so that our answer is immediately recognisable as an ellipse—it is not an obvious substitution at this stage! After a little manipulation, Eqn. (8) can be written as

$$\dot{r} = L\,/m\,[\;2/\,r\,2\,0 - (\rho - 1/\,r0\,)\,2\,]\,1/2\,. \qquad (11)$$

Substituting this into Eqn. (7) we get

$$\theta = -\int 1 \Big/ \sqrt{(e/r0)^2 - (\rho - 1/r0)^2}\,(\mathrm{d}\rho) \qquad (12)$$

$$= \cos^{-1}\left(- 1/r0 \Big/ e/r0 \right) . \qquad (13)$$

This can be rearranged to give

$$r = r0 /(1 + e \cos \theta), \qquad (14)$$

which is the equation of an ellipse in polar coordinates, with the origin at a focus. We can now identify $r0$ as the semi-latus rectum of the ellipse and e as its eccentricity

KEPLER'S THIRD LAW

Now for Kiii: The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit. We'll use some of the results from the lectures for this one. The total area of an ellipse of semimajor and semiminor axes a and b is

$$\boldsymbol{A_{tot} = \pi ab = \pi a^2\sqrt{1 - e^2}}, \qquad \textbf{(e)}$$

where e is again the eccentricity (we derived $b = a\sqrt{1 - e^2}$ in the lectures using the 'string' definition of an ellipse). From Eqn. (17), the rate of sweeping out area is

$$\dot{\boldsymbol{A}} = 1/2\, \boldsymbol{r} \times \dot{\boldsymbol{r}} = 1/2\, \boldsymbol{r} \times \boldsymbol{v} = \boldsymbol{L}/2m, \qquad (f)$$

where $\boldsymbol{L}$ is the planet's orbital angular momentum around the Sun and m is its mass. The orbital period T is simply the time taken to sweep out an area A_{tot},

i.e.,

$T = \pi ab\ L/(2m) = m\ /L(\ 2\pi a2\surd 1 - e2,)$ (g)

So $2 = m2/\ L2\ (4\pi 2a\ 4\ (1 - e2\ ))$. (h)

We are nearly there, but we need to address the L and e terms in this expression. Using Fig. 3, and that $d + r0 = 2a$ for an ellipse, we have (by Pythagoras)

$(2a - r0\ )\ 2 = 4a2\ e\ 2 + r2\ 0$ (i)

$r0 = (1 - e2\ )$. (j)

Inserting $r0$ from Eqn. (9) we get

$m2\ L2 = 1\ (1 - e2)$. (k)

We can therefore write Eqn. (g)

As

$T\ 2 = 4\pi 2/\ GM\ (a\ 3)$, (L)

which is Kiii. As a bonus we get the constant of proportionality.

By this way we proof and show the last law of kepler.

The end

summary

Johannes Kepler is now chiefly remembered for discovering the three laws of planetary motion that bear his name published in 1609 and 1619). He also did important work in optics (1604, 1611), discovered two new regular polyhedra (1619), gave the first mathematical treatment of close packing of equal spheres (leading to an explanation of the shape of the cells of a honeycomb, 1611), gave the first proof of how logarithms worked (1624), and devised a method of finding the volumes of solids of revolution that (with hindsight!) can be seen as contributing to the development of calculus (1615, 1616). Moreover, he calculated the most exact astronomical tables hitherto known, whose continued accuracy did much to establish the truth of heliocentric astronomy (*Rudolphine Tables*, Ulm, 1627).

A large quantity of Kepler's correspondence survives. Many of his letters are almost the equivalent of a scientific paper (there were as yet no scientific journals), and correspondents seem to have kept them because they were interesting. In consequence, we know rather a lot about Kepler's life, and indeed about his character. It is partly because of this that Kepler has had something of a career as a more or less fictional character.

Kepler’s impact on society is still observable for example :

He was the first to determine that refraction drives vision in the eye, and that using two eyes enables depth perception. He created eyeglasses for both near and farsightedness, and explained how a telescope worked. He described images and magnification, and understood the properties of reflection and so more use full things and ideas.

References used:

https://www.discoverwalks.com/blog/top-10-interesting-facts-about-johannes-kepler/

https://byjus.com/jee/keplers-laws/

https://radio.astro.gla.ac.uk/a1dynamics/keplerproofs.pdf

https://solarsystem.nasa.gov/resources/310/orbits-and-keplers-laws/

https://mathshistory.st-andrews.ac.uk/Biographies/Kepler/