St John Baptist De La Salle Catholic School

Mathematics Olympiad

Grade 11

March, 2023

1. Which one of the following is not true?

   A. $\exists x \, \exists y \, (\frac{x}{y} = \pi)$

   B. $\forall x \, \forall y \, (x = 0)$

   C. $\exists x \, \forall y \, (y > y^2 - 1)$

   D. $\forall x \, \forall y \, (x(x-1) > y(1-y))$

2. The computation of $\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}}$ gives
   A. 16 B. $2\sqrt{5}$ C. 2 D. $\sqrt{3}$ E. $\sqrt{6-2\sqrt{5}}$

3. Let $ABCD$ be a convex quadrilateral. Suppose that P is a point in the interior of $ABCD$ such that $\angle PAD : \angle PDA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BCP \angle BPC$. The internal bisectors of the angles ADP and PCB meet at a point O inside the triangle ABP. Which of the following is true?
   A. $AO = BO$ B. $DP = CP$ C. $OD = OC$ D. None

4. If $a^3 + 12ab^2 = 679$ and $9a^2b + 12b^3 = 978$. What is the value of $a^2 - 4ab + 4b^2$?
   A. 1 B. 9 C. 25 D. 49

5. How many ordered pairs (x, y) of positive integers satisfy the equation $\sqrt{y} = \sqrt{17} + \sqrt{x}$?
   A. 0 B. 1 C. 2 D. infinite

6. Given that $0 < b < a$ and $a^2 + b^2 = 6ab$, what is the value of $\frac{a-b}{a+b}$?
   A. $\sqrt{2}$ B. $1 + \sqrt{2}$ C. $\frac{1}{2}\sqrt{2}$ D. $-1 + \sqrt{2}$

7. Let $F : R \to R$ be a function such that $F(a+b) = F(a) + F(b)$. If $F(2008) = 3012$, what is the value of $F(2009)$?
   A. 3012.5 B. 3013 C. 3013.5 D. 3014

8. If $F(x) = x^2 + px + 1$ such that $F(0)$ and $F(1)$ have opposite signs so that F has a root $(0, 1)$, what is the value of p?

9. Simplify $\dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}$

10. Find all the real numbers x that satisfy the equation $(x^3-x)^{x^3+x^2-2x}=0$