ST JOHN BAPTIST DE LA SALLE CATHOLIC SCHOOL ,ADDIS ABABA

PHYSICS ASSIGNMENT,GROUP 8 AND 2

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MATHEMATICAL PROOF OF KEPLER'S LAWS

GRADE 10D

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1 Introduction

In the past time people thought the earth was the center of the solar system with all the other planets, satellites and other celestial bodies revolving around it. They just thought that the earth as the sun.(which celestial bodies revolve around it).The other thing during the time of Kepler they thought that all planetary orbit were circular.

Currently ,We know that all planets revolve around the sun. the Danish astronomer Tycho Brahe followed the paths of the planets for many years. Johannes Kepler made good use of Brahe's WORK, extracting from his carefully collected data the three laws of orbital motion we know today as Kepler's laws. The data for Mars presented the greatest challenge to this view and that eventually motivated Kepler to give up the popular idea.

Kepler's laws are experimental laws that describe the rotation of satellites about their parent body. They apply to all the satellites around the earth, but kepler's data were collected in the late 1500s before the telescope had been invented by closely observing the paths of the planets as they went around the sun.kepler began with many sheets of figures, and amazingly succeeded in spotting three unexpected patterns to them all. These are now known as kepler's laws.(kepler's first law, kepler's second law ,and kepler's third law).

Generally ,kepler's laws describe the motion of planet around the sun which we are going to see in this literature in detail about the mathemathecal proof of the kepler's laws.

2 MATHEMATICAL PROOF OF KEPLER'S LAWS

2.1 Kepler's first law

Kepler's first law States that every planet moves along an ellipse ,with the sun located at a focus of the ellipse .An ellipse is defined as the set of all points such that the sum of distance from each point to two foci is constant.

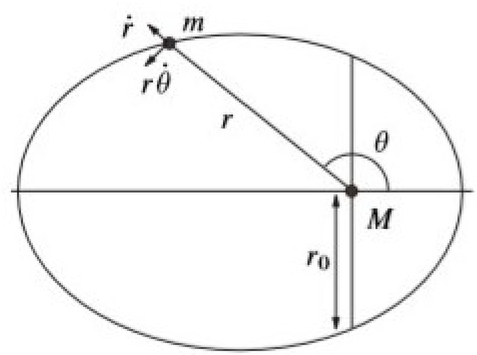

Figure 1: The geometry used in the proof.

Take a mass m in a general elliptical orbit around a much more massive body M. When the separation of the masses is r, the total energy of the orbit is

$$E = 1/2mv^2 - GMm/r - - - - - equation1$$

where v is the speed of the orbiting mass and G the constant of gravitation. The velocity of m has two components: a radial component written r and a component perpendicular to r which is the 'circular' component of the velocity, equal to $r\omega$ where $\omega = \theta$ is the instantaneous angular velocity of the body, with θ as shown in the diagram. Because these components are orthogonal, the square of the total velocity equals the sum of the squares of these components. We can now write this energy equation in polar coordinates:

$$E = 1/2m(r^2 + r^2\theta^2) - - - - - equation2$$

similarly, we can write the angular momentum of m as

$$L = mr^2/theta$$

as $r\theta$ is the component of v perpendicular to r. Now make the substitution $p = 1/r$, so that $\theta = Lp^2/m$ We can evaluate θ as

$$\theta = Lp^2/m$$

$$p = 1/r$$

$$so, \theta = L/m$$

Rearranging equation 2 we can see that r is

$$r^2 = 2E/m + GMp - L^2p^2/m^2$$

we replaced $r^2\theta^2$ as

$$\theta = Lp^2/m$$

and

$$r = 1/p$$

so

$$\theta^2 r^2 = (Lp^2/m)^2 * (1/p)^2$$
$$= L^2p^2/m^2$$

Now a further manipulation .we make the substitutions

$$r0 = L^2/GMm^2$$

$$e^2 = 1 + 2Er0/GMm$$

Clearly, both r0 and e are constants. We choose them in this way so that our answer is immediately recognisable as an ellipse – it is not an obvious substitution at this stage! Which even our group hasn't understood it.After a Little manipulation, equation can be written as

$$L/m\sqrt{\frac{e^2}{r0^2} - (p - \frac{1}{r0})^2}$$

then after substitution in the third equation we can find θ as

$$cos^-1(\frac{p - 1/r0}{e/r0})$$

This can be rearranged to give

$$r = [\frac{r0}{(1 + ecos\theta)}$$

Which is the equation of an ellipse in polar coordinates with the origin at focus. we can now identity r0 as the semi-latus rectum of the ellipse and e as its eccentricity .

2.2 Kepler's second law

Kepler's second law states that a planet sweeps out equal times, is the area divided by time,called the areal velocity ,is constant.It is the result of the conservation of angular momentum since the angular momentum is constant ,the areal velocity must also be constant.

In order for the areas to be equal ,the planet must speed up as it gets closer to the sun and slow down as it moves away. The planets all have orbits round the sun that are close to being circles. These are some natural satellites of the sun that are very different ,though the comets. These are only visible to us as they pass quickly through the part of their orbit Which is close to the sun.Most of the time they are remote from the sun ,in the darkness of space and travelling more slowly.

The path of the moon as it travels round the earth is very nearly a circle ,though there are times in its orbit when it is closer to us and so appears slightly larger in the sky.

we know that angular momentum $L = rp$ which L is the angular momentum of the planet about the sun, r is the position vector of the planet measured from the sun and $p = mv$ which is the linear momentum at a point in the orbit. because the planet moves along the ellipse p is always tangent to the ellipse .

The linear momentum Can be resolved into radial component(Pr) along the line to the sun,and another component called perpendicular to r(Pp).so we can have angular momentum as

$$L = rp = r(Pr` + Pp`) = rPr + rPp$$

The first term is zero Because r is parallel to Pr and in the second term r is perpendicular to Pp so the magnitude will be $L = rPp = rmv$ which v is perpendicular to r.

The areal velocity is given by

$$arealV = \frac{chnageinarea}{changeintime} = \frac{L}{2m}$$

Generally , keplers second law is trying to address if the angular momentum is constant then the areal velocity remains constant.

2.3 Kepler's third law

Kepler's third law states that the square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the sun.

simply we know that the centrepetal force is equal to gravitational force of the planet because both are acted towards the sun .but we assume the path they follow as circular orbit for simplicity.so we can see the proof as Follows:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

then after cross multiplication

$$GmMr = mv^2r^2$$

then we can cancel out m and r then we find

$$GM = v^2r$$

but we know that for on period(T) which is the time it takes for the planet to travel 1 revolution which is equal to the circumfrance of the orbit. so we can find v for one revolution as:

$$V = \frac{2\pi r}{T}$$

so by substituting it into the first equation we find:

$$GM = (\frac{2\pi r}{T})^2r$$

then

$$GMT^2 = 4\pi^2r^3$$

we have said that T^2 is directly proportional to r^3. so their must be a constant that fulfill this proportionality.which is:

$$T^2 = \frac{4\pi^2}{GM} * r^3$$

so $4\pi^2/GM$ is the constant for this proportionality. generally from this we can conclude that

$$\frac{T1^2}{R1^3} = \frac{4\pi^2}{GM}$$

and also

$$\frac{T2^2}{R2^3} = \frac{4\pi^2}{GM}$$

finally we can finalize it as:

$$\frac{T1^2}{R1^3} = \frac{T2^2}{R2^3}$$

3 conclusion

In the past time people thought the earth was the center of the solar system with all the other planets, satellites and other celestial bodies revolving around it.Kepler's laws describe the motion of planet around the sun .

Kepler's first law States that every planet moves along an ellipse ,with the sun located at a focus of the ellipse .An ellipse is defined as the set of all points such that the sum of distance from each point to two foci is constant.

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