

Consumption growth persistence and the stock-bond correlation*

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March 2023

JEL classification: G10, G12, E32, E44

Keywords: Consumption persistence, long-run risk, stock-bond correlation

*We thank Thierry Foucault, Allaudeen Hameed, Scott Joslin, Mete Kilic, Lars Lochstoer, Miguel Palacios, Thomas Sargent, Johan Sulaeman, Selale Tuzel, participants at the Fourteenth Risk Management Conference, NFA Annual Meetings, Korea University, NUS, and USC. All errors are our own.

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Abstract

We consider a model in which the correlation between shocks to consumption and to expected future consumption growth is nonzero and varies over time. We validate this assumption empirically using the model's implication that time-variation in consumption growth persistence drives the correlation between stock and bond returns. Consistent with model predictions, we find that the stock-bond correlation is also related to the correlation between expected consumption growth and uncertainty and the predictive relation between bond yields and future stock returns.

1. Introduction

While the correlation between stock and bond returns has been the subject of research for some time, the abrupt change in the sign of this correlation, which Figure 1 (a) shows turned from positive to negative in the late 1990s, has spurred renewed interest in its determinants. One explanation of this shift is an apparent regime change in the behavior of inflation, as demonstrated in David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020). However, this explanation is at best incomplete, because as pointed out by Duffee (2018a), expected inflation shocks are responsible for just 10-20% of the variation in nominal yields, implying that it is unlikely that inflation is the dominant driver of nominal bond returns or their correlation with stocks.

In this paper, we propose a new explanation of this shift and of variation in the correlation between stock and bond returns (SB correlation) more generally. Our explanation relies on a shift in macroeconomic dynamics, but in a channel omitted by extant models. Specifically, we show that the stock-bond correlation is related to variation in consumption growth persistence (CGP), which we define as the tendency of positive shocks to current consumption growth to raise expected future consumption growth. The logic is straightforward: Changes in current realized growth affect cash flows, while changes in expected growth drive real interest rates via intertemporal smoothing. When CGP increases, the correlation between real rates and cash flows rises, resulting in a lower (and likely negative) SB correlation. When CGP is negative, higher consumption growth forecasts lower growth in the future, and the SB correlation rises.

The persistence of consumption growth indeed appears to have changed over time. Figure 1 (b) shows that autocorrelations in consumption growth were moderate through 1998 but significantly higher in the period starting in 1999, which is around when the SB correlation changed sign. The goal of this study is to determine whether this suggestive evidence is indicative of a more systematic effect that CGP has on the SB correlation and other asset return moments.

We generalize the long-run risk (LRR) model of Bansal and Yaron (2004) by allowing a time-varying correlation between current and expected consumption growth shocks. As in the standard LRR model, a highly persistent expected consumption growth process induces modest but very long-run dependence in consumption growth. Adding variation in the correlation between current and expected consumption growth induces time variation in the degree of serial dependence while maintaining the long-run positive autocorrelation critical for matching the moments of asset returns.

We validate the model-implied relationship between CGP and the SB correlation by showing that the serial correlation in consumption growth is significantly higher when the SB correlation is low. This result holds at multiple horizons and is obtained whether we use returns on short- or long-term bonds and whether the bonds are nominal or inflation-indexed. Second, we show that the contemporaneous relationship between consumption growth and changes in survey forecasts of long-run consumption growth is more negative when the SB correlation is higher. This result is also consistent with a negative relation between CGP and the stock-bond correlation.

An additional feature of our model is the negative correlation between shocks to con-

sumption growth and its volatility. This correlation is empirically motivated and is thought to arise from a precautionary savings motive.¹ With precautionary savings, a higher CGP implies that uncertainty shocks will not only lead to a lower current consumption, but also to a lower future expected consumption. We confirm that the relationship between uncertainty shocks and expected consumption growth becomes more negative when the SB correlation is low.

An implication of this additional feature is a conditional predictive relation between yields and future stock returns. The model implies that this negative relationship should be stronger when CGP is high or the SB correlation is low. As documented in prior work, we report a weak unconditional relationship between yields and future stock returns, but we observe a strong conditional relationship when the SB correlation is negative. Therefore, the insignificant unconditional relation, which is at odds with the predictions of many models, is a natural result of the SB correlation being positive over much of our sample.

There are a number of other explanations for why the SB correlation varies over time, and we believe it is unlikely that any single theory can explain all fluctuations. Aside from the other predictions that we confirm from our empirical analysis, we believe that consumption growth persistence has certain merits that distinguish it from other explanations. Baele, Bekaert, and Inghelbrecht (2010), for example, claim that “macroeconomic fundamentals contribute little to explaining stock and bond return correlations” and conclude that flights to quality/liquidity are the likely explanation for negative correlations. While Pástor and Stambaugh (2003), Connolly, Stivers, and Sun (2005), and others provide additional evidence

¹See, for example, Carroll (1997) and Basu and Bundick (2017)

for this channel, the mechanism seems inadequate in the period starting in 1999, during which the stock-bond correlation has remained negative even during periods of relative market stability.

Variation in the stock-bond correlation has also been attributed to changes in the dynamics of inflation. David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020) present models in which the relation between inflation and real economic activity changes signs. Campbell, Pflueger, and Viceira (2020), for example, show that the correlation between inflation and the output gap was negative between 1979 and 2001 but positive in the following decade. If inflation shocks are the primary driver of bond returns, this result would appear to provide a clear explanation for the shift in correlation that occurred around that time.²

In contrast, the SB correlations shown in our model are entirely driven by variation in real interest rates and not in inflation. While changing properties of inflation are undoubtedly a reason for changes in SB correlations, a model based on real rates may be better positioned to explain interest rate behavior in environments such as that of the last 20 years, in which both inflation levels and inflation risk have mostly been low. From 2003 to 2019, the part of our sample for which reliable TIPS data are available, real and nominal bond yields have tracked each other closely, with a correlation above 90%. More importantly, the real bond-stock correlation and nominal bond-stock correlation are themselves closely related over the sample period in which real rates are available.³ While inflation is undoubtedly more

²Hasseltoft (2012), Ilmanen (2003), Campbell, Sunderam, and Viceira (2017), and Swanson (2019) also advance inflation-based explanations of the stock-bond return correlation.

³When these two correlations are measured using non-overlapping monthly subsamples, the correlation between them is 86%.

important in earlier decades, it almost certainly does not tell the entire story.

This is not the only paper to propose that variation in real yields is an important driver of changes in the SB correlation. For example, Duffee (2018 a,b) argues that the inflation expectations that underlie long-term bond yields vary too little to explain much variation in such yields and that the stock-bond correlation is therefore primarily driven by changes in real yields. Kozak (2021) proposes a production model with two technologies that generates a time-varying correlation, attributing the shift in correlation in the late 1990s to a decline in high-risk capital.

In contemporaneous and complementary work, Chernov, Lochstoer, and Song (2021) also highlight the role of consumption growth persistence in explaining the stock-bond correlation. Their model is substantially different from ours, relying on a regime-switching model to generate time variation in the relative importance of permanent and transient shocks. Similar to ours, it succeeds in explaining the stock-bond correlation using a mechanism based on consumption persistence rather than inflation dynamics. Besides differences in modeling, our paper focuses more on the stock-bond correlation and on the predictability of equity returns, while Chernov, Lochstoer, and Song (2021) concentrate on explaining the real and nominal term structures. The different empirical predictions derived in their paper show that time-varying consumption growth persistence can explain a broader range of phenomena than those we address here.

In the next section, we present and calibrate our model. Section III describes our data and strategies for measuring latent processes. Section IV contains our empirical results, and Section V concludes.

2. A Model of the Real Stock-Bond Correlation

2.1. Model dynamics

Our model is a generalization of the standard framework of Bansal and Yaron (2004). In our baseline specification, the representative agent has Epstein and Zin (1991) preferences, and consumption growth (Δc_{t+1}) has a persistent time-varying component x_t and time-varying uncertainty σ_t^2 :

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \epsilon_{c,t+1} \tag{1}$$

$$x_{t+1} = \xi_1 x_t + \phi_x \sigma_t \epsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = s_0 + s_1 \sigma_t^2 + \sigma_v \sigma_t \epsilon_{v,t+1},$$

where $\epsilon_{c,t+1}$, $\epsilon_{x,t+1}$, and $\epsilon_{v,t+1}$ are $N(0, 1)$.

In addition to the consumption process, a dividend growth process is specified as

$$\Delta d_{t+1} = \mu_d + \phi_d x_t + \sigma_t \varphi_{cd} \epsilon_{c,t+1} + \sigma_t \varphi_d \epsilon_{d,t+1}, \tag{2}$$

where $\epsilon_{d,t+1}$ is *i.i.d.* $N(0, 1)$. Thus, dividend growth shares similarities with consumption due to its dependence on the long-run growth process x_t and its sensitivity to the consumption growth shock $\epsilon_{c,t}$.

Thus far, the only change made to the original model of Bansal and Yaron (2004) is the use of a “square root” process for consumption variance. While the three state variables are uncorrelated in this baseline model, this is not the case below, and this modification will continue to allow for analytical solutions.

Our generalized model deviates from Bansal and Yaron in several important dimensions. We first allow shocks to consumption growth ($\epsilon_{c,t+1}$) and expected long-run consumption growth ($\epsilon_{x,t+1}$) to be stochastically correlated. We refer to this correlation as consumption growth persistence, or CGP, given that it determines whether a shock to current consumption growth is associated with more or less consumption growth in the future. This correlation, which we denote ρ_t , involves a stochastic process that is specified below.

A stochastic correlation can be viewed as a reduced-form approach to modeling time variation in the relative importance of permanent and transitory shocks. For example, in the production economy described by Kaltenbrunner and Lochstoer (2010), the assumption of permanent productivity shocks results in a positive CGP, while transitory shocks generate a negative CGP. This results from differences in how investment (and therefore consumption) responds to changing productivity and in how adjustment costs and mean reversion induce trends in future output. Given that both types of shocks are likely important, either effect could dominate depending on which type of shock is currently more volatile. Furthermore, this phenomenon is not limited to shocks to productivity. Permanent and transitory shocks to income generate similar responses, as discussed, for example, by Hall and Mishkin (1982) and Campbell and Deaton (1989).

We also allow consumption growth shocks to be correlated with consumption variance shocks. A negative correlation is the expected result of a precautionary savings motive, confirmed empirically in several studies, including Carroll and Samwick (1998) and Basu and Bundick (2017). For simplicity, we assume that this correlation, denoted ϱ_{cv} , is constant.

Given the negative correlation between consumption growth and consumption volatility

shocks, it is natural to expect a nonzero correlation between shocks to expected consumption growth and consumption volatility. For example, an increase in precautionary savings induced by greater uncertainty should reduce current consumption as households increase their savings, leading to a rise in expected long-run consumption growth as uncertainty wanes and consumption returns to normal. Empirically, a nonzero correlation between σ_t and x_t is found by Nakamura, Sergeyev, and Steinsson (2017), who show that it tends to be more negative during economic contractions. In another work, Parker and Preston (2005) find significant evidence from household survey data that the precautionary savings motive explains the predictable component of consumption growth.

In the interest of parsimony, we avoid introducing unnecessary additional parameters by assuming that this correlation between shocks to consumption volatility and expected consumption growth is equal to the product $\rho_t \varrho_{ps}$.⁴

To obtain closed-form solutions, we parameterize the conditional covariance (as opposed to correlation) between c_{t+1} and x_{t+1} , which we label q_t . This covariance, which we call the CGP covariance, is related to CGP (ρ_t) by

$$q_t = \sigma_t^2 \rho_t.$$

Its dynamics are given as

$$q_{t+1} = \omega_0 + \omega_1 q_t + \sigma_q \sigma_t \epsilon_{q,t+1}, \quad (6)$$

⁴This correlation structure is consistent with the assumption that there are three orthonormal shocks, $[u_{c,t} \ u_{x,t} \ u_{v,t}]$, that drive the shocks to the three state variables via

$$\epsilon_{c,t} = u_{c,t} \quad (3)$$

$$\epsilon_{x,t} = \rho_t u_{c,t} + \sqrt{1 - \rho_t^2} u_{x,t} \quad (4)$$

$$\epsilon_{v,t} = \varrho_{cv} u_{c,t} + \sqrt{1 - \varrho_{cv}^2} u_{v,t} \quad (5)$$

where $\epsilon_{q,t+1}$ is *i.i.d.* $N(0, 1)$. The baseline model is the special case in which $q_t = \rho_t = 0$.

Similar to other models in the LRR framework, the wealth-to-consumption ratio z_t can be approximated as a linear function of long-run expected consumption growth (x_t), the variance of consumption growth (σ_t^2), and the CGP covariance (q_t). That is,

$$z_t = A_0 + A_x x_t + A_v \sigma_t^2 + A_q q_t, \quad (7)$$

where $A_x > 0$, $A_v < 0$, and $A_q < 0$ under conventional parameter assumptions ($\gamma > 1$ and $\psi > 1$), as shown in the appendix.

Bond yields of all maturities are also linear functions of the three state variables. The appendix derives an analytic formula for the one-period bond, which is increasing in x_t and decreasing in σ_t^2 and q_t , and provides a solution method for longer-term bonds.

Similar to the wealth-consumption ratio, we approximate the return on the market portfolio using the Campbell-Shiller decomposition and verify that the price-dividend ratio is also a linear function of the three state variables.

Given closed-form linear expressions for stock returns and for bond yields of any maturity, it is straightforward to solve for the stock and bond return variances and covariance. We show in the appendix that all three may be expressed as linear and increasing functions of σ_t^2 and q_t . Furthermore, the stock-bond return correlation is a univariate (though nonlinear) function of ρ_t .

2.2. Calibration

We calibrate the model to examine its quantitative implications. The consumption and consumption variance parameters mirror those of Bansal and Yaron (2004). We set a high bar for the exercise by choosing the parameters that govern the persistent component of consumption growth and the CGP covariance to match macro data and survey forecasts, not asset returns. Coherence between these values requires that we deviate from Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), but the differences are minor. The parameter values we assume are summarized in Panel A of Table 1.

We proxy for the long-run growth process using the four-quarter-ahead forecast of real consumption from the Survey of Professional Forecasters (SPF), as detailed in the data section. The persistence (ξ_1) and volatility (ϕ_x) of the long-run growth process are set to match the persistence and volatility of the SPF forecast at the annual level, and the forecasts are only slightly less persistent than the values implied by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). The correlation between the shocks to the survey forecasts and realized consumption is slightly negative (-0.12). For our primary specification, we set the correlation that determines the relationship between consumption growth and uncertainty shocks ϱ_{cv} to -0.2 following the results of Basu and Bundick (2017), but we also show results for two alternative specifications by setting ϱ_{cv} to -0.1 or -0.4 . We set the parameters of the q_t process such that the realized correlation between consumption growth and changes in the SPF forecast matches the correlation between Δc and x in the model in terms of the average, standard deviation, and persistence.

Panel B compares the asset moments generated by the two specifications. These include the baseline model, in which the CGP covariance is set to zero, and the full model, in which the CGP covariance is time varying. For each specification, we generate one million monthly observations and evaluate the first two moments of stock and bond returns as well as several other relevant asset pricing moments. The table shows that the unconditional moments generated by the simulations are generally comparable to those of other standard LRR models, aside from the correlations between stock and bond returns and between returns and volatility changes, which our new model matches better.

Finally, Panel C compares the dynamic behavior of estimated correlations implied by the models to those observed in the data. From both data and simulations, we compute various correlation measures using non-overlapping 60-month windows. The panel displays the means and standard deviations of these values. Monthly serial correlations are inferred by assuming a first-order autoregressive process and taking the 60th root of each autocorrelation. The results show that the level of persistence in the model matches the data quite well, though the correlations are somewhat less volatile in the full model compared to the data.

2.3. The stock-bond return correlation

This section shows the relationship between CGP and the SB correlation. Establishing the link between is essential because it provides a new explanation for why the correlation varies over time. Furthermore, since it can be computed easily as long as stock and bond returns are available, the SB correlation can be used as an empirical proxy for CGP (ρ_t).

To establish the relationship between CGP and the SB correlation, we first explain how

bond yields, stock returns, and stock variances are determined in our model. For bonds, we examine both a short-term bond with a one-year maturity and a long-term bond with a ten-year maturity. Table 2 summarizes the relationships between these quantities and the model's state variables.

There are two channels that drive SB correlations under the baseline model, in which all shocks in (1) are assumed to be uncorrelated. The first channel occurs via shocks to expected consumption growth ($\epsilon_{x,t+1}$). If this shock is positive, the higher expected future cash flows result in higher stock prices. Bond yields will also increase, as the demand for money rises due to the intertemporal consumption smoothing motive. Since stock and bond returns will have opposite responses, this channel implies a negative SB correlation.

The second channel is the result of shocks to consumption growth uncertainty ($\epsilon_{v,t+1}$). Stock market variance will rise following a positive uncertainty shock, raising the risk premium and lowering equity valuations. At the same time, bond yields will drop due to a precautionary savings effect. Therefore, an increase in uncertainty leads to stock and bond prices moving in opposite directions. The flight-to-quality phenomenon often refers to the negative SB correlation arising from this second channel.

While both channels in the baseline model imply a negative correlation between stock and bond returns, regardless of bond maturity, Panel A of Table 2 shows that the first channel is generally much stronger than the second. This is especially true for short-term yields, for which the correlation between yield changes and shocks to expected consumption growth is greater than 0.97 for both models. For long-term yields, the correlation between yield changes and shocks to the x_t process is approximately 0.94 for the baseline and 0.92

for the full model. In contrast, the correlation between yield changes and volatility shocks is -0.10 for short-term yields and -0.30 for long-term yields for the full model. These results establish the close connection between interest rates and the expected consumption growth process that underlies our empirical analyses. The table also shows that shocks to the market portfolio are highly correlated with realized consumption growth for both models.

While Table 1 shows that our generalized model exhibits a similar average SB correlation, CGP causes this correlation to vary over time. For example, the SB correlation should increase when CGP (ρ_t) decreases. To illustrate this point, suppose there is a positive shock to expected consumption growth ($\epsilon_{x,t+1} > 0$), which is likely to coincide with a decline in current consumption when ρ_t is negative. In this case, bond yields will increase as the economy expects higher levels of future growth, while the negative shock to current consumption will lower equity values. While the net effect may be that equity values rise due to higher expected long-run growth, the rise will be moderated by the negative shock to current consumption. Therefore, a negative ρ_t will lead to the SB correlation being less negative than usual and perhaps even positive.

Figure 2 shows how the SB correlation varies with CGP (ρ_t). Panel (a) shows the relationship for two different bond maturities. Other panels consider only the correlation based on the short-term bond but examine sensitivity to key parameters. Panel (b) examines different values of the correlation between consumption growth and uncertainty (ϱ_{cv}), while Panel (c) shows results for different values of risk aversion. Panel (d) considers different values of the intertemporal elasticity of substitution, and Panel (e) examines different values of the persistence of the CGP covariance process (ω_1). For comparison, the horizontal solid

lines in each panel depict the corresponding values under the baseline model, in which CGP is assumed to be constant at zero.

Overall, this figure confirms the negative relation between ρ_t and the SB correlation is slightly convex in ρ_t . In comparing the panels, the value of risk aversion coefficient (γ), the intertemporal elasticity of substitution ψ , and the bond maturity all have relatively weak effects. Results are also similar when $\psi < 1$, although the model will in that case have a countercyclical wealth-consumption ratio. The figure confirms the robustness of the model's implication that low CGP values are associated with high SB correlations.

We also examine this relationship using simulation. Panel B of Table 2 examines the “correlation of correlations.”⁵ While the figure shows that CGP (ρ_t) and the SB correlation have a nonlinear relationship, the table helps assess the goodness of fit of the linear projection of the SB correlation onto ρ_t . We find that ρ_t and the SB correlation is almost perfectly negatively related for both models, with correlations below -0.99 . Thus, our model suggests that the SB correlation is a very good proxy for the less easily observed ρ_t process.

2.4. Conditional moments of consumption growth

The key assumption of our generalized model is the time-varying persistence of consumption growth shocks. In this section, we show how this assumption affects the conditional distribution of consumption growth for different values of ρ_t .

While higher CGP should clearly increase the serial correlation in consumption growth, it is difficult to assess the strength of this effect analytically. We therefore simulate 10

⁵For each simulation, we derive the value of SB correlation using the analytic formula and compute the correlation to ρ_t .

million months of data from our full model and compute approximate conditional moments by separating the simulated sample into narrow bins (e.g., $[-0.05, 0)$, $[0, 0.05)$, etc.) according to the value of ρ_t .⁶ We then compute moments of interest using all the observations in each bin. The simulation applied is at a monthly frequency, but we aggregate consumption growth to the quarterly level by taking the sum of three consecutive monthly growth rates.

Panel (a) of Figure 3 describes how the first-order serial correlation of quarterly and annual consumption growth relates to CGP. Due to the presence of the LRR process, both quarterly and annual serial correlation are positive even for moderately negative values of ρ_t . For higher values of ρ_t the serial correlation is higher. The simulated ρ_t process has a mean of -0.128 and a standard deviation of 0.23 , so it tends to fluctuate in the range from about -0.6 to 0.4 . The panel shows that when ρ_t increases from the bottom to the top of this range, the serial correlation of consumption growth increases from -0.01 to 0.09 at the quarterly frequency and from -0.01 to 0.18 at the annual frequency.

To further understand the driving forces of the variation in serial correlation, we examine the conditional slope coefficient of the regression of shocks to expected consumption growth on contemporaneous realized consumption growth. These slope coefficients are plotted, as a function of CGP (ρ), in Panel (b). The figure shows that a 1% increase in consumption growth leads to a -0.09% decrease in the expected growth when $\rho_t = -0.5$, but it leads to an increase of $+0.15\%$ when $\rho_t = 0.5$.⁷

⁶In simulating the CGP process, the correlations are outside the boundary of -1 and 1 less than 0.05% of the time. If these boundaries are reached, we set the value to -1 or 1 .

⁷In the regressions, we match the horizons with which consumption growth and shocks to expected growth are measured.

2.5. Stock return predictability of bond yields

A final implication of the model is that CGP should modulate the strength of the predictive relationship between bond yields and future stock returns. This result enriches standard consumption-based asset pricing models, which imply that bond yields should negatively predict future stock returns. This occurs because the equity risk premium increasing while bond yields are decreasing in consumption volatility, implying a negative relationship between the equity risk premia and bond yields.

While several studies, starting with Fama and Schwert (1977), find a negative relation between stock returns and lagged bond yields, the negative relationship appears to be sample dependent. Additionally, as evidenced by Welch and Goyal (2008) the significance level of this relation is well below that of other predictors, such as the aggregate dividend yield.

Our model implies that the strength of this form of stock market return predictability depends critically on the level of CGP. Bond yields are the inverse of the expected marginal utility of investors, which is closely related to the level of expected consumption growth. Meanwhile, the stock risk premium is increasing in volatility. Therefore, the negative relationship between bond yields and stock risk premia should be stronger when expected consumption growth is more negatively related to volatility.

As a result, given the negative correlation between uncertainty shocks and consumption, we expect the negative predictive relationship between future stock returns and bond yields to be stronger when CGP is high, or equivalently, when the SB correlation is low. Using the simulations described above, we examine the correlation between the stock risk premia and

bond yields. This analysis uses the analytic formula for the market risk premium.

Panel (c) of Figure 3 shows that for values $\rho_t > -0.5$, there is a negative relationship between bond yields and the market risk premium, as generally implied by many asset pricing models. But whereas other models imply that the degree of predictability is constant, our model suggests that it is highly time-varying. The figure implies that bond yields will be poor predictors of market returns when CGP is low or the SB correlation is high, but that predictability will increase as CGP rises and the SB correlation drops.

3. Data

Quarterly consumption data is obtained from the National Income and Product Accounts (NIPA), provided by the Bureau of Economic Analysis. We measure consumption at the quarterly frequency as the sum of the real personal consumption expenditure on non-durables and services on a per capita basis. Specifically, we take the quantity index of NIPA Table 2.3.3 and divide it by the total population obtained from NIPA Table 7.1. Consumption growth is defined as the first log difference and is computed from 1962 to 2019.

We supplement the NIPA data with other consumption measures that are arguably less noisy and/or less affected by time aggregation. First, we consider fourth-quarter to fourth-quarter consumption growth, as analyzed by Jagannathan and Wang (2007). They conjecture that a disproportionate fraction of the population is likely to review their consumption decisions in the fourth quarter, making fourth quarter measurements more reflective of economic conditions. Second, we use the unfiltered consumption series of Kroencke (2017). Kroencke argues that the filtering and smoothing process implemented in the NIPA data

adds noise to the consumption measures that obscures their relationship to asset prices, and he proposes a method to reverse the effects of these transformations. Third, since the fourth-quarter consumption and unfiltered consumption series are both at the annual frequency, we also implement the empirical analysis using NIPA annual consumption. Each of these annual consumption measures is obtained from Tim Kroencke’s website.

To proxy for expected consumption growth, we use data from the Survey of Professional Forecasters (SPF), obtained from the Federal Reserve Bank of Philadelphia. The sample for the survey data begins in the third quarter of 1981. We use the four-quarter-ahead median forecast in real consumption expenditures.

For economic uncertainty, we use the 12-month macro and real uncertainty measures from Jurado, Ludvigson, and Ng (2015). These are obtained from Sydney Ludvigson’s website and are available from 1961 to 2019. To reduce the noise that comes from the imprecise timing of these measurements, we analyze growth expectation and uncertainty estimates at the yearly frequency, using observations from the last quarter of the year.

Bond yields are obtained from the Federal Reserve Bank of St. Louis’ website. The nominal one-year and ten-year yields are available from 1962 to 2019, while we analyze real yields over the period from 2003 to 2019. Real yields are constructed from ten-year Treasury Inflation-Protected Securities (TIPS). We use them starting in 2003 to avoid well-known illiquidity problems in the early years of that market (e.g., Dudley, Roush, and Ezer 2009, Gürkaynak, Sack, and Wright 2010, D’Amico, Kim, and Wei 2018). Excess market returns and total market returns are from Ken French’s website.

The calibration in Section 2.2 required us to compute moments of real returns and yields.

To compute averages of real variables, we subtract the average changes in the Consumer Price Index, obtained from the Bureau of Labor Statistics, over the entire calibration period. To compute the standard deviation of real bond yields and the stock-bond return correlation, we make several assumptions. One is that the relative variances of shocks to inflation and nominal yields remains constant over the entire sample period. Another is that inflation follows a unit-root process. That is, the change in expected inflation equals the unexpected price change in the previous period.

We first calculate the variance ratio (VR) of inflation, defined as in Duffee (2018a), which is the relative ratio of the variance of inflation shocks to the variance of nominal yields. The variance of real yields is then computed by multiplying the variance of nominal yields by $(1 - VR)$. In computing the real SB covariance $Cov(\Delta y_{t+1}^r, R_{m,t+1}^r)$, we assume that the inflation expectation equals past realized inflation and compute the covariance by:

$$Cov(\Delta y_t^r, R_{m,t}^r) = Cov(\Delta y_{t+1} - \Delta \pi_{t+1}, R_{m,t+1} - \Delta \pi_{t+1}) = Cov(\Delta y_{t+1}, R_{m,t+1}) - Var(\Delta \pi_{t+1}),$$

where y_{t+1} is the nominal bond yield, y_{t+1}^r is the real bond yield, $R_{m,t+1}$ is the nominal stock return, and $R_{m,t+1}^r$ is the real stock return. The variance of real stock returns is $Var(R_{m,t}^r) = Var(R_{m,t}) - Var(\Delta \pi_t)$, which is very close to the variance of nominal stock returns.

We also use several measures of wealth. In addition to the value-weighted stock market index, these are the value of assets from Sydney Ludvigson's website and used in Lettau and Ludvigson (2001), the All-Transactions Housing Price Index of the U.S. Federal Housing Finance Agency, and the net worth of households and nonprofit organizations from the Federal Reserve Bank of St. Louis.

4. Empirical Results

The primary prediction of our model is the negative relationship between consumption growth persistence and the stock-bond correlation. In this section we test this prediction using quarterly consumption growth, several annual consumption measures, and an expected consumption growth series from the Survey of Professional Forecasters.

We estimate the stock-bond return correlation as the negative of the correlation between first differences in yields and stock returns, computed on a rolling basis using daily observations over the last 365 calendar days. This estimate approximates the true SB correlation, as it ignores the effect of convexity, but it nevertheless should be highly accurate.

Since the SB correlation can be measured using different bond maturities, we compute several such correlation series. Specifically, we report results based on one-year and ten-year constant maturity bonds, although using other maturities produces very similar results. Given that inflation likely contaminates our measures of the SB correlation, we also calculate the SB correlation using real yields drawn from TIPS prices. Real yields are the difference between the ten-year nominal yields and the ten-year break-even inflation rate. We refer to this correlation, also estimated using a one-year rolling window, as the real SB correlation.

Establishing the connection between CGP, which is difficult to measure, and the SB correlation, which can be measured accurately using high-frequency asset price data, allows us to test additional implications of our model that would otherwise prove difficult. The later parts of this section test these implications using the SB correlation as a proxy for CGP.

4.1. Serial correlation of consumption growth

A direct implication of our model is that the persistence of consumption growth shocks should be reflected in the level of the SB correlation. We test the relationship by examining autocorrelations in consumption growth at different horizons, where our model predicts that serial correlation will be larger in periods when the SB correlation is more negative. In interpreting these results, we note that first-order autocorrelations obtained from consumption growth data are likely high due to time-aggregation effects and measurement issues that are absent from our theoretical model. As shown by Breeden, Gibbons, and Litzenberger (1989) and Heaton (1993), if investors make consumption decisions more frequently than the interval over which consumption is measured, first-order autocorrelation in growth rates may be as high as 0.25 in quarterly data even if higher frequency changes are unpredictable. However, serial correlations at longer lags should be immune to this effect.

We first estimate a predictive regression of quarterly consumption growth on its own lag. We then test whether this relationship is stronger during high or low SB correlation periods by adding an interaction term. The regression we estimate is

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{t+k}, \quad (8)$$

for $k = 1, 2, 3, 4$, where Δc_t is quarterly consumption growth and $\hat{\rho}_{SB,t}$ is one of the SB correlation series described previously. If, as implied by our model, the serial correlation is stronger for periods in which the SB correlation is negative, we should see a negative slope on the interaction term ($\alpha_2 < 0$).

Table 3 summarizes the results of these regressions. Panel A of the table first shows

the simple regressions in which the only explanatory variable is a single lag of consumption growth. We observe significant positive autocorrelations at up to a four-quarter horizon, and the first-order autocorrelation is much larger than the value implied by time aggregation. These results are consistent with the base assumption of long-run risk in consumption growth, and the results are comparable to values reported by previous studies (e.g., Savov 2011).

However, our primary interest is in testing the sign and the significance of the interactive coefficient, α_2 . The results of this regression are summarized in Panel B for multiple horizons. The regression results in the first two columns show that the first-order serial correlation increases when the SB correlation is more negative. The interaction coefficients are negative and statistically significant for both the one-year and ten-year SB correlations. Quantitatively, a 0.1 increase in the SB correlation leads to a 0.03 – 0.04 decrease in the first-order serial correlation, which is slightly higher than what is implied by our model. We then increase the forecast horizon by replacing the one-quarter-ahead dependent variable with one that is between two and four quarters ahead. Overall, the results in this panel are all consistent with our model predictions.

A potential concern is that a significant fraction of the variation in nominal yields may be driven by inflation, which falls outside the scope of our model. Whether nominal yields change more due to expected inflation or real yields is controversial.⁸ To study whether the above results originate from the correlation of stock returns with changes in the inflation rate or real bond yields, we repeat the previous analysis using the real SB correlation, which

⁸For example, Fama (1975) finds a strong relationship between the nominal interest rate and future inflation for the pre-1970 sample. More recently, Ang, Bekaert, and Wei (2008) also find that most of the monthly variation in nominal interest rates results from fluctuations in expected inflation. However, Mishkin (1992), Barr and Campbell (1997), and Duffee (2018a) suggest that most of the variation in the nominal interest rate is due to the real interest rate.

is based on real yields from 10-year TIPS. We also estimate the correlation between stock returns and changes in the breakeven inflation rate. We multiply this correlation by negative one to be consistent with the signs in the first two panels.⁹

Panel C of Table 3 shows the results of estimating regression (8) after replacing the SB correlation with one of these two alternative measures. While no time variation in consumption persistence is found at the one-quarter horizon, higher-order correlations do appear to be significantly lower when the real SB correlation ($\hat{\rho}_{SR,t}$) is higher. This finding is again consistent with a negative relation between the SB correlation and CGP.

The panel also shows that the correlation between stock returns and changes in breakeven inflation ($\hat{\rho}_{S\pi,t}$) is generally uninformative about future consumption and is not significantly related to consumption persistence. If anything, a lower stock-inflation correlation is associated with more persistence in consumption growth, which again confirms the hypothesis that inflation effects are not responsible for the results given in Table 3. These results support our earlier conclusion that nominal SB correlations are informative in this setting because they are related to the corresponding correlation based on real yields.

While the regressions shown in Table 3 examine non-overlapping growth rates at different horizons, Figure 4 examines consumption growth autocorrelation using overlapping longer-horizon growth rates. These regressions are identical to equation (8), except that the dependent variable is the average consumption growth rate from quarter $t + 1$ to quarter $t + k$. Each panel plots the coefficient on the interaction term (α_2) for different horizons (k),

⁹In computing the stock-bond correlation, we approximate the value by taking the negative of the correlation between stock returns and first differences in bond yields. Since changes in nominal bond yields can be decomposed into the sum of changes in real bond yields and changes in expected inflation, we also take the negative value of the correlation between stock returns and inflation shocks.

as well as 68%, 90%, and 95% confidence intervals, where the panels differ with respect to the SB correlation series used and the sample period. While we present results only for the ten-year nominal SB correlation, corresponding results based on 1-year nominal yields are very similar.

Panel (a) of Figure 4 reports the results obtained using the full sample period at horizons of one to ten quarters. The graph shows that predictability is observed even over very long horizons, consistent with the premise that the SB correlation is associated with the correlation between long-run growth and current consumption growth.

Panel (b) shows the same result, still based on nominal yields, for the shorter sample in which TIPS data are available, while Panel (c) shows the corresponding results using real yields. While the results based on TIPS are somewhat stronger, both graphs indicate more long-term persistence in consumption growth in low SB correlation environments.

The final panel of Figure 4 examines the role of the stock-inflation correlation at longer horizons. As in Table 3, a lower stock-inflation correlation decreases the persistence of consumption growth¹⁰, an effect that becomes statistically significant over longer horizons. This is again inconsistent with the hypothesis that inflation effects are responsible for the relation between CGP and the SB correlation. While the interpretation of this result is difficult given that inflation falls outside the scope of our model, the results reinforce the conclusion that the SB correlation is related to consumption persistence due to the behavior of real rates.

¹⁰Note that we take the negative sign of inflation to compute the correlation. If our results are driven by the correlation between stock returns and the expected inflation component of bond yields, we would expect the opposite of what we find.

Although NIPA consumption measures are widely used both in the macroeconomics and macro-finance literature, several problems with these data have been identified. The aggregation bias mentioned above implies that the measured consumption growth at the quarterly level may have a serial correlation of 0.25 or more even in the absence of any true persistence in growth rates. Furthermore, Breeden, Gibbons, and Litzenberger (1989) suggests that the effect of time-aggregation bias is worsened by the use of interpolation in the construction of NIPA consumption data. Finally, consumption decisions made at different fixed intervals across agents, as suggested by Grossman and Laroque (1990) and Lynch (1996), generate serial correlation even in the absence of any correlation between current and expectations of consumption growth.

While some of these concerns are alleviated by the evidence showing longer-term persistence, we perform additional analysis using alternative measures of consumption for added robustness. First, we measure consumption only at the fourth quarter, following Jaganathan and Wang (2007). Given that the tax year ends in December, they argue that investors are more likely to make joint consumption/investment decisions in the fourth quarter (Q4) of each year. In addition, as noted by Savov (2011) and Kroencke (2017), using fourth-quarter consumption also mitigates time-aggregation bias. Second, we use the unfiltered consumption measure of Kroencke (2017), who applies a backward recursion to the filtered NIPA consumption process. The resulting unfiltered series also mitigates time-aggregation bias by adding a correction factor to the recursion.

Table 4 summarizes the results of regression (8) using these alternative consumption series. Since these measures are constructed at the annual frequency, we compare them

results with the NIPA consumption measured at the annual frequency. Panel A shows results based on the nominal SB correlation, while Panel B uses the real SB correlation as well as the correlation between stock returns and the breakeven inflation rate.

Overall, the results using these alternative consumption data are qualitatively similar to those of Table 3. Focusing on Panel A, there is a positive serial correlation of consumption growth measured either using NIPA annual consumption or NIPA Q4 consumption. The positive first-order serial correlation disappears, however, if the unfiltered data of Kroencke (2017) is used to compute consumption growth.

The main focus of this paper is whether this serial correlation varies with the SB correlation. Similar to previous results, we test this relation using the interactive regression in Equation (8). For all three annual consumption measures considered, the coefficient on the interaction term is negative and statistically significant. These results confirm the hypothesis that consumption growth is more persistent when the SB correlation is negative.

Panel B of Table 4 provides results based instead on the real SB correlation or the inflation component of the SB correlation. We find comparable results to Panel C of Table 3, in that inflation does not appear to be the primary driver of our main findings. The difference in this table is that the inflation component of the SB correlation is now statistically significant, though with a positive sign, which is again inconsistent with an inflation driving our results based on nominal yields.

Table 5 presents an alternative and arguably more direct test of the model. Here we examine whether the relationship between shocks to the long-run consumption growth forecast ($\Delta\hat{x}_t$) and to current consumption growth (Δc_t) depends on the level of the SB correlation.

Since the timing of SPF survey forecasts during the quarter is vague, we use annual data to test this relationship. Each panel differs by how consumption is measured. Focusing on the first regression, when changes in expected growth are regressed on contemporaneous consumption growth, we find a weak negative relationship between the two shocks.

We next test whether the growth forecast is more positively correlated for the period beginning 1999, during which time the SB correlation was usually negative. If there was a regime shift around 1999 that increased the overall level of CGP, we expect a more positive relationship between consumption growth and its forecast in the later period. We therefore estimate the regression

$$\Delta \hat{x}_t = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 1_{99+,t} \times \Delta c_t + \alpha_3 1_{99+,t} + \epsilon_{2,t}, \quad (9)$$

where 1_{99+} is a dummy variable that takes a value of 1 starting in 1999 and 0 before, and where \hat{x}_t is the long-run SPF forecast of consumption growth.

The second column of each panel shows the results of this regression, where our primary focus is on whether $\alpha_2 > 0$, as our hypothesis suggests. All three regressions show that the relationship between long-run SPF forecasts and current consumption growth is more negative in the earlier sample.

Given the negative relation between CGP and the SB correlation, the model predicts that the α'_2 coefficient in the regression

$$\Delta \hat{x}_t = \alpha'_0 + \alpha'_1 \Delta c_t + \alpha'_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha'_3 \hat{\rho}_{SB,t} + \epsilon_{2,t} \quad (10)$$

to be negative. We report these regression results in the last three columns of each panel. The results show that the relationship between current consumption growth and long-run

expectations is more positive when the SB correlation is lower, confirming our theoretical prediction presented in Figure 3 (b). Similar results are obtained for SB correlations based on the one-year and ten-year nominal yields and the ten-year real yield.

4.2. Expected consumption growth and uncertainty

Several recent studies (e.g., Nakamura, Sergeyev, and Steinsson 2017, Bollerslev, Xu, and Zhou 2015) document the unconditionally negative relationship between economic uncertainty and future expected consumption growth. Our model implies that this relationship also varies with CGP.

We test this hypothesis in Table 6 using expected consumption growth from the SPF and the macro and real uncertainty measures of Jurado, Ludvigson, and Ng (2015). Each panel in the table uses a different measure of uncertainty. Similar to Table 5, we use fourth-quarter data for this analysis.

We first test whether the relationship between expected consumption growth and uncertainty is more negative during the period beginning 1999. If CGP increases in this sample, we expect a stronger negative relationship between expected consumption growth and uncertainty. We use the contemporaneous regression

$$\Delta \hat{x}_t = \beta_0 + \beta_1 \Delta UNC_t + \beta_2 1_{99+,t} \times \Delta UNC_t + \beta_3 1_{99+,t} + \epsilon_t, \quad (11)$$

where 1_{99+} is a dummy variable that takes a value of 1 starting in 1999 and 0 before, \hat{x}_t is the long-run SPF forecast of consumption growth, and UNC_t is a measure of economic uncertainty. If our hypothesis is true, we expect β_2 to be negative.

The first two columns of each panel summarize the results and provide strong support

for our hypothesis. For both uncertainty measures, we find that the relationship between expected consumption growth and uncertainty is more negative in the later sample.

We also test the hypothesis by replacing the dummy variable with the SB correlation, or

$$\Delta \hat{x}_t = \beta'_0 + \beta'_1 \Delta UNC_t + \beta'_2 \hat{\rho}_{SB,t} \times \Delta UNC_t + \beta'_3 \hat{\rho}_{SB,t} + \epsilon_t, \quad (12)$$

where $\hat{\rho}_{SB,t}$ is one of the SB correlation estimates. If the SB correlation is negatively related to CGP, we should obtain positive estimates for the β'_2 parameter.

Overall, the table provides reasonably strong support for our hypothesis. Using the nominal one-year or ten-year SB correlation in Panels A and B, we find a positive β_2 in every regression, which are statistically significant in most cases. The last two columns of the panels instead use the real SB correlation. These results are somewhat weaker, which is likely due to the shorter sample period and collinearity.

Taken together, these results paint a consistent picture that the negative relationship between consumption growth and economic uncertainty is stronger when the SB correlation is negative or when CGP is positive, confirming a key prediction of our model.

4.3. Stock return predictability

A final implication of the model is that the relationship between bond yields and future stock returns should vary over time. A number of papers have studied Treasury bond yields as a stock market return predictor. Fama and Schwert (1977) estimate a simple predictive regression of future stock returns on lagged bond yields and find a negative slope, which they interpret as the result of stocks being inflation hedges. Breen, Glosten, and Jagannathan (1989) further confirm the economic significance of this predictability. More

recently, Ang and Bekaert (2007) find that short-term Treasury yields and dividend yields jointly predict stock returns in many international markets. They argue that the yields represent a component of the discount rate used by investors to value equities. Campbell and Thompson (2008) also document statistically significant in-sample predictability, but Welch and Goyal (2008) report weak in- and out-of-sample performance.

While equation (7) suggests a negative relationship between discount rates and the SB correlation, we do not report results for regressions in which the market return is regressed directly on $\hat{\rho}_{SB,t}$. As shown by Kozak (2021), there is no clear evidence of any such predictive relation, though he argues that the negative relation between changes in the SB correlation and stock return represents indirect evidence. In contrast, the results we show here provide direct evidence of stock market predictability, but it is driven by the interaction between the SB correlation and bond yields rather than the SB correlation alone.

Our model suggests that the extent to which the bond yield predicts stock returns depends on CGP. Specifically, a higher CGP is associated with a more negative predictive slope coefficient between bond yields and future returns. Given the relation between CGP and the stock-bond correlation, we test this hypothesis in monthly regressions of the form

$$R_{S,t,t+\tau}^e = b_0 + b_1 y_t + b_2 y_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1}, \quad (13)$$

where $R_{S,t,t+\tau}^e$ is the τ -month excess market return, y_t is a bond yield, and $\hat{\rho}_{SB,t}$ is a SB correlation estimated from a one-year rolling window of daily returns. Panels A and B of Table 7 consider regressions based on nominal yields, while Panel C examines real yields over the post-2002 subsample.

Panel A first considers the simple predictive regression in which leading stock returns are

regressed on bond yields alone, separately for samples before and after 1999, using either the one-year or ten-year Treasury yield. Comparing the two subsamples, we see that the regression slope coefficient is uniformly negative and significant in the latter sample, with high R^2 s, but generally insignificant in the earlier period. These results are consistent with consumption growth being more persistent for the sample beginning in 1999 and the sign change in the SB correlation that occurred around that time.

The novel implication of our model is that the slope should be more negative when the SB correlation is lower, implying $b_2 > 0$ in equation (13). The results, summarized in Panel B, show strong support for this prediction, with b_2 coefficient estimates that are positive and significant across all horizons for both yields. To understand the strength of the relation, consider the relation between one-month excess market returns and one-year Treasury yields that would hold if the SB correlation were equal to 0.4. In this case, the conditional slope coefficient would be a paltry -0.045 ($-0.267 + 0.554 \times 0.4$), implying that yields have essentially no predictive power for future returns. Similar conclusions hold for longer investment horizons as well. However, were the SB correlation instead equal to -0.5 , the conditional slope coefficient would increase in magnitude to around -0.544 . A one percentage point increase in the one-year Treasury yield would then be associated with a 0.5% decline in monthly stock returns and, following the same logic, a 1.7% decline in three-month returns, a 3.2% decline in six-month returns, and a 4.7% decline in 12-month returns. Economic magnitudes are similarly large when based on ten-year yields.

Panel C repeats these regressions using real yields and the real SB correlation rather than nominal values. Overall, we see similar results, albeit with noticeably higher interaction

coefficients. The coefficients are all statistically significant, echoing previous panels. A potential reason for the large interaction effects is the shorter sample period considered, in which the real SB correlation does not vary as much as it does over the full sample.

Many asset pricing models imply a negative relationship between bond yields and the stock risk premium, as high levels of uncertainty means lower bond yields and a higher risk premium. Therefore, it is puzzling that the empirical relationship is so weak. Our results show that the predictive relationship is stronger than it may appear, but only during periods when proxies indicate that CGP is high.

4.4. The recent shift in consumption growth persistence

The results presented so far suggest that an increase in the persistence of consumption growth was largely responsible for the shift in signs of the SB correlation that happened around 1999. But what was the reason for the significant increase in CGP observed around this time?

Production-based models, such as Kaltenbrunner and Lochstoer (2010), suggest that an increase in the magnitude of permanent productivity shocks will cause CGP to rise when consumers face adjustment costs. More volatile persistent shocks will also increase the variability in valuation ratios to a much greater extent than more volatile transient shocks. Consistent with this, the period starting in 1999 is notable for its inclusion of several major asset market “bubbles” and crashes (e.g., the “dot-com” crash, the real estate boom, and the Great Financial Crisis).

Regardless of the reason for such fluctuations, any permanent shock to asset values may

be expected to produce some level of consumption persistence. While this claim cannot be demonstrated within our model, which features an exogenous consumption process, it is intuitive. According to the permanent income hypothesis, a positive wealth shock will raise the consumption level in perpetuity. However, if agents face adjustment costs, this higher level will not be reached immediately. Instead, they will experience a sequence of consecutive positive growth rates as consumption rises to its new steady state level. If wealth falls, a sequence of negative consumption growth rates will result. In a period with multiple booms and busts, consumption is constantly trending towards some target value, but that target is moving, sometimes higher and sometimes lower than current consumption, raising CGP.

In this section, we present some evidence, which we view as suggestive, that is consistent with the greater consumption persistence of this period being driven by fluctuating asset values. In particular, we show that consumption growth rates have become more responsive to past asset returns since 1999. These results are in line with recent papers by Laibson and Mollerstrom (2010), Mian and Sufi (2011), and Chen, Michaux, and Roussanov (2020), which show a stronger tendency for consumption to be driven by fluctuating asset valuations over this period. Our new finding is that asset returns affect consumption growth at longer horizons than documented previously, particularly in the latter period.

We first demonstrate the changing relation between asset returns and consumption growth by computing the predictive correlation ($\text{Corr}(R_t, \Delta c_{t+k})$) for different horizons k and using returns R_t on different wealth proxies, as defined in Section 3. We analyze horizons of one to 12 quarters and compute the correlations separately for the period before 1999 and starting in 1999.

The results, shown in Figure 5, suggest a major shift in the predictive relationship, particularly at longer horizons. Prior to 1999, the predictive relationship between asset returns and future consumption growth was relatively weak. Statistical significance, which is indicated by the estimated correlation exceeding the corresponding dashed line, is observed in some cases, but mainly at short horizons. For the sample starting in 1999, correlations are, in many cases, twice as large, if not more, and highly significant even at multi-year lags. The largest shift occurs in response to housing returns, to which consumption responded little before 1999 and very significantly thereafter.

The relationship between wealth and consumption growth also features critically in the model of Lettau and Ludvigson (2004). They find that fluctuations in the wealth-to-consumption ratio only weakly predict consumption growth but are strongly negatively related to future changes in aggregate wealth. These findings explain the success of the *cay* variable of Lettau and Ludvigson (2001) in predicting returns on the stock market. Because Lettau and Ludvigson's (2004) sample ends in 2003, their analysis focuses mainly on a period in which CGP is low, meaning that shocks tend to be transient. The effect of wealth shocks on consumption is therefore immediate, occurring mostly during the current period. Since the expected growth rate (x) responds little to wealth shocks, deviations in the wealth-to-consumption ratio must revert by a decline in the value of aggregate wealth.

In contrast, during the high-CGP period starting around 1999, our previous analysis shows that growth in aggregate wealth has a substantial positive effect on the long-term consumption growth rate. Since shocks are persistent during this period, any growth in wealth relative to current consumption is also likely to increase future consumption. When

transient deviations in the wealth-consumption ratio resolve by changing future consumption, such deviations no longer need forecast wealth changes or asset returns. This shift in the predictive information contained in the consumption-wealth ratio may underlie the poor post-sample performance of *cay* that has been documented by Goyal, Welch, and Zafirov (2021) and others.

We estimate the cointegrated vector error correction model examined by Lettau and Ludvigson (2004). We use the data from Sydney Ludvigson’s website and estimate the following model separately for samples before and after 1999:

$$\Delta \mathbf{X}_{t+1} = \mathbf{u} + \zeta \text{cay}_t + \mathbf{\Gamma} \Delta \mathbf{X}_t + \mathbf{e}_t, \quad (14)$$

where $\mathbf{X}_t = [c_t \ a_t \ y_t]$ is a vector of log consumption, wealth, and human capital, and *cay*_{*t*} is the deviation from common trend, which we estimate separately for each subsample.¹¹

Panel A of Table 8 shows the estimation results. For the pre-1999 sample, we find similar results to Lettau and Ludvigson. *cay* positively predicts asset growth, and there is no predictability of consumption growth. However, focusing on the later sample, there is no predictability of future asset growth. Instead, *cay* negatively predicts future consumption growth.

In panel B, we re-examine the predictive regression, where quarterly excess market returns are regressed on *cay*. We construct different *cay* series from each of the split samples analyzed in Panel A. For completeness, we also report the results from using the *cay* series directly taken from Ludvigson, which is estimated over a single extended sample period.

¹¹In order to follow standard notation, this section uses *y_t* to denote human capital, which is different from the rest of this article.

The results show that *cay* only predicts market excess returns for the pre-1999 sample, which approximately coincides with the sample studied by Lettau and Ludvigson (2001). For the sample beginning in 1999, we find statistically insignificant relationship between the consumption wealth ratio and the leading excess market returns. These results confirm our hypothesis that the degree of CGP determines the shocks' influence on future asset growth.

These results are not meant to be definitive, and a more careful analysis would likely require a further enhancement of the theoretical framework that we adopt here, but they suggest a mechanism that may underlie the changes in CGP and related changes in the stock-bond correlation that we have documented.

5. Conclusion

While the consumption process examined by Bansal and Yaron (2004) is highly successful in replicating key moments of asset returns, its assumption of independent shocks is inconsistent both with macroeconomic theory and with consumption data. In particular, the model does not account for the relationship between shocks to current consumption growth and expected future consumption growth, which we term consumption growth persistence.

Because of these assumptions, the model cannot match several well-documented features of financial markets. Most significantly, the correlation between stocks and bonds is highly time-varying in the data and varies with the level of stock market volatility. These effects are absent in the model of Bansal and Yaron, which features a constant stock-bond correlation.

We propose a model that allows for a significantly more realistic dependence structure.

Shocks to current and expected future consumption growth are stochastically correlated, which we view as a reduced form approach to modeling the relative importance of transitory and permanent shocks. Shocks to current consumption and consumption growth are negatively correlated at a fixed value, which maintains parsimony and reflects the likely importance of the precautionary savings motive.

The model implies that the correlation between stock and bond returns is decreasing in CGP. Empirically, we see that consumption growth tends to become more serially correlated in periods of more negative stock-bond correlations. This result provides evidence of time variation in CGP and also links CGP to correlations that are readily estimable from high-frequency asset price data.

Our model also predicts the negative relation between stock market volatility and the SB correlation observed in prior studies, such as Connolly, Stivers, and Sun (2005) and Baele, Bekaert, and Inghelbrecht (2010). This is the case because high consumption persistence makes cash flows and discount rates negatively correlated, which amplifies the effects of these shocks. Empirically, we find strong evidence for this relation.

Finally, we show in our model that bond yields should negatively predict future market excess returns, but only if CGP is sufficiently high. Again using the SB correlation to proxy for the unobserved CGP process, we find strong confirmation of this prediction in the significance of the coefficient on the interaction of bond yields and the SB correlation. We also show that the source of this predictability is the real SB correlation rather than the component related to inflation.

Combined with the observation that consumption growth persistence has increased markedly

since 1999, our model provides a new explanation for the dramatic downward shift in the stock-bond correlation occurring around this time. Additional evidence suggests that the increase in persistence may be the result of a greater role of asset valuations in driving long-run consumption growth rates. The result may be a marked change in the cointegrating relationship, studied by Lettau and Ludvigson (2004), between consumption, labor income, and aggregate wealth.

Overall, time-varying consumption growth persistence accounts for a number of stylized facts that are typically not linked together and whose explanations are still not fully understood. Furthermore, it uses an intuitive and relatively simple generalization of the standard LRR framework. As researchers examine the conditional implications of LRR more closely, it seems natural that time-varying correlations should play an important role.

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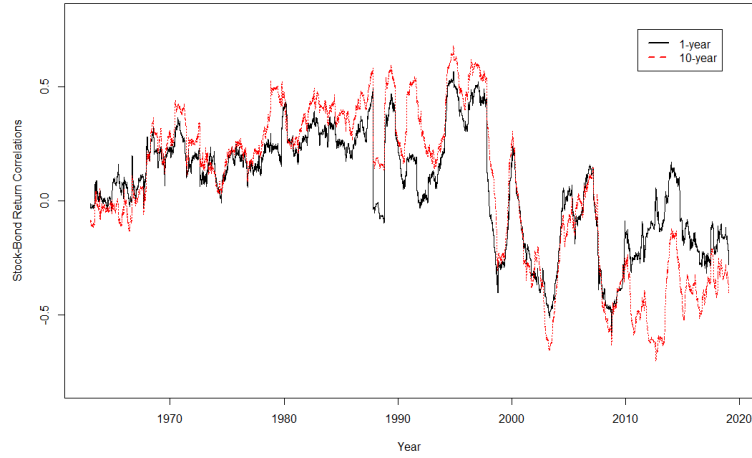
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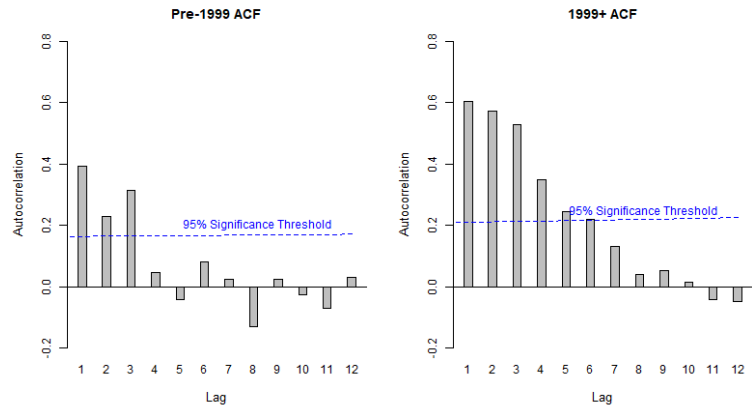
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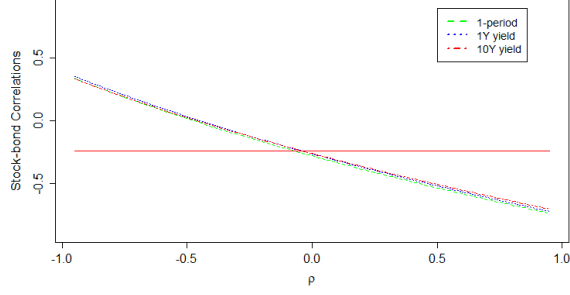
(a) The stock-bond correlation over time



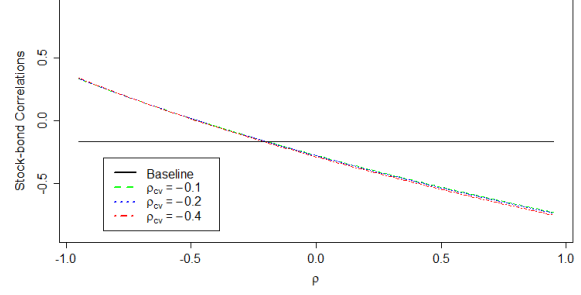
(b) Autocorrelations of consumption growth

Figure 1. Pre-1999 vs. 1999+ Comparison

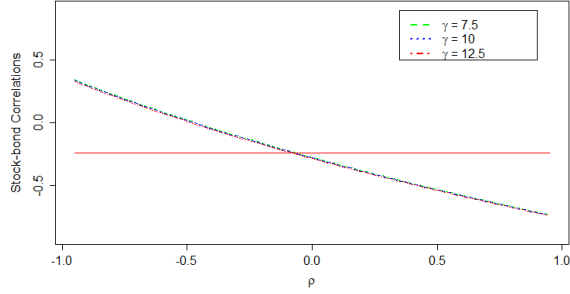
This figure shows the time-variation in the stock-bond return (SB) correlation in Panel (a) and the autocorrelations in consumption growth in Panel (b) for the pre-1999 (1962-1998) and 1999+ (1999-2019) sample periods. The SB correlation is estimated using first differences in daily one-year or ten-year bond yields over a rolling window of 365 calendar days.



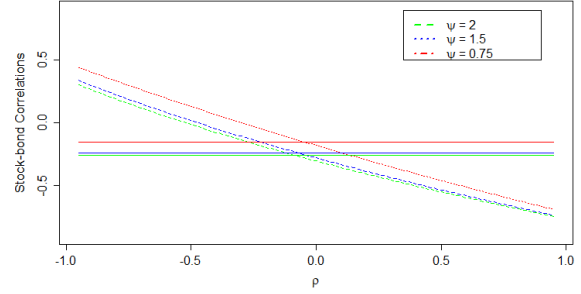
(a) SB correlations for different maturities



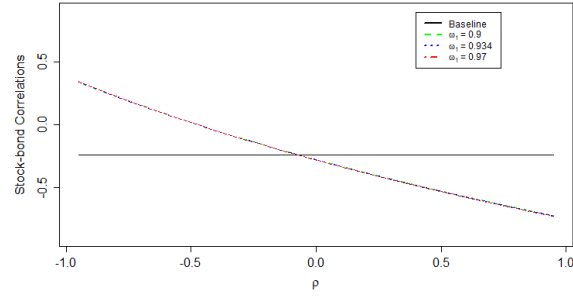
(b) SB correlations for different values of ρ_{cv}



(c) SB correlations for different values of γ



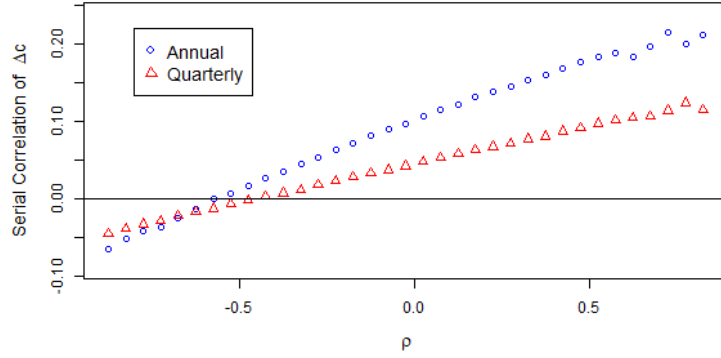
(d) SB correlations for different values of ψ



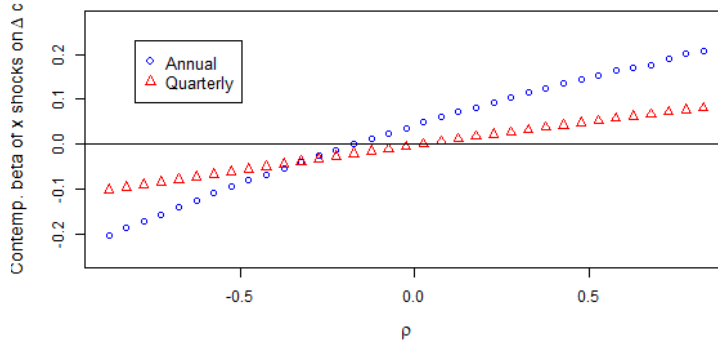
(e) SB correlations for different values of ω_1

Figure 2. Consumption Persistence and Model-Based Correlations

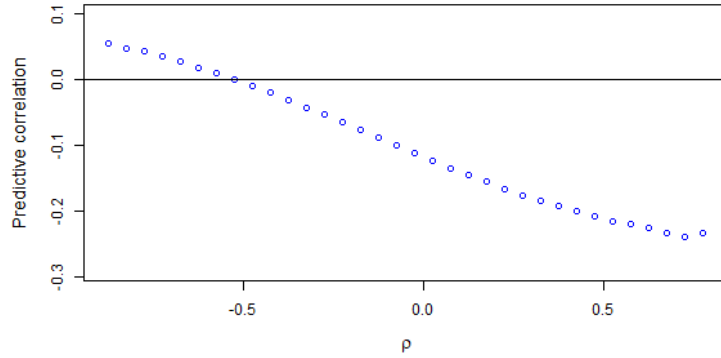
This figure shows the relationships between CGP and the stock-bond return correlation for different bond maturities and parameter assumptions. The value of the baseline model is shown in solid horizontal lines. The relationship for the full model is drawn in dashed lines. The panels show the relationship for different maturities (a), correlations between consumption and consumption volatility (b), risk aversion coefficients (c), intertemporal elasticities of substitution (d), and values of the persistence of the CGP process (e). In panel (e), we also vary the standard deviation parameter σ_ω so that the unconditional standard deviation of the CGP process is identical to its value under the baseline parameterization.



(a) Regression of Δc_{t+1} on Δc_t



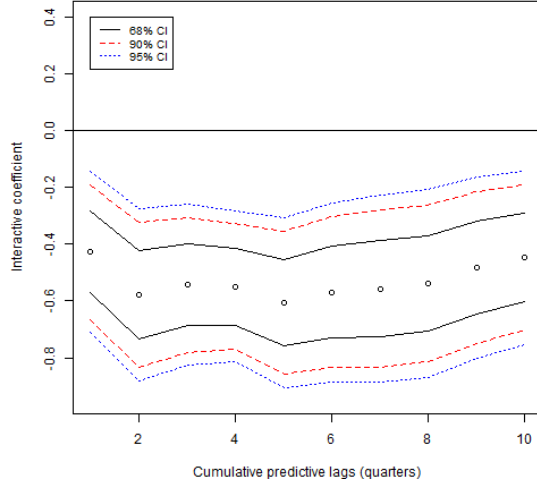
(b) Regression of x_t shocks on Δc_t



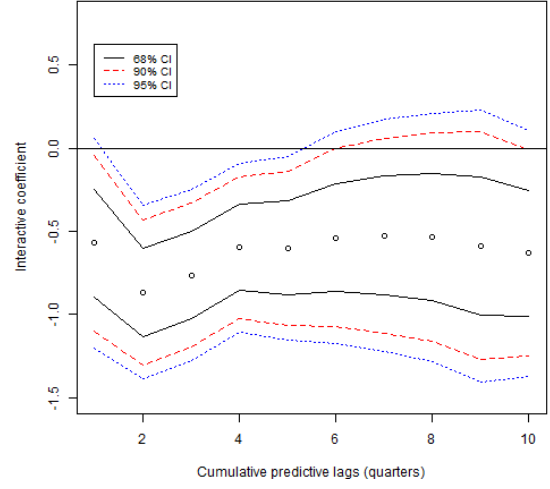
(c) Correlation between market risk premium and one-period yields

Figure 3. Simulation-Based Regression Betas and Correlations Conditional on CGP

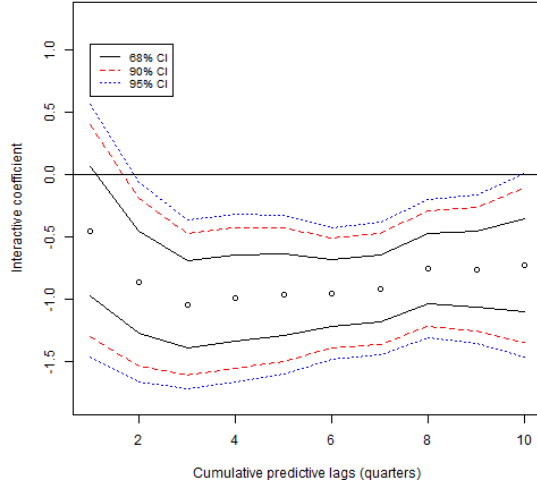
This figure depicts the relationship between CGP and the serial correlation of consumption growth (Panel (a)), the conditional slope coefficient of shocks to expected consumption growth regressed on the contemporaneous consumption growth (Panel (b)), and the conditional predictive correlation between the market risk premium and the lagged one-period bond yield (Panel (c)). For the first two panels, quarterly values are in triangles, and the annual values implied by the model are shown in circles.



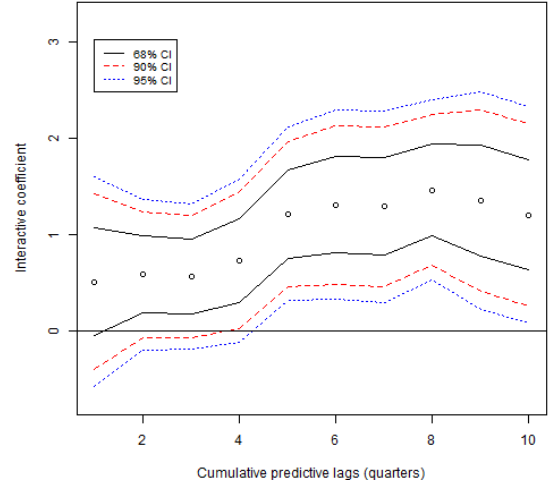
(a) Stock-bond correlations



(b) Stock-bond correlations (shorter sample)



(c) Real stock-bond correlation



(d) Stock-inflation correlation

Figure 4. Interactive Beta of Consumption Growth Regressions For Multiple Lags

This figure plots the slope coefficient estimates ($\hat{\alpha}_{3,k}$) from the interactive regression

$$\sum_{k=1}^K \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} \Delta c_t + \alpha_{2,K} \hat{\rho}_{SB,t} + \alpha_{3,K} \hat{\rho}_{SB,t} \times \Delta c_t + \epsilon_{t+K}$$

for different values of the interval (K). In Panels (a) and (b), $\hat{\rho}_{SB,t}$ represents the correlation between stock returns and nominal 10-year bond returns. Panel (c) uses the correlation with real 10-year bond returns instead, while Panel (d) uses the negative of the correlation between stock returns and inflation shocks. The lines show the 68%, 90%, and 95% confidence intervals computed using Newey-West standard errors with 12 lags. Panel (a) is based on the full 1962-2019 sample period, while other panels use the 2003-2019 sample.

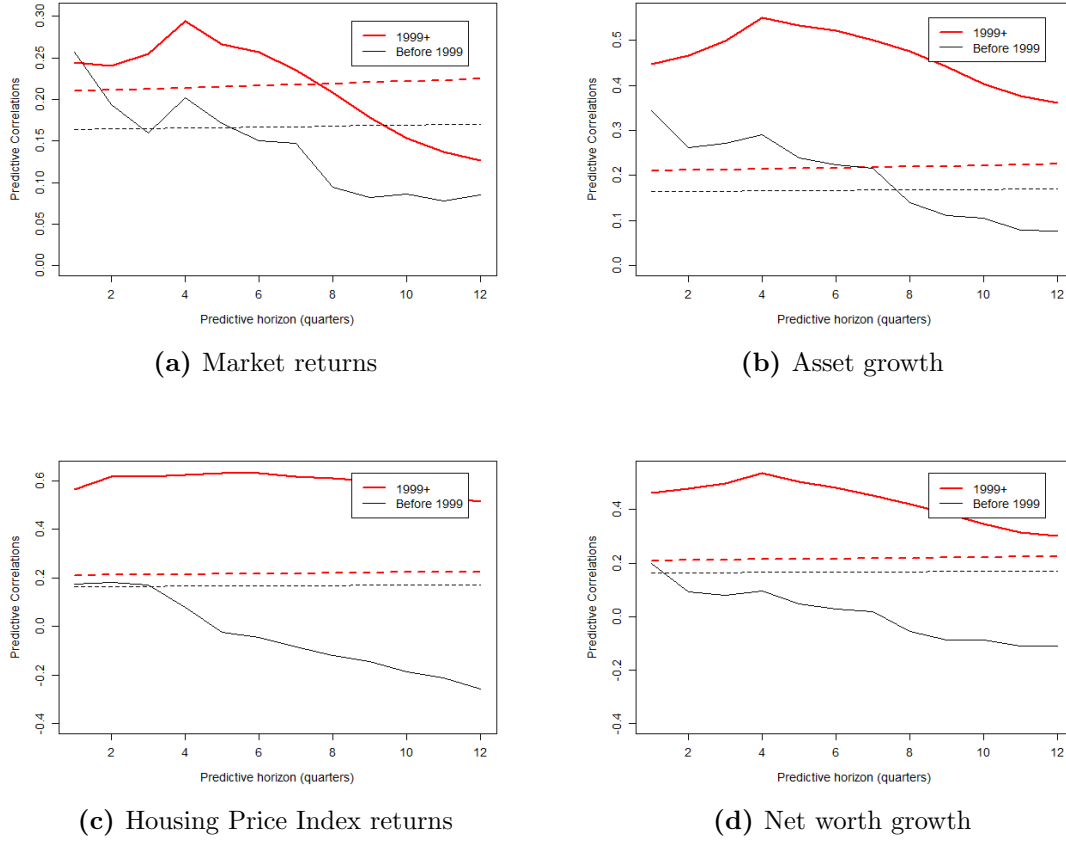


Figure 5. Consumption Growth Response to Past Asset Returns: Pre-1999 vs. 1999+

This figure computes the predictive correlation $\text{Corr}(\Delta c_{t+k}, R_{.,t})$ for $k = 1, \dots, 12$, where $R_{.,t}$ is the market return (Panel (a)), asset growth (Panel (b)), the housing price index return (Panel (c)), or net worth growth (Panel (d)) for the pre-1999 and 1999+ sample.

Table 1
Model Calibration

This table summarizes the parameters that describe the representative investor's preferences and the consumption and dividend growth, volatility, and covariance processes used in the main specification, as well as asset pricing moments implied by these parameters. Panel A shows the values of the parameters, and Panel B shows the moments obtained by simulating the model dynamics. y denotes a bond yield, R_m is the return on the market portfolio, σ_m is the volatility of the market portfolio, and ρ_{SB} denotes the stock-bond return correlation. Values in Panel B are annualized. We also simulate stock returns, bond yields, and stock market variance using the consumption/dividend dynamics assumed in the model. Panel C displays model-implied simulated moments of 60-month rolling-window correlations of stock returns with one- or ten-year bond yields. The simulated moments are compared with the data, in which we perform the same estimations.

Panel A. Parameters

Preferences		Consumption		Covariance	
γ	10	μ	0.0015	ω_0	-5.18×10^{-7}
ψ	1.5	ξ_1	0.95	ω_1	0.934
β	0.9985	ϕ_x	0.046	σ_q	6.3×10^{-4}
				ϱ_{cv}	-0.2
Variance		Dividend			
s_0	8.52×10^{-7}	μ_d	0.0015		
s_1	0.986	ϕ_d	2.5		
σ_v	2.6×10^{-4}	φ_{cd}	3.50		
		φ_d	4.50		

Panel B. Unconditional means (annualized)

Variable	Maturity	Model		Data	
		Baseline	Full	Real	Nominal
R_m		5.80%	5.53%	6.64%	10.44%
y	1Y	2.09%	2.15%	1.36%	5.10%
	10Y	2.05%	2.14%	2.45%	6.20%
σ_m		15.88%	15.69%	15.26%	15.26%
$SD(y)$	1Y	2.31%	2.34%	2.37%	3.39%
	10Y	0.62%	0.63%	1.86%	2.89%
$SD(\Delta y)$	1Y	0.73%	0.74%	0.83%	1.65%
	10Y	0.17%	0.18%	0.95%	1.15%
ρ_{SB}	1Y	-0.241	-0.184	-0.183	-0.092
	10Y	-0.241	-0.184	-0.121	-0.100

Panel C. Rolling-window simulated correlations

Variable	Statistic	Model		Data	
		Baseline	Full	Nominal	
ρ_{SB} (1Y)	Mean	-0.241	-0.187	-0.092	
	AC(1)	0.000	0.971	0.978	
	STD	0.000	0.133	0.179	
ρ_{SB} (10Y)	Mean	-0.241	-0.187	-0.100	
	AC(1)	0.000	0.971	0.971	
	STD	0.000	0.128	0.173	

Table 2
Relationships Between Simulated Values

This table summarizes the correlations between macroeconomic and asset pricing variables based on the simulation of different models. Panel A shows the relationship between Δc_{t+1} , the shocks to x_{t+1} and σ_{t+1} , the first differences in one-year ($y_{1,t+1}$) and ten-year ($y_{10,t+1}$) bond yields, the return on the market portfolio (R_m), and the first difference in the variance of the market portfolio ($\sigma_{m,t+1}$). Panel B shows the relationship between CGP (ρ_t) and model-based correlations of stock returns with one-year ($\rho_{SB,1}$) or 10-year ($\rho_{SB,10}$) bond returns. For all calculations, we simulate 1,000,000 periods and drop the first 100,000 before calculating correlations.

Panel A. Correlations between simulated variables					
	Model	$\Delta y_{1,t+1}$	$\Delta y_{10,t+1}$	$R_{m,t+1}$	$\Delta \sigma_{m,t+1}^2$
Δc_{t+1}	Baseline	-0.023	-0.021	0.591	0.001
	Full	-0.148	-0.141	0.565	0.001
$x_{t+1} - E_t[x_{t+1}]$	Baseline	0.978	0.935	0.238	-0.001
	Full	0.970	0.918	0.194	0.002
$\sigma_{t+1}^2 - E_t[\sigma_{t+1}^2]$	Baseline	-0.098	-0.304	-0.048	0.992
	Full	-0.097	-0.308	-0.051	0.793
$p_{t+1} - E_t[p_{t+1}]$	Baseline	—	—	—	—
	Full	-0.160	-0.194	-0.045	0.598

Panel B. Correlations between CGP (ρ_t) and ρ_{SB}			
	Model	$\rho_{SB,1}$	$\rho_{SB,10}$
ρ_t	Baseline	—	—
	Full	-0.997	-0.997

Table 3
CGP and the SB Correlation

This table summarizes the slopes and Newey-West-adjusted (12 lags) t -statistics from quarterly regressions of the form

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{S\cdot,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{S\cdot,t} + \epsilon_{1,t+k}$$

for $k = 1, 2, 3, 4$, where $\hat{\rho}_{S\cdot,t}$ is the stock-bond return correlation (or stock-inflation correlation) estimated in one of several ways. Panel A summarizes the results under the restriction that $\alpha_2 = \alpha_3 = 0$, while Panel B uses the one-year or ten-year nominal Treasury yield to compute the stock-bond correlation. The real stock-bond correlation and the stock-inflation correlation are used in Panel C after decomposing 10-year nominal yields into real and inflation components based on TIPS yields. The sample is quarterly from 1963 to 2019 (228 observations) in Panel A and from 2003 to 2019 (68 observations) in Panel B.

Panel A. Serial correlation in consumption growth

Dependent Variable:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.494 (7.39)	0.370 (4.30)	0.420 (6.54)	0.186 (2.22)
Adj- R^2	0.219	0.134	0.173	0.030

Panel B. Persistence in consumption growth and the SB correlation based on nominal yields

Dependent Variable:	Δc_{t+1}		Δc_{t+2}		Δc_{t+3}		Δc_{t+4}	
	1Y	10Y	1Y	10Y	1Y	10Y	1Y	10Y
Maturity:								
Δc_t	0.480 (10.80)	0.482 (10.48)	0.367 (7.34)	0.371 (6.78)	0.397 (8.03)	0.406 (7.62)	0.150 (2.47)	0.164 (2.79)
$\Delta c_t \times \hat{\rho}_{SB,t}$	-0.400 (-2.79)	-0.312 (-2.17)	-0.690 (-3.76)	-0.520 (-2.87)	-0.372 (-1.92)	-0.294 (-1.94)	-0.475 (-2.39)	-0.164 (-3.38)
$\hat{\rho}_{SB,t}$	0.004 (3.38)	0.003 (3.46)	0.005 (4.39)	0.004 (4.00)	0.005 (3.19)	0.003 (2.59)	0.005 (2.96)	0.004 (2.88)
Adj- R^2	0.242	0.243	0.173	0.171	0.202	0.194	0.069	0.068

Panel C. Persistence in consumption growth and real SB or stock-inflation correlation

Dependent Variable:	Δc_{t+1}		Δc_{t+2}		Δc_{t+3}		Δc_{t+4}	
	$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$	$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$	$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$	$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$
$\hat{\rho}_t$ used:								
Δc_t	0.451 (2.85)	0.684 (3.40)	0.240 (1.69)	0.742 (4.20)	0.218 (3.55)	0.804 (3.12)	0.125 (1.68)	0.738 (4.29)
$\Delta c_t \times \hat{\rho}_{S\cdot,t}$	-0.451 (-0.87)	0.412 (0.92)	-1.138 (-2.42)	0.739 (1.73)	-1.394 (-4.46)	0.721 (0.91)	-0.895 (-2.29)	1.193 (1.85)
$\hat{\rho}_{S\cdot,t}$	0.005 (3.49)	0.002 (0.53)	0.007 (5.14)	-0.002 (-0.88)	0.008 (3.08)	-0.004 (-1.00)	0.007 (2.99)	-0.006 (-1.89)
Adj- R^2	0.232	0.231	0.346	0.275	0.424	0.338	0.186	0.153

Table 4
CGP and the SB Correlation using Alternative Consumption Measures

This table summarizes the slopes and Newey-West-adjusted (3 lags) t -statistics of the regression considered in Table 3 but estimated with annual consumption data. “NIPA Annual” is the annual consumption on non-durables and services, “Q4” uses the Q4-to-Q4 consumption growth series of Jagannathan and Wang (2007), while “Unfiltered” is the unfiltered consumption series of Kroencke (2017). Consumption data used this table is from Tim Kroencke’s website. The sample period used for Panel A is from 1962 to 2018 (56 observations). The sample in Panel B begins in 2003 (16 observations).

Panel A. Persistence in consumption growth and the SB correlation based on nominal yields									
Consumption series		Dependent Variable: Δc_{t+1}							
		NIPA Annual			Q4			Unfiltered	
		Maturity:			Maturity:			Maturity:	
	1Y	10Y		1Y	10Y		1Y	10Y	
Δc_t	0.497 (4.25)	0.480 (7.05)	0.476 (6.63)	0.393 (3.21)	0.390 (4.88)	0.386 (5.10)	0.030 (0.25)	0.070 (1.16)	0.239 (3.89)
$\Delta c_t \times \hat{\rho}_{SB,t}$		-0.844 (-2.60)	-0.647 (-3.06)		-0.844 (-2.74)	-0.674 (-2.58)		-1.069 (-3.71)	-0.691 (-2.88)
$\hat{\rho}_{SB,t}$		0.023 (2.41)	0.016 (3.28)		0.022 (2.23)	0.017 (2.90)		0.014 (1.15)	0.007 (1.41)
Adj- R^2	0.234	0.262	0.266	0.139	0.160	0.172	-0.018	0.006	0.033

Panel B. Persistence in consumption growth and real SB or stock-inflation correlation									
Dependent Variable: Δc_{t+1}									
Consumption series	NIPA Annual			Q4			Unfiltered		
$\hat{\rho}_{S\cdot,t}$ used:	$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$		$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$		$\hat{\rho}_{SR,t}$	$-\hat{\rho}_{S\pi,t}$	
Δc_t	0.583 (8.51)	-0.021 (-0.43)	0.844 (1.59)	0.393 (3.21)	-0.083 (-1.52)	1.852 (13.92)	0.030 (0.27)	-0.963 (-6.78)	1.968 (6.07)
$\Delta c_t \times \hat{\rho}_{S\cdot,t}$		-5.143 (-4.23)	0.630 (0.44)		-3.568 (-4.70)	4.581 (7.48)		-5.854 (-5.20)	5.087 (5.66)
$\hat{\rho}_{S\cdot,t}$		0.075 (4.14)	-0.011 (-0.37)		0.055 (10.72)	-0.094 (-6.94)		0.112 (4.78)	-0.117 (-4.44)
Adj- R^2	0.338	0.683	0.250	0.139	0.551	0.400	-0.018	0.472	0.337

Table 5
Consumption Growth and Survey Forecasts

This table summarizes the slopes and Newey-West-adjusted (3 lags) t -statistics from regressing the first-difference in the SPF long-run consumption growth forecast on contemporaneous consumption growth. Some regressions also include interactions with a 1999+ year dummy or with the SB correlation, which is estimated using 1-year or 10-year nominal yields or with the 10-year real yield, as well as main effects for these variables. “NIPA Annual” is the annual consumption on non-durables and services, “Q4” uses the Q4-to-Q4 consumption growth series of Jagannathan and Wang (2007), while “Unfiltered” is the unfiltered consumption series of Kroencke (2017). There are 38 observations (1981-2018), except for when the 10-year real SB correlation is used (2003-2018, 16 observations).

Panel A. NIPA Annual Consumption					
Maturity:	Dependent Variable: $\Delta \hat{x}_t$				
			1Y	10Y	10Y Real
Δc_t	-0.161 (-2.12)	-0.450 (-3.46)	-0.217 (-3.81)	-0.204 (-2.51)	-0.265 (-7.57)
$\Delta c_t \times 1_{99+}$		0.460 (3.47)			
$\Delta c_t \times \hat{\rho}_{SB,t}$			-1.013 (-4.56)	-0.650 (-3.13)	-1.101 (-3.61)
1_{99+}		-0.010 (-3.03)			
$\hat{\rho}_{SB,t}$			0.020 (5.33)	0.012 (2.70)	0.013 (3.22)
Adj- R^2	0.103	0.325	0.371	0.324	0.247

Panel B. Q4 Consumption					
Maturity:	Dependent Variable: $\Delta \hat{x}_t$				
			1Y	10Y	10Y Real
Δc_t	-0.121 (-1.86)	-0.343 (-2.75)	-0.016 (-2.86)	-0.145 (-2.09)	-0.276 (-5.71)
$\Delta c_t \times 1_{99+}$		0.371 (2.98)			
$\Delta c_t \times \hat{\rho}_{SB,t}$			-0.736 (-3.47)	-0.473 (-2.52)	-0.940 (-4.81)
1_{99+}		-0.008 (-2.45)			
$\hat{\rho}_{SB,t}$			0.015 (3.58)	0.009 (2.22)	0.010 (4.87)
Adj- R^2	0.057	0.212	0.241	0.193	0.201

Panel C. Unfiltered Consumption					
Maturity:	Dependent Variable: $\Delta \hat{x}_t$				
			1Y	10Y	10Y Real
Δc_t	-0.073 (-1.76)	-0.177 (-2.89)	-0.101 (-4.27)	-0.085 (-2.52)	-0.067 (-2.61)
$\Delta c_t \times 1_{99+}$		0.192 (3.13)			
$\Delta c_t \times \hat{\rho}_{SB,t}$			-0.470 (-8.18)	-0.315 (-3.63)	-0.408 (-2.54)
1_{99+}		-0.005 (-2.43)			
$\hat{\rho}_{SB,t}$			0.010 (3.83)	0.006 (2.56)	0.004 (1.48)
Adj- R^2	0.062	0.175	0.271	0.229	-0.023

Table 6
Uncertainty and Expected Growth

This table summarizes the slopes and Newey-West-adjusted (3 lags) t -statistics from regressing the first-difference in the SPF long-run consumption growth forecast on the first difference of uncertainty. Some regressions also include interactions with a 1999+ year dummy or with the SB correlation, which is estimated using 1-year or 10-year nominal yields or with the 10-year real yield, as well as main effects for these variables. In Panel A, UNC denotes macro uncertainty, while in panel B it is real uncertainty, both from Jurado, Ludvigson, and Ng (2015). There are 38 observations (1981-2018), except for when the 10-year real SB correlation is used (2003-2018, 16 observations) for the analysis.

Panel A. Using Macro Uncertainty for UNC

Bond maturity:	Dependent Variable: $\Delta \hat{x}_t$							
			1Y		10Y		10Y Real	
ΔUNC_t	-0.042 (-1.92)	0.007 (0.22)	-0.023 (-1.61)	-0.023 (-1.62)	-0.027 (-0.61)	-0.028 (-1.70)	-0.055 (-4.28)	-0.017 (-1.42)
$\Delta \text{UNC}_t \times 1_{99+}$		-0.069 (-2.05)						
$\Delta \text{UNC}_t \times \hat{\rho}_{SB,t}$			0.133 (4.00)	0.130 (4.36)	0.089 (1.98)	0.085 (2.03)	0.052 (0.87)	0.061 (0.82)
1_{99+}		0.001 (0.57)						
$\hat{\rho}_{SB,t}$				-0.003 (-1.81)		-0.001 (-0.89)		-0.004 (-1.09)
Adj- R^2	0.175	0.250	0.301	0.305	0.260	0.242	0.516	0.499

Panel B. Using Real Uncertainty for UNC

Bond maturity:	Dependent Variable: $\Delta \hat{x}_t$							
			1Y		10Y		10Y Real	
ΔUNC_t	-0.083 (-2.32)	-0.008 (-0.18)	-0.056 (-2.80)	-0.063 (-3.71)	-0.064 (2.88)	-0.067 (-3.17)	-0.059 (-1.05)	-0.058 (-0.04)
$\Delta \text{UNC}_t \times 1_{99+}$		-0.110 (-2.49)						
$\Delta \text{UNC}_t \times \hat{\rho}_{SB,t}$			0.205 (3.94)	0.197 (4.44)	0.138 (2.46)	0.129 (2.53)	0.385 (1.62)	0.419 (2.83)
1_{99+}		0.001 (0.68)						
$\hat{\rho}_{SB,t}$				-0.003 (-1.62)		-0.001 (-0.90)		0.003 (0.66)
Adj- R^2	0.197	0.249	0.296	0.302	0.262	0.244	0.546	0.518

Table 7
Market Return Predictability using Bond Yields

This table summarizes the results of the regression

$$R_{S,t,t+\tau}^e = \beta_0 + \beta_1 y_t + \beta_2 y_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1},$$

where R_S^e is the value-weighted market excess return over a one-month, three-month, six-month, or twelve-month horizon (τ), and y_t is either the one-year nominal ($y_{1,t}$), ten-year nominal ($y_{10,t}$), or ten-year real yield ($y_{r,10,t}$). $\hat{\rho}_{SB,t}$ is the estimated correlation between stock and bond returns, which is constructed using the same yield used for y_t . Panel A shows the results in which future stock returns are regressed on nominal bond yields separately for two periods before and after the end of 1998 (with $\beta_2 = 0$). Panel B estimates the full model using the entire sample, again using nominal yields, for the sample period of 1963-2019. Panel C shows regression results for simple and interactive regressions based on 10-year real yields for the sample period of 2004-2019. T -statistics, in parentheses, are adjusted for heteroscedasticity and autocorrelation using Newey-West standard errors with 12 lags.

Panel A. Simple predictive regressions using nominal yields

Horizon:	1962-1998				1999-2019			
	1-Month	3-Month	6-Month	12-Month	1-Month	3-Month	6-Month	12-Month
$y_{1,t}$	-0.154 (-1.94)	-0.296 (-1.71)	-0.368 (-0.97)	-0.212 (-0.28)	-0.315 (-2.20)	-0.972 (-3.07)	-2.081 (-3.48)	-4.553 (-4.05)
Adj- R^2	0.007	0.008	0.006	-0.001	0.015	0.052	0.111	0.254
$y_{10,t}$	-0.083 (-0.98)	-0.126 (-0.59)	-0.095 (-0.21)	0.310 (0.36)	-0.584 (-2.90)	-1.709 (-3.79)	-3.462 (-3.98)	-6.503 (-4.28)
Adj- R^2	0.000	-0.001	-0.002	0.000	0.027	0.079	0.150	0.250

Panel B. Interactive predictive regressions using nominal yields (684 observations)

Horizon:	One-month		Three-month		Six-month		Twelve-month	
	k=1	k=10	k=1	k=10	k=1	k=10	k=1	k=10
$y_{k,t}$	-0.267 (-2.86)	-0.436 (-3.74)	-0.770 (-3.72)	-1.176 (-4.11)	-1.440 (-3.70)	-2.120 (-3.81)	-2.142 (-2.86)	-2.759 (-2.78)
$y_{k,t} \times \hat{\rho}_{SB,t}$	0.554 (2.35)	0.581 (3.40)	1.762 (2.94)	1.616 (3.52)	3.465 (3.13)	2.980 (3.40)	5.138 (2.43)	4.006 (2.70)
Adj- R^2	0.016	0.020	0.044	0.052	0.079	0.085	0.088	0.075

Panel C. Predictive regressions using 10-year real yields (192 observations)

Horizon:	One-month		Three-month		Six-month		Twelve-month	
$y_{r,10,t}$	-0.788 (-2.54)	-0.207 (-0.75)	-2.119 (-2.52)	-0.308 (-0.69)	-3.306 (-2.34)	-0.063 (-0.04)	-5.458 (-2.01)	-1.741 (-0.61)
$y_{r,10,t} \times \hat{\rho}_{SR,t}$		4.834 (2.81)		15.079 (3.63)		27.005 (3.13)		30.953 (2.18)
Adj- R^2	0.012	0.075	0.060	0.256	0.065	0.343	0.095	0.284

Table 8
Consumption, Wealth, and Human Capital Dynamics
Pre-1999 vs. 1999+

Panel A of this table summarizes the results of estimating the vector error correction model

$$\Delta \mathbf{X}_{t+1} = \mathbf{u} + \zeta \text{cay}_t + \mathbf{\Gamma} \Delta \mathbf{X}_t + \mathbf{e}_t,$$

where $\mathbf{X}_t = [c_t \ a_t \ y_t]$ includes the log consumption, wealth, and human capital series from Sydney Ludvigson's website and *cay* is the deviation in their common trend, estimated separately for the pre-1999 (1962-1998) and 1999+ (1999-2019) sample. Panel B shows results for the predictive regression in which quarterly excess market returns are regressed on the lagged value of *cay*. In the first row of the panel, *cay* is taken directly from Sydney Ludvigson's website and is estimated using a single extended sample. In the second row, *cay* is estimated separately for each subsample, as in Panel A. The t-statistics reported in parenthesis uses the Newey-West standard errors with 12 lags.

Panel A. Vector error correction model

	Pre-1999			1999+		
	Δc_{t+1}	Δa_{t+1}	Δy_{t+1}	Δc_{t+1}	Δa_{t+1}	Δy_{t+1}
Δc_t	0.099 (1.44)	0.204 (1.20)	0.248 (2.11)	0.790 (3.92)	0.700 (1.77)	0.378 (1.76)
Δa_t	0.165 (3.53)	0.175 (1.62)	0.083 (1.08)	0.040 (1.63)	0.149 (0.97)	0.062 (1.50)
Δy_t	0.156 (1.32)	-0.196 (-1.16)	-0.072 (-0.34)	0.034 (0.61)	0.142 (0.49)	-0.199 (-2.06)
<i>cay</i> _t	-0.026 (-0.81)	0.181 (1.98)	0.027 (0.62)	-0.213 (-3.11)	0.200 (0.34)	0.423 (3.31)

Panel B. Predicting excess stock market returns

	Pre-1999	1999+
<i>cay</i> _t estimated over extended sample	0.708 (2.81)	-0.831 (-1.42)
Adj-R ²	0.026	0.012
<i>cay</i> _t estimated separately for each subsample	0.946 (2.15)	-0.823 (-0.58)
Adj-R ²	0.024	-0.008

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A. Technical appendix

A.1. The wealth-consumption ratio

Following the Campbell-Shiller decomposition, returns to total wealth portfolio can be represented by

$$R_{TW,t+1} = \kappa_0 + \Delta c_{t+1} + A_0(\kappa_1 - 1) + A_x(\kappa_1 x_{t+1} - x_t) + A_v(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_q(\kappa_1 q_{t+1} - q_t).$$

The intertemporal marginal rate of substitution (IMRS) is

$$\begin{aligned} m_{t+1} = & \theta \log \beta - \gamma \Delta c_{t+1} + (\theta - 1) [\kappa_0 + A_0(\kappa_1 - 1) + A_x(\kappa_1 x_{t+1} - x_t) \\ & + A_v(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_q(\kappa_1 q_{t+1} - q_t)]. \end{aligned}$$

The unexpected component of the IMRS is represented by

$$m_{t+1} - E_t[m_{t+1}] = \lambda_c \sigma_t \epsilon_{c,t+1} + \lambda_x \sigma_t \epsilon_{x,t+1} + \lambda_v \sigma_t \epsilon_{v,t+1} + \lambda_\delta \sigma_t \epsilon_{q,t+1},$$

where $\lambda_c = -\gamma$, $\lambda_x = (\theta - 1)\kappa_1 A_x \phi_x$, $\lambda_v = (\theta - 1)\kappa_1 A_v \sigma_v$, and $\lambda_\delta = (\theta - 1)\kappa_1 A_q \sigma_q$.

We solve for A_0 , A_x , A_v , and A_q using the Euler equation $E_t[m_{t+1} + R_{TW,t+1}] + \text{Var}_t[m_{t+1} + R_{TW,t+1}] = 0$. For A_x , we collect all terms associated with x_t :

$$A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \xi_1}.$$

Collecting the terms from the Euler equation that are functions of σ_t^2 and q_t , it can be seen

that A_v and A_q must jointly satisfy the conditions

$$2A_v(\kappa_1 s_1 - 1) + \theta((A_x \kappa_1 \varphi_x)^2 + (A_v \kappa_1 \sigma_v)^2 + (A_q \kappa_1 \sigma_q)^2 + (1 - \frac{1}{\psi})^2) + 2(1 - \gamma)\kappa_1 A_v \sigma_v \varrho_{cv} = 0$$

$$A_q = Q_0 + Q_1 A_v,$$

where $Q_0 = \frac{(1-\gamma)\kappa_1 A_x \varphi_x}{1-\kappa_1 \omega_1} < 0$ and $Q_1 = \frac{\theta \varrho_{cv} \kappa_1^2 A_x \varphi_x \sigma_v}{1-\kappa_1 \omega_1} > 0$.

A_v can be obtained by solving a quadratic equation after plugging the second equation into the first. It can also be shown that $A_v < 0$ when $\gamma > 1$ and $\psi > 1$ by evaluating the characteristics of the quadratic equation. We obtain two values for A_v . We choose the value that is closer to the baseline model. The second value generates unrealistic moments of asset returns. The negative sign of A_v also implies $A_q < 0$.

$$\text{Finally, } A_0 \text{ satisfies } A_0 = \frac{1}{1-\kappa_1} \left[\log \beta + \kappa_0 + (1 - \frac{1}{\psi})\mu + k_1(A_v s_0 + A_q \omega_0) \right].$$

A.2. The price-dividend ratio

Similar to the wealth-consumption ratio we assume that the price-dividend ratio is a linear function, $A_{m,0} + A_{m,x}x_t + A_{m,v}\sigma_t^2 + A_{m,q}\delta_t$, and we again solve for the coefficients using the Euler equation $E_t[m_{t+1} + R_{m,t+1}] + 0.5\text{Var}_t[m_{t+1} + R_{m,t+1}] = 0$. Collecting the terms associated with x_t , σ_t^2 , and q_t , we can solve for $A_{m,0}$, $A_{m,x}$, $A_{m,v}$, and $A_{m,q}$. First, we have

$$A_{m,x} = \frac{\phi_d - \frac{1}{\psi}}{1 - \kappa_1 \xi_1}.$$

As in the wealth-consumption ratio, $A_{m,v}$, and $A_{m,q}$ must jointly satisfy the conditions

$$\begin{aligned}
& 2A_{m,v}(\kappa_{m,1}s_1 - 1) + 2(\theta - 1)(\kappa_1s_1 - 1)A_v + 2(\varphi_{cd} + \lambda_c)(\kappa_{m,1}A_{m,v}\sigma_v + \lambda_v)\varrho_{cv} \\
& + (\kappa_{m,1}A_{m,x}\varphi_x + \lambda_x)^2 + (\kappa_{m,1}A_{m,v}\sigma_v + \lambda_v)^2 + (\kappa_{m,1}A_{m,q}\sigma_q + \lambda_\delta)^2 + (\varphi_{cd} + \lambda_c)^2 + \varphi_d^2 = 0 \\
& A_{m,q} = Q_{m,0} + Q_{m,1}A_{m,v},
\end{aligned}$$

where $Q_{m,0} = \frac{1}{1-\kappa_1\omega_1}((\varphi_{cd} + \lambda_c)(\kappa_1A_{m,x}\varphi_x + \lambda_x) + (\theta - 1)(\kappa_1\omega_1 - 1)A_q + \lambda_v(\kappa_1A_{m,x}\varphi_x + \lambda_x)\varrho_{cv})$ and $Q_{m,1} = \frac{1}{1-\kappa_1\omega_1}\kappa_1\sigma_v(\kappa_1A_{m,x}\varphi_x + \lambda_x)\varrho_{cv}$. Evaluating the characteristics of the quadratic function, similar to the earlier case, we find that $A_{m,v} < 0$ when $\gamma > \varphi_{cd} > 1$, which is consistent with the general long-run risk specification. Also, one can show that $A_{m,30} < 0$ and $A_{m,32} > 0$ under the condition of $\gamma > \phi_d$ and $\varphi_{cd} > 1$, which implies that $A_{m,q} < 0$.

Finally, $A_{m,0}$ satisfies

$$\begin{aligned}
A_{m,0} &= \frac{1}{1 - \kappa_{m,1}}(\theta \log \beta + (\theta - 1)\kappa_0 + \kappa_{m,0} + (1 - \gamma)\mu \\
&+ \kappa_1A_v s_0(\theta - 1) + \kappa_{m,1}A_{m,v}s_0 + \kappa_1A_q\omega_0(\theta - 1) + \kappa_{m,1}\omega_0A_{m,q} + (\theta - 1)(\kappa - 1)A_0).
\end{aligned}$$

A.3. Bond yields

Denote the state vector as

$$\Sigma_t = \begin{bmatrix} \Delta C_t & x_t & \sigma_t^2 & q_t \end{bmatrix}'$$

We can write the conditional mean as

$$\mathbb{E}_t[\Sigma_{t+1}] = K_0 + K\Sigma_t,$$

where

$$K_0 = \begin{bmatrix} \mu & 0 & s_0 & \omega_0 \end{bmatrix}'$$

and

$$K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \xi_1 & 0 & 0 \\ 0 & 0 & s_1 & 0 \\ 0 & 0 & 0 & \omega_1 \end{bmatrix}$$

The conditional covariance matrix is

$$\text{Cov}_t(\Sigma_{t+1}, \Sigma'_{t+1}) = \begin{bmatrix} \sigma_t^2 & \phi_x q_t & \varrho_{cv} \sigma_v \sigma_t^2 & 0 \\ \phi_x q_t & \phi_x^2 \sigma_t^2 & \sigma_v \varrho_{cv} q_t & 0 \\ \varrho_{cv} \sigma_v \sigma_t^2 & \sigma_v \varrho_{cv} q_t & \sigma_v^2 \sigma_t^2 & 0 \\ 0 & 0 & 0 & \sigma_q^2 \sigma_t^2 \end{bmatrix} = \Omega_1 \sigma_t^2 + \Omega_2 q_t,$$

where

$$\Omega_1 = \begin{bmatrix} 1 & 0 & \varrho_{cv} \sigma_v & 0 \\ 0 & \phi_x^2 & 0 & 0 \\ \varrho_{cv} \sigma_v & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_q^2 \end{bmatrix} \quad \text{and} \quad \Omega_2 = \begin{bmatrix} 0 & \phi_x & 0 & 0 \\ \phi_x & 0 & \phi_x \varrho_{cv} \sigma_v & 0 \\ 0 & \phi_x \varrho_{cv} \sigma_v & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In vector notation, we can write the log pricing kernel as

$$m_{t+1} = m_0 + M'_1 \Sigma_{t+1} - M'_2 \Sigma_t$$

with

$$m_0 = \theta \log \beta + (\theta - 1) (\kappa_0 + A_0 (\kappa_1 - 1)),$$

$$M_1 = \begin{bmatrix} -\gamma & (\theta - 1) \kappa_1 A_x & (\theta - 1) \kappa_1 A_v & (\theta - 1) \kappa_1 A_q \end{bmatrix}',$$

and

$$M_2 = \begin{bmatrix} 0 & (\theta - 1)A_x & (\theta - 1)A_v & (\theta - 1)A_q \end{bmatrix}',$$

where A_x , A_v , and A_q are as defined earlier.

The log price of a riskless one-period bond ($B_{1,t}$) is given by

$$\begin{aligned} B_{1,t} &= E_t[m_{t+1}] + 0.5\text{Var}_t(m_{t+1}) \\ &= m_0 + M_1'K_0 + (M_1'K - M_2')\Sigma_t + 0.5M_1'\text{Cov}_t(\Sigma_{t+1}, \Sigma_{t+1}')M_1 \\ &= m_0 + M_1'K_0 + (M_1'K - M_2')\Sigma_t + 0.5M_1'\Omega_1M_1\sigma_t^2 + 0.5M_1'\Omega_2M_1q_t \\ &= m_0 + M_1'K_0 + (M_1'K - M_2')\Sigma_t + 0.5\Psi'\Sigma_t \\ &= m_0 + M_1'K_0 + (M_1'K - M_2' + 0.5\Psi')\Sigma_t, \end{aligned}$$

where

$$\Psi' = \begin{bmatrix} 0 & 0 & M_1'\Omega_1M_1 & M_1'\Omega_2M_1 \end{bmatrix}'.$$

Therefore, the yield of a one-period bond is equal to

$$y_t = Y_0 + Y\Sigma_t,$$

where

$$Y_0 = -m_0 - M_1'K_0$$

and

$$Y = -M_1'K + M_2' - 0.5\Psi'.$$

It can be shown that for

$$Y = \begin{bmatrix} 0 & Y_x & Y_v & Y_p \end{bmatrix}'$$

we have $Y_x > 0$ and $Y_v, Y_p < 0$.

Now suppose that the n -period bond has a log price

$$B_{n,t} = D_{n,0} + D'_n \Sigma_t.$$

Then the $(n+1)$ -period bond has a price that is equal to the conditional expectation of

$$E_t [m_{t+1} + B_{n,t+1}] + 0.5 \text{Var}_t (m_{t+1} + B_{n,t+1}),$$

where

$$m_{t+1} + B_{n,t+1} = m_0 + D_{n,0} + (M_1 + D_n)' \Sigma_{t+1} - M'_2 \Sigma_t.$$

The log price of the bond can be solved as

$$\begin{aligned} B_{n,t+1} &= m_0 + D_{n,0} + (M_1 + D_n)'(K_0 + K \Sigma_t) - M'_2 \Sigma_t + 0.5(M_1 + D_n)' \text{Cov}_t (\Sigma_{t+1}, \Sigma'_{t+1}) (M_1 + D_n) \\ &= m_0 + D_{n,0} + (M_1 + D_n)' K_0 + ((M_1 + D_n)' K - M'_2) \Sigma_t + 0.5 \Psi'_n \Sigma_t, \end{aligned}$$

where

$$\Psi_n = \begin{bmatrix} 0 & 0 & (M_1 + D_n)' \Omega_1 (M_1 + D_n) & (M_1 + D_n)' \Omega_2 (M_1 + D_n) \end{bmatrix}'.$$

The log of $(n+1)$ -period bond price is therefore

$$B_{n+1,t} = D_{n+1,0} + D'_{n+1} \Sigma_t,$$

where

$$D_{n+1,0} = m_0 + D_{n,0} + (M_1 + D_n)' K_0$$

and

$$D_{n+1} = K'(M_1 + D_n) - M_2 + \frac{1}{2} \Psi_n.$$

The $(n + 1)$ -period yield is therefore equal to

$$y_{n+1,t} = Y_{n+1,0} + Y_{n+1}\Sigma_t,$$

where $Y_{n+1,0} = -D_{n+1,0}$ and $Y_{n+1} = -D'_{n+1}$.

A.4. The stock-bond return correlation

The stock-bond return correlation is the negative of the correlation between stock returns and changes in the bond yield. Unexpected stock market returns are derived using the Campbell-Shiller decomposition:

$$R_{m,t+1} - E_t[R_{m,t+1}] = \kappa_{m,1}\phi_x A_{m,x}\sigma_t\epsilon_{x,t+1} + \kappa_{m,1}\sigma_v A_{m,v}\sigma_t\epsilon_{v,t+1} + \kappa_{m,1}\sigma_q A_{m,q}\epsilon_{q,t+1} + \varphi_{cd}\sigma_t\epsilon_{c,t+1} + \varphi_d\sigma_t\epsilon_{d,t+1}.$$

We can then compute the stock-bond return correlation by taking the negative of the conditional correlation between market returns and bond yields.

The conditional covariance between a n -period bond yield and stock returns can be expressed as

$$\begin{aligned} \text{Cov}_t(R_{m,t+1}, y_{n,t+1}) &= (Y_{n,x}S_x\varphi_x + Y_{n,v}S_v\sigma_v + Y_{n,q}S_q\sigma_q + Y_{n,v}S_c\sigma_v\varrho_{cv})\sigma_t^2 \\ &\quad + ((Y_{n,x}\varphi_x S_v + Y_{n,v}S_x\sigma_v)\varrho_{cv} + Y_{n,x}S_c\varphi_x)q_t, \end{aligned}$$

in which the terms $Y_{n,\cdot}$ are elements of the 1×4 vector Y_{n+1} :

$$Y_{n+1} = \begin{bmatrix} 0 & Y_{n+1,x} & Y_{n+1,v} & Y_{n+1,p} \end{bmatrix},$$

and S_x , S_v , S_v , and S_q are defined as:

$$S_x = \kappa_{m,1}\phi_x A_{m,x}, \quad S_v = \kappa_{m,1}\sigma_v A_{m,v}, \quad S_q = \kappa_{m,1}\sigma_q A_{m,q}, \quad S_c = \varphi_{cd}, \quad S_d = \varphi_d.$$

The conditional variance of the bond yield is

$$\text{Var}_t(y_{n,t+1}) = (Y_n \Omega_1 Y_n' + Y_n \Omega_2 Y_n' \rho_t) \sigma_t^2.$$

Similarly, the conditional variance of the wealth portfolio/market returns is

$$\text{Var}_t(R_{m,t+1}) = \sigma_{m,t}^2 = (V_v + V_q \rho_t) \sigma_t^2,$$

where $V_v = S_x^2 + S_v^2 + S_q^2 + S_c^2 + S_d^2 + 2S_c S_v \varrho_{cv}$ and $V_q = 2S_x S_v \varrho_{cv} + 2S_c S_x$.

A.5. The market risk premium

The risk premium of the wealth/market portfolio can be expressed as

$$\begin{aligned} \text{Cov}_t(-m_{t+1}, R_{j,t+1}) &= (-\lambda_c(S_c + S_v \varrho_{cv}) - \lambda_x S_x - \lambda_v S_v - \lambda_\delta S_q - S_c \lambda_v \varrho_{cv}) \sigma_t^2 \\ &\quad + (-\lambda_x S_v \varrho_{cv} - \lambda_v S_x \varrho_{cv} - \lambda_c S_x - \lambda_x S_c) q_t. \end{aligned}$$