Kepler's laws of planetary motion

Grade 10th B

Presented by Group 2&8

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Tyco Brahe and Johannes Kepler

These two scientists showed that the universe was not some ideal perfection as Ptolemy proposed and worked toward acceptance of Copernicus' heliocentric model.

Proof of Kepler's first law

- **We can proof kepler's first law from Newtonian dynamics.**
- **E=** $\frac{1}{2}mv^2 - \frac{GMm}{r}$ **…………………. (1)**
- $\frac{1}{2}m\,(\dot{r}^2+r^2\,\dot{\theta}^2)$ **……………. …..(2)**
- **L=**$mr^2\,\dot{\theta}$ **…………………….. (3)**
- $\theta=\int\frac{L}{m}\rho 2$ **…………………………(4)**
- $=\int\frac{L}{m}\rho 2$ **…………………………(5)**
- **But** $\dot{r}= -\frac{1}{\rho 2}\frac{d\rho}{dt}$ **……………………(6)**
- **So that,**$\theta = -\int\frac{L}{m\dot{r}}d\rho$ **………(7)**

Proof of Kepler's first law

- Rearranging equation 2 we can see that $\dot{r}$ is
- $\dot{r}^2 = \frac{2e}{m} + \mathbf{2GM\rho} - \frac{L^2}{m^2}$
- Now for further manipulation we make the following substitutions
- $r_o = \frac{L^2}{GMm\char`^2}$……………………………………(9)

Proof of Kepler's first law

- $e^2 = 1 + \frac{2er_o}{GMm}$...(10)
- $\dot{r} = \frac{\dot{L}}{m}[e^2/{r_o}^2 - (\rho - 1/r\_o\ \ )\verb|^|2\ ]^{1/2}$.......................(11)
- $\theta = -\int \frac{1}{\sqrt{\left({}^{e}/{}_{r_o}\right)^2 - (\rho - 1/r_0)^2}} d\rho$ ....(12)
- $cos^{-1}\left(\frac{\rho - \frac{1}{r_o}}{\frac{e}{r_o}}\right)$...(13)
- r $= r_o/(1 + e\cos\theta)$...(14)

Proof of Kepler’s first law

- **Equation 14 is the equation of an ellipse in polar coordinates, with the origin at a focus. We can now identify r_o as the semi-major axis of the ellipse and e as its eccentricity.**
- **So you can proof Kepler’s first way from Newtonian dynamics in this way.**

Proof of Kepler's second law

- **You can proof kepler's second law using the law of conservation of momentum.**
- **r min + r max = 2a × (length of major axis of an ellipse) .**
- $\Delta A = \frac{1}{2}[r\ (r\Delta\theta)] = \frac{1}{2}r^2\Delta\theta$
- $\Delta A/\Delta t = [\frac{1}{2}r^2]\ d\theta/dt$
- $\frac{dA}{dt} = \frac{1}{2}r^2\omega$
- $\frac{dA}{dt}=\frac{L}{2m}$
- **Thus,** $\frac{dA}{dt}$ **= constant.**

Proof of Kepler's third law

The following four formulas can be used to proof Kepler's third law.

1. $a = v^2/r$
2. $v = \omega r = 2\pi/T$
3. $F = GMm/r^2$
4. $F = ma$

- $Mv^2/r = GMm/r^2$
- $V^2/GM = 1/r$

Proof of kepler’s Third Law

- **$4\pi^2/GMT^2 = 1/r^3$**

Multiplying by T^2: $T^2/r^3 = 4\pi^2/GM$ as required!