$C = Q/V$

$\boxed{C = \frac{\varepsilon A}{d}}$, $\varepsilon = K\varepsilon_0$

$\boxed{C = \frac{K\varepsilon_0 A}{d}}$

K - dielectric constant

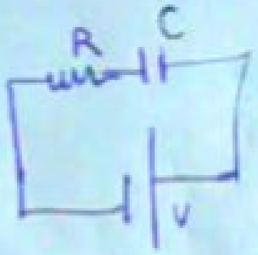
ε - dielectric permittivity

ε_0 - vacuum permittivity

$PE = QV$

$E = \frac{QV}{2}$ $\quad Q = CV$

$E = \frac{CV^2}{2}$ → Capacitance
→ potential difference



$V = V_C + V_R$

$V = \frac{\Delta Q}{\Delta C} + RI$

→ differential equation

$\boxed{V = \frac{\Delta Q}{\Delta C} + R\left(\frac{\Delta Q}{\Delta t}\right)}$

$Q(t) = Q(1 - e^{-t/RC})$ $\quad Q$ - full charge

$Q(t) = Qe^{-t/RC}$ $\quad e$ - euler number

$e^{-t/RC}$, $t = RC \rightarrow \tau$ time constant

Charging

$Q(RC) = Q(1 - e^{-RC/RC})$
$= Q(1 - 1/e)$
$= 0.63Q$
$\approx 0.63Q$

Discharging

$Q(RC) = Qe^{-RC/RC}$
$= \frac{Q}{e}$
≈ 0.37

$F \propto \frac{q_1 q_2}{r^2}$

$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2}$ k

$F = \frac{k q_1 q_2}{r^2}$

Electric field: the force acting on a charge per unit charge

Source charge (Q)

test charge (q)

$\boxed{F = k\frac{Qq}{r^2}}$

$F = \boxed{\frac{kQ}{r^2}} \cdot q$ E

$\boxed{F = Eq}$

Escape → $PE = KE$

Orbital velocities → $F_g = F_c$

$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$

$V_o = \sqrt{\frac{GM}{r}} = \sqrt{gr}$

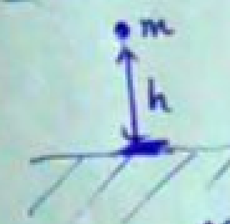
→ Energy stored in a system due to masses in a gravitational field

$\Delta PE = -W$ (PE in a system is the work done by conservative forces)

$U = (\underbrace{F_g}_{\text{conservative}})(\text{distance})$

$= \left(\frac{GMm}{r^2}\right)(r)$

$= \frac{GMm}{r}$



ground

$\Delta PE = mgh$ (on a height h the ground) & it is positive

PE at infinity → 0

$PE = 0$ as $r \to \infty$

PE within a finite from a mass is negative

$\Delta PE \to$ positive

$U = \frac{GMm}{r}$

$U(r) = \frac{GMm}{r}$ F_g → initial & final position

$F_g = \frac{GMm}{r^2}$

$F_g = \left(\frac{GM}{r^2}\right)(m)$ a_g

$F_g = m a_g$

$a_g = \frac{GM}{r^2}$

moment of inertia (I): distribution of mass in a body/around axis of rotation

$I = mr^2$ → point mass system

→ mass

→ geometry

$$I = m_1 r_1^2 + m_2 r_2^2 \dots + m_n r_n^2$$
$$= \sum_{i=1}^{n} m_i r_i^2$$

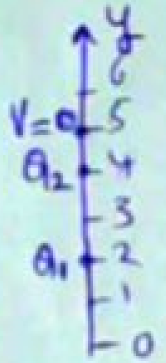
uniform object (Center of mass)

→ rod $= \frac{1}{12} ml^2$

→ hollow Sphere $= \frac{2}{3} mr^2$

→ solid Sphere $= \frac{2}{5} mr^2$

6



Potential at $y=5$

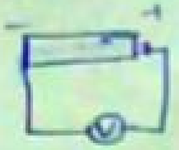
↓

Sum of the potential due to Q_1 & Q_2 → scalar quantities

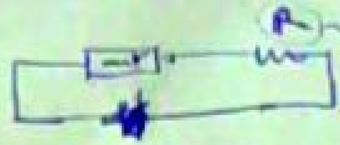
$r_1 = 3$

$r_2 = 1$

$$V = V_1 + V_2$$
$$0 = V_1 + V_2$$
$$V_1 = -V_2$$
$$\frac{KQ_1}{r_1} = -\frac{KQ_2}{r_2}$$
$$\frac{Q_1}{3} = -\frac{Q_2}{1}$$
$$\boxed{Q_1 = -3Q_2}$$



↓ terminal voltage



R ~ Circuit's total resistance before connecting to the battery

Emf: ideal voltage of a battery

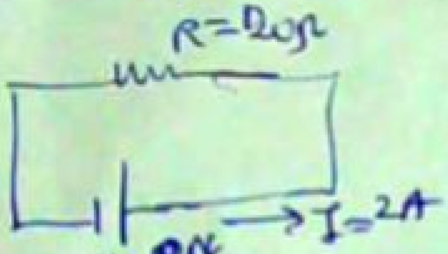
$\mathcal{E} = I(\text{total resistance})$

$\boxed{\mathcal{E} = I(r+R)}$

$\mathcal{E} = Ir + IR$ ~ terminal voltage (V)

$\mathcal{E} = Ir + V$

↳ voltage dissipated due to internal resistance



$\boxed{\mathcal{E} - V}$

What is the internal resistance

$Ir = \mathcal{E} - V$

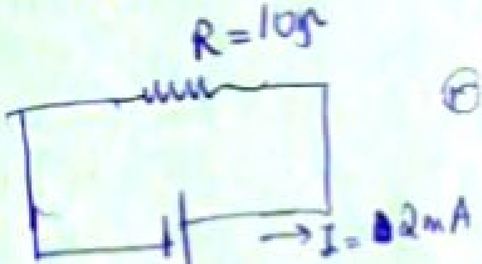
$V = IR$

$= (2A)(20\Omega)$

$V = 40V$

$\boxed{\mathcal{E} = V + Ir}$

usually, r is very small



$\mathcal{E} = 3V$

terminal voltage (v) → $V = IR$

$V = 2mA \times (10\Omega)$

$V = 20mV$

$r = \frac{\mathcal{E} - V}{I}$

$= \frac{3V - 20mV}{2mA}$

→ units of Energy

1eV

9V

$(1.6\times10^{-19})(1\ J/C)$ 1eV

$\Delta V = 1V$

$V_e = \sqrt{\frac{2GM}{R}}$

→ Joule (SI unit)

→ Calorie → 1Cal = 4.2J

→ 1eV → 1.6×10^{-19} J

/h → kwh → 1kwh

$1t \times 60^3 \times (\ldots)$

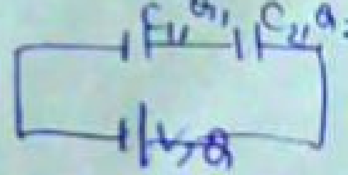
km/h

→ → 5.6×10^6 J

Capacitors

↳ series → $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C} \ldots$

↳ parallel → $C = C_1 + C_2 + C_3$



$C = Q/V$

$V = Q/C$

$Q = CV$

$Q = Q_1 = Q_2$

$V = V_1 + V_2$

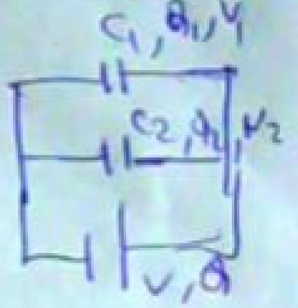
$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$

$Q/C = \frac{Q}{C_1} + \frac{Q}{C_2}$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

20μF, 60μF, 3μF

$E = \frac{1}{2} CV^2$



$Q = Q_1 + Q_2$

→ $V_2 = V_1 = V$

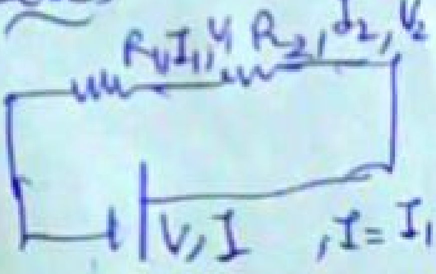
→ $CV = C_1V_1 + C_2V_2$

$CV = C_1V + C_2V$

$\boxed{C = C_1 + C_2}$

Resistors

Series



$, I = I_1 = I_2$

$V = RI$

$I = \frac{V}{R}$

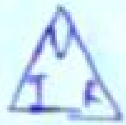
$V = V_1 + V_2$

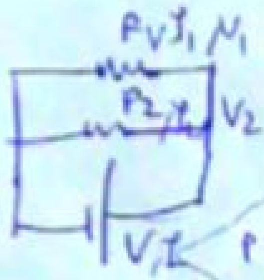
$RI = R_1I_1 + R_2I_2$

$RI = R_1I + R_2I$

$R = R_1 + R_2$

Parallel





$V = V_1 = V_2$

$I = I_1 + I_2$

$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$P = \frac{\text{Energy transf.}}{\text{time}}$

$P = \frac{QV}{t}$

$P = IV$ $\left\{ \begin{array}{l} I = V/R \\ V = IR \end{array} \right.$

$P = (\frac{V}{R})V = \frac{V^2}{R}$

$P = I(IR) = I^2R$

(potential)(Absolute potential): the potential energy per unit charge

$V = \frac{PE}{q} = \frac{\frac{kQq}{r}}{q}$

$V = \frac{kQ}{r}$

$V_A = \frac{kQ}{r_A}$, $V_B = \frac{kQ}{r_B}$

PE → work done by conservative forces

$PE = W$

$W = -\Delta PE$

Conservative → ME conserved
→ path → independent
(initial & final states)

dissipative → ME not conserved
→ path dependent

$W = \Delta(q)(V_B - V_A)$

$= -\Delta PE$

$W = -(PE_B - PE_A)$

$W = -(q(V_B) - q(V_A))$

$= -(q(V_B - V_A))$ → potential difference (voltage)

$\boxed{W = -q(\Delta V)}$

equipotential surface

$W = -q(\Delta V)$

$W = -q(V_B - V_A)$

$V_B = V_A$ → equipotential surface

$W = -q(0)$

$= 0$

equipotential surfaces $\perp$ electric field lines

$W = Fd\cos\theta$

$0 = Fd\cos\theta$

$\cos\theta = 0$

$\theta = 90^\circ / \frac{\pi}{2}$

uniform

→ Resistivity (ρ)
→ Conductivity (σ)

$$\boxed{\rho\sigma = 1}$$
$$\frac{1}{\rho} = \sigma$$
$$\frac{1}{\sigma} = \rho$$

$\tau_{net} = \frac{\Delta L}{\Delta t}$

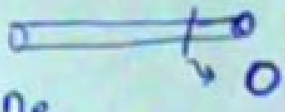
$\tau_{net} \Delta t = \Delta L$

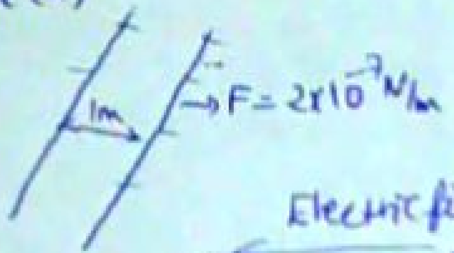
angular impulse ⇒ Change in angular momentum

14. $I = \frac{\Delta Q}{\Delta t}$: the rate flow of Charges

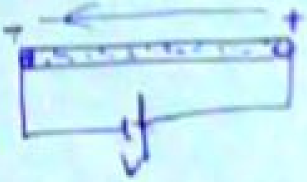
SI unit: Ampere (A)

1 A:





$n = \frac{N_e}{V}$ $\quad \underline{I = nAev}$



Conventional Current → flow of positive Charges → the same direction as the electric field

Current ∝ Electric field

$\vec{J} \propto \vec{E}$

Electron Current → flow of electrons

microscopic ohms Law → $\vec{J} = \sigma\vec{E}$ $\quad \vec{J}$ → Current density

$J = \frac{\Delta Q / A}{\Delta t} = \frac{I}{A}$ $\quad \vec{J}$: the time rate of flow charges in a given area

J

$\vec{F}_{net} = m\vec{a}$

$\vec{\tau}_{net} = I\alpha$

$L = I\omega$

$\boxed{\tau_{net} = I\alpha}$ — Newton's Second Law

$\tau_{net} = I\left(\frac{\omega_f - \omega_i}{t}\right)$ $\quad \cancel{\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}}$

$\tau_{net} = \frac{I\omega_f - I\omega_i}{t}$ $\quad \alpha = \frac{\omega_f - \omega_i}{t}$

$\tau_{net} = \frac{L_f - L_i}{t}$ $\quad \boxed{\tau_{net} = \frac{\Delta L}{\Delta t}}$ angular momentum

16. macroscopic form : $J \propto V$, $I \propto V$, $I = \frac{1}{R} V$ → at constant temperature

$I = V/R$

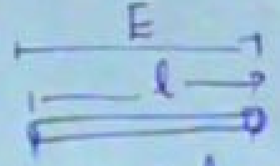
microscopic form : $\vec{J} = \sigma \vec{E}$

$\vec{J} = \frac{\vec{E}}{\rho}$

ohm's law isn't necessarily alway true

↓

Ohmic material → when it obeys ohm's law.



$\vec{J} = \frac{\vec{E}}{\rho}$

$\frac{I}{A} = \frac{E}{\rho} \Rightarrow \frac{V}{RA} = \frac{E}{\rho}$

$V = El$

$= El$

$\frac{El}{RA} = \frac{E}{\rho} \Rightarrow \rho l = RA$

$R = \frac{\rho l}{A}$

<u>Temperature</u>

R_0

$\Delta R \propto \Delta T R_0$

$\Delta R = \alpha R_0 \Delta T$

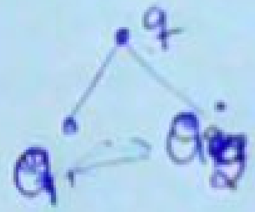
$\Delta R = R - R_0$

$R = \Delta R + R_0$

$= \alpha R_0 \Delta T + R_0$

$= \underline{R_0(1 + \alpha \Delta T)}$

<u>Superposition principle</u>



Earth $= 6 \times 10^{24}$ kg

M

$r = \frac{2 \times G \times M_E}{c^2}$

$= 1$ cm

Swarzschild radius

18 <u>Blackhole</u>

$V = \sqrt{\frac{2GM}{r}}$

$c = \sqrt{\frac{2GM}{r}}$

$c^2 = \frac{2GM}{r}$

$\boxed{r = \frac{2GM}{c^2}}$

19. 0^{16} km