Consumption growth persistence and the stock/bond correlation*

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Abstract

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Abstract

We consider a model in which the correlation between shocks to consumption and to expected future consumption growth is nonzero and varies over time. We validate this assumption empirically using the model's implication that time-variation in consumption growth persistence drives the correlation between stock and bond returns. Consistent with model predictions, we find that the stock/bond correlation is also related to the volatility of stock returns, the so-called stock market "leverage effect," and the predictive relation between bond yields and future stock returns.

1. Introduction

While the correlation between stock and bond returns has been the subject of research for some time, the abrupt change in the sign of this correlation in the late 1990s, shown in Figure 1 (a), has spurred renewed interest in its determinants. One explanation of this shift is an apparent regime change in the behavior of inflation, as demonstrated in David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020). In contrast, Duffee (2018b) finds that the stock/bond return correlation is primarily driven by changes in the behavior of real yields. He does not find, however, any shift in macro fundamentals that would be responsible for this change, concluding that "macroeconomic dynamics appear to have almost nothing to do with this time-varying comovement."

In this paper, we propose a new explanation of this shift and of variation in the correlation between stock and bond returns (SB correlation) more generally. We show that this correlation is related to variation in consumption growth persistence (CGP), which we define as the tendency of positive shocks to current consumption growth to raise expected future consumption growth. The logic is straightforward: Changes in current realized growth affect cash flows, while changes in expected growth drive real interest rates via intertemporal smoothing. When CGP increases, the correlation between real rates and cash flows rises, resulting in a lower (and likely negative) SB correlation. When CGP is negative, higher consumption growth forecasts lower growth in the future, and the SB correlation rises.

The persistence of consumption growth indeed appears to have changed over time. Figure 1 (b) shows that autocorrelations in consumption growth were moderate through 1997

but significantly higher in the period starting in 1999, which is around when the SB correlation changed sign. The goal of this study is to determine whether this suggestive evidence is indicative of a more systematic effect that CGP has on the SB correlation and other asset return moments.

Macroeconomic theory tells us that the persistence of consumption growth reflects the relative importance of transitory versus permanent shocks. Transient shocks to productivity (Kaltenbrunner and Lochstoer 2010), income (Hall and Mishkin 1982), or uncertainty (Basu and Bundick 2017) can all increase consumption in the short run while at the same time decreasing long-term consumption growth. In contrast, permanent shocks generally imply persistence in growth rates, as is the case for the production economy discussed by Kaltenbrunner and Lochstoer (2010), resulting from the frictions that induce gradual adjustment to such shocks.

Starting with Friedman (1957), abundant empirical evidence shows that both permanent and transitory shocks are necessary to explain observed patterns of consumption persistence.

The more recent literature on macroeconomic volatility indicates that multiple sources of uncertainty affect macroeconomic and financial variables to different degrees (e.g. Jurado, Ludvigson, and Ng 2015), implying that the relative importance of different shocks varies over time. Intuitively, if the most volatile shocks are transitory (e.g., pandemics and oil shocks), CGP will become negative. When permanent shocks (e.g., technology and climate change) dominate, CGP becomes positive.

Our model represents a stylized way to capture the net effect of these mechanisms. As

¹ See Beveridge and Nelson (1981), Watson (1986), and Clark (1987), among numerous others.

in the long-run risk (LRR) model of Bansal and Yaron (2004), a highly persistent expected consumption growth process induces modest but very long-run dependence in consumption growth. We generalize this model by allowing a time-varying correlation between current and expected consumption growth shocks. By including both processes, it is possible to induce time variation in CGP while maintaining the long-run positive autocorrelation critical for matching the moments of asset returns.

In addition to the relation between CGP and the stock/bond return correlation, allowing for time-varying consumption persistence generates a number of new predictions. One is that more CGP will raise stock market volatility. This is the case because current and expected future cash flows are two primary drivers of equity valuation. When these shocks are positively correlated, their effects will be amplified, and market volatility will rise.

Another implication of our model is a relationship between CGP and the stock market "leverage effect" – the generally negative correlation between stock returns and volatility changes. The connection arises from an additional feature of our model, a negative correlation between shocks to consumption growth and its volatility. This correlation is empirically motivated and is thought to arise from a precautionary savings motive.² With precautionary savings, a higher CGP will strengthen the predictive relation between volatility shocks and future consumption growth. One implication for asset prices is that the leverage effect will be magnified (i.e., made more negative) by higher CGP.

Finally, our model implies that the relationship between interest rates and future stock market returns should depend on CGP. While many macro-finance models suggest a negative

²See, for example, Carroll (1997) and Basu and Bundick (2017)

relationship between these variables, the weak empirical relationship is puzzling. Since bond yields are closely related to the level of expected consumption growth, they are more strongly related to consumption volatility – which drives the market risk premium – when CGP is high. The predictive relationship should be weak when CGP is low.

Testing these predictions is challenging because CGP, which is assumed to vary over time, is unobserved. Moreover, standard estimation methods are unlikely to be effective given that CGP describes the correlation between consumption growth and a latent variable, expected consumption growth, whose measurement is itself difficult (e.g., Schorfheide, Song, and Yaron 2018). We, therefore, devise an empirical strategy based on our model's implication that CGP is inversely related to the stock/bond return correlation, which can be measured accurately due to the availability of high-frequency asset price data.

We validate the model-implied relationship between CGP and the SB correlation by showing that the serial correlation in consumption growth is significantly higher when the correlation is low. This result holds at multiple horizons and is obtained whether we use returns on short- or long-term bonds and whether the bonds are nominal or inflation-indexed. Second, we show that the contemporaneous relationship between consumption growth and changes in bond yields is more negative when the SB correlation is higher. Given that bond yields and expected consumption growth are closely related in the LRR framework, this result is also consistent with a negative relation between CGP and the stock/bond correlation.

With this justification for using the SB correlation as a proxy for CGP, we confirm all of our model predictions in the data. First, we find strong evidence that the SB correlation is positively related to the leverage effect of the stock market. We test this relationship in

multiple ways, using a volatility forecast model and using monthly non-overlapping estimates of the correlations.

Second, since CGP should also be positively related to market volatility, the predicted relationship between market volatility and the stock/bond correlation should be negative. More precisely, the correlation should have a negative interactive effect on macroeconomic uncertainty. We confirm this relationship using several uncertainty measures, namely the macro uncertainty of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty of Baker, Bloom, and Davis (2016), and a simple measure of consumption growth variance.

The final implication is for the conditional predictive relation between yields and future stock returns. The model says that this negative relationship should be stronger when CGP is high or when the SB correlation is low. As documented in prior work, we report a weak unconditional predictive relationship but a strong conditional relationship when the SB correlation is negative. Therefore, the insignificant unconditional relation, which is at odds with the predictions of many models, is a natural result of the SB correlation being positive over much of our sample.

There are a number of other explanations for why the SB correlation varies over time, and we believe it is unlikely that any single theory can explain all fluctuations. Aside from the other predictions that we confirm from our empirical analysis, we believe that consumption growth persistence has certain merits that distinguish it from other explanations.

Other studies attribute the variation in SB correlations to other factors. Baele, Bekaert, and Inghelbrecht (2010), claiming that "macroeconomic fundamentals contribute little to explaining stock and bond return correlations, conclude that flights to quality/liquidity are

the likely explanation for negative correlations. While Pástor and Stambaugh (2003), Connolly, Stivers, and Sun (2005), and others provide additional evidence for this channel, such evidence seems incomplete given the prolonged period since 1999, over which the stock/bond correlation has remained negative even during periods of relative market stability.

Variation in the stock/bond correlation has also been attributed to changes in the dynamics of inflation. David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020) present models in which the relation between inflation and real economic activity changes signs. Campbell, Pflueger, and Viceira (2020), for example, show that the correlation between inflation and the output gap was negative between 1979 and 2001 but positive in the following decade. If inflation shocks are the primary driver of bond returns, this result would appear to provide a clear explanation for the shift in correlation that occurred around that time.³

In contrast, the SB correlations shown in our model are entirely driven by variation in real interest rates and not in inflation. While changing properties of inflation are undoubtedly a reason for changes in SB correlations, a model based on real rates may be better positioned to explain interest rate behavior in environments such as that of the last 20 years, in which both inflation levels and inflation risk have been low. Since 2003, in periods where reliable TIPS data are available, real and nominal bond yields have tracked each other closely, with a correlation above 90%. More importantly, the real bond/stock correlation and nominal bond/stock correlation are themselves closely related over the post-2003 sample period.⁴

³Hasseltoft (2012), Ilmanen (2003), Campbell, Sunderam, and Viceira (2017), and Swanson (2019) also advance inflation-based explanations of the stock/bond return correlation.

⁴When these two correlations are measured using non-overlapping monthly subsamples, the correlation between them is 86%.

While inflation is undoubtedly more important in earlier decades, it almost certainly does not tell the entire story.

This is not the only paper to propose that variation in real yields is the primary driver of changes in the SB correlation. For example, Duffee (2018 a,b) argues that the inflation expectations that underlie long-term bond yields vary too little to explain much variation in such yields and that the stock/bond correlation is therefore primarily driven by changes in real yields. Kozak (2021) proposes a production model with two technologies that generates a time-varying correlation, attributing the shift in correlation in the late 1990s to a decline in high-risk capital. Chernov, Lochstoer, and Song (2021) also highlight the role of consumption growth persistence in explaining the stock/bond correlation. While their model is substantially different from ours and more focused on explaining the nominal term structure, it succeeds in explaining the stock/bond correlation using a mechanism based on consumption persistence rather than inflation dynamics. The different empirical predictions derived in their paper show that time-varying consumption growth persistence can explain a broader range of phenomena than those we address here.

In the next section, we describe and calibrate our model. Section III describes our data and strategies for measuring latent processes. Section IV presents our empirical results, and Section V concludes.

2. A real model of the stock-bond correlation

2.1. Model dynamics

Our model is a generalization of the standard framework of Bansal and Yaron (2004). In our baseline specification, the representative agent has Epstein and Zin (1991) preferences, and consumption growth (Δc_{t+1}) has a persistent time-varying component x_t and time-varying uncertainty σ_t^2 :

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \epsilon_{c,t+1}$$

$$x_{t+1} = p_1 x_t + \phi_x \sigma_t \epsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = s_0 + s_1 \sigma_t^2 + \sigma_v \sigma_t \epsilon_{v,t+1},$$

$$(1)$$

where $\epsilon_{c,t+1}$, $\epsilon_{x,t+1}$, and $\epsilon_{v,t+1}$ are i.i.d. N(0,1). The only change made to the original model of Bansal and Yaron (2004) is the use of a "square root" process for consumption variance. While the three state variables are uncorrelated in this baseline model, this is not the case below, and this modification will continue to allow for analytical solutions.

Our generalized model deviates from Bansal and Yaron in several important dimensions. First, we allow shocks to consumption growth $(\epsilon_{c,t+1})$ and expected long-run consumption growth $(\epsilon_{x,t+1})$ to be stochastically correlated. We refer to this correlation as consumption growth persistence, or CGP, given that it determines whether a shock to current consumption growth is associated with more or less consumption growth in the future. This correlation, which we denote as ρ_t , involves a stochastic process specified below.

As discussed in the introduction, a stochastic correlation can be viewed as a reduced-form

approach to modeling time variation in the relative importance of permanent and transitory shocks. For example, in the production economy described by Kaltenbrunner and Lochstoer (2010), the assumption of permanent productivity shocks results in a positive CGP, while transitory shocks generate a negative CGP. This results from differences in how investment (and therefore consumption) responds to changing productivity and in how adjustment costs and mean reversion induce trends of future output. Given that both types of shocks are likely, either effect could dominate depending on which type of shock is currently more volatile. Furthermore, this phenomenon is not limited to shocks to productivity. Permanent and transitory shocks to income generate similar responses, as discussed, for example, by Hall and Mishkin (1982) and Campbell and Deaton (1989).

We also deviate from Bansal and Yaron by allowing consumption growth shocks to be correlated with consumption variance shocks. A negative correlation is a natural result of a precautionary savings motive, confirmed empirically in several studies, including Carroll and Samwick (1998) and Basu and Bundick (2017). For simplicity, we assume that this correlation, denoted ϱ_{ps} , is constant.

Finally, given that consumption growth shocks are correlated with shocks to expected growth rates (ρ_t) and to consumption volatility (ϱ_{ps}) , it is natural to expect a nonzero correlation between shocks to expected consumption growth and consumption volatility. For example, an increase in precautionary savings induced by greater uncertainty should reduce current consumption since households increase their savings, leading to a rise in expected long-run consumption growth as uncertainty wanes and consumption returns to normal. Empirically, a nonzero correlation between σ_t and x_t is found by Nakamura, Sergeyev, and

Steinsson (2017), who show that it tends to be more negative during economic contractions. In another work, Parker and Preston (2005) find significant evidence from household survey data that the precautionary savings motive explains the predictable component of consumption growth.

In the interest of parsimony, we avoid introducing unnecessary additional parameters by assuming that this correlation between shocks to consumption volatility and expected consumption growth is equal to the product $\rho_t \varrho_{ps}$.⁵

Closing the model requires a specification of the dynamics of CGP, or ρ_t . To obtain closed-form solutions, we parameterize the conditional covariance between $\sigma_t \epsilon_{c,t}$ and $\sigma_t \epsilon_{x,t}$ (as opposed to that between $\epsilon_{c,t}$ and $\epsilon_{x,t}$) as an autoregressive process. This covariance, which is related to ρ_t by

$$p_t = \sigma_t^2 \rho_t,$$

follows

$$p_{t+1} = \omega_0 + \omega_1 p_t + \sigma_p \sigma_t \epsilon_{p,t+1}. \tag{5}$$

For simplicity, we assume that $\epsilon_{p,t+1}$ is uncorrelated with other shocks.

Given the same preference assumptions as those used by Bansal and Yaron, the price-toconsumption ratio z_t can be represented as a linear function of long-run expected consump-

$$\epsilon_{c,t} = u_{c,t} \tag{2}$$

$$\epsilon_{x,t} = \rho_t u_{c,t} + \sqrt{1 - \rho_t^2} u_{x,t} \tag{3}$$

$$\epsilon_{v,t} = \varrho_{ps} u_{c,t} + \sqrt{1 - \varrho_{ps}^2} u_{v,t} \tag{4}$$

⁵This correlation structure is consistent with the assumption that there are three orthonormal shocks, $[u_{c,t} u_{x,t} u_{v,t}]$, that drive the shocks to the three state variables via

tion growth (x_t) , the variance of consumption growth (σ_t^2) , and the covariance process (p_t) . That is,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 p_t, (6)$$

where $A_1 > 0$, $A_2 < 0$, and $A_3 < 0$ under conventional parameter assumptions ($\gamma > 1$ and $\psi > 1$), as shown in the appendix.

Bond yields of all maturities are linear functions of the three state variables. The appendix derives an analytic formula for the one-period bond, which is increasing in x_t and decreasing in σ_t^2 and p_t , and provides a solution method for longer-term bonds.

As is standard, we extend this model further by adding a dividend process. In this model, which we label the "full" model, dividend growth is specified as

$$\Delta d_{t+1} = \mu_d + \phi_d x_t + \sigma_t \varphi_{cd} \epsilon_{c,t+1} + \sigma_t \varphi_d \epsilon_{d,t+1}, \tag{7}$$

where $\epsilon_{d,t+1}$ is assumed to be uncorrelated with other shocks. Thus, dividend growth shares similarities with consumption due to its dependence on the long-run growth process x_t and because of its sensitivity to consumption growth shock $\epsilon_{c,t}$. The magnitudes of ϕ_d and φ_{cd} , relative to φ_d , determine the strength of this commonality.

We approximate the return on the market portfolio using Campbell-Shiller decomposition. Similar to the wealth-consumption ratio, we can verify the conjecture that the pricedividend ratio is a linear function of the three state variables.

Given closed-form linear expressions for stock returns and for bond yields of any maturity, it is straightforward to solve for the stock and bond return variances and covariance. We show in the appendix that all three may be expressed as linear and increasing functions of σ_t^2

and p_t . Furthermore, the stock/bond return correlation is a univariate (though nonlinear) function of ρ_t .

2.2. Calibration

We calibrate the model to examine its quantitative implications. The consumption and consumption variance parameters mirror those of Bansal and Yaron (2004). We set a high bar for the exercise by choosing the parameters that govern the persistent component of consumption growth and the covariance process that drives CGP to match macro data and survey forecasts, not asset returns. Coherence between these values requires that we deviate from Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), but the differences are minor. The parameter values we assume are summarized in Panel A of Table 1.

We proxy for the long-run growth process using the four-quarter-ahead forecast of real consumption from the Survey of Professional Forecasters (SPF), as detailed in the data section. The persistence (p_x) and volatility (ϕ_x) of the long-run growth process are set to match the persistence and volatility of the SPF forecast at the annual level, and the forecasts are only slightly less persistent than the values implied by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). The correlation between the shocks to the survey forecasts and realized consumption is slightly negative (-0.12). We set the parameters of the p_t process such that the realized correlation between consumption growth and changes in the SPF forecast matches the correlation between Δc and x in the model in terms of the average, standard deviation, and persistence.

For our primary specification, we set the precautionary savings parameter ϱ_{ps} to -0.2

following the results of Basu and Bundick (2017), but we also show results for two alternative specifications by setting ϱ_{ps} to -0.3 or -0.5.

Panel B compares the asset moments generated by our three specifications. These include the baseline model, in which correlations are set to zero; the consumption-only model, in which the total wealth portfolio is assumed to be the stock market portfolio; and the full model, which incorporates a dividend process. For each specification, we generate one million monthly observations and evaluate the first two moments of stock and bond returns as well as several other relevant asset pricing moments. The table shows that the unconditional moments generated by the simulations are generally comparable to those of other standard LRR models, aside from the correlations between stock and bond returns and between returns and volatility changes, which our new models better match.

Finally, Panel C of Table 1 compares the dynamic behavior of estimated correlations implied by the models to those observed in the data. From both data and simulations, we compute various correlation measures using non-overlapping 60-month windows. The table displays the means and standard deviations of these values. Monthly serial correlations are inferred by assuming an autoregressive process of order one and taking the 60^{th} root of each autocorrelation. The results show that the level of persistence in the model matches the data quite well, though the correlations are somewhat less volatile in the full model compared to the data.

⁶When computing correlations between stock returns and stock volatility in the data, we use the long-run volatility from the two-factor EGARCH model of Brandt and Jones (2006), which is described in more detail in Section 4.3.

2.3. The stock/bond return correlation

This section shows the relationship between CGP and the SB correlation. Establishing this link is essential because it provides a new explanation for why the correlation varies over time. Furthermore, since it can be computed easily as long as stock and bond returns are available, the SB correlation can be used as an empirical proxy for ρ_t .

To establish the relationship between CGP and the SB correlation, we first explain how the yields of a short-term bond, represented by the one-year bond, those of a long-term bond, represented by the ten-year bond, stock returns, and stock variances are determined in our model. Table 2 summarizes the relationship between yields and state variables.

We consider two channels that drive SB correlations under the baseline model, in which all shocks in (1) are assumed to be uncorrelated. The first channel occurs through shocks to the expected consumption growth $(\epsilon_{x,t+1})$. If this shock is positive, higher expected future cash flows will lead to higher stock returns. Bond yields will also increase as the demand for money rises due to the intertemporal consumption smoothing motive. Since stock and bond returns will have opposite responses, this channel implies a negative SB correlation.

The second channel occurs via shocks to consumption growth uncertainty $(\epsilon_{v,t+1})$. Stock market variance will rise following a positive uncertainty shock, raising the risk premium and lowering valuation. At the same time, bond yields will drop due to a precautionary savings effect. Therefore, an increase in uncertainty leads to stock and bond prices moving in opposite directions. The flight-to-quality phenomenon often refers to the negative SB correlation arising from this second channel.

While both channels imply a negative correlation between bond and stock returns for the baseline model, regardless of bond maturity, Panel A of Table 2 shows that the first channel is generally much stronger than the second. This is especially true for short-term yields, for which the correlation between yield changes and shocks to expected consumption growth is greater than 0.96 for all models. For long-term yields, the correlation between yield changes and shocks to the x_t process is approximately 0.93 for the baseline and 0.91 for the generalized models. In contrast, the correlation between yield changes and volatility shocks is around -0.08 for short-term yields and -0.31 for long-term yields for the generalized models. These results establish the close connection between interest rates and the expected consumption growth process that underlies our empirical analyses. The table also shows that shocks to the total wealth portfolio or the stock index are highly correlated with realized consumption growth, even in the "full" model, which introduces a separate dividend process.

While Table 1 shows that our generalized models exhibit a similar average negative SB correlation, CGP causes this correlation to vary over time. For example, this correlation should increase when ρ_t decreases. To illustrate this point, suppose there is a positive expected consumption growth shock ($\epsilon_{x,t+1} > 0$). This shock is likely to coincide with a decline in current consumption when ρ_t is negative. In this case, bond yields will increase as the economy expects higher levels of future growth, while the negative shock to current consumption will lower equity values. While the net effect may be that equity values rise due to higher expected long-run growth, the rise will be moderated by the negative shock to current consumption. Therefore, a negative ρ_t will lead to the SB correlation being less negative than usual and perhaps even positive.

This intertemporal smoothing effect is amplified by the negative relationship between uncertainty shocks $(\epsilon_{v,t+1})$ and consumption growth shocks $(\epsilon_{c,t+1})$. Unconditionally, a positive uncertainty shock will lower stock valuation and bond yields, regardless of maturity. When ρ_t is positive, precautionary savings will further reduce the SB correlation, as positive uncertainty shocks are also likely to be associated with lower expected consumption growth. In contrast, when ρ_t is negative, this shock is more likely to increase expected future growth, which would ambiguously affect stock and bond prices. In this case, a positive uncertainty shock could even increase the stock/bond return correlation.

The first four panels of Figure 2 show how the SB correlation varies as a function of the state variable that represents persistence (ρ_t). Panels (a) and (c) show the relationship between the SB correlation and the ρ_t process for the consumption-only model, while Panels (b) and (d) present corresponding results for the full model. Each line of Panels (a) and (b) characterizes the relationship for different values of precautionary savings parameter ϱ_{ps} using one-year bond yields, while different lines in Panel (c) and (d) represent the relationship for different bond maturities. For comparison, each panel includes a flat line indicating the constant SB correlation obtained under the baseline model, in which all shocks are uncorrelated.

Overall, this figure confirms the negative relation between ρ_t and the SB correlations, which are slightly convex in ρ_t in all cases. In comparing the four panels, the value of precautionary savings parameter ϱ_{ps} and bond maturity has a relatively weak effect. Last, while low CGP values are associated with positive SB correlations in both models, positive SB correlations are rarer in the full model.

This relationship is also examined from the simulation results of Panel B of Table 2, which examines the "correlation of correlations." While the figure shows that the relation between ρ_t and the SB correlation is slightly nonlinear, the table also helps assess the goodness of fit of the linear projection of the SB correlation onto ρ_t . The relationship between ρ_t and the SB correlation is almost perfectly negatively related for both the consumption-only and full models, with correlations below -0.99. Thus, our model suggests that the SB correlation is a very good proxy for the less easily observed ρ_t process.

2.4. Stock market variance and the leverage effect

Time-varying CGP also has implications for stock market variance and the time-varying "leverage effect" in the stock market, which refers to the negative relationship between stock returns and their variance shocks. While robust, this relationship is nevertheless time-varying, as demonstrated by Pyun (2019).

First, high CGP will raise stock return variance. Stock returns depend positively on both current consumption and future expected consumption shocks. A positive correlation between these shocks, therefore, magnifies these risks, while a negative correlation reduces risk due to a hedging effect. This pattern is formalized with an analytical result, which we prove in the appendix, which is that

$$Var_{t}(R_{m,t+1}) = V_{m,v}\sigma_{t}^{2} + V_{m,p}\sigma_{t}^{2}\rho_{t},$$
(8)

where values for $V_{m,v}, V_{m,p} > 0$ are given in the appendix.

Second, higher CGP will strengthen the negative relationship between stock market returns and variance shocks. This is the case because variance shocks affect stock prices

through their relation to both current consumption and future consumption growth. The standard precautionary savings motive implies a stable negative relationship between current consumption and volatility. When CGP is positive, positive volatility shocks are also likely to be associated with a decrease in expected future consumption growth, which causes stock returns to react more to the same variance shocks. In contrast, when CGP is negative, stock returns will react less to variance shocks, thereby decreasing the magnitude of the leverage effect.

The last two panels of Figure 2 show the relationship between CGP and stock market leverage, defined as the correlation between market returns and their variance shocks. The results for the consumption-only model and our full specification are provided in Panels (e) and (f), respectively. The negative and constant flat lines shown in these figures indicate the values under the baseline model. The figure shows that stock market leverage is negatively related to CGP. Additionally, as the intuition above suggests, the leverage effect is sensitive to the value chosen for precautionary savings parameter ϱ_{ps} .

While the relationship is almost linear, with a correlation of below -0.95, as reported in Panel B of Table 2, it does not vary as much with CGP as with the SB correlation and is sensitive to precautionary savings parameter ϱ_{ps} . This result means that with a moderate level of measurement error, the stock return/volatility correlation is likely a much weaker empirical proxy for CGP relative to the SB correlation.

2.5. Conditional moments of consumption growth

The key assumption of our generalized model is the time-varying persistence of consumption growth shocks. In this section, we address how this assumption affects the conditional distribution of consumption growth for different values of ρ_t .

While more CGP should clearly increase the serial correlation in consumption growth, it is difficult to assess the strength of this effect analytically. We instead simulate 10 million months of data from our full model and compute approximate conditional moments by separating the simulated sample into narrow bins (e.g., [-0.05, 0), [0, 0.05), [0.05, 0.1), etc.) according to the value of ρ_t . We then compute several moments of interest using all the observations in each bin. The simulation applied is at a monthly frequency, but we aggregate consumption growth to the quarterly level by taking the sum of three consecutive realizations of monthly growth rates.

Panel (a) of Figure 3 quantifies the contemporaneous relationship between shocks to consumption growth and expected long-run growth. For example, our figure suggests that when ρ_t is at the first quartile (-0.29), a one standard deviation shock to x_t (+0.04%) implies a -0.36% change in quarterly consumption.

Panel (b) shows how the first-order serial correlation of quarterly consumption growth relates to CGP. Due to the presence of the LRR process, the serial correlation is positive even for very negative values of ρ_t , even in the absence of time aggregation, and rises with ρ_t .

 $^{^{7}}$ In simulating the CGP correlations, the correlations are outside the boundary of -1 and 1 less than 0.05% of the time. If these boundaries are reached, we set the value to -1 or 1.

Because we assume that the correlation between shocks to expected consumption growth and volatility is equal to $\rho_t \varrho_{ps}$, our model implies that this correlation will be more negative when CGP is high. Panel (c) of Figure 3 shows that the same relation holds in levels in addition to holding in shocks. The level of expected future consumption growth is more negatively related to the level of consumption variance when CGP is high, where we measure the relation by the slope coefficient of the regression of x_t on σ_t^2 . We examine levels in order to be consistent with our empirical analysis, which recognizes that first differences in observable proxies for x_t on σ_t^2 are likely to be dominated by measurement error.

2.6. Stock return predictability of bond yields

Most consumption-based asset pricing models with time-varying consumption volatility imply that bond yields should negatively predict future stock returns. This prediction results from the stock market risk premium increasing but bond yields decreasing in consumption volatility. These relationships imply a negative relationship between the equity risk premia and bond yields.

While several studies, starting with Fama and Schwert (1977), find a negative relation between stock returns and lagged bond yields, the negative relationship appears to be sample-dependent. Additionally, as evidenced by Welch and Goyal (2008) the statistical significance level is well below the levels of other predictors, such as the aggregate dividend yield.

The generalized model implies that the strength of this form of stock market return predictability depends on CGP. Bond yields are the inverse of the expected marginal utility of investors, which is closely related to the level of expected consumption growth. Meanwhile, the stock risk premium is higher when volatility is higher. Therefore, the negative relationship between bond yields and stock risk premia should be stronger when expected consumption growth is more negatively related to volatility,

Therefore, given the close negative relationship between the SB correlation and CGP, we expect the negative predictive relationship between future stock returns and bond yields to be stronger when the SB correlation is low. Using the simulations described above, we examine the correlation between the stock risk premia and bond yields. The analysis uses the exact formula for the market risk premium, derived in the appendix.

Panel (d) of Figure 3 shows that $\rho_t > -0.5$ implies a negative relationship between bond yields and the market risk premium, as implied by many other asset pricing models. But whereas other models imply that the degree of predictability is constant, our model suggests that it is highly time-varying. The figure implies that bond yields will be poor predictors of market returns when the SB correlation is high, but that predictability will increase as the SB correlation drops.

3. Data

Quarterly consumption data is obtained from the national income and product accounts (NIPA) provided by the Bureau of Economic Analysis. We measure consumption at the quarterly frequency as the sum of the real personal consumption expenditure on non-durables and services. We take the quantity index of NIPA Table 2.3.3 and divide it by the total population obtained from NIPA Table 7.1. Consumption growth is defined as the first log difference and is computed from 1962 to 2019.

To proxy expected consumption growth, we use data from the Survey of Professional Forecasters (SPF), obtained from the Federal Reserve of Philadelphia. The sample for the survey data begins in the third quarter of 1981. We use the four-quarter-ahead median forecast in real consumption expenditure. Because the surveys are conducted throughout each quarter, it is unclear whether the forecast from quarter t better represents the expectation at the end of quarter t or the end of quarter t-1. We assume the latter given that only quarter t-1's consumption would have been known when forecasts were made.

Bond yields are obtained from the Federal Reserve Bank of St. Louis website. The nominal one-year and ten-year yields are available from 1962 to 2019, while the ten-year TIPS data used to compute the ten-year real yield is only sufficiently liquid starting in 2003. Excess market returns and total market returns are from Ken French's data library.

In Section 2.2, the averages of real variables are computed by subtracting the average changes in the consumer price index, obtained from the Bureau of Labor Statistics, over the entire calibration period. To compute the standard deviation of real bond yields and the stock/bond return correlation, we make several assumptions. We assume that the relative fraction of the shocks to inflation to the nominal yields remains constant over the entire sample period. We also assume that inflation follows a unit root process. That is, the change in expected inflation equals the unexpected price change in the previous period. This assumption implies that nominal dividend is represented as the sum of real dividends and shocks to expected inflation (π) .

$$d_{t+1}^n = d_{t+1} + \pi_{t+1},$$

where d_{t+1}^n is the nominal dividend at time t+1. Finally, a zero nominal-real correlation is

assumed.

We first calculate the variance ratio (VR) of the inflation shocks (e.g., Duffee 2018a) using the TIPS data between 2003 – 2019. The variance of the real yields is computed by multiplying the variance of nominal yields by (1 - VR). Similarly, the real SB covariance $Cov(r_t, R_{m,t}^r)$ is computed as

$$Cov(\Delta r_t, R_{m,t}^r) = Cov(\Delta y_t - \Delta \pi_t, R_{m,t} - \Delta \pi_t) = Cov(\Delta y_t, R_{m,t}) - Var(\Delta \pi_t),$$

where y_t is the nominal bond yield, r_t is the real bond yield, $R_{m,t}$ is the nominal stock return, and the real return of a stock investment is $R_{m,t}^r$. The variance of the inflation shock $Var(\pi_t)$ is computed as $VR \times Var(y_t)$. The variance of real stock returns is $Var(R_{m,t}^r) = Var(R_{m,t}) - Var(\Delta \pi_t)$, which is quantitatively similar to the variance of nominal stock returns.

We measure macroeconomic uncertainty in three different ways. First, we use the 12-month macro uncertainty measure from Jurado, Ludvigson, and Ng (2015), obtained from Sydney Ludvigson's website is available from 1961 to 2019. These data are available monthly, and we convert to quarterly by choosing the last value of each quarter. Second, we use the monetary policy uncertainty from Baker, Bloom, and Davis (2016). This uncertainty index is estimated using textual analysis of newspaper articles and is substantially different from those estimated from macroeconomic aggregates. The data covers the period from 1985 to 2019 and can be downloaded at the authors' Economic Policy Uncertainty website. Third, we use the volatility estimate of expected consumption growth estimated using the long-run risk model of Schorfheide, Song, and Yaron (2018). That series, which was provided by the authors of the paper, is available from 1962 to 2014.

We also use several different measures of stock market volatility. The first is a monthly measure, the so-called "realized variance" computed as the squared daily excess market returns. The second volatility measure is the VXO index of the Chicago Board Options Exchange (CBOE). VXO is the predecessor of the VIX and measures the implied volatility of options on the S&P 100 Index (as opposed to the VIX, which is the model-free implied volatility of S&P 500 Index options). We choose it because it is available going back to 1986, while the VIX starts in 1990. Finally, we estimate measure of equity market volatility using the two-factor EGARCH model of Brandt and Jones (2006), which is closely related to the model of Engle and Lee (1999). Specifically, we use the long-run factor from the most general specification of Brandt and Jones, which we fit using daily market returns from 1950 to 2019. By focusing on the long-run factor, we are excluding volatility fluctuations with very low persistence, which we believe are less relevant for explaining macroeconomic dynamics at horizons of one quarter or more.

We also use several measures of wealth. The value weighted stock market index is from French data library, the value of assets is from Lettau and Ludvigson (2001), the Housing Price Index is the All-Transactions Housing Price Index of the U.S. Federal Housing Finance Agency, and the net worth is the net worth of Households and Nonprofit Organizations is from the Federal Reserve of St. Louis.

4. Empirical results

4.1. Empirical proxies for CGP

The direct measurement of time variation in the relationship between current and expected consumption growth shocks is hampered by the difficulties of measuring the latent expected consumption growth process (e.g., Schorfheide, Song, and Yaron 2018) and the relatively low frequency of consumption growth data. The direct measurement of CGP is likely to be particularly unsuccessful if it varies over time.

Therefore, we examine CGP using an indirect approach based on high-frequency asset price data. The stock/bond return correlation is estimated as the negative correlation between the first-order difference in nominal bond yields and stock returns. This estimate approximates the true SB correlation, as it ignores the effect of convexity, but it nevertheless should be highly accurate. As a baseline, we estimate the correlations with a rolling basis using daily observations for the past 365 calendar days. Since the SB correlation can be measured using different bond maturities, we compute several such correlation series. This paper reports the results of the one-year and ten-year constant maturity bonds, although using other maturities produces very similar results.

Given that inflation likely contaminates our measures of the SB correlation, we also calculate the SB correlation using real yields drawn from Treasury Inflation-Protected Securities (TIPS) prices. Real yields are the difference between the ten-year nominal yields and the 10-year break-even inflation rate. We refer to this correlation, also estimated using a one-year rolling window, as the real SB correlation. We use the sample from 2003 due to the

well-known illiquidity problem of the TIPS market affecting the early period (e.g., Dudley, Roush, and Ezer 2009, Gürkaynak, Sack, and Wright 2010, D'Amico, Kim, and Wei 2018).

4.2. Serial correlation of consumption growth

A direct implication of the model is that the persistence of consumption growth shocks should be reflected in the level of the SB correlation. Establishing this relationship is critical because it justifies using the SB correlation as an empirical proxy for the latent CGP process.

We test the relationship by examining the correlation between current and future consumption growth at different horizons, where our model predicts that serial correlation will be larger in periods when the SB correlation is more negative. In interpreting these results, we note that first-order autocorrelations obtained from consumption growth data are likely high due to time-aggregation effects absent from our theoretical model. As shown by Breeden, Gibbons, and Litzenberger (1989) and Heaton (1993), if investors make consumption decisions more frequently than the interval over which consumption is measured, first-order autocorrelation in growth rates may be as high as 0.25 in quarterly data even if higher frequency changes are unpredictable. However, serial correlations at longer lags should be immune to this effect.

We first estimate a predictive regression of quarterly consumption growth on its own lag. We then test whether this relationship is stronger during high or low SB correlation periods by adding an interaction term. The regression we estimate is

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{t+k}$$
(9)

for k = 1, 2, 3, 4, where Δc_t is quarterly consumption growth and $\hat{\rho}_{SB,t}$ is one of the SB

correlation series described previously. If, as implied by our model, the serial correlation is stronger for periods in which the SB correlation is negative, we should see a negative slope on the interaction term ($\alpha_2 < 0$).

Table 3 summarizes the results of these regressions. Panel A of the table first shows the simple regressions in which the only explanatory variable is a single lag of consumption growth. We observe significant positive autocorrelations at up to a four-quarter horizon, and the first-order autocorrelation is much larger than the value implied by time aggregation. These results are consistent with the base assumption of long-run risk in consumption growth, and the results are comparable to values reported by previous studies (e.g., Savov 2011).

However, our primary interest is to test the sign and the significance of the interactive coefficient, α_2 , for multiple horizons. The results of this regression are summarized in Panel B. The regression results in the first two columns show, consistent with the model's predictions, that the first-order serial correlation increases when the SB correlation is more negative. The interaction coefficients are negative and statistically significant for both one-year and tenyear SB correlations. Quantitatively, a 0.1 increase in the SB correlation leads to a 0.03 – 0.04 decrease in the first-order serial correlation. We then increase the forecast horizon by replacing the one-quarter-ahead dependent variable with one that is between two and four quarters ahead. These results, which are reported in the last six columns in Panel B, are consistent.

Panel C of Table 3 presents an alternative test of the time-varying relationship between shocks to the long-run consumption growth forecast $(\Delta \hat{x}_t)$ and current consumption growth (Δc_t) . Given the negative relation between CGP and the SB correlation, the model predicts that the α_2' coefficient in the regression

$$\Delta \hat{x}_t = \alpha_0' + \alpha_1' \Delta c_t + \alpha_2' \hat{\rho}_{SB,t} \times \Delta c_{1,t} + \alpha_3' \hat{\rho}_{SB,t} + \epsilon_{2,t}, \tag{10}$$

will be negative. Using SPF forecasts as a proxy for the x process, the regression results show that the relationship between current consumption growth and expectations is positive when the SB correlation is more negative, confirming our theoretical prediction. Similar results are obtained for SB correlations based on the one- and ten-year yields.

A potential concern is that a significant fraction of the variation in nominal yields may be driven by inflation, which falls outside the scope of our model. Whether nominal yields change more due to changes in expected inflation or real yields is controversial. For example, Fama (1975) finds a strong relationship between the nominal interest rate and future inflation for the pre-1970 sample, claiming that the real interest rate for the sample is close to constant. More recently, Ang, Bekaert, and Wei (2008) also find that most of the monthly variation in nominal interest rates results from fluctuations in expected inflation.

However, Mishkin (1992) argues that a close relationship between interest rate and inflation observed in an earlier sample is likely due to a missing stochastic trend. Barr and Campbell (1997) also find that, over short horizons, real interest rates are highly timevarying. Using more recent data, Duffee (2018a) finds that most of the variation in the nominal term structure is either due to the term premium or real interest rates.

To study whether the SB correlation captures information about the correlation between stock returns and changes in the inflation rate, we repeat the previous analysis using yields from 10-year TIPS to calculate the real SB correlation and the change in real yields. These results are shown in Table 4.

In Panel A, we estimate regression (9) by replacing the SB correlation with the real SB correlation. While no time variation in consumption persistence is found at the one-quarter horizon, higher-order correlations do appear to be significantly lower when the real SB correlation is higher. This finding is again consistent with a negative relation between the SB correlation and CGP.

Panel B shows, however, that the correlation between stock returns and changes in breakeven inflation is generally uninformative of future consumption and is not significantly related to consumption persistence. If anything, a higher stock/inflation correlation is associated with more persistence in consumption growth, which is inconsistent with the hypothesis that inflation effects are responsible for the results given in Table 3. This again supports the conclusion that nominal SB correlations are informative in this setting because they are correlated with the corresponding correlation based on real yields.

Overall, the results of the regressions given in Panels A and B suggest that the stock/bond correlation is related to consumption persistence mainly because of how real rate changes affect that correlation. However, to further rule out inflation as the reason for our results based on nominal yields, we perform an additional analysis.

Specifically, in addition to the regression (10) using the stock/real yield (SR) correlation, we also examine a specification in which the stock/bond correlation is replaced with the correlation between stock returns and changes in breakeven inflation (the $S\pi$ correlation). If variation in the SB correlation estimated with nominal yields is mainly driven by inflation, then isolating the inflation component of yield changes should result in a stronger relation with consumption persistence. However, the result in Panel C suggests that the relationship

between consumption growth and long-run expectations in the growth are more positively related as the SR correlation becomes more negative. We find no relationship between the two using $S\pi$ correlation.

While the regressions shown in Tables 3 and 4 examine non-overlapping growth rates at different horizons, Figure 4 examines consumption growth autocorrelation using overlapping longer-horizon growth rates. These regressions are identical to equation (9), except that the dependent variable is the average consumption growth rate from quarter t + 1 to quarter t + k. Each panel plots the coefficient on the interaction term (α_2) for different horizons (k), as well as 68%, 90%, and 95% confidence intervals, where the panels differ with respect to the SB correlation series used and the sample period. While we present results only for the ten-year nominal SB correlation, corresponding results based on 1-year nominal yields are very similar.

Panel (a) of Figure 4 reports the results obtained using the full sample period at horizons of one to ten quarters. The graph shows that predictability is observed even over very long horizons, consistent with the premise that the SB correlation is associated with the correlation between long-run growth and current consumption growth.

Panel (b) shows the result, based on nominal yields, for the shorter sample for which TIPS data are available, while Panel (c) shows the corresponding results for the same sample period using TIPS yields. While the results based on TIPS are somewhat stronger, both graphs indicate more long-term persistence in consumption growth in high correlation environments.

The final panel of Figure 4 examines the role of stock/inflation correlation at longer horizons. As in Table 4, a higher stock/inflation correlation increases the persistence of

consumption growth, and over longer horizons, this effect becomes statistically significant. While the interpretation of this result is difficult given that inflation falls outside the scope of our model, the results reinforce the conclusion that the SB correlation is not related to consumption persistence because of that correlation's dependence on inflation.

4.3. The stock/bond correlation and stock market leverage

Given the well-known theoretical motivation and empirical observation that consumption decreases when there is more uncertainty, the model provided in the previous section also shows that CGP is associated with the stock market leverage effect, the negative relationship between stock market returns and their variance shocks. In this section, we test the hypothesis that a negative SB correlation is associated with a more substantial leverage effect, which we estimate as the correlation between stock returns and variance shocks (the "SV correlation"). We use daily stock market returns and changes in the square of the VXO index, obtained from the Chicago Board Options Exchange, to estimate the correlation.

Testing the relationship between CGP and the leverage effect is challenging, as both must be estimated from rolling samples. If the sample period is too long, artificial persistence in both series may lead to the spurious regression problem described by Granger and Newbold (1974). Therefore, we estimate the SB and the SV correlations using one-month non-overlapping samples.⁸

We evaluate the relationship between the two series using time-series regression. To control for the flight-to-quality phenomenon (e.g., Baele, Bekaert, and Inghelbrecht 2010),

 $^{^8}$ Pyun (2019) shows that stock market leverage betas can be estimated with reasonable accuracy using just one month of daily data.

which may induce a negative relation between the SB correlation and the level of volatility, some regressions also include the lagged market variance, proxied by the square of the VXO index.

Table 5 summarizes the results of regressions in which monthly SB correlations are regressed on monthly SV correlations, with or without a market variance control. The results for the sample based on nominal yields start in 1986, which is when the VXO index becomes available, while the results for real yields start in 2003. Overall, the table shows a strong positive relationship between SB correlations and SV correlations, particularly for the nominal correlations, which are available for a longer sample period. In all cases, the significance remains even after controlling for market volatility and is consistent for different measures of SB correlations.

Because the previous analysis requires using the VXO index, we consider an alternative approach that allows us to use a longer sample period, at least for the SB correlations based on nominal yields. Specifically, we embed a time-varying leverage effect into a two-component exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model:

$$\ln h_t - \ln h_{t-1} = \kappa_h (\ln q_{t-1} - \ln h_{t-1}) + \phi_h X_{t-1} + \delta_h \frac{R_{t-1}}{h_{t-1}}$$

$$\ln q_t - \ln q_{t-1} = \kappa_h (\theta - \ln q_{t-1}) + \phi_q X_{t-1} + \delta_q \frac{R_{t-1}}{h_{t-1}} + \delta_{qc} \hat{\rho}_{SB,t-1} \frac{R_{t-1}}{h_{t-1}}$$

$$X_t = \left(\left| \frac{R_{t-1}}{h_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) / \sqrt{1 - \frac{2}{\pi}},$$

where $R_t \sim N(0, h_t^2)$. In this model, q_t represents the persistent component of stock market volatility, around which the short-term volatility h_t mean reverts. X_t can be interpreted as a volatility surprise and will be mean zero with unit variance if returns are Gaussian. $\hat{\rho}_{SB,t-1}$ is

an estimate of the stock/bond correlation based on a one-year rolling window of daily data, where we use the 1-year, 10-year, or real 10-year bond yield.

Aside from the introduction of a time-varying leverage effect, the model is identical to the one given in Brandt and Jones (2006), which combines the EGARCH specification of Nelson (1991) with the two-component structure of Engle and Lee (1999). As in Nelson (1991), log volatility increases with high past absolute returns, and a leverage effect is induced by negative values for δ_h and δ_q .

The novel feature of the current model is the δ_{qc} term, which allows the strength of the leverage effect to vary with the stock/bond correlation. We include this term only in the more persistent q_t process for parsimony, because the persistence in h_t prices is quite low, with a half-life of less than two weeks, making it less likely to be associated with the macro-level forces we are concerned with.

Table 6 summarizes the maximum likelihood estimates of the model for different SB correlations, where asymptotic t-statistics are reported in parenthesis. The first two lines show the baseline model estimates, where we do not allow the leverage effect to vary with the SB correlation. Consistent with previous literature, we find both δ_h and δ_q to be negative and highly statistically significant, indicating a strong unconditional leverage effect.

Other models shown in the table allow the leverage effect to depend on the SB correlation. In all three cases, we find a positive estimate of δ_{qc} that is highly statistically significant, providing strong support for the model's prediction that the stock market leverage effect is stronger when the SB correlation is negative.

4.4. Stock market variance

As shown in (8), our model implies that market variance is related to consumption growth uncertainty and the correlation between current and future expected consumption growth shocks. The latter effect, which relates to the stock/bond return correlation, suggests a link to the "flight-to-quality" hypothesis.

There are two key differences between our predictions and the flight-to-quality hypothesis. The first difference concerns the direction of causality. The flight-to-quality hypothesis suggests that the SB correlation becomes negative in high volatility states as investors shift their portfolios from more risky stocks to safer bonds in response to heightened uncertainty. In contrast, our model implies that the stock market becomes more volatile when consumption growth shocks become more persistent.

Second, flight-to-quality implies a simple negative relationship between stock market variance and the SB correlation. In contrast, in our model, the SB correlation should have an interactive effect with macroeconomic uncertainty. In other words, stock market variance will respond more to economic uncertainty when consumption growth is more persistent. Additionally, the relationship between SB correlation and stock market variance will become more negative when consumption shocks are more persistent.

In this section, we test whether the relationship between SB correlation and stock market variance fluctuates in the manner predicted by the CGP channel, or if the traditional flightto-quality effect is more consistent with those fluctuations. As described in more detail in Section 3, we use several macroeconomic uncertainty measures to proxy for consumption growth variance.

The main predictive regression we study is

$$RV_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 UNC_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1}, \tag{11}$$

where RV_{t+1} is the realized market variance (estimated as the sum of daily squared returns estimated over month t+1) and $\hat{\rho}_{SB,t}$ is a one-year rolling SB correlation. UNC_t is either the macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty measure of Baker, Bloom, and Davis (2016), or the consumption volatility estimated by Schorfheide, Song, and Yaron (2018).

The first regression in each panel of Table 7 shows estimates of the main specification. Overall, the results are consistent with our model. We find strong statistical significance for the negative β_2 across all three uncertainty measures when using the one-year, ten-year, or real SB correlations to proxy for CGP. These results suggest that economic uncertainty predicts stock market variance with a higher slope when SB correlations are negative.

We consider two additional specifications, in which we control the lagged realized variance of the stock market and the SB correlation. The control for lagged realized variance is to show that the interactive term predicts the *innovation* in stock market variance, rather than just explaining its current level. Adding the SB correlation control is to show that the interactive predictive relationship offers incremental predictive power relative to that implied by the flight-to-quality hypothesis. In these additional specifications, all β_2 coefficients are negative, and most are statistically significant, confirming that time-varying CGP drives the negative relationship between stock market variance and SB correlation.

4.5. Expected consumption growth and uncertainty

Several recent studies (e.g., Nakamura, Sergeyev, and Steinsson 2017, Bollerslev, Xu, and Zhou 2015) document the unconditional negative relationship between economic uncertainty and future expected consumption growth. Our model implies that this relationship varies with CGP. Because shocks to expected consumption growth and consumption volatility are difficult to measure, we instead examine the relationship in levels. This is justified by the calibrated model, which we demonstrated in Panel (c) of Figure 3 implies a higher correlation between x_t and σ_t when ρ_t is low. Equivalently, the correlation between x_t and σ_t will be higher when the SB correlation is high.

Because Δc_{t+1} is equal to x_t in expectation, we test this hypothesis using full and restricted versions of the predictive regression

$$\Delta c_{t+1} = \beta_0 + \beta_1 U N C_t + \beta_2 \hat{\rho}_{SB,t} \times U N C_t + \beta_3 \hat{\rho}_{SB,t} + \beta_4 \Delta c_t + \epsilon_{t+1}, \tag{12}$$

where $\hat{\rho}_{SB,t}$ is one of the SB correlation estimates. UNC_t is a measure of uncertainty, which is either one of the three measures used in Table 7 or the long-run volatility (q_t) estimated from the two-component model of Brandt and Jones (2006). If the SB correlation is negatively related to CGP, we should obtain positive estimates for the β_2 parameter.

Table 8 summarizes the results of these regressions, where each panel uses a different measure of SB correlation. We include regressions with and without controls for SB correlation and lagged consumption growth. Overall, the table provides reasonably strong support for our hypothesis. Using the one-year or the ten-year SB correlation in Panel A and B, we find a positive β_2 in every regression. It is statistically significant (at the 10 percent

level) in most cases. Panel C shows the result using the real SB correlation. The results are somewhat weaker, particularly if we control for the SB correlation, which is likely due to the shorter sample period and collinearity.

Since consumption growth is more persistent since around 1999, we examine the simple relationship between future consumption growth and macroeconomic uncertainty for the pre-1999 and 1999+ periods separately. Furthermore, the relationship between uncertainty and expected consumption growth implies that the relationship between macroeconomic uncertainty and future consumption growth should hold beyond the one-quarter horizon analyzed in the table. We therefore examine the simple predictive regression

$$\frac{1}{K} \sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} UNC_t + \epsilon_{t+K}$$

separately for the pre-1999 and 1999+ periods for values of K (measured in quarters) between one and eight.

Figure 5 shows estimates of the slope coefficients, along with 68%, 90%, and 95% confidence intervals. Overall, the plots show that the relationship between uncertainty and future consumption growth is more negative for the 1999+ sample period. This is particularly true when we use the monetary policy uncertainty of Baker, Bloom, and Davis (2016), for which slope coefficients are positive before 1999 and negative after. While less dramatic, the differences between the two sample periods are substantial for other uncertainty measures as well, either economically or statistically. And for both sample periods, there is a moderate tendency for slope coefficients to attenuate for longer horizon regressions.

The complementary Figure 6 reports the slope of the full interactive regression of (12), except that the dependent variable is replaced with a cumulative average over multiple

periods. The values plotted are the slope coefficients $\alpha_{3,K}$ of the regression

$$\frac{1}{K} \sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} UNC_t + \alpha_{2,K} \hat{\rho}_{SB,t} + \alpha_{3,K} \hat{\rho}_{SB,t} \times UNC_t + \epsilon_{t+K}$$

for different values of the interval (K). We again consider four measures of uncertainty, but each panel now includes three plots, each using a different SB correlation, and we use all available data for each set of results.

For all but one specification in Figure 6, we find a significant positive slope for the interactive variable for at least some horizons. Combined with earlier results in Table 8 and Figure 5, these results paint a consistent picture that the negative relationship between consumption growth and economic uncertainty is stronger when the SB correlation is negative or when CGP is positive, confirming a key prediction of our model.

4.6. Stock return predictability

A final implication of the model is the time-varying negative relationship between bond yields and future stock returns. A number of papers have studied Treasury bond yields as a stock market return predictor. Fama and Schwert (1977) estimate a simple predictive regression of future stock returns on lagged bond yields and find a negative slope, which they interpret as the result of stocks being inflation hedges. Breen, Glosten, and Jagannathan (1989) further confirm the economic significance of this predictability. More recently, Ang and Bekaert (2007) find that short-term Treasury yields and dividend yields jointly predict stock returns in many international markets. They argue that the yields represent a component of the discount rate used by investors to value equities. Campbell and Thompson (2008) also document statistically significant in-sample predictability, but Welch and Goyal

(2008) report weak in and out-of-sample performance.

While equation (6) suggests a negative relationship between discount rates and the SB correlation, we do not report results for regressions in which the market return is regressed directly on $\hat{\rho}_{SB,t}$. As shown by Kozak (2021), there is no clear evidence of any such predictive relation, though he argues that the negative relation between changes in the SB correlation and stock return represents indirect evidence. In contrast, the results we show here provide direct evidence of stock market predictability, but it is driven by the interaction between the SB correlation and bond yields rather than the SB correlation alone.

Our model suggests that the extent to which the bond yield predicts stock returns depends on CGP. Specifically, a higher CGP is associated with a more negative predictive slope between bond yields and future returns. Given the relation between CGP and the stock/bond correlation, we test this hypothesis in monthly regressions of the form

$$R_{S,t,t+\tau}^{e} = \beta_0 + \beta_1 y_t + \beta_2 y_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1}, \tag{13}$$

where $R_{S,t,t+\tau}^e$ is the τ -month excess market return, y_t is a bond yield, and $\hat{\rho}_{SB,t}$ is a SB correlation estimated from a one-year rolling window. Panels A and B of Table 9 consider regressions based on nominal yields, while Panel C examines real yields over the post-2003 subsample.

Panel A first considers the simple predictive regression in which leading stock returns are regressed on bond yields alone, separately for the pre- and post-1999 periods, using either the one-year and ten-year Treasury yield. Comparing the two subsamples, we see that the regression slope coefficient is uniformly negative and significant in the latter sample, with high R^2 s, but generally insignificant in the earlier period. These results are consistent with

consumption growth being more persistent during the 1999+ sample and the sign change in the SB correlation that occurred around that time.

The novel implication of our model is that the slope should be more negative when the SB correlation or stock market leverage is lower, implying $\beta_2 > 0$ in equation (13). The results, summarized in Panel B, show strong support for this prediction, with are positive and significant across all horizons for both yields. To understand the strength of the relation, consider the relation between one-month excess market returns and one-year Treasury yields that would hold if the SB correlation were equal to 0.4. In this case, the conditional slope coefficient would be a paltry -0.048 ($-0.295 + 0.616 \times 0.4$), implying that yields have essentially no predictive power for future returns. Similar conclusions hold for longer investment horizons as well. However, were the SB correlation instead equal to -0.5, the conditional slope coefficient would increase in magnitude to around -0.6. A one percentage point increase in the one-year Treasury yield would then be associated with a 0.6% decline in monthly stock returns and, following the same logic, a 1.7% decline in three-month returns, a 3.3% decline in six-month returns, and a 4.8% decline in 12-month returns. Economic magnitudes are similarly large when based on ten-year yields.

Panel C repeats these regressions using real yields and real SB correlation rather than nominal values. Overall, we see similar results, albeit with noticeably higher interaction coefficients. The coefficients are all statistically significant, echoing previous panels. A potential reason for the large interaction effects is the shorter sample period considered, in which the real SB correlation does not vary as much as it does over the full sample.

Many asset pricing models imply a negative relationship between bond yields and stock

risk premium, as high levels of uncertainty means lower bond yields and a higher risk premium. Therefore, it is puzzling that the empirical relationship is so weak. Our results show that the predictive relationship is stronger than it appears, but only during periods when proxies indicate that CGP is high.

4.7. The recent shift in consumption growth persistence

As discussed in the introduction, rolling 1-year estimates of the SB correlation turned negative around 1999 after more than three decades of positive values. Our model and empirical results suggest that an increase in the persistence of consumption growth was largely responsible for this shift. However, what then could be the reason for the significant increase in CGP observed around this time, as shown in Figure 1(b)?

Production-based models, such as Kaltenbrunner and Lochstoer (2010), suggest that an increase in the magnitude of permanent productivity shocks will cause CGP to rise when consumers face adjustment costs. More volatile persistent shocks will also increase the variability in valuation ratios to a much greater extent than more volatile transient shocks. Consistent with this, the period starting in 1999 is notable for its inclusion of several major asset market "bubbles" and crashes (e.g., the "dot-com" crash, the real estate boom, and the Great Financial Crisis).

Regardless of the reason for such fluctuations, any exogenous change in asset values may be expected to produce some level of consumption persistence. While this claim cannot be demonstrated within our model, which features an exogenous consumption process, it is intuitive. According to the permanent income hypothesis, a positive wealth shock will raise the consumption level in perpetuity. However, if agents face adjustment costs, this higher level will not be reached immediately. Instead, they will experience a sequence of consecutive positive growth rates as consumption rises to its new steady state level. If later wealth falls, a sequence of negative consumption growth rates will result. In a period with multiple booms and busts, consumption is constantly trending towards some target value, but that target is moving and is sometimes higher and sometimes lower than current consumption, raising CGP.

In this section, we present some evidence, which we view as suggestive, that is consistent with the greater consumption persistence of this period being driven by fluctuating asset values. In particular, we show that consumption growth rates have become more responsive to past asset returns since 1999. These results are in line with recent papers by Laibson and Mollerstrom (2010), Mian and Sufi (2011), and Chen, Michaux, and Roussanov (2020), which show a stronger tendency for consumption to be driven by fluctuating asset valuations over this period. Our new finding is that asset returns affect consumption growth at longer horizons than documented previously, particularly in the latter period.

We demonstrate the changing relation between asset returns and consumption growth by computing the predictive correlation

$$\operatorname{Corr}\left(R_{t}, \Delta c_{t+k}\right)$$

for different horizons k and using returns R_t on different wealth proxies, as defined in Section 3. We analyze horizons of one to 12 quarters and compute the correlations separately for the period before 1999 and starting in 1999.

The results, shown in Figure 7, suggest a major shift in the predictive relationship,

particularly at longer horizons. Prior to 1999, the predictive relationship between asset returns and future consumption growth was relatively weak. Statistical significance, which is indicated by the estimated correlation exceeding the corresponding dashed line, is observed in some cases, but mainly at short horizons. For the sample starting in 1999, correlations are, in many cases, twice as large, if not more, and highly significant even at multi-year lags. The largest shift occurs in response to housing returns, to which consumption responded little before 1999 and very significantly thereafter.

These results are not meant to be definitive, and a more careful analysis would likely require a further enhancement of the theoretical framework that we adopt here, but they suggest a mechanism that may underlie the changes in CGP and related changes in the stock/bond correlation that we have documented.

5. Conclusion

While the consumption process examined by Bansal and Yaron (2004) is highly successful in replicating key moments of asset returns, its assumption of independent shocks is inconsistent both with macroeconomic theory and with consumption data. In particular, the model does not account for the relationship between shocks to current consumption growth and expected future consumption growth, which we term consumption growth persistence, or CGP. In theory, this relationship may be positive or negative, depending on whether permanent or transient income or productivity shocks are more prevalent. The model also does not account for the negative correlation between consumption growth and consumption volatility shocks, which likely arises from the precautionary savings motive.

Because of these assumptions, the model cannot match several well-documented features of financial markets. Most significantly, the correlation between stocks and bonds is highly time-varying in the data and appears to vary with the level of stock market volatility. These effects are absent in the model of Bansal and Yaron, which features a constant stock/bond correlation.

We propose a model that allows for a significantly more realistic dependence structure. Shocks to current and expected future consumption growth are stochastically correlated, which we view as a reduced form approach to modeling the relative importance of transitory and permanent shocks. Shocks to current consumption and consumption growth are negatively correlated at a fixed value, which maintains parsimony and reflects the likely importance of the precautionary savings motive.

The model implies that the correlation between stock and bond returns is decreasing in CGP. The same applies to the stock market leverage effect. Empirically, we see that consumption growth tends to become more serially correlated in periods of more negative stock/bond correlations. This result provides evidence of time variation in CGP and also links CGP to correlations that are readily estimable from high-frequency asset price data. We also see strong evidence that the SB correlation is positively related to the stock market leverage effect, which is implied by our model and a novel finding of this paper.

Our model also predicts the negative relation between stock market volatility and the SB correlation observed in prior studies, such as Connolly, Stivers, and Sun (2005) and Baele, Bekaert, and Inghelbrecht (2010). This is the case because high consumption persistence makes cash flows and discount rates negatively correlated, which amplifies the effects of

these shocks. Empirically, we find strong evidence for this relation.

We also find evidence of a time-varying relation between current uncertainty and future consumption growth. Nakamura, Sergeyev, and Steinsson (2017) show that this relation is generally negative, particularly during economic contractions. We confirm our model's implication that the correlation should be more negative when CGP is high or, equivalently, when the SB correlation is negative.

Finally, the model implies that the slope coefficient of the predictive relationship between current bond yields and future stock returns also varies as a function of CGP. Using our CGP proxies, we confirm this prediction from the data. Stock returns are strongly related to lagged bond yields, but only in environments where the SB correlation is negative. We also show that the source of this predictability is the real yield rather than the inflation component.

Combined with the observation that consumption growth persistence has increased markedly since 1999, our model provides a new explanation for the dramatic downward shift in the stock/bond correlation occurring around this time. Additional evidence suggests that the increase in persistence may be the result of a greater role of asset valuations in driving long-run consumption growth rates. While a more detailed analysis of changes in the response of consumption to asset values is beyond the scope of this study, we believe that it is an intriguing result that merits further attention.

References

- Ang, Andrew, and Geert Bekaert, 2007, Stock Return Predictability: Is it There?, Review of Financial Studies 20, 651–707.
- Ang, Andrew, Geert Bekaert, and Min Wei, 2008, The Term Structure of Real Rates and Expected Inflation, *The Journal of Finance* 63, 797–849.
- Baele, Lieven, Geert Bekaert, and Koen Inghelbrecht, 2010, The Determinants of Stock and Bond Return Comovements, *Review of Financial Studies* 23, 2374–2428.
- Baker, Scott R., Nicholas Bloom, and Steven J. Davis, 2016, Measuring Economic Policy Uncertainty, *The Quarterly Journal of Economics* 131, 1593–1636.
- Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An Empirical Evaluation of the Long-Run Risks Model for Asset Prices, *Critical Finance Review* 1, 183–221.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *The Journal of Finance* 59, 1481–1509.
- Barr, David G., and John Y. Campbell, 1997, Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-linked Government Bond Prices, *Journal of Monetary Economics* 39, 361–383.
- Basu, Susanto, and Brent Bundick, 2017, Uncertainty Shocks in a Model of Effective Demand, *Econometrica* 85, 937–958.
- Beveridge, Stephen, and Charles R. Nelson, 1981, A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle', *Journal of Monetary Economics* 7, 151–174.

- Bollerslev, Tim, Lai Xu, and Hao Zhou, 2015, Stock Return and Cash Flow Predictability: The Role of Volatility Risk, *Journal of Econometrics* 187, 458–471.
- Brandt, Michael W., and Christopher S. Jones, 2006, Volatility Forecasting With Range-Based EGARCH Models, *Journal of Business and Economic Statistics* 24, 470–486.
- Breeden, Douglas T., Michael R. Gibbons, and Robert H. Litzenberger, 1989, Empirical Test of the Consumption-Oriented CAPM, *The Journal of Finance* 44, 231.
- Breen, William, Lawrence R. Glosten, and Ravi Jagannathan, 1989, Economic Significance of Predictable Variations in Stock Index Returns, *The Journal of Finance* 44, 1177.
- Campbell, John, and Angus Deaton, 1989, Why is Consumption So Smooth?, The Review of Economic Studies 56, 357–373.
- Campbell, John Y., Carolin Pflueger, and Luis M. Viceira, 2020, Macroeconomic Drivers of Bond and Equity Risks, *Journal of Political Economy* 128, 3148–3185.
- Campbell, John Y., Adi Sunderam, and Luis M. Viceira, 2017, Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds, *Critical Finance Review* 6, 263–301.
- Campbell, John Y., and Samuel B. Thompson, 2008, Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?, Review of Financial Studies 21, 1509–1531.
- Carroll, Christopher D., 1997, Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis, *The Quarterly Journal of Economics* 112, 1–55.
- Carroll, Christopher D., and Andrew A. Samwick, 1998, How Important Is Precautionary Saving?, Review of Economics and Statistics 80, 410–419.

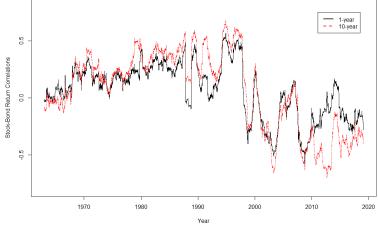
- Chen, Hui, Michael Michaux, and Nikolai Roussanov, 2020, Houses as ATMs: Mortgage Refinancing and Macroeconomic Uncertainty, *The Journal of Finance* 75, 323–375.
- Chernov, Mikhal, Lars A. Lochstoer, and Dongho Song, 2021, The Real Explanation of Nominal Bond-Stock Puzzle, *Unpublished working paper*.
- Clark, Peter K., 1987, The Cyclical Component of U. S. Economic Activity, *The Quarterly Journal of Economics* 102, 797–814.
- Connolly, Robert, Chris Stivers, and Licheng Sun, 2005, Stock Market Uncertainty and the Stock-Bond Return Relation, *Journal of Financial and Quantitative Analysis* 40, 161–194.
- D'Amico, Stefania, Don H. Kim, and Min Wei, 2018, Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices, *Journal of Financial and Quantitative Analysis* 53, 395–436.
- David, Alexander, and Pietro Veronesi, 2013, What Ties Return Volatilities to Price Valuations and Fundamentals?, *Journal of Political Economy* 121, 682–746.
- Dudley, William, Jennifer Roush, and Michelle Steinberg Ezer, 2009, The Case for TIPS: An Examination of the Costs and Benefits, Working paper, Federal Reserve Economic Policy Review New York.
- Duffee, Gregory R., 2018a, Expected Inflation and Other Determinants of Treasury Yields, *The Journal of Finance* 73, 2139–2180.
- Duffee, Gregory R, 2018b, Expected inflation, real rates, and stock-bond comovement, *Unpublished working paper*.

- Engle, Robert F, and Gary G J Lee, 1999, A Permanent and Transitory Component Model of Stock Return Volatility, in *Cointegration Causality and Forecasting A Festschrift in Honor of Clive WJ Granger* (Oxford University Press., New York).
- Epstein, Larry G., and Stanley E. Zin, 1991, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis, *Journal of Political Economy* 99, 263–286.
- Fama, Eugene F., 1975, Short-Term Interest Rates as Predictors of Inflation, American Economic Review 65, 269–282.
- Fama, Eugene F., and G.William Schwert, 1977, Asset returns and Inflation, *Journal of Financial Economics* 5, 115–146.
- Friedman, Milton, 1957, The Permanent Income Hypothesis, in *Theory of the Consumption Function* (Princeton University Press,).
- Granger, C.W.J., and P. Newbold, 1974, Spurious Regressions in Econometrics, *Journal of Econometrics* 2, 111–120.
- Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2010, The TIPS Yield Curve and Inflation Compensation, *American Economic Journal: Macroeconomics* 2, 70–92.
- Hall, Robert E., and Frederic S. Mishkin, 1982, The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households, *Econometrica* 50, 461–481.
- Hasseltoft, Henrik, 2012, Stocks, Bonds, and Long-Run Consumption Risks, *Journal of Financial and Quantitative Analysis* 47, 309–332.
- Heaton, John, 1993, The Interaction Between Time-Nonseparable Preferences and Time Aggregation, *Econometrica* 61, 353–385.

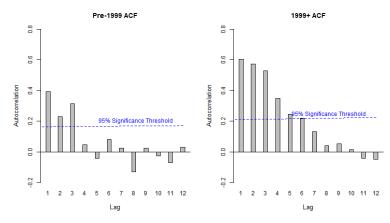
- Ilmanen, Antti, 2003, Stock-Bond Correlations, The Journal of Fixed Income 13, 55–66.
- Jurado, Kyle, Sydney C. Ludvigson, and Serena Ng, 2015, Measuring Uncertainty, American Economic Review 105, 1177–1216.
- Kaltenbrunner, Georg, and Lars A. Lochstoer, 2010, Long-Run Risk through Consumption Smoothing, *Review of Financial Studies* 23, 3190–3224.
- Kozak, Serhiy, 2021, Dynamics of bond and stock returns, Journal of Monetary Economics.
- Laibson, David, and Johanna Mollerstrom, 2010, Capital Flows, Consumption Booms and Asset Bubbles: A Behavioural Alternative to the Savings Glut Hypothesis, *The Economic Journal* 120, 354–374.
- Lettau, Martin, and Sydney Ludvigson, 2001, Consumption, Aggregate Wealth, and Expected Stock Returns, *The Journal of Finance* 56, 815–849.
- Mian, Atif, and Amir Sufi, 2011, House Prices, Home EquityBased Borrowing, and the US Household Leverage Crisis, *American Economic Review* 101, 2132–2156.
- Mishkin, Frederic S., 1992, Is the Fisher effect for real?, *Journal of Monetary Economics* 30, 195–215.
- Nakamura, Emi, Dmitriy Sergeyev, and Jón Steinsson, 2017, Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence, American Economic Journal: Macroeconomics 9, 1–39.
- Nelson, Daniel B., 1991, Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica* 59, 347.
- Parker, Jonathan A., and Bruce Preston, 2005, Precautionary Saving and Consumption Fluctuations, *American Economic Review* 95, 1119–1143.

- Pástor, Luboš, and Robert F. Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, *Journal of Political Economy* 111, 642–685.
- Pyun, Sungjune, 2019, Variance Risk in Aggregate Stock Returns and Time-varying Return Predictability, *Journal of Financial Economics* 132, 150–174.
- Savov, Alexi, 2011, Asset Pricing with Garbage, The Journal of Finance 66, 177–201.
- Schorfheide, Frank, Dongho Song, and Amir Yaron, 2018, Identifying Long-Run Risks: A Bayesian Mixed-Frequency Approach, *Econometrica* 86, 617–654.
- Song, Dongho, 2017, Bond Market Exposures to Macroeconomic and Monetary Policy Risks,

 The Review of Financial Studies 30, 2761–2817.
- Swanson, Eric, 2019, A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt, *Unpublished working paper*.
- Watson, Mark W., 1986, Univariate Detrending Methods with Stochastic Trends, *Journal of Monetary Economics* 18, 49–75.
- Welch, Ivo, and Amit Goyal, 2008, A Comprehensive Look at The Empirical Performance of Equity Premium Prediction, *Review of Financial Studies* 21, 1455–1508.



(a) The stock/bond correlation over time



(b) Autocorrelation plot of consumption growth

Figure 1. Pre-1999 vs. 1999+ Comparison

This figure shows the time-variation in the stock/bond return (SB) correlation (Panel a) and the autocorrelations in consumption growth (Panel b) in the pre-1999 and 1999+ sample periods. The SB correlation is estimated using the first difference in daily one-year or ten-year bond yields over a rolling window of twelve months.

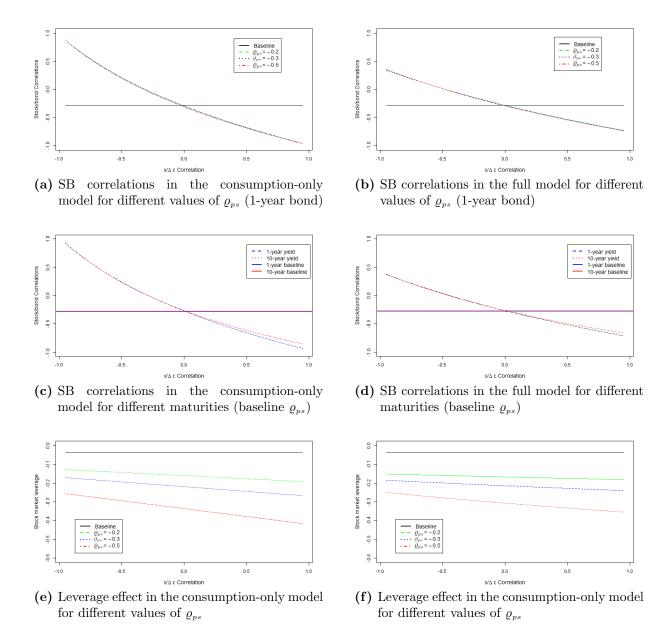


Figure 2. Consumption Persistence and Model-based Correlations

This figure shows the relationships between CGP and the stock/bond return correlation or leverage effect for different values of ϱ_{ps} and, for the SB correlation, different bond maturities. Panels a and b show how the relation between CGP and the SB correlation is affected by ϱ_{ps} , which measures the strength of the precautionary savings effect. Panels c and d show how the relation between CGP and the SB correlation differs across maturities. Panels e and f examine the relation between CGP and the stock market leverage effect, which is the correlation between stock returns and volatility shocks, for different values of ϱ_{ps} . Panels on the left side show results for the consumption-only model, while panels on the right are based on the full model. For comparison, each plot includes a solid line depicting the fixed value of each correlation under the baseline model.

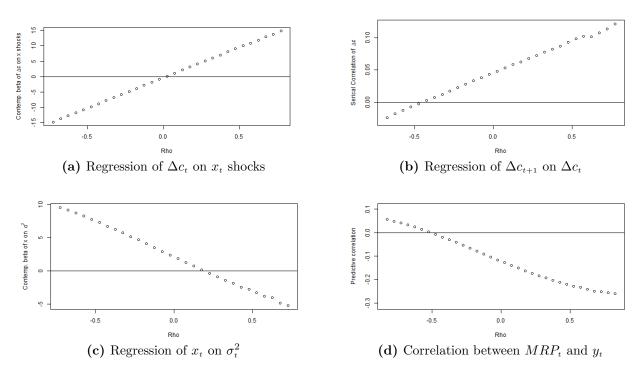


Figure 3. Simulation-based Regression Betas and Correlations Conditional on CGP

This figure describes the relationship between CGP and the conditional slope coefficients of various univariate relations. MRP_t and y_t denotes the market risk premium and one-period bond yield at time t.

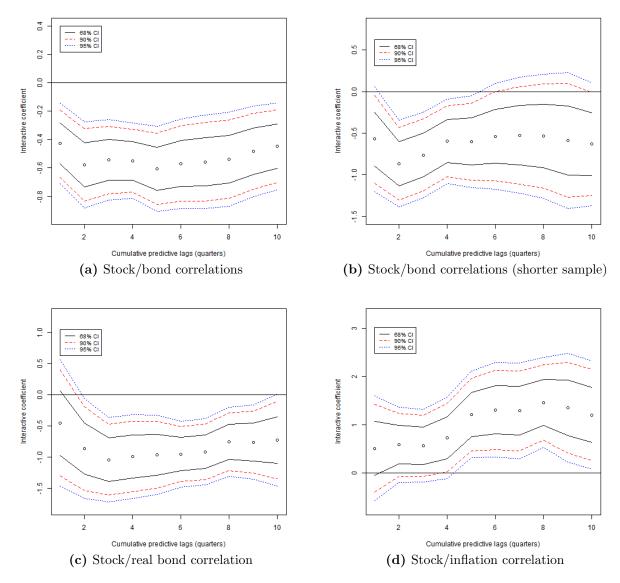


Figure 4. Interactive Beta of Consumption Growth Regressions For Multiple Lags This figure plots the slope estimates $(\hat{\alpha}_{3,k})$ of the interactive regressions

$$\sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} \Delta c_{t} + \alpha_{2,K} \hat{\rho}_{SB,t} + \alpha_{3,K} \hat{\rho}_{SB,t} \times \Delta c_{t} + \epsilon_{t+K},$$

for different values of the interval (K), where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation. The lines show the 68%, 90%, and 95% confidence intervals computed using Newey-West standard errors using 12 lags. Panel a uses the full 1962-2019 sample period, while other panels use the 2003-2019 sample.

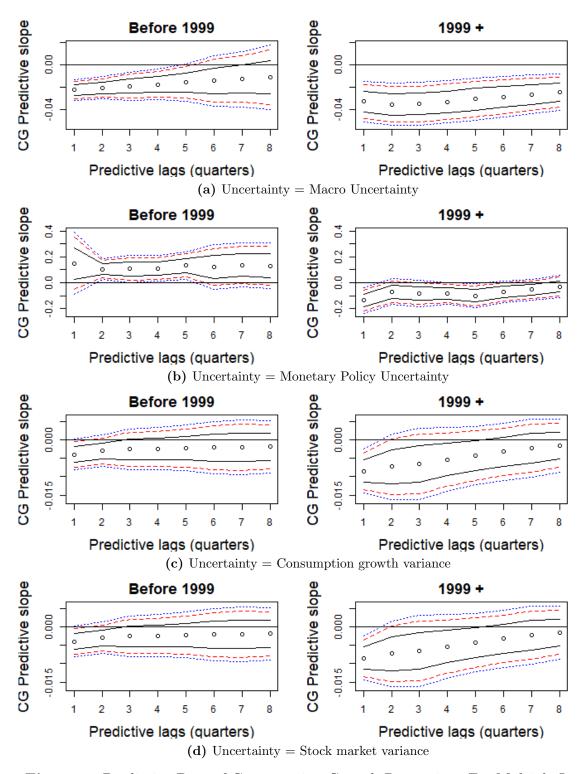


Figure 5. Predictive Beta of Consumption Growth Regressions For Multiple Lags This figure plots the slope estimates $(\hat{\alpha}_{1,K})$ of the interactive regressions

$$\frac{1}{K} \sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} UNC_t + \epsilon_{t+K},$$

for different interval values (K), where UNC_t is one of the uncertainty measures described in the main text. The figures for the pre-1999 and 1999+ periods are provided in separate plots. The lines show the 68%, 90%, and 95% confidence intervals, respectively, computed using Newey-West standard errors using 12 lags.

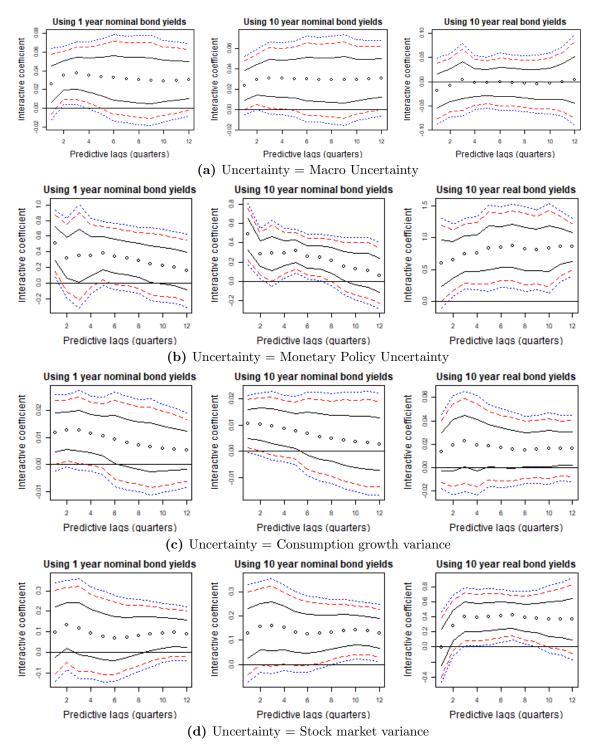


Figure 6. Interactive Beta of Consumption Growth Regressions For Multiple Lags This figure plots the slope estimates $(\hat{\alpha}_{3,K})$ of the interactive regressions

$$\frac{1}{K} \sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} UNC_t + \alpha_{2,K} \hat{\rho}_{SB,t} + \alpha_{3,K} \hat{\rho}_{SB,t} \times UNC_t + \epsilon_{t+K},$$

for different interval values (K), where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation, UNC_t is one of the uncertainty measures described in the main text. The lines show the 68%, 90%, and 95% confidence intervals computed using Newey-West standard errors using 12 lags, respectively.

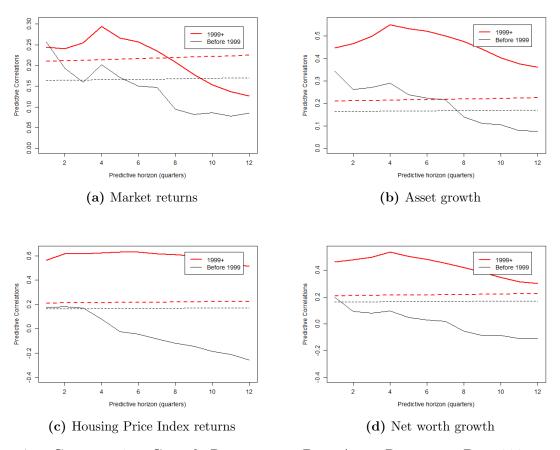


Figure 7. Consumption Growth Response to Past Asset Returns : Pre-1999 vs. 1999+ Comparison

This figure computes the predictive correlation $Cor(\Delta c_{t+k}, R_{.,t})$ for k = 1, ..., 12, where $R_{.,t}$ is market returns (Panel a), asset growth (Panel b), Housing Pricing Index returns (Panel c), and Net worth growth (Panel d) as defined in the data appendix.

Table 1 Model Calibration

This table summarizes the parameters that describe the representative investor's preferences and the consumption and dividend growth, volatility, and covariance processes used in the main specification, as well as asset pricing moments implied by these parameters. Panel A shows the values of the parameters, and Panel B shows the moments obtained by simulating the dynamics. y denotes a bond yield, $R_{TW/m}$ is the return on the total wealth (consumption only) or the market portfolio (full model), σ is the volatility of the wealth/market portfolio, ρ_{SB} denotes the stock/bond return correlation, and ρ_{SV} is the correlation between stock returns and stock market variance shocks. Values in Panel B are scaled to the annual level. We also simulate the stock returns, bond yields, and stock market variance using the consumption/dividend dynamics assumed in the model. Panel C displays model-implied simulated moments of 60-month rolling-window correlations of stock returns with one- or ten-year bond yields as well as with stock market variance. It then compares them with moments similarly estimated from the data.

Panel A. Parameters	Panel	A.	Parameters
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	Parameters	Parameters		Parameters	
$\frac{\text{Prefere}}{\gamma}$ ψ β	10 1.5 0.9986	$\begin{array}{c} \text{Consum} \\ \mu \\ p_x \\ \phi_x \end{array}$	0.0015 0.950 0.046	$\begin{array}{c} \underline{\text{Covari}} \\ \omega_0 \\ \omega_1 \\ \sigma_p \end{array}$	ance Parameters -7.84×10^{-6} 0.934 6.3×10^{-4}
$rac{ ext{Variance}}{s_0} \ s_1 \ \sigma_v$	ee Parameters 8.52×10^{-7} 0.986 2.6×10^{-4}	$\begin{array}{c} \underline{\text{Dividend}} \\ \mu_d \\ \phi_d \\ \varphi_{cd} \\ \varphi_d \end{array}$	0.0015 2.5 3.50 4.50	$\frac{\operatorname{Precau}}{\varrho_{\scriptscriptstyle ps}}$	tionary Savings -0.2

Panel B. Unconditional Means

			Model		D	ata
	Maturity	Baseline Model	Consumption Only	Full Model	Real	Nominal
$R_{\scriptscriptstyle TW/m}$		3.18%	3.21%	5.65%	6.64%	10.44%
y	1Y	2.06%	2.13%	2.13%	1.35%	5.15%
	10Y	1.99%	2.08%	2.08%	2.38%	6.18%
$\sigma_{_{TW/m}}$		2.82%	2.75%	15.78%	15.24%	15.26%
$SD(\Delta y)$	1Y	2.33%	2.36%	2.36%	1.42%	1.67%
	10Y	0.63%	0.63%	0.63%	0.72%	1.13%
$ ho_{\scriptscriptstyle SB}$	1Y	-0.278	-0.133	-0.182	-0.118	-0.092
	10Y	-0.272	-0.129	-0.182	-0.171	-0.100
$ ho_{\scriptscriptstyle SV}$		-0.038	-0.161	-0.164	-0.132	-0.132

Panel C. Rolling-window Simulated Correlations

			Model		Data (Nominal)
Variable	Statistic	Baseline Model	Consumption Only	Full Model	
$\rho_{\scriptscriptstyle SB}$ (1Y)	Mean	-0.278	-0.134	-0.184	-0.092
	AC(1)		0.971	0.971	0.978
	STD		0.229	0.137	0.179
$\rho_{SB} \ (10 { m Y})$	Mean	-0.272	-0.130	-0.183	-0.100
	AC(1)		0.971	0.971	0.971
	STD		0.221	0.132	0.173
$ ho_{\scriptscriptstyle SV}$	Mean	-0.038	-0.164	-0.167	-0.132
	AC(1)		0.971	0.970	0.976
	STD		0.102	0.137	0.281

Table 2 Relationships Between Simulated Values

This table summarizes the correlations between macroeconomic and asset pricing variables based on the simulation of different models. Panel A shows the relationship between Δc_{t+1} , the shocks to x_{t+1} and σ_{t+1} , the first differences in one-year $(y_{1,t+1})$ and ten-year $(y_{10,t+1})$ bond yields, the returns on the total wealth/market portfolio $(R_{TW/m})$, and the first difference in the variance of the total wealth/market portfolio $(\sigma_{TW/m,t+1})$. Panel B shows the relationship between CGP (ρ_t) and model-based correlations of stock returns with one-year $(\rho_{SB,1})$ or 10-year $(\rho_{SB,10})$ bond returns or variance shocks (ρ_{SV}) . For all calculations, we simulate 1,000,000 periods and drop the first 100,000 before calculating correlations.

Panel A. Relationships between simulated variables

	Model	$\Delta y_{\scriptscriptstyle 1,t+1}$	$\Delta y_{_{10,t+1}}$	$R_{TW/m,t+1}$	$\Delta \sigma^2_{_{TW/m,t+1}}$
Δc_{t+1}	Baseline	-0.023	-0.022	0.959	0.001
	Consumption	-0.123	-0.071	0.958	-0.122
	Full Model	-0.123	-0.071	0.572	-0.154
$x_{t+1} - E_t[x_{t+1}]$	Baseline	0.982	0.933	0.275	-0.001
	Consumption	0.965	0.906	0.154	0.016
	Full Model	0.965	0.906	0.192	0.019
$\sigma_{t+1}^2 - E_t[\sigma_{t+1}^2]$	Baseline	-0.102	-0.324	-0.037	0.996
	Consumption	-0.079	-0.309	-0.230	0.609
	Full Model	-0.079	-0.309	-0.168	0.788
$p_{t+1} - E_t[p_{t+1}]$	Baseline	-	-	-	-
	Consumption	-0.178	-0.210	-0.023	0.778
	Full Model	-0.178	-0.210	-0.049	0.604

Panel B. Relationships between correlations

	Model	$ ho_{{\scriptscriptstyle SB},1}$	$ ho_{\scriptscriptstyle SB,10}$	$ ho_{\scriptscriptstyle SV}$
$ ho_{\scriptscriptstyle t}$	Baseline	0.000 -0.993	0.000 -0.991	0.000 -0.951
	Consumption Full Model	-0.993 -0.998	-0.991	-0.999

This table summarizes the slopes and the Newey-West adjusted (12 lags) t-statistics of quarterly regressions that examine the relationship between CGP and the SB correlation. Panel A and B summarize the results of

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{1,t+k},$$

for k=1,2,3,4, where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation estimated using either the one-year or ten-year Treasury yield. Panel C shows the results of the regression

$$\Delta \hat{x}_t = \alpha_0' + \alpha_1' \Delta c_t + \alpha_2' \hat{\rho}_{SB,t} \times \Delta c_{1,t} + \alpha_3' \hat{\rho}_{SB,t} + \epsilon_{2,t},$$

where $\Delta \hat{x}_t$ is the median change in the long-run SPF consumption growth forecast.

Panel A. Serial correlation of consumption growth

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.494 (7.39)	0.370 (4.30)	0.420 (6.54)	0.186 (2.22)
$\text{Adj-}R^2$	0.219	0.134	0.173	0.030

Panel B. Serial correlation of consumption growth and SB correlation

Dep. Var.:	Δ	ΔC_{t+1}	Δ	C_{t+2}	Δ	Δc_{t+3}	Δ	ΔC_{t+4}
Maturity:	1Y	10Y	1Y	10Y	1Y	10Y	1Y	10Y
Δc_t	0.480	0.482	0.367	0.371	0.397	0.406	0.150	0.164
	(10.80)	(10.48)	(7.34)	(6.78)	(8.03)	(7.62)	(2.47)	(2.79)
$\Delta c_{\scriptscriptstyle t} imes \hat{ ho}_{\scriptscriptstyle SB,t}$	-0.400	-0.312	-0.690	-0.520	-0.372	-0.294	-0.475	-0.164
	(-2.79)	(-2.17)	(-3.76)	(-2.87)	(-1.92)	(-1.94)	(-2.39)	(-3.38)
$\hat{ ho}_{{\scriptscriptstyle SB},t}$	0.004	0.003	0.005	0.004	0.005	0.003	0.005	0.004
	(3.38)	(3.46)	(4.39)	(4.00)	(3.19)	(2.59)	(2.96)	(2.88)
$\mathrm{Adj}\text{-}R^2$	0.242	0.243	0.173	0.171	0.202	0.194	0.069	0.068

Panel C. CG and shocks to survey expectations

Dep. Var.:		$\Delta \hat{x}_t$	
$\hat{\rho}_{SB,t}$ based on:		1Y	10Y
Δc_t	-0.043	0.065	-0.007
	(-0.45)	(0.66)	(-0.07)
$\Delta c_{\scriptscriptstyle t} imes \hat{ ho}_{\scriptscriptstyle SB,t}$		-0.607	-0.497
		(-1.70)	(-2.74)
$\hat{ ho}_{SB,t}$		0.003	0.001
. ,		(1.65)	(0.87)
$\mathrm{Adj}\text{-}R^2$	-0.004	0.832	0.022

This table summarizes the slopes and the Newey-West adjusted (12 lags) t-statistics of quarterly regressions that examine the relationship between CGP and SB correlations computed using 10-year real yields. Panels A and B show the results of the regression

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{,,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{.,t} + \epsilon_{1,t+k},$$

for k = 1, 2, 3, 4, where $\hat{\rho}_{.,t}$ is either the stock/bond return correlation computed using 10-year real yields $(\hat{\rho}_{SR,t})$ or the correlation between stock returns and the 10-year breakeven inflation rate $(\hat{\rho}_{S\pi})$. Panel C shows the results of the regression

$$\Delta \hat{x}_t = \alpha_0' + \alpha_1' \Delta c_t + \alpha_2' \hat{\rho}_{SR,t} \times \Delta c_{1,t} + \alpha_3' \rho_{.,t} + \epsilon_{2,t},$$

where $\Delta \hat{x}_t$ is the median change in the long-run SPF consumption growth forecast and $\rho_{..t}$ is defined as above.

Panel A. Consumption growth persistence and Real stock/bond correlation

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_{t}	0.451	0.240	0.218	0.125
	(2.85)	(1.69)	(3.55)	(1.68)
$\Delta c_t \times \hat{\rho}_{SR,t}$	-0.451	-1.138	-1.394	-0.895
	(-0.87)	(-2.42)	(-4.46)	(-2.29)
$\hat{ ho}_{SR,t}$	0.005	0.007	0.008	0.007
	(3.49)	(5.14)	(3.08)	(2.99)
$\mathrm{Adj}\text{-}R^2$	0.232	0.346	0.424	0.186

Panel B. Consumption growth persistence and stock/inflation correlation

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.684	0.742	0.804	0.738
	(3.40)	(4.20)	(3.12)	(4.29)
$\Delta c_{\scriptscriptstyle t} \times \hat{ ho}_{\scriptscriptstyle S\pi,t}$	0.412	0.739	0.721	1.193
	(0.92)	(1.73)	(0.91)	(1.85)
$\hat{ ho}_{S\pi,t}$	0.002	-0.002	-0.004	-0.006
	(0.53)	(-0.88)	(-1.00)	(-1.89)
$Adj-R^2$	0.231	0.275	0.338	0.153

Panel C. Changes in survey forecasts and consumption growth

Dep. Var.:		$\Delta \hat{x}_t$	
ρ_t based on:		$\hat{ ho}_{\scriptscriptstyle SR}$	$\hat{ ho}_{S\pi}$
Δc_t	-0.035	-0.257	0.140
	(-0.59)	(-2.94)	(0.51)
$\Delta c_{\scriptscriptstyle t} \times \rho_{\scriptscriptstyle t}$		-1.392	0.740
		(-4.96)	(0.82)
$ ho_t$		0.005	-0.001
		(1.98)	(-0.17)
$\mathrm{Adj}\text{-}R^2$	-0.014	0.086	-0.023

Table 5 Stock/Bond Return Correlations and the Stock Market Leverage Effect

This table summarizes the slopes and Newey-West adjusted standard errors (12 lags) of the contemporaneous regressions of stock/bond correlations on the correlation between stock returns and stock market variance innovations, where the variance innovation is defined as daily changes in VXO². One month of daily data is used to estimate the correlations, and the end-of-the-month observation is used for VXO²_t.

		SB Correlation									
Bond maturity:	-	lΥ	1	.0Y	10Y Real						
$VXO_t^2/10000$		-1.485		-0.755		-1.041					
$\hat{ ho}_{\scriptscriptstyle SV,t}$	1.033 (5.25)	(-2.99) 0.982 (5.32)	1.602 (5.11)	(-0.98) 1.576 (5.07)	0.775 (2.09)	(-2.79) 0.979 (2.11)					
$Adj-R^2$	0.315	0.378	0.355	0.362	0.067	0.169					

Table 6 Stock Market Volatility Estimation

The table summarizes the parameter estimates for the two-factor EGARCH model of Brandt and Jones (2006), with and without an extension that links the leverage effect to the stock/bond return correlation. The extended model is

$$\ln h_{t} - \ln h_{t-1} = \kappa_{h} (\ln q_{t-1} - \ln h_{t-1}) + \phi_{h} X_{t-1} + \delta_{h} \frac{R_{t-1}}{h_{t-1}}$$

$$\ln q_{t} - \ln q_{t-1} = \kappa_{h} (\theta - \ln q_{t-1}) + \phi_{q} X_{t-1} + \delta_{q} \frac{R_{t-1}}{h_{t-1}} + \delta_{qc} \hat{\rho}_{SB,t-1} \frac{R_{t-1}}{h_{t-1}}$$

$$X_{t} = \left(\left| \frac{R_{t-1}}{h_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) / \sqrt{1 - \frac{2}{\pi}},$$

where $R_t \sim N(0, h_t^2)$. δ_{qc} is set to 0 in the baseline case. The stock/bond return correlation is estimated using the one-year nominal, ten-year nominal, or ten-year real yield using a rolling window of 365 calendar days. Asymptotic t-statistics are reported in parentheses.

	κ_h	ϕ_h	$\delta_{\scriptscriptstyle h}$	κ_q	θ	ϕ_q	δ_q	δ_{qc}
Baseline	0.100 (12.09)	0.042 (22.81)	-0.073 (-33.49)	0.004 (6.97)	-4.423 (-71.93)	0.022 (15.96)	-0.009 (-5.40)	
SB Corr (1Y yield)	0.104 (12.42)	0.042 (23.20)	-0.074 (-33.29)	0.004 (7.48)	-4.461 (-81.43)	0.019 (15.41)	-0.013 (-7.60)	0.028 (6.92)
SB Corr (10Y Yield)	0.108 (12.43)	0.042 (22.99)	-0.075 (-33.40)	0.005 (7.61)	-4.458 (-82.25)	0.020 (15.87)	-0.012 (-7.49)	0.018 (6.67)
SB Corr (Real)	$0.095 \\ (8.32)$	0.024 (5.19)	-0.119 (-20.98)	0.010 (6.13)	-4.537 (-79.10)	0.029 (10.47)	0.008 (1.07)	0.073 (3.29)

Table 7 Stock Market Variance Predictability

This table summarizes the relationship between stock/bond correlations and market variance. In all regressions, the dependent variable is the realized variance of stock returns (RV_{t+1}) estimated using the sum of daily squared returns in the following month. Independent variables include measures of macro uncertainty of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty of Baker, Bloom, and Davis (2016), and the consumption growth volatility of Schorfheide, Song, and Yaron (2018). Uncertainty measures are interacted with daily stock/bond return correlations.

Panel A. UNC = Macro Uncertainty

				Depend	dent Varial	ble: RV_{t+1}				
Bond Maturity:		1Y			10Y			10Y Real		
UNC_t	0.016	0.008	0.024	0.016	0.008	0.024	0.055	0.024	0.014	
	(2.60)	(2.77)	(3.75)	(2.44)	(2.66)	(3.72)	(3.09)	(2.59)	(1.05)	
$UNC_t \times \hat{\rho}_{SB,t}$	-0.005	-0.003	-0.084	-0.003	-0.002	-0.070	-0.006	-0.004	-0.165	
	(-2.54)	(-2.69)	(-2.85)	(-2.45)	(-2.67)	(-2.91)	(-1.89)	(-1.60)	(-1.69)	
RV_{t}		0.441			0.457			0.527		
		(3.12)			(3.15)			(8.12)		
$\hat{ ho}_{{\scriptscriptstyle SB},t}$			0.074			0.061			0.147	
			(2.77)			(2.87)			(1.67)	
$Adj-R^2$	0.153	0.301	0.282	0.128	0.305	0.271	0.400	0.564	0.462	

Panel B. UNC = Monetary Policy Uncertainty

		Dependent Variable: RV_{t+1}									
Bond Maturity:		1Y			10Y			10Y Real			
UNC_t	-0.008	-0.069	-0.114	0.099	-0.201	0.019	0.022	-0.113	-0.248		
	(-0.14)	(-1.28)	(-1.26)	(1.36)	(-2.48)	(0.35)	(0.19)	(-1.09)	(-1.19)		
$UNC_t \times \hat{\rho}_{\scriptscriptstyle SB,t}$	-0.619	-0.477	-1.350	-0.218	-0.021	-0.561	-0.910	-0.844	-2.174		
	(-2.42)	(-2.41)	(-2.13)	(-2.11)	(-0.35)	(-1.63)	(-1.82)	(-1.78)	(-1.82)		
RV_{t}		0.323			0.366			0.417			
		(2.46)			(2.55)			(2.46)			
$\hat{ ho}_{SB,t}$			0.009			0.004			0.019		
			(1.67)			(1.26)			(1.65)		
$Adj-R^2$	0.128	0.213	0.161	0.072	0.186	0.083	0.135	0.284	0.160		

Panel C. UNC = Consumption Growth Volatility

				Depend	dent Varial	ble: RV_{t+1}				
Bond Maturity:		1Y			10Y			10Y Real		
UNC_t	0.004	0.002	0.005	0.004	0.002	0.005	0.013	0.003	0.003	
	(2.79)	(2.49)	(2.44)	(2.48)	(2.24)	(2.05)	(2.40)	(1.77)	(0.75)	
$UNC_t \times \hat{\rho}_{SB,t}$	-0.006	-0.003	-0.016	-0.003	-0.001	-0.010	-0.005	-0.004	-0.062	
	(-2.40)	(-2.40)	(-1.59)	(-2.08)	(-2.11)	(-1.30)	(-1.26)	(-1.24)	(-1.99)	
RV_t		0.441			0.475			0.630		
		(3.05)			(3.08)			(7.53)		
$\hat{ ho}_{SB,t}$			0.011			0.008			0.062	
			(1.26)			(1.08)			(1.95)	
$\mathrm{Adj}\text{-}R^2$	0.157	0.313	0.181	0.103	0.303	0.120	0.236	0.549	0.342	

Table 8 Predictability of Consumption Growth (II)

This table summarizes the results of the predictive regression

$$\Delta c_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 \hat{\rho}_{SB,t} \times UNC_t + \beta_3 \hat{\rho}_{SB,t} + \beta_4 \Delta c_t + \epsilon_{t+1},$$

where $\hat{\rho}_{SB,t}$ is the stock/bond correlation estimated in various ways, the proxies of uncertainty (UNC) are defined as in previous tables, with the addition of the long-run stock market variance (q) estimate from the baseline model presented in Table 6.

Panel A. One-year stock/bond return correlation

	Dependent Variable: Δc_{t+1}												
	UNC = MU			U	UNC = MPU			$\mathrm{UNC} = \mathrm{SD}(\Delta c)$			UNC = q		
UNC_t	-0.025	-0.016	-0.028	0.016	0.034	0.020	-0.005	-0.002	-0.006	0.071	-0.020	-0.068	
	(-5.51)	(-4.16)	(-6.16)	(0.20)	(0.65)	(0.48)	(-2.19)	(-2.25)	(-3.31)	(-1.71)	(-0.73)	(-1.51)	
$UNC_t \times \hat{\rho}_{SB}$	$_{t}$ 0.005	0.004	0.026	0.361	0.299	0.506	0.005	0.003	0.011	0.071	0.056	0.098	
	(2.91)	(3.13)	(1.35)	(1.76)	(2.49)	(2.38)	(2.50)	(2.40)	(1.68)	(1.36)	(1.85)	(0.81)	
$\hat{ ho}_{SB,t}$			-0.024			-0.001			-0.007			-0.001	
			(-0.99)			(-0.51)			(0.99)			(-0.23)	
$\Delta c_{\scriptscriptstyle t}$		0.342			0.406			0.413			0.419		
		(5.46)			(4.86)			(7.55)			(8.06)		
Adj- R^2	0.205	0.287	0.208	0.093	0.252	0.103	0.135	0.274	0.139	0.102	0.242	0.099	

Panel B. Ten-year stock/bond return correlation

					D	ependent '	Variable: 4	Δc_{t+1}					
	UNC = MU			U	UNC = MPU			$\mathrm{UNC} = \mathrm{SD}(\Delta c)$			UNC = q		
UNC_t	-0.260	-0.017	-0.028	0.051	0.059	0.094	-0.005	-0.003	-0.006	-0.073	-0.024	-0.068	
	(-5.68)	(-4.59)	(-6.94)	(0.70)	(1.04)	(1.29)	(-2.63)	(-2.18)	(-2.98)	(-1.89)	(-0.95)	(-1.57)	
$UNC_t \times \hat{\rho}_{SB}$	t 0.004	0.003	0.023	0.316	0.241	0.487	0.003	0.002	0.011	0.060	0.045	0.129	
	(3.12)	(3.37)	(1.68)	(2.19)	(2.83)	(2.94)	(2.64)	(2.68)	(1.89)	(1.81)	(2.26)	(1.27)	
$\hat{ ho}_{{\scriptscriptstyle SB},t}$			-0.018			-0.002			-0.007			-0.003	
			(-1.30)			(-1.19)			(-1.27)			(-0.68)	
Δc_t		0.329			0.406			0.413			0.419		
		(5.41)			(4.39)			(7.38)			(8.11)		
$\mathrm{Adj}\text{-}R^2$	0.223	0.295	0.228	0.124	0.252	0.125	0.133	0.274	0.144	0.113	0.247	0.114	

Panel C. Real stock/bond return correlation

					De	pendent '	Variable: 4	Δc_{t+1}					
		UNC = MU			UNC = MPU			$UNC = SD(\Delta c)$			UNC = q		
UNC_t	-0.029	-0.017	-0.036	0.084	0.015	0.082	-0.005	-0.001	-0.004	-0.082	-0.027	-0.102	
	(-3.58)	(-3.08)	(-4.01)	(0.10)	(0.36)	(0.76)	(-1.05)	(-0.39)	(-0.55)	(-2.39)	(-0.64)	(-3.12)	
$UNC_t \times \hat{\rho}_S$	$_{B,t}$ 0.007	0.006	-0.019	0.613	0.344	0.599	0.007	0.005	0.013	0.111	0.098	-0.003	
	(1.96)	(2.80)	(-0.54)	(3.85)	(2.52)	(1.66)	(2.50)	(2.57)	(0.83)	(1.97)	(2.62)	(-0.01)	
$\hat{ ho}_{SB,t}$			0.024			0.000			-0.007			0.006	
			(0.69)			(0.03)			(-0.40)			(0.43)	
Δc_t		0.365			0.513			0.497			0.394		
		(3.36)			(3.70)			(3.03)			(2.85)		
$Adj-R^2$	0.251	0.303	0.243	0.098	0.256	0.084	0.296	0.432	0.282	0.215	0.256	0.208	

Table 9 Market Return Predictability

This table summarizes the results of the regression

$$R_{S,t,t+\tau}^{e} = \beta_{0} + \beta_{1}y_{t} + \beta_{2}y_{t} \times \hat{\rho}_{SB,t} + \epsilon_{t+1},$$

where R_S^e is the leading value-weighted market excess return over a one-month, three-month, six-month, or twelve-month horizon, and y_t is either the one-year nominal $(y_{1,t})$, ten-year nominal $(y_{10,t})$, or ten-year real yield $(r_{10,t})$. $\hat{\rho}_{SB,t}$ is the estimated correlation between stock and bond returns, which is constructed using the same yield used as a regressor. Panel A shows the results where future stock returns are regressed on nominal bond yields only for the pre- and post-1999 periods. Panel B estimates the full model using the entire sample, again using nominal yields. Panel C shows regression results for simple and interactive regressions based on 10-year real yields. T-statistics are adjusted for heteroscedasticity and autocorrelation using Newey-West standard errors with a lag of 12.

Panel A. Simple predictive regressions using nominal yields

		Pre	e-1999		1999+					
	1-Month	3-Month	6-Month	12-Month	1-Month	3-Month	6-Month	12-Month		
$y_{1,t}$	-0.135 (-1.69)	-0.246 (-1.37)	-0.349 (-0.92)	-0.294 (-0.40)	-0.367 (-2.52)	-1.108 (-3.46)	-2.282 (-3.78)	-4.760 (-4.27)		
$\mathrm{Adj}\text{-}R^2$	0.005	0.005	0.009	0.007	0.019	0.062	0.128	0.266		
$y_{\scriptscriptstyle 10,t}$	-0.065 (-0.77)	-0.076 (-0.35)	-0.088 (-0.19)	0.183 (0.22)	-0.616 (-2.88)	-1.819 (-4.06)	-3.581 (-4.35)	-6.616 (-4.85)		
$\mathrm{Adj}\text{-}R^2$	-0.001	-0.001	0.009	-0.001	0.032	0.095	0.009	0.285		

Panel B. Interactive predictive regressions using nominal yields

	One	-month	Thre	e-month	Six-	month	Twel	Twelve-month		
	k=1	k=10	k=1	k=10	k=1	k=10	k=1	k=10		
$y_{k,t}$	-0.295	-0.429	-0.818	-1.204	-1.512	-2.175	-2.252	-2.976		
	(-3.45)	(-3.63)	(-3.95)	(-4.24)	(-3.89)	(-3.99)	(-3.06)	(-3.14)		
$y_{k,t} \times \hat{\rho}_{SB,t}$	0.616	0.571	1.857	1.655	3.539	3.016	5.144	4.152		
	(2.63)	(3.36)	(3.11)	(3.65)	(3.20)	(3.48)	(2.46)	(2.85)		
	0.016	0.022	0.047	0.054	0.082	0.089	0.092	0.086		

Panel C. Predictive regressions using 10-year real yields

	One	One-month		e-month	Six-	month	Twelv	Twelve-month		
$r_{\scriptscriptstyle 10,t}$	-0.835	-0.254	-2.341	-0.462	-3.749	-0.367	-6.026	-2.131		
	(-2.48)	(-0.82)	(-2.88)	(-0.69)	(-2.70)	(-4.69)	(-2.41)	(-0.81)		
$r_{\scriptscriptstyle 10,t} imes \hat{ ho}_{\scriptscriptstyle SB,t}$		4.624		15.180		27.328		31.477		
		(2.68)		(3.73)		(3.23)		(2.23)		
$\mathrm{Adj}\text{-}R^2$	0.027	0.075	0.076	0.242	0.092	0.346	0.126	0.303		

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A. Technical appendix

A.1. The wealth-consumption ratio

Following the Campbell-Shiller decomposition, the returns to total wealth portfolio can be represented by

$$R_{TW,t+1} = \kappa_0 + \Delta c_{t+1} + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t).$$

The intertemporal marginal rate of substitution (IMRS) is

$$m_{t+1} = \theta \log \beta - \gamma \Delta c_{t+1} + (\theta - 1) \left[\kappa_0 + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t) \right].$$

The unexpected component of the IMRS is represented by

$$m_{t+1} - \mathbf{E}_t[m_{t+1}] = \lambda_c \sigma_t \epsilon_{c,t+1} + \lambda_x \sigma_t \epsilon_{x,t+1} + \lambda_v \sigma_t \epsilon_{v,t+1} + \lambda_\delta \sigma_t \epsilon_{p,t+1},$$

where
$$\lambda_c = -\gamma$$
, $\lambda_x = (\theta - 1)\kappa_1 A_1 \phi_x$, $\lambda_v = (\theta - 1)\kappa_1 A_2 \sigma_v$, and $\lambda_\delta = (\theta - 1)\kappa_1 A_3 \sigma_p$.

We solve for A_0 , A_1 , A_2 , and A_3 using the Euler equation $E_t[m_{t+1} + R_{TW,t+1}] + Var_t[m_{t+1} + R_{TW,t+1}] = 0$. For A_1 , we collect all terms associated with x_t :

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 p_1}.$$

Collecting the terms from the Euler equation that are functions of σ_t^2 and p_t , it can be seen that A_2 and A_3 must jointly satisfy the conditions

$$2A_2(\kappa_1 s_1 - 1) + \theta((A_1 \kappa_1 \varphi_x)^2 + (A_2 \kappa_1 \sigma_v)^2 + (A_3 \kappa_1 \sigma_p)^2 + (1 - \frac{1}{\psi})^2) + 2(1 - \gamma)\kappa_1 A_2 \sigma_v \varrho_{ps} = 0$$

$$A_3 = A_{30} + A_{32}A_2$$

where
$$A_{30} = \frac{(1-\gamma)\kappa_1 A_1 \varphi_x}{1-\kappa_1 \omega_1} < 0$$
 and $A_{32} = \frac{\theta \varrho_{ps} \kappa_1^2 A_1 \varphi_x \sigma_v}{1-\kappa_1 \omega_1} > 0$.

 A_2 can be obtained by solving a quadratic equation after plugging the second equation into the first. It can also be shown that $A_2 < 0$ when $\gamma > 1$ and $\psi > 1$ by evaluating the characteristics of the quadratic equation. We obtain two values for A_2 . We choose the value that is closer to the baseline model. The second value generates unrealistic moments of asset returns. The negative sign of A_2 also implies $A_3 < 0$.

Finally,
$$A_0$$
 satisfies $A_0 = \frac{1}{1-\kappa_1} \left[\log \beta + \kappa_0 + (1 - \frac{1}{\psi})\mu + k_1(A_2s_0 + A_3\omega_0) \right]$.

A.2. The price-dividend ratio

Similar to the wealth-consumption ratio, market returns can be expressed as

$$R_{m,t+1} = \kappa_0 + \Delta d_{t+1} + A_{m,0}(\kappa_1 - 1) + A_{m,1}(\kappa_1 x_{t+1} - x_t) + A_{m,2}(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_{m,3}(\kappa_1 p_{t+1} - p_t).$$

We again solve for the coefficients using the Euler equation $E_t[m_{t+1} + R_{m,t+1}] + 0.5 Var_t[m_{t+1} + R_{m,t+1}] = 0$. Collecting the terms associated with x_t , σ_t^2 , and p_t , we can solve for $A_{m,0}$, $A_{m,1}$, $A_{m,2}$, and $A_{m,3}$. First, we have

$$A_{m,1} = \frac{\phi_d - \frac{1}{\psi}}{1 - \kappa_1 p_1}.$$

As in the wealth-consumption ratio, $A_{m,2}$, and $A_{m,3}$ must jointly satisfy the conditions

$$2A_{m,2}(\kappa_1 s_1 - 1) + 2(\theta - 1)(\kappa_1 s_1 - 1)A_2 + 2(\varphi_{cd} + \lambda_c)(\kappa_1 A_{m,2} \sigma_v + \lambda_v)\varrho_{ps}$$
$$+ (\kappa_1 A_{m,1} \varphi_x + \lambda_x)^2 + (\kappa_1 A_{m,2} \sigma_v + \lambda_v)^2 + (\kappa_1 A_{m,3} \sigma_p + \lambda_\delta)^2 + (\varphi_{cd} + \lambda_c)^2 + \varphi_d^2 = 0$$
$$A_{m,3} = A_{m,30} + A_{m,32} A_{m,2},$$

where $A_{m,30} = \frac{1}{1-\kappa_1\omega_1} \left((\varphi_{cd} + \lambda_c)(\kappa_1 A_{m,1} \varphi_x + \lambda_x) + (\theta - 1)(\kappa_1\omega_1 - 1)A_3 + \lambda_v(\kappa_1 A_{m,1} \varphi_x + \lambda_x) \varrho_{ps} \right)$ and $A_{m,32} = \frac{1}{1-\kappa_1\omega_1} \kappa_1 \sigma_v(\kappa_1 A_{m,1} \varphi_x + \lambda_x) \varrho_{ps}$. Evaluating the characteristics of the quadratic function, similar to the earlier case, $A_{m,2} < 0$ when $\gamma > \varphi_{cd} > 1$, which is consistent with a general long-run risk specification. Also, one can show that $A_{m,30} < \text{and } A_{m,32} > 0$ under the condition of $\gamma > \phi_d$ and $\varphi_{cd} > 1$, which implies $A_{m,3} < 0$.

Finally, $A_{m,0}$ satisfies

$$A_{m,0} = \frac{1}{1 - \kappa_1} (\theta \log \beta + \theta \kappa_0 + (1 - \gamma)\mu + \kappa_1 s_0 (A_2(\theta - 1) + A_{m,2}) + \kappa_1 \omega_0 (A_3(\theta - 1) + A_{m,3}) + (\theta - 1)(\kappa - 1)A_0)).$$

A.3. Bond yields

Denote the state vector as

$$\Sigma_t = \left[\begin{array}{ccc} \Delta C_t & x_t & \sigma_t^2 & p_t \end{array} \right]'$$

We can write the conditional mean as

$$E_t \left[\Sigma_{t+1} \right] = K_0 + K \Sigma_t,$$

where

$$K_{\scriptscriptstyle 0} = \left[egin{array}{cccc} \mu & & 0 & & s_{\scriptscriptstyle 0} & & \omega_{\scriptscriptstyle 0} \end{array}
ight]'$$

and

$$K = \left[egin{array}{ccccc} 0 & 1 & 0 & 0 \ 0 & p_1 & 0 & 0 \ 0 & 0 & s_1 & 0 \ 0 & 0 & 0 & \omega_1 \end{array}
ight]$$

The conditional covariance matrix is

$$\operatorname{Cov}_t\left(\Sigma_{t+1}, \Sigma_{t+1}'\right) = \begin{bmatrix} \sigma_t^2 & \phi_x p_t & \varrho_{ps} \sigma_v \sigma_t^2 & 0 \\ \phi_x p_t & \phi_x^2 \sigma_t^2 & \sigma_v \varrho_{ps} p_t & 0 \\ \varrho_{ps} \sigma_v \sigma_t^2 & \sigma_v \varrho_{ps} p_t & \sigma_v^2 \sigma_t^2 & 0 \\ 0 & 0 & 0 & \sigma_p^2 \sigma_t^2 \end{bmatrix} = \Omega_1 \sigma_t^2 + \Omega_2 p_t,$$

where

In vector notation, we can write the log pricing kernel as

$$m_{t+1} = m_0 + M_1' \Sigma_{t+1} - M_2' \Sigma_t$$

with

$$m_0 = \theta \log \beta + (\theta - 1) \left(\kappa_0 + A_0(\kappa_1 - 1) \right),$$

$$M_1 = \begin{bmatrix} -\gamma & (\theta - 1)\kappa_1 A_1 & (\theta - 1)\kappa_1 A_2 & (\theta - 1)\kappa_1 A_3 \end{bmatrix}'$$

and

$$M_2 = \begin{bmatrix} 0 & (\theta - 1)A_1 & (\theta - 1)A_2 & (\theta - 1)A_3 \end{bmatrix}'$$

where A_1 , A_2 , and A_3 are defined earlier.

The log price of a riskless one-period bond $(B_{1,t})$ is derived by

$$B_{1,t} = \mathbb{E}_{t} [m_{t+1}] + 0.5 \text{Var}_{t} (m_{t+1})$$

$$= m_{0} + M'_{1} K_{0} + (M'_{1} K - M'_{2}) \Sigma_{t} + 0.5 M'_{1} \text{Cov}_{t} (\Sigma_{t+1}, \Sigma'_{t+1}) M_{1}$$

$$= m_{0} + M'_{1} K_{0} + (M'_{1} K - M'_{2}) \Sigma_{t} + 0.5 M'_{1} \Omega_{1} M_{1} \sigma_{t}^{2} + 0.5 M'_{1} \Omega_{2} M_{1} p_{t}$$

$$= m_{0} + M'_{1} K_{0} + (M'_{1} K - M'_{2}) \Sigma_{t} + 0.5 \Psi' \Sigma_{t}$$

$$= m_{0} + M'_{1} K_{0} + (M'_{1} K - M'_{2} + 0.5 \Psi') \Sigma_{t},$$

where

$$\Psi' = \left[\begin{array}{ccc} 0 & 0 & M_1' \Omega_1 M_1 & M_1' \Omega_2 M_1 \end{array} \right]'.$$

Therefore, the yield of a one-period bond is represented by

$$y_t = Y_0 + Y\Sigma_t,$$

where

$$Y_0 = -m_0 - M_1' K_0$$

and

$$Y = -M_1'K + M_2' - 0.5\Psi'.$$

It can be shown that for

$$Y = \begin{bmatrix} 0 & Y_x & Y_v & Y_p \end{bmatrix}'$$

we have $Y_x > 0$ and $Y_{\sigma^2}, Y_p < 0$.

Now suppose that the n-period bond has a log price

$$B_{n,t} = D_{n,0} + D'_n \Sigma_t.$$

Then the (n+1)-period bond has a price that is equal to the conditional expectation of

$$E_t[m_{t+1} + B_{n,t+1}] + 0.5 Var_t(m_{t+1} + B_{n,t+1}),$$

where

$$m_{t+1} + B_{n,t+1} = m_0 + D_{n,0} + (M_1 + D_n)' \Sigma_{t+1} - M_2' \Sigma_t.$$

The log price of the bond can be solved as

$$B_{n,t+1} = m_0 + D_{n,0} + (M_1 + D_n)'(K_0 + K\Sigma_t) - M_2'\Sigma_t + 0.5(M_1 + D_n)'\operatorname{Cov}_t\left(\Sigma_{t+1}, \Sigma_{t+1}'\right)(M_1 + D_n)$$

$$= m_0 + D_{n,0} + (M_1 + D_n)'K_0 + ((M_1 + D_n)'K - M_2')\Sigma_t + 0.5\Psi_n'\Sigma_t,$$

where

$$\Psi_n = \left[0 \quad 0 \quad (M_1 + D_n)'\Omega_1(M_1 + D_n) \quad (M_1 + D_n)'\Omega_2(M_1 + D_n) \right]'$$

The log of (n+1)-period bond price is therefore

$$B_{n+1,t} = D_{n+1,0} + D'_{n+1} \Sigma_t,$$

where

$$D_{n+1,0} = m_0 + D_{n,0} + (M_1 + D_n)' K_0$$

and

$$D_{n+1} = K'(M_1 + D_n) - M_2 + \frac{1}{2}\Psi_n.$$

The (n+1)-period yield is derived as

$$y_{n+1,t} = Y_{n+1,0} + Y_{n+1} \Sigma_t,$$

where $Y_{n+1,0} = -D_{n+1,0}$ and $Y_{n+1} = -D_{n+1}$. Therefore, Y_{n+1} can further be represented as

$$Y_{n+1} = \begin{bmatrix} 0 & Y_{n+1,x} & Y_{n+1,v} & Y_{n+1,p} \end{bmatrix}'$$

A.4. The stock/bond return correlation

The stock/bond return correlation is the negative of the correlation between stock return and changes in bond yield. The unexpected return of the total wealth portfolio and the market return are derived using the Campbell-Shiller decomposition:

$$R_{TW,t+1} - E_t[R_{TW,t+1}] = \kappa_1 \phi_x A_1 \sigma_t \epsilon_{x,t+1} + \kappa_1 \sigma_v A_2 \sigma_t \epsilon_{v,t+1} + \kappa_1 \sigma_p A_3 \epsilon_{p,t+1} + \sigma_t \epsilon_{c,t+1}$$

$$R_{m,t+1} - E_t[R_{m,t+1}] = \kappa_1 \phi_x A_{m,1} \sigma_t \epsilon_{x,t+1} + \kappa_1 \sigma_v A_{m,2} \sigma_t \epsilon_{v,t+1} + \kappa_1 \sigma_p A_{m,3} \epsilon_{p,t+1} + \varphi_{cd} \sigma_t \epsilon_{c,t+1} + \varphi_d \sigma_t \epsilon_{d,t+1}$$

We represent the above relationship by:

$$S_{j,x}\sigma_t\epsilon_{x,t+1} + S_{j,v}\sigma_t\epsilon_{v,t+1} + S_{j,p}\sigma_t\epsilon_{p,t+1} + S_{j,c}\sigma_t\epsilon_{c,t+1} + S_{j,d}\sigma_t\epsilon_{d,t+1},$$

where j is either TW for the wealth portfolio or m for the market portfolio. From the above equation, we can derive the stock/bond return correlation by taking the negative of conditional correlation between wealth portfolio/market returns and bond yields.

The conditional covariance between a n-period bond yield and stock returns can be expressed as

$$Cov_{t}(R_{j,t+1}, y_{n,t+1}) = (Y_{n,x}S_{j,x}\varphi_{x} + Y_{n,v}S_{j,v}\sigma_{v} + Y_{n,p}S_{j,p}\sigma_{p} + Y_{n,v}S_{j,c}\sigma_{v}\varrho_{ps})\sigma_{t}^{2} + ((Y_{n,x}\varphi_{x}S_{j,v} + Y_{n,v}S_{j,x}\sigma_{v})\varrho_{ps} + Y_{n,x}S_{j,c}\varphi_{x})p_{t}.$$

The conditional variance of the bond yield is

$$\operatorname{Var}_{t}(y_{n,t+1}) = (Y_{n}'\Omega_{1}Y_{n} + Y_{n}'\Omega_{2}Y_{n}\rho_{t}) \sigma_{t}^{2}.$$

Similarly, the conditional variance of the wealth portfolio/market returns is

$$\operatorname{Var}_{t}(R_{j,t+1}) = \sigma_{j,t}^{2} = (V_{j,v} + V_{j,p}\rho_{t})\sigma_{t}^{2}$$

for $j = \{TW, m\}$, where $V_{j,v} = S_{j,x}^2 + S_{j,v}^2 + S_{j,p}^2 + S_{j,c}^2 + S_{j,d}^2 + 2S_{j,c}S_{j,v}\varrho_{ps}$ and $V_{j,p} = 2S_{j,x}S_{j,v}\varrho_{ps} + 2S_{j,c}S_{j,x}S_{j,v}\varrho_{ps}$.

A.5. The stock market leverage effect

The leverage correlation is the conditional covariance between the returns and variance shocks of the wealth portfolio divided by the conditional standard deviations of each. The covariance can be represented by

$$\operatorname{Cov}_{t}(R_{j,t+1}, \sigma_{i,t+1}^{2}) = \left[(S_{v} + S_{c}\varrho_{ps})V_{j,v}\sigma_{v} + S_{p}V_{j,p}\sigma_{p} + S_{x}V_{j,v}\sigma_{v}\varrho_{ps}\rho_{t} \right]\sigma_{t}^{2},$$

for $j = \{TW, m\}$. Dividing the above by the variance of variance shocks yields the stock market leverage effect. The variance of the market variance shocks is

$$((V_v\sigma_v)^2 + (V_p\sigma_p)^2)\sigma_t^2.$$

A.6. The market risk premium

The risk premium of the wealth/market portfolio can be expressed as

$$Cov_t(-m_{t+1}, R_{j,t+1}) = \left(-\lambda_c(S_{j,c} + S_{j,v}\varrho_{ps}) - \lambda_x S_{j,x} - \lambda_v S_{j,v} - \lambda_\delta S_{j,p} - S_{j,c}\lambda_v \varrho_{ps}\right)\sigma_t^2 + \left(-\lambda_x S_{j,v}\varrho_{ps} - \lambda_v S_{j,x}\varrho_{ps} - \lambda_c S_{j,x} - \lambda_x S_{j,c}\right)p_t$$

for $j = \{TW, m\}$.