Mathematical Proof of Kepler's Third Law

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1 Johannes Kepler

Johannes Kepler (27 December 1571 – 15 November 1630) was a German astronomer, mathematician, astrologer, natural philosopher and writer on music. He is a key figure in the 17th-century Scientific Revolution, best known for his laws of planetary motion, and his books Astronomia nova, Harmonice Mundi, and Epitome Astronomiae Copernicanae. These works also provided one of the foundations for Newton's theory of universal gravitation.

Kepler was a mathematics teacher at a seminary school in Graz, where he became an associate of Prince Hans Ulrich von Eggenberg. Later he became an assistant to the astronomer Tycho Brahe in Prague, and eventually the imperial mathematician to Emperor Rudolf II and his two successors Matthias and Ferdinand II. He also taught mathematics in Linz, and was an adviser to General Wallenstein. Additionally, he did fundamental work in the field of optics, invented an improved version of the refracting (or Keplerian) telescope, and was mentioned in the telescopic discoveries of his contemporary Galileo Galilei. He was a corresponding member of the Accademia dei Lincei in Rome.

Kepler lived in an era when there was no clear distinction between astronomy and astrology, but there was a strong division between astronomy (a branch of mathematics within the liberal arts) and physics (a branch of natural philosophy). Kepler also incorporated religious arguments and reasoning into his work, motivated by the religious conviction and belief that God had created the world according to an intelligible plan that is accessible through the natural light of reason. Kepler described his new astronomy as "celestial physics", as "an excursion into Aristotle's Metaphysics",[and as "a supplement to Aristotle's On the Heavens", transforming the ancient tradition of physical cosmology by treating astronomy as part of a universal mathematical physics.

2 Kepler laws of planetary motion

2.1 kepler's first law

Kepler's first law States that every planet moves along an ellipse ,with the sun located at a focus of the ellipse .An ellipse is defined as the set of all points such that the sum of distance from each point to two foci is constant.

2.2 kepler's second law

Kepler's second law states that a planet sweeps out equal times, is the area divided by time,called the areal velocity ,is constant.It is the result of the conservation of angular momentum since the angular momentum is constant ,the areal velocity must also be constant.

In order for the areas to be equal ,the planet must speed up as it gets closer to the sun and slow down as it moves away. The planets all have orbits round the sun that are close to being circles. These are some natural satellites of the sun that are very different ,though the comets. These are only visible to us as they pass quickly through the part of their orbit Which is close to the sun.Most of the time they are remote from the sun ,in the darkness of space and travelling more slowly.

The path of the moon as it travels round the earth is very nearly a circle ,though there are times in its orbit when it is closer to us and so appears slightly larger in the sky.
we know that angular momentum L = rp which L is the angular momentum of the planet about the sun, r is the position vector of the planet measured from the sun and p = mv which is the linear momentum at a point in the orbit. because the planet moves along the ellipse p is always tangent to the ellipse .
The linear momentum Can be resolved into radial component(Pr) along the line to the sun,and another component called perpendicular to r(Pp).so we can have angular momentum as

L = rp = r(P r' + P p') = rP r + rP p

The first term is zero Because r is parallel to Pr and in the second term r is perpendicular to Pp so the magnitude will be L = rP p = rmv which v is perpendicular to r.

area lV = change in area/change in time =L/2m

Generally , keplers second law is trying to address if the angular momentum is constant then the areal velocity remains constant.

2.3 kepler's third law

The squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axis of their orbits.

The period, T, of a planet increases as its mean distance from the sun, R, raised to the 3/2 power

3 Mathematical proof of kepler's third law

Kepler's Third Law states that the square of the time period of orbit is directly proportional to the cuber of the semi-major axis of that respective orbit. (the semi-major axis for a circular orbit is of course the radius) Mathematically this can be represented as: k = T$_2/r_3 wherekisaconstant.Thevaluekisrelatedtophysicalconstantssuchthatk = 4pi^2/GMwhereGisthegravitationalconstantandMthemassoftheobjectatthecentreoftheorbit.$

$GMm/r^2 = mv^2/r$
$thenaftercrossmultiplication$
$GmMr = mv^2r^2$
$thenwecancanceloutmandrthenwefind$
$GM = v^2r$
$butweknowthatforonperiod(T)whichisthetimeittakesfortheplanettotravel1revolutionwhichisequaltothecircumfranceoftheo$
$sowecanfindvforonerevolutionas:$

$V = 2pir/T$
$sobysubstitutingitintothefirstequationwefind:$

$GM = (2pir/T)^2r$
$then$

$GMT^2 = 4^2r^3$
$wehavesaidthatT_2isdirectlyproportionaltor_3.sotheirmustbeaconstant$
$thatfulfillthisproportionality.whichis:$

$T^2 = (4pi^2/GM)r^3$
$so4pi^2/GMistheconstantforthisproportionality.generallyfromthiswecanconcludethat$
$T_1^2/R_1^3 = 4pi^2/GM$
$andalso$
$T_2^2/R_2^3 = 4pi^2/GM$
$finallywecanfinalizeitas:$
$T_1^2/R_1^3 = T_2^2/R_2^3$

conclusion

In the past time people thought the earth was the center of the solar system with all the other planets, satellites and other celestial bodies revolving around it.Kepler's laws describe the motion of planet around the sun.

Kepler's first law States that every planet moves along an ellipse ,with the sun located at a focus of the ellipse .An ellipse is defined as the set of all points such that the sum of distance from each point to two foci is constant.

Kepler's second law states that a planet sweeps out equal times, is the area divided by time,called the areal velocity ,is constant.It is the result of the conservation of angular momentum since the angular momentum is constant ,the areal velocity must also be constant.

Kepler's third law states that the square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the sun.

4 reference

physics.kebede.org
https://en.wikipedia.org/wiki/Johannes-Kepler