Consumption growth persistence and the stock/bond correlation*

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July 2021

ABSTRACT

We consider a model in which the correlation between shocks to consumption and to expected future consumption growth is nonzero and varies over time. We validate this assumption empirically using the model's implication that the time-variation in consumption growth persistence drives the correlation between stock and bond returns. Consistent with model predictions, we find that the stock/bond correlation is also related to the volatility of stock returns, the so-called stock market "leverage effect," and the predictive relation between bond yields and future stock returns.

Keywords: Consumption persistence, long-run risk, stock/bond correlation

*We thank Allaudeen Hameed, Scott Joslin, Mete Kilic, Lars Lochstoer, Johan Sulaeman, Selale Tuzel, participants at Fourteenth Risk Management Conference, Korea University, NUS, and USC. All errors are our own.

I. Introduction

While the correlation between stock and bond returns has been the subject of research for some time, the abrupt change in the sign of that correlation in the late 1990s, shown in Figure 1 (a), has spurred a renewed interest in its determinants. One explanation of this shift is an apparent regime change in the behavior of inflation, as in David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020). In contrast, Duffee (2018b) finds that the stock/bond return correlation is primarily driven by changes in the behavior of real yields. He does not find, however, any shift in macro fundamentals that would be responsible for this change, concluding that "macroeconomic dynamics appear to have almost nothing to do with this time-varying comovement."

In this paper, we propose a new explanation of this shift and of variation in the correlation between stock and bond returns (SB correlation) more generally. We show that this correlation is related to variation in consumption growth persistence (CGP), which we define as the tendency of positive shocks to current consumption growth to raise expected future consumption growth. The logic is straightforward: Changes in current realized growth affect cash flows, while changes in expected growth drive real interest rates via intertemporal smoothing. When CGP increases, the correlation between real rates and cash flows rises, resulting in a lower (and likely negative) SB correlation. When CGP is negative, higher consumption growth forecasts lower growth in the future, and the SB correlation rises.

The persistence of consumption growth indeed appears to have changed over time. Figure 1 (b) shows that autocorrelations in consumption growth were moderate through 1997 but significantly higher in the period starting in 1998, which is when the SB correlation changed sign. The goal of this study is to determine whether this suggestive evidence is indicative of a more systematic effect that CGP has on the SB correlation and other asset return moments.

Macroeconomic theory tells us that the persistence of consumption growth reflects the relative importance of transitory versus permanent shocks. Transient shocks to productivity (Kaltenbrunner and Lochstoer 2010), income (Hall and Mishkin 1982), or uncertainty (Basu and Bundick 2017) can all drive consumption higher in the short run, while at the same time decrease long-term consumption growth. In contrast, permanent shocks generally imply persistence in growth rates, as is the case in the production economy of Kaltenbrunner and Lochstoer (2010), resulting from the frictions that induce gradual adjustment to those shocks.

Starting with Friedman (1957), there is abundant empirical evidence that shows that both permanent and transitory shocks are necessary to explain observed patterns of consumption persistence.[1] The more recent literature on macroeconomic volatility shows that multiple sources of uncertainly affect macroeconomic and financial variables to different degrees (e.g. Jurado, Ludvigson, and Ng 2015), implying that the relative importance of different shocks varies over time. Intuitively, if the most volatile shocks are transitory (e.g., pandemics, oil shocks), CGP will become negative. When permanent shocks (e.g., technology, climate change) dominate, CGP turns positive.

Our model represents a stylized way to capture the net effect of these mechanisms.

[1] See Beveridge and Nelson (1981), Watson (1986), and Clark (1987), among numerous others.

As in the long-run risk model of Bansal and Yaron (2004), a highly persistent expected consumption growth process induces modest but very long-run dependence in consumption growth. We generalize this model by allowing a time-varying correlation between current and expected consumption growth shocks. By including both processes, it is possible to induce time variation in CGP while maintaining the long-run positive autocorrelation that is critical for matching the moments of asset returns.

In addition to the relation between CGP and the stock/bond return correlation, allowing for time-varying consumption persistence generates a number of new predictions. One is that higher CGP will raise stock market volatility. This is because current and expected future cash flows are two primary drivers of equity valuation. When these shocks are positively correlated, their effects will be amplified, and market volatility will rise.

Another implication of our model is a relationship between CGP and the stock market "leverage effect" – the generally negative correlation between stock returns and volatility changes. The connection arises from an additional feature of our model, a negative correlation between shocks to consumption growth and its volatility. This correlation is empirically motivated and is thought to arise from a precautionary savings motive.[2] With precautionary savings, higher CGP will strengthen the predictive relation between volatility shocks and future consumption growth. One implication for asset prices is that the leverage effect will be magnified (i.e., made more negative) by higher CGP.

Finally, our model implies that the relationship between interest rates and future stock market returns should depend on CGP. While many macro-finance models suggest a negative

[2] See, for example, Carroll (1997) and Basu and Bundick (2017)

relationship between them, a weak empirical relationship observed is a puzzle. Since bond yields are closely related to the level of expected consumption growth, they are more strongly related to consumption volatility – which drives the market risk premium – when CGP is high. The predictive relationship should be weak when CGP is low.

Testing these predictions is challenging because CGP, which is assumed to vary over time, is unobserved. Moreover, standard estimation methods are unlikely to be effective given that CGP describes the correlation between consumption growth and a latent variable, expected consumption growth, whose measurement is itself difficult (e.g., Schorfheide, Song, and Yaron 2018). We, therefore, devise an empirical strategy based on our model's implication that CGP is inversely related to the stock/bond return correlation, which can be measured accurately due to the availability of high-frequency asset price data.

We validate the model-implied relationship between CGP and the SB correlation by showing that the serial correlation in consumption growth is significantly higher when the correlation is low. This result holds at multiple horizons and is obtained whether we use returns on short-term or long-term bonds and whether the bonds are nominal or inflation-indexed. Secondarily, we show that the contemporaneous relationship between consumption growth and changes in bond yields is more negative when the SB correlation is higher. Given that bond yields and expected consumption growth are closely related in the LRR framework, this result is also consistent with a negative relation between CGP and the stock/bond correlation.

With this justification for using the SB correlation as a proxy for CGP, we confirm all of our model predictions in the data. First, we find strong supporting evidence that the SB correlation is positively related to the stock market leverage effect. We test this relationship in multiple ways, using a volatility forecast model and using monthly non-overlapping estimates of the correlations.

Second, since CGP should also be positively related to market volatility, the predicted relationship between market volatility and the stock/bond correlation should be negative. More precisely, the correlation should have a negative interactive effect on macroeconomic uncertainty. We confirm this relationship using several uncertainty measures, namely the macro uncertainty of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty of Baker, Bloom, and Davis (2016), and a simple measure of consumption growth variance.

The final implication is for the conditional predictive relation between yields and future stock returns. The model says that this negative relationship should be stronger when CGP is high or when the SB correlation is low. We report a weak unconditional predictive relationship, as documented in prior work, but a strong conditional relationship when the SB correlation is negative. The insignificant unconditional relation, which is at odds with the predictions of many models, is, therefore, a natural result of the SB correlation being positive over much of our sample.

There are a number of other explanations of why the SB correlation varies over time, and we believe it is unlikely that any single theory explains all fluctuations. Aside from the other predictions that we confirm in our empirical analysis, we believe that consumption growth persistence has certain merits that distinguish it from other explanations.

Other studies attribute the variation in SB correlations to other factors. Baele, Bekaert, and Inghelbrecht (2010), claiming that "macroeconomic fundamentals contribute little to explaining stock and bond return correlations, conclude that flights to quality/liquidity are the likely explanation of negative correlations. While Pástor and Stambaugh (2003), Connolly, Stivers, and Sun (2005), and others provide additional evidence for this channel, it seems incomplete given the prolonged period since 1998 over which the stock/bond correlation has remained negative, which includes many years in which markets remained stable.

Variation in the stock/bond correlation has also been attributed to changes in the dynamics of inflation. David and Veronesi (2013), Song (2017), and Campbell, Pflueger, and Viceira (2020) all present models in which the relation between inflation and real economic activity changes signs. Campbell, Pflueger, and Viceira (2020), for example, show that the correlation between inflation and the output gap was negative between 1979 and 2001 but positive in the following decade. If inflation shocks are the primary driver of bond returns, this result would appear to provide a clear explanation of the shift in correlation that occurred around that time.[3]

In contrast, the SB correlations in our model are entirely driven by variation in real interest rates, not inflation. While changing properties of inflation are undoubtedly one reason why SB correlations change, a model based on real rates may be better positioned to explain interest rate behavior in environments like the last 20 years, in which both inflation levels and inflation risk are low. Since 2003, when reliable TIPS data are available, real and nominal bond yields have tracked each other closely, with a correlation in levels above 90%. More importantly, the real bond/stock correlation and the nominal bond/stock correlation are themselves closely related over the post-2003 sample.[4] While inflation is undoubtedly

[3]Hasseltoft (2012), Ilmanen (2003), Campbell, Sunderam, and Viceira (2017), and Swanson (2019) also advance inflation-based explanations of the stock/bond return correlation.

[4]When these two correlations are measured using non-overlapping monthly subsamples, the correlation

more important in earlier decades, it is almost certainly not the entire story.

We are not the only paper to propose that variation in real yields is the primary driver of the changes in the SB correlation. For example, Duffee (2018 a,b) argues that the inflation expectations that underly long-term bond yields vary too little to explain much variation in those yields and that the stock/bond correlation is therefore primarily driven by changes in real yields. The role of consumption growth persistence is also highlighted in the recent work by Chernov, Lochstoer, and Song (2021). While their model is substantially different from ours and more focused on explaining the nominal term structure, it succeeds in explaining the stock/bond correlation using a mechanism based on consumption persistence rather than inflation dynamics. The different empirical predictions derived in that paper show that time-varying consumption growth persistence can explain a broader range of phenomena than those we address here.

In the next section, we describe and calibrate our model. Section III describes our data and strategies for measuring latent processes. Section IV presents our empirical results, and Section V concludes.

II. A real model of the stock-bond correlation

1. Model dynamics

Our model is a generalization of the standard framework of Bansal and Yaron (2004). In our baseline specification, the representative agent has Epstein and Zin (1991) preferences,

between them is 86%.

and consumption growth (Δc_{t+1}) has a persistent time-varying component x_t and time-varying uncertainty σ_t^2:

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \epsilon_{c,t+1} \tag{1}$$

$$x_{t+1} = p_1 x_t + \phi_x \sigma_t \epsilon_{x,t+1}$$

$$\sigma_{t+1}^2 = s_0 + s_1 \sigma_t^2 + \sigma_v \sigma_t \epsilon_{v,t+1},$$

where $\epsilon_{c,t+1}$, $\epsilon_{x,t+1}$, and $\epsilon_{v,t+1}$ are i.i.d. N(0,1). The only change to the original model of Bansal and Yaron (2004) is the use of a "square root" process for consumption variance. While the three state variables are uncorrelated in this baseline model, this will not be the case below, and this modification will continue to allow for analytical solutions.

Our generalized model deviates from Bansal and Yaron in several important dimensions. First, we allow shocks to consumption growth ($\epsilon_{c,t+1}$) and expected long-run consumption growth ($\epsilon_{x,t+1}$) to be stochastically correlated. We refer to this correlation as consumption growth persistence, or CGP, given that it determines whether a shock to current consumption growth is associated with higher or lower consumption growth in the future. This correlation, which we denote as ρ_t, is a stochastic process specified below.

As discussed in the introduction, a stochastic correlation can be viewed as a reduced-form approach to modeling time variation in the relative importance of permanent and transitory shocks. For example, in the production economy of Kaltenbrunner and Lochstoer (2010), the assumption of permanent productivity shocks results in a positive CGP, while transitory shocks generate a negative CGP. This results from differences in how investment (and therefore consumption) responds to changing productivity and how adjustment costs and mean reversion induce trends in future output. Given that both types of shocks are likely, either effect could dominate depending on which type of shock is currently more volatile. Furthermore, this phenomenon is not limited to shocks to productivity. Permanent and transitory shocks to income generate similar responses, as discussed, for example, by Hall and Mishkin (1982) and Campbell and Deaton (1989).

We also deviate from Bansal and Yaron by allowing consumption growth shocks to be correlated with consumption variance shocks. A negative correlation is a natural result of a precautionary savings motive, which has been confirmed empirically in several studies, including Carroll and Samwick (1998) and Basu and Bundick (2017). For simplicity, we assume that this correlation, denoted ϱ_{ps}, is constant.

Finally, given that consumption growth shocks are correlated with shocks to expected growth rates (ρ_t) and to consumption volatility (ϱ_{ps}), it is natural to expect a nonzero correlation between shocks to expected consumption growth and consumption volatility. For example, an increase in precautionary savings induced by greater uncertainty should reduce current consumption since households increase their savings and lead to a rise in expected long-run consumption growth as uncertainty wanes and consumption returns to normal. Empirically, a nonzero correlation between σ_t and x_t is found by Nakamura, Sergeyev, and Steinsson (2017), who show that it tends to be more negative during economic contractions. In another work, Parker and Preston (2005) find significant evidence, using household survey data, that the precautionary savings motive explains the predictable component of consumption growth.

In the interest of parsimony, we avoid introducing unnecessary additional parameters by assuming that this correlation between shocks to consumption volatility and expected consumption growth is equal to the product $\rho_t \varrho_{ps}$.[5]

Closing the model requires a specification of the dynamics of CGP or ρ_t. To obtain closed-form solutions, we parameterize the conditional covariance between $\sigma_t \epsilon_{c,t}$ and $\sigma_t \epsilon_{x,t}$, as opposed to between $\epsilon_{c,t}$ and $\epsilon_{x,t}$, as an autoregressive process. This covariance, which is related to ρ_t by

$$p_t = \sigma_t^2 \rho_t,$$

follows

$$p_{t+1} = \omega_0 + \omega_1 p_t + \sigma_p \sigma_t \epsilon_{p,t+1}. \tag{5}$$

For simplicity, we assume that $\epsilon_{p,t+1}$ is uncorrelated with other shocks.

Given the same preference assumptions as Bansal and Yaron, the price-to-consumption ratio z_t can be represented as a linear function of long-run expected consumption growth (x_t), the variance of consumption growth (σ_t^2), and the covariance process (p_t). That is,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 p_t. \tag{6}$$

Under conventional parameter assumptions ($\gamma > 1$ and $\psi > 1$), the appendix shows that $A_1 > 0$ and $A_2 < 0$, which is consistent with the model of Bansal and Yaron. In addition, we find that $A_3 < 0$, meaning that the price-consumption ratio is lower when consumption growth is more persistent.

[5]This correlation structure is consistent with the assumption that there are three orthonormal shocks, $[u_{c,t}\ u_{x,t}\ u_{v,t}]$, that drive the shocks to the three state variables via

$$\begin{aligned} \epsilon_{c,t} &= u_{c,t} & (2)\\ \epsilon_{x,t} &= \rho_t u_{c,t} + \sqrt{1-\rho_t^2}\, u_{x,t} & (3)\\ \epsilon_{v,t} &= \varrho_{ps} u_{c,t} + \sqrt{1-\varrho_{ps}^2}\, u_{v,t} & (4) \end{aligned}$$

Bond yields of all maturities are linear functions of the three state variables. The appendix derives an analytic formula for the one-period bond, which is increasing in x_t and decreasing in σ_t^2 and p_t, and provides a solution method for longer-term bonds.

As is standard, we extend this model further by adding a dividend process. In this model, which we label as the "full" model, dividend growth is specified as

$$\Delta d_{t+1} = \mu_d + \phi_d x_t + \sigma_t \varphi_{cd} \epsilon_{c,t+1} + \sigma_t \varphi_d \epsilon_{d,t+1}, \tag{7}$$

where $\epsilon_{d,t+1}$ is assumed to be uncorrelated with other shocks. Thus, dividend growth shares similarities with consumption both due to its dependence on the long-run growth process x_t and because of its sensitivity to the consumption growth shock $\epsilon_{c,t}$. The values ϕ_d and φ_{cd} relative to φ_d determine the strength of this commonality.

We approximate the return on the market portfolio using the Campbell-Shiller decomposition. Similar to the wealth-consumption ratio, we can verify the conjecture that the price-dividend ratio is a linear function of the three state variables. We show in the appendix that the signs of the coefficients match those of equation (6).

Given closed-form linear expressions for stock returns and for the bond yield for any maturity, it is straightforward to solve for the stock and bond return variances and covariance. We show in the appendix that all three may be expressed as linear and increasing functions of σ_t^2 and p_t. Furthermore, the stock/bond return correlation is a univariate (though nonlinear) function of just ρ_t.

2. Calibration

We perform a calibration of the model to examine its quantitative implications. In doing so, we match the level of the stock/bond return correlation and the stock market return/variance correlation. We also match the levels of expected market returns, market volatility, and real interest rates while maintaining the parameter values of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) closely as possible.

The generalized specifications require several correlation parameters. These include the three parameters of the covariance (p_t) process in equation (5), the constant correlation parameter ϱ_{ps}, as well as two parameters (φ_c and φ_{cd}) that determine the correlation between consumption and dividend shocks. Because of the difficulty in estimating unconditional means, we assume that the mean of the p_t process is zero, implying that $\omega_0 = 0$.

The slope (ω_1) and volatility (σ_p) parameters are chosen to match the volatility and first-order serial correlation of the covariances between stock returns and bond yields. This is justified by the strong relationship between these covariances and the p_t process, as shown in the following section. For our primary specification, we set the precautionary savings parameter ϱ_{ps} equal to -0.3 following the results of Basu and Bundick (2017), but we also show results for two alternative specifications by setting ϱ_{ps} equal to -0.2 or -0.5. We choose the persistence of long-run growth to match the correlation between stock and bond returns. For the volatility-of-variance parameter (σ_v), we set this parameter to the value that matches the stock return/variance correlation in the data while maintaining the unconditional volatility-of-variance with the constant value within the range assumed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). Estimates from the data are based on annual data and use real yields and stock returns, as described in Appendix B. The assumed parameters are summarized in Panel A of Table I.

Panel B compares the asset moments generated by our three specifications. These include the baseline model, in which correlations are set to zero, the consumption-only model, in which the total wealth portfolio is assumed to be the stock market portfolio, and the full model, which incorporates a dividend process. For each specification, we generate one million observations and evaluate the first two moments of stock and bond returns, as well as several other relevant asset pricing moments. The table shows that the unconditional moments generated by the simulations are generally comparable to those of other standard LRR models, aside from the correlation between stock returns and volatility changes, which is better matched by our new models.

3. The stock/bond return correlation

This section shows the relationship between CGP and the SB correlation. Establishing this link is essential because it provides a new explanation for for why that correlation varies over time. Futhermore, since it can be computed easily as long as stock and bond returns available, the SB correlation can be used as an empirical proxy for ρ_t.

To establish the relationship between CGP and the SB correlation, we first explain how the yields of a short-term bond, represented by the one-year bond, those of a long-term bond, represented by the ten-year bond, stock returns, and stock variances are determined in our model. Table II summarizes the relationship between yields and state variables.

We first consider the two channels that drive SB correlations under the baseline model, in which all shocks in (1) are assumed to be uncorrelated. The first is through shocks to the expected consumption growth ($\epsilon_{x,t+1}$). If this shock is positive, higher expected future cash flows will lead to higher stock returns. Bond yields will also increase as the demand for money rises due to the intertemporal consumption smoothing motive. Since stock and bond returns will have opposite responses, this channel implies a negative SB correlation.

The second channel is through shocks to consumption growth uncertainty ($\epsilon_{v,t+1}$). The stock market variance will rise following a positive uncertainty shock, raising the risk premium and lowering valuation. At the same time, bond yields will drop due to a precautionary savings effect. Therefore, an increase in uncertainty leads to stock and bond prices moving in the opposite direction. The flight-to-quality phenomenon often refers to the negative SB correlation arising from this second channel.

While both channels imply a negative correlation between bond and stock returns for the baseline model, regardless of the bond maturity, Panel A of Table II shows that the first channel is generally much stronger than the second. This is especially for short-term yields, for which the correlation between yield changes and shocks to expected consumption growth is around 0.98 for the baseline model and 0.93 for the generalized models. For long-term yields, the correlation between yield changes and shocks to the x_t process is around 0.83 for the baseline model and 0.77 for the generalized models. In contrast, the correlation between yield changes and volatility shocks is around -0.15 for short-term yields and -0.5 for long-term yields for all models. These results establish the close connection between interest rates and the expected consumption growth process that will underlie some of our empirical analysis. The table also shows that shocks to the total wealth portfolio or the stock index are highly correlated with realized consumption growth, even in the "full" model, which introduces a separate dividend process.

While Table 1 shows that our generalized models exhibit a similar average negative SB correlation, CGP causes this correlation to vary over time. For example, this correlation should increase when ρ_t decreases. To see this, suppose there is a positive expected consumption growth shock ($\epsilon_{x,t+1} > 0$). This shock is likely to coincide with a decline in current consumption. In this case, bond yields will increase as the economy expects higher future growth, while the negative shock to current consumption will lower equity values. While the net effect may be that equity values rise due to higher expected long-run growth, the rise will be moderated by the negative shock to current consumption. Therefore, a negative ρ_t will lead to SB correlation to be less negative than usual, perhaps even positive.

This intertemporal smoothing effect is amplified by the negative relationship between uncertainty shocks ($\epsilon_{v,t+1}$) and consumption growth shocks ($\epsilon_{c,t+1}$). Unconditionally, a positive uncertainty shock will lower stock valuation and bond yields, regardless of maturity. When ρ_t is positive, precautionary savings will further reduce the SB correlation, as positive uncertainty shocks are likely to be associated with lower expected consumption growth. In contrast, when ρ_t is negative, this shock is more likely to increase expected future growth, which would ambiguously affect stock and bond prices. A positive uncertainty shock could even increase the stock/bond return correlation in this case.

The first four panels of Figure 2 show how the SB correlation varies as a function of the state variable that represents persistence (ρ_t). Panel (a) and (c) show the relationship between the SB correlation and the ρ_t process for the consumption-only model, while Panel

(b) and (d) present corresponding results for the full model. In all panels, we show the results computed using one-period bond yields. Each line in Panels (a) and (b) characterizes the relationship under a different value of the precautionary savings parameter ϱ_{ps}, while different lines in Panel (c) and (d) represent the relationship for different bond maturities. For comparison, each panel includes a flat line indicating the constant SB correlation obtained under the baseline model, in which all shocks are uncorrelated.

Overall, this figure confirms the negative relation between ρ_t and the SB correlations, which is slightly convex in ρ_t in all cases. Comparing the four panels, the value of the precautionary savings parameter ϱ_{ps} and bond maturity has relatively little effect. Lastly, while low values of CGP are associated with positive SB correlations in both models, positive SB correlations are rarer in the full model.

This relationship is also examined in the simulation results of Panel B of Table II, which examines the "correlation of correlations." While the figure shows that the relation between ρ_t and the SB correlation is slightly nonlinear, the table also helps assess the goodness of fit of the linear projection of the SB correlation onto ρ_t. The relationship between ρ_t and SB correlation is almost perfectly negatively related for both the consumption-only model and the full model, with correlations below -0.98. Thus, our model suggests that the SB correlation is a very good proxy for the less easily observed ρ_t process.

4. Stock market variance and the leverage effect

Time-varying CGP also has implications for stock market variance and the time-varying "leverage effect" in the stock market, which refers to the negative relationship between stock returns and their variance shocks. While robust, this relationship is nevertheless time-varying, as demonstrated by Pyun (2019).

First, high CGP will raise stock return variance. Stock returns depend positively on both current consumption and future expected consumption shocks. A positive correlation between these shocks therefore magnifies these risks, while a negative correlation reduces risk due to a hedging effect. This is formalized with an analytical result, which we prove in the appendix, which is that

$$\mathrm{Var}_t(R_{m,t+1}) = V_{m,v}\sigma_t^2 + V_{m,p}\sigma_t^2\rho_t, \tag{8}$$

where for values for $V_{m,v}, V_{m,p} > 0$ are given in the appendix.

Second, higher CGP will strengthen the negative relationship between stock market returns and variance shocks. This is because variance shocks affect stock prices through their relation to both current consumption and future consumption growth. The standard precautionary savings motive implies a stable negative relationship between current consumption and volatility. When CGP is positive, positive volatility shocks are also likely to be associated with a decrease in expected future consumption growth, which causes stock returns to react more to the same variance shocks. In contrast, when CGP is negative, stock returns will react less to variance shocks, thereby decreasing the magnitude of the leverage effect.

The last two panels of Figure 2 show the relationship between CGP and stock market leverage, defined as the correlation between market returns and their variance shocks. Results for the consumption-only model and our full specification are provided in Panels (e) and (f), respectively. The negative and constant flat lines in these figures indicate the values under the baseline model. The figure shows that stock market leverage is negatively related to

CGP. Also, the leverage effect is sensitive to the value chosen for the precautionary savings parameter ϱ_{ps}, as the intuition above suggests.

While the relationship is almost linear, with a correlation below -0.93, as reported in Panel B of Table II, it does not vary as much with the CGP as the SB correlation and is sensitive to the precautionary savings parameter ϱ_{ps}. This means that with a moderate level of measurement error, the stock return/volatility correlation would likely make a much weaker empirical proxy for CGP relative to the SB correlation.

5. Conditional moments of consumption growth

The key assumption of our generalized model is the time-varying persistence of consumption growth shocks. In this section, we address how this assumption affects the conditional distribution of consumption growth for different values of ρ_t.

While greater CGP should clearly increase the serial correlation in consumption growth, it is difficult to assess the strength of this effect analytically. We instead simulate 10 million months of data from our full model and compute approximate conditional moments by separating the simulated sample into narrow bins (e.g., [-0.05, 0), [0, 0.05), [0.05, 0.1), etc.) according to the value of ρ_t. We then compute several moments of interest using all the observations in each bin. The simulation is at the monthly frequency, but we aggregate consumption growth to the quarterly level by taking the **sum** of three consecutive realizations of consumption before calculating consumption growth at the quarterly level. Then, we only keep one in every three simulated value after aggregating all shocks to the quarterly level.

Panel (a) of Figure 3 quantifies the contemporaneous relationship between shocks to consumption growth and expected growth. For example, our figure suggests that when ρ_t is at the first quartile (-0.26), a $+0.05\%$ quarterly shock to x_t implies a -0.11% change in quarterly consumption growth.[6] Panel (b) shows how the first-order serial correlation of quarterly consumption growth relates to CGP. The serial correlation is positive even for very negative values of ρ_t due to the presence of the LRR process and rises with ρ_t.

Because we have assumed that the correlation between shocks to expected consumption growth and volatility is equal to $\rho_t \varrho_{ps}$, our model implies that this correlation will be more negative when CGP is high. Panel (c) of Figure 3 shows that the same relation holds in levels in addition to holding in shocks. The level of expected future consumption growth is more negatively related to the level of consumption variance when CGP is high, where we measure the relation by the slope coefficient of the regression of x_t on σ_t^2. We examine levels to be consistent with our empirical analysis, where first differences in observable proxies for x_t on σ_t^2 are likely to be dominated by measurement error.

6. Stock return predictability of bond yields

Most consumption-based asset pricing models with time-varying consumption volatility imply that bond yields should negatively predict future stock returns. This prediction results from the stock market risk premium increasing but bond yields decreasing in consumption volatility. These relationships imply a negative relationship between the equity risk premia and bond yields.

There is, at best, weak empirical evidence for such a relationship. While several studies

[6] If consumption variance is at its long-run level, then 0.05% represents a one standard deviation increase in x_t.

starting with Fama and Schwert (1977) find a negative relation between stock returns on lagged bond yields, the negative relationship appears sample-dependent. Also, as evidenced by Welch and Goyal (2008) the statistical significance is well below other predictors such as the aggregate dividend yield.

The final implication of the generalized model is that the strength of this form of stock market return predictability depends on CGP. Bond yields are the inverse of the expected marginal utility of the investors, which is closely related to the level of the expected consumption growth. Meanwhile, the stock risk premium is higher when volatility is higher. Therefore, the negative relationship between bond yields and stock risk premia should be stronger when expected consumption growth is more negatively related to volatility,

Therefore, given the close negative relationship between the SB correlation and CGP, we expect the negative predictive relationship between future stock returns and bond yields to be stronger when the SB correlation is low. Using the simulations described earlier, we examine the correlation between the stock risk premia and bond yields. This analysis uses the exact formula for the market risk premium, derived in the appendix.

Panel (d) of Figure 3 shows that the relationship between bond yields and the market risk premium is stronger when ρ_t is high, suggesting that bond yields should be poor predictors of future stock returns when CGP is low. Unconditionally, when the market risk premium can be perfectly observed, the maximum R^2 that is attainable is 8.3% (when $\rho_t = 0$). However, the R^2 is 12.0% when ρ_t is at its third quartile and is 4.9% when it is at the first quartile. This result implies that market return predictability should improve during periods when the SB correlation is more negative if bond yields are used as a return predictor.

III. Empirical results

1. Empirical proxies for CGP

Direct measurement of time variation in the relationship between current and expected consumption growth shocks is hampered by the difficulty in measuring the latent expected consumption growth process (e.g., Schorfheide, Song, and Yaron 2018) and the relatively low frequency of consumption growth data. The direct measurement of CGP is likely to be particularly unsuccessful if it varies over time.

Therefore, we examine CGP using an indirect approach using high-frequency asset price data. The stock/bond return correlation is estimated as the negative correlation between the first-order difference in nominal bond yields and stock returns. This estimate approximates the true SB correlation, as it ignores the effect of convexity, but it nevertheless should be highly accurate. As a baseline, we estimate the correlations using a rolling basis using daily observations over the past 365 calendar days. Since the SB correlation can be measured using different bond maturities, we compute several such correlation series. This paper reports the results of the one-year and ten-year constant maturity bonds, though using other maturities produces very similar results.

Given that that inflation likely contaminates our measures of the SB correlation, we also calculate the SB correlation using real yields from Treasury Inflation-Protected Securities (TIPS) prices. Real yields are defined as the difference between the ten-year nominal yields and the 10-year breakeven inflation rate. We refer to this correlation, also estimated using a one-year rolling window, as the real SB correlation. We use the sample from 2003 due to the well-known illiquidity problem of the TIPS market for the early period (e.g., Dudley, Roush, and Ezer 2009, Gürkaynak, Sack, and Wright 2010, D'Amico, Kim, and Wei 2018).

2. Serial correlation of consumption growth

A direct implication of the model is that the persistence of consumption growth shocks should be reflected in the level of the SB correlation. Establishing this relationship is critical because it justifies using the SB correlation as an empirical proxy for the latent CGP process.

We test the relationship by examining the correlation between current and future consumption growth at different horizons, where our model predicts that serial correlation will be larger during periods when the SB correlation is more negative. In interpreting these results, it is essential to note that first-order autocorrelations from consumption growth data are likely high due to time-aggregation effects absent from our theoretical model. As shown both by Breeden, Gibbons, and Litzenberger (1989) and Heaton (1993), if investors make consumption decisions more frequently than the interval over which consumption is measured, then first-order autocorrelation in growth rates may be as high as 0.25 in quarterly data even if higher frequency changes are unpredictable. However, serial correlations at longer lags should be immune to this effect.

We first estimate a predictive regression of quarterly consumption growth on its lag. We test whether this relationship is stronger or weaker during the high or low SB correlation period by adding an interactive term. The regression we estimate is

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{t+k} \tag{9}$$

for $k = 1, 2, 3, 4$, where Δc_t is quarterly consumption growth and $\hat{\rho}_{SB,t}$ is one of the SB correlation series described above. If, as implied by our model, the serial correlation is stronger during periods in which the SB correlation is negative, then we should see a negative slope on the interaction term ($\alpha_2 < 0$).

Table III summarizes the results of these regressions. Panel A of the table first shows the simple regression result of using only lagged consumption growth as an explanatory variable. We observe significant positive autocorrelations at up to a four-quarter horizon, and the first-order autocorrelation is much larger than the value implied by time aggregation. These results are consistent with the base assumption of long-run risk in consumption growth, and the results are comparable to numbers reported by previous studies (e.g., Savov 2011).

Our primary interest, however, is to test the sign and the significance of the interactive coefficient, α_2, for multiple horizons. The results of this regression are summarized in Panel B. In the first two columns, the regression results show that, consistent with the model's predictions, the first-order serial correlation increases when the SB correlation is more negative. The interactive coefficients are negative and statistically significant for both one-year and ten-year SB correlation. Quantitatively, a 0.1 increase in the SB correlation leads to a 0.03 – 0.04 decrease in the first-order serial correlation. We then increase the forecast horizon, replacing the one-quarter-ahead dependent variable with one that is between two and four quarters ahead. These results, in the last six columns in Panel B, are consistent.

Panel C of the table presents an alternative test that exploits the high correlation implied by our model between short-term bond yields (y_t) and expected consumption growth (x_t), which was shown in Table II. We use the first difference in one-year yields to represent shocks to the expected consumption growth. Then, a direct implication is that higher CGP will be reflected in a higher contemporaneous correlation between consumption growth and changes in yields. Given the negative relation between CGP and the SB correlation, the model, therefore, predicts that the α_2 coefficient in the regression

$$\Delta c_t = \alpha_0 + \alpha_1 \Delta y_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta y_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{2,t} \tag{10}$$

will be negative. Note that even though one-year bond yield is used for y_t, both the one-year and ten-year yields are used to compute $\hat{\rho}_{SB,t}$.

The table shows that bond yield changes are unconditionally positively related to consumption growth, as implied by all LRR models, suggesting that short-term yield changes may reasonably proxy for shocks to long-run growth. Furthermore, the significance of the interactive coefficient α_2 conforms our theoretical prediction of a stronger relationship when SB correlations are negative. Similar results are obtained for SB correlations based on the one-year and ten-year yield.

A potential concern is that a significant fraction of the variation in nominal yields may be driven by inflation, which is outside our model. Whether nominal yields change more due to changes in expected inflation or real yields is a controversial subject. For example, Fama (1975) finds a strong relationship between nominal interest rate and future inflation for the pre-1970 sample, claiming that the real interest rate in the sample is close to constant. More recently, Ang, Bekaert, and Wei (2008) also find that most of the monthly variation in nominal interest rates is the result of fluctuations in expected inflation.

However, Mishkin (1992) argues that a close relationship between interest rate and inflation observed in earlier sample is likely due to a missing stochastic trend. Barr and Campbell (1997) also find that, over short horizons, real interest rates are highly time-varying. Using more recent data, Duffee (2018a) finds that most of the variation in the nominal term structure is either due to the term premium or real interest rates.

To study the possibility that the SB correlation is capturing information about the correlation between stock returns and changes in the inflation rate, we repeat the previous analysis using yields from 10-year TIPS to calculate the real SB correlation and the change in real yields. These results are shown in Table IV.

In Panel A, we estimate the regression (9) by replacing the SB correlation with the real SB correlation. While no time-variation in consumption persistence is found at the one-quarter horizon, higher-order correlations do appear to be significantly lower when the real SB correlation is higher. This is again consistent with a negative relation between the SB correlation and CGP.

Panel B reports the results of regression (10), except that the nominal yield is replaced by the real yield (r_t). Unlike nominal yields, shocks to real yields are unconditionally uncorrelated with consumption growth, which is inconsistent with the LRR framework. However, if we interact real yield shocks with the real SB correlation, then we find that the interactive coefficient is negative and highly statistically significant.

Overall, the results of the regressions in Panels A and B suggest that the stock/bond correlation is related to consumption persistence mainly because of how real rate changes affect that correlation. However, to further rule out inflation as the reason for our results based on nominal yields we perform one additional analysis, again running regression (9) but now replacing the SB correlation with the correlation between stock returns and changes in breakeven inflation rates from TIPS. If variation in the SB correlation estimated with nominal yields is mainly driven by inflation, then isolating the inflation component of yield changes should result in a stronger relation with consumption persistence.

Panel C shows, however, that the correlation between stock returns and changes in breakeven inflation is generally uninformative about future consumption and is not significantly related to consumption persistence. If anything, a higher stock/inflation correlation is associated with more persistence in consumption growth, which is inconsistent with the hypothesis that inflation effects were responsible for the results in Table III. This again supports the conclusion that nominal SB correlations are informative in this setting because they are correlated with the corresponding correlation based on real yields.

While the regressions in Tables III and IV examine non-overlapping growth rates at different horizons, Figure 4 examines consumption growth autocorrelation using overlapping longer-horizon growth rates. These regressions are identical to equation (9), except that the dependent variable is the average consumption growth rate from quarter $t+1$ to quarter $t+k$. Each panel plots the coefficient on the interaction term (α_2) for different horizons (k) as well as 68%, 90%, and 95% confidence intervals, where the panels differ with respect to the SB correlation series used and the sample period. While we present results only for the ten-year nominal SB correlation, corresponding results based on 1-year nominal yields are very similar.

Panel (a) of Figure 4 reports results using the full sample period at horizons from one to ten quarters. The graph shows that predictability is observed even at very long horizons, consistent with the premise that the SB correlation is associated with the correlation between long-run growth and current consumption growth.

Panel (b) shows the result, still based on nominal yields, for the shorter sample during which TIPS data is available, while Panel (c) shows the corresponding results over the same sample period using TIPS yields. While results based on TIPS are somewhat stronger, both graphs indicate greater long-term persistence in consumption growth in high correlation environments.

The final panel of Figure 4 examines the role of stock/inflation correlation at longer horizons. As in Table IV, a higher stock/inflation correlation increases the persistence of consumption growth, but at longer horizons this effect becomes statistically significant. While interpretation of this result is difficult given that inflation is outside our model, the results reinforce the conclusion that the SB correlation is not related to consumption persistence because of that correlation's dependence on inflation.

3. The stock/bond correlation and stock market leverage

Given the well-known theoretical motivation and empirical observation that consumption decreases when there is more uncertainty, the model provided in the previous section also shows that CGP is associated with the stock market leverage effect. In this section, we test the hypothesis that a negative SB correlation is associated with a more substantial stock market leverage effect, the negative relationship between stock market returns and their variance shocks. We estimate the stock market leverage effect as the correlation between stock returns and variance shocks (the "SV correlation"). We use daily stock market returns and changes in the square of the VXO index, obtained from the Chicago Board Options Exchange, to estimate the correlation.

Testing the relationship between the two series is challenging, as both must be estimated from rolling samples. If the samples are too long, artificial persistence in both series may lead to the spurious regression problem of Granger and Newbold (1974). Therefore, we estimate the SB correlation and the SV correlation using one-month non-overlapping samples.[7]

We evaluate the relationship between the two series using time-series regression. To control for the flight-to-quality phenomenon (e.g., Baele, Bekaert, and Inghelbrecht 2010), which may induce a negative relation between that SB correlation and the level of volatility, some regressions also include the lagged market variance, proxied by the square of the VXO index.

Table V summarizes the results of regressions in which monthly SB correlations are regressed on monthly SV correlations, with or without the market variance control. Results for the sample based on nominal yields start in 1986, which is when the VXO index becomes available, while results for real yields start in 2003. Overall, the table shows a strong positive relationship between SB correlations and SV correlations, particularly for the nominal correlations, which are available over a longer sample. In all cases, the significance remains even after controlling for market volatility and is consistent for different measures of SB correlations.

Because the previous analysis requires data on the VXO index, we consider an alternative approach that allows us to use a longer sample, at least for the SB correlations based on nominal yields. Specifically, we embed a time-varying leverage effect into a two-component

[7]Pyun (2019) shows that stock market leverage betas can be estimated with reasonable accuracy using just one month of daily data.

exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model:

$$\ln h_t - \ln h_{t-1} = \kappa_h(\ln q_{t-1} - \ln h_{t-1}) + \phi_h X_{t-1} + \delta_h \frac{R_{t-1}}{h_{t-1}}$$
$$\ln q_t - \ln q_{t-1} = \kappa_h(\theta - \ln q_{t-1}) + \phi_q X_{t-1} + \delta_q \frac{R_{t-1}}{h_{t-1}} + \delta_{qc}\hat{\rho}_{SB,t-1}\frac{R_{t-1}}{h_{t-1}}$$
$$X_t = \left(\left|\frac{R_{t-1}}{h_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) / \sqrt{1 - \frac{2}{\pi}},$$

where $R_t \sim \mathrm{N}(0, h_t^2)$. In this model, q_t represents the persistent component of stock market volatility, around which the short-term volatility h_t mean reverts. X_t can be interpreted as a volatility surprise and will be mean zero with unit variance. $\hat{\rho}_{SB,t-1}$ is an estimate of the stock/bond correlation based on a rolling one-year window of daily data, where we use either the 1-year, 10-year, or real 10-year bond yield.

Aside from the introduction of a time-varying leverage effect, the model is identical to one in Brandt and Jones (2006), which combined the EGARCH specification of Nelson (1991) with the two-component structure of Engle and Lee (1999). As in Nelson (1991), log volatility increases with high past absolute returns, and a leverage effect is induced by negative values for δ_h and δ_q.

The novel feature in the current model is the δ_{qc} term, which allows the strength of the leverage effect to vary with the stock/bond correlation. For parsimony, we include this term only in the more persistent q_t process. This is because the persistence in the h_t prices is quite low, with a half-life of less than two weeks, making it less likely to be associated with the macro forces we are concerned with.

Table VI summarizes the maximum likelihood estimates of the model for different SB correlations, where asymptotic t-statistics are in parenthesis. The first two lines show the baseline model estimates, where we do not allow the leverage effect to vary with the SB correlation. Consistent with previous literature, we find both δ_h and δ_q to be negative and highly statistically significant, indicating a strong unconditional leverage effect.

Other models in the table allow the leverage effect to depend on the SB correlation. In all three cases, we find a positive estimate of δ_{qc} that is highly statistically significant, providing strong support for the model's prediction that the stock market leverage effect is stronger when the SB correlation is negative.

4. Stock market variance

As shown in (8), our model implies that market variance is related to consumption growth uncertainty and the correlation between shocks to current and future expected consumption growth. The latter effect, which relates to the stock/bond return correlation, suggests a link to the "flight-to-quality" hypothesis.

There are two key differences between our predictions and the flight-to-quality hypothesis. The first is the direction of causality. The flight-to-quality hypothesis suggests that the SB correlation becomes negative in high volatility states as investors shift their portfolios from more risky stocks to safer bonds in response to heightened uncertainty. In contrast, our model implies that the stock market becomes more volatile when consumption growth shocks become more persistent.

Second, flight-to-quality implies a simple negative relationship between stock market variance and the SB correlation. In contrast, in our model, the SB correlation should have an effect that is interactive with macroeconomic uncertainty. In other words, stock market variance will respond more to economic uncertainty when consumption growth is more persistent. Also, the relationship between SB correlation and stock market variance will become more negative when consumption shocks are more persistent.

In this section, we test whether the relationship between SB correlation and stock market variance fluctuates in the manner predicted by the CGP channel, or if the in traditional flight-to-quality channel is more consistent with those fluctuations. We use several macroeconomic uncertainty measures, as described in Appendix B, to proxy for consumption growth variance.

The main predictive regression we study is

$$RV_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 UNC_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1}, \tag{11}$$

where RV_{t+1} is realized variance (estimated as the sum of daily squared returns estimated over month $t+1$), $\rho_{SB,t}$ is the SB correlation, and UNC_t is either the macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty measure of Baker, Bloom, and Davis (2016), or estimated consumption volatility from Schorfheide, Song, and Yaron (2018), as detailed in the data appendix.

The first regression of each panel in Table VII shows the results of the main specification. Overall, the results are consistent with our model. We find strong statistical significance of the negative β_2 across all three uncertainty measures, using either the one-year, ten-year, or real SB correlations to proxy for CGP. These results suggest that economic uncertainty predicts stock market variance with a higher slope when SB correlations are negative.

We consider two additional specifications, where we control the lagged realized variance of the stock market and the SB correlation. The control for lagged realized variance is to show that the interactive term *predicts* future stock market variance. The control for the SB correlation is to show that the interactive predictive relationship adds to the variance predictability in addition to what is implied by the flight-to-quality hypothesis. In these additional specifications, the coefficients β_2 are all negative and mostly statistically significant, confirming that time-varying CGP drives the negative relationship between stock market variance and SB correlation.

5. Expected consumption growth and uncertainty

Our model implies that the correlation between shocks to expected consumption growth and consumption volatility also varies with CGP. Because shocks to expected consumption growth and consumption volatility are difficult to measure, we instead examine the relationship in levels. In panel (c) of Figure 3, we showed that our model implies a higher correlation between x_t and σ_t when ρ_t is low. Equivalently, the correlation between x_t and σ_t will be higher when the SB correlation is high.

Because Δc_{t+1} is equal to x_t in expectation, we test this hypothesis using full and restricted versions of the predictive regression

$$\Delta c_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 \hat{\rho}_{SB,t} \times UNC_t + \beta_3 \hat{\rho}_{SB,t} + \beta_4 \Delta c_t + \epsilon_{t+1}, \tag{12}$$

where $\hat{\rho}_{SB,t}$ is one of the SB correlation estimates. UNC_t is a measure of uncertainty, which is either one of the three measures used in Table VII or the long-run volatility (q_t) estimated from the two-component model of Brandt and Jones (2006). If the SB correlation is negatively related to CGP, we should obtain positive estimates for the β_2 parameter.

Table VIII summarizes the results of these regressions, where each panel uses a different measure of SB correlation. We include regressions with and without controls for SB correlation and lagged consumption growth. Overall, the table provides reasonably strong support for our hypothesis. Using the one-year or the ten-year SB correlation in Panel A and B, we find a positive β_2 in every regression. It is statistically significant (at the 10 percent level) in most cases. Panel C shows the result using the real SB correlation. The results are somewhat weaker, particularly if we control for the SB correlation, which is likely due to the shorter sample period and collinearity.

The persistence of the expected consumption growth process implies that the relationship between macroeconomic uncertainty, the SB correlation, and future consumption growth should hold beyond the one-quarter horizon analyzed in the table. Figure 5 therefore reports the slope (β_2) of the full interactive regression (12), including all control variables, but where the dependent variable is the cumulative average over multiple periods. The slope estimates are shown as dotted lines, along with 68%, 90%, and 95% confidence intervals.

Each panel consists of three figures where the one-year, ten-year, and the real SB correlation are used as the interactive variable. Each panel represents slopes for different measures of uncertainty. In all but one specification, we find a significant positive slope for the interactive variable. These panels confirms that the negative relationship between long-run consumption growth and economic uncertainty is stronger when the SB correlation is negative or when CGP is positive.

6. Stock return predictability

One final implication of the model is the time-varying negative relationship between bond yields and future stock returns. Several papers have studied the Treasury bond yields as a stock market return predictor. Fama and Schwert (1977) estimate a simple predictive regression of future stock returns on lagged bond yields and find a negative slope, which they interpret as the result of stocks being inflation hedges. Breen, Glosten, and Jagannathan (1989) further confirm the economic significance of this predictability. More recently, Ang and Bekaert (2007) find that short-term Treasury yields and dividend yields jointly predict stock returns in many international markets. They argue that the yields represent a component of the discount rate used by investors to value equities. Campbell and Thompson (2008) also document statistically significant in-sample predictability and but Welch and Goyal (2008) report weak in and out-of-sample performance.

Our model suggests that the extent to which bond yield predicts stock returns depends on the CGP. Specifically, a higher CGP is associated with a more negative predictive slope between bond yields and future returns. We test this hypothesis in the monthly regression

$$R^e_{S,t,t+\tau} = \beta_0 + \beta_1 y_t + \beta_2 y_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1}, \tag{13}$$

where $R^e_{S,t,t+\tau}$ is the τ-month excess market return, y_t is either the one-year or the ten-year Treasury yield, and $\hat{\rho}_{SB,t}$ is the SB correlation estimated in several ways. We show the result for one, three, six, and 12-month forecast horizons (τ) across three different measures of SB correlation. The yield y_t corresponds to the bond used to to compute the SB correlation. For the real SB correlation, this is the real yield computed from TIPS.

Table IX summarizes the results of these regressions. Panel A shows the results using the one-year SB correlation and yield, and Panel B summarizes the results using the ten-year SB correlation and yield. We first consider a simple regression where leading stock returns are regressed on bond yields alone. Although the regression coefficients are all negative, they are only statistically significant for the one-year yield and and only up to a three-month horizon. For ten-year yields, the coefficients are statistically insignificant. These results are qualitatively consistent with but notably weaker than the results of early studies by Fama and Schwert (1977) and Breen, Glosten, and Jagannathan (1989).

The novel implication of our model is that the slope should be more negative when the SB correlation or the stock market leverage is lower, implying $\beta_2 > 0$. We find evidence of this hypothesis in both panels of the table, as evidenced by the consistently positive coefficients on the $y_{j,t} \times \hat{\rho}_{SB,t}$ terms. To understand the degree to which return predictability varies, consider forecasts based on one-year Treasury yields. If the SB correlation were 0.4, the conditional slope of one-month market excess returns on yields would be a paltry -0.048 $(-0.295 + 0.616 \times 0.4)$, implying that yields have essentially no predictive power for future returns. Similar conclusions hold for longer investment horizons as well. However, were the return correlation instead -0.5, a 1% increase in the one-year Treasury yield would be associated with a 0.6% decline in monthly stock returns, a 1.7% decline in three-month returns, 3.3% decline in six-month returns, and 4.8% decline in 12-month returns. Economic magnitudes are similar when based on the ten-year SB correlation.

In Panel C, we repeat the exercise with our estimated real yields and real SB correlation rather than nominal values. Overall, we see similar results, albeit with noticeably higher interactive coefficients. The coefficients are all statistically significant, which is comparable to the previous panels. A potential reason is the shorter sample period, in which the real SB correlation does not vary as much as it does over the full sample.

Many asset pricing models imply a negative relationship between bond yields and stock risk premium, as high uncertainty means lower bond yields and higher risk premium. Therefore, it is puzzling why the empirical relationship is so weak. Our results show that a stronger predictive relationship is observed than it appears, but only during periods when proxies indicate that CGP is high.

7. The recent shift in consumption growth persistence

As discussed in the introduction, rolling 1-year estimates of the SB correlation turned negative in mid-1998 after more than three decades of positive values. Our model and empirical results suggest that an increase in the persistence of consumption growth was largely responsible for this shift. But what, then, could be the reason for the significant increase in CGP observed around that time, as shown in Figure 1(b)?

Production-based models, such as Kaltenbrunner and Lochstoer (2010), suggest that an increase in the magnitude of permanent productivity shocks will cause CGP to rise when consumers face adjustment costs. More volatile persistent shocks will also increase the variability in valuation ratios to a much greater extent then more volatile transient shocks. Consistent with this, the period starting in 1998 is notable for its inclusion of several major asset market "bubbles" and crashes (e.g., the "dot-com" crash, the real estate boom, and the Great Financial Crisis).

Regardless of the reason for such fluctuations, any exogenous change in asset values may be expected to produce some level of consumption persistence. While this claim cannot be demonstrated within our model, which features an exogenous consumption process, it is intuitive. According to the permanent income hypothesis, a positive wealth shock will raise the level of consumption in perpetuity. However, if agents face adjustment costs, then this higher level will not be reached immediately, and agents will instead experience a sequence of consecutive positive growth rates as consumption rises to its new steady state level. If, later, wealth falls, then a sequence of negative consumption growth rates will be the result. In a period with multiple booms and busts, consumption is constantly trending towards some target value, but that target value keeps changing and is sometimes higher and sometimes lower than current consumption, raising consumption growth persistence.

In this section we present some evidence, which we view as suggestive, that is consistent with the greater consumption persistence of this period being driven by fluctuating asset values. In particular, we show that consumption growth rates have become more responsive to past asset returns in the period since 1998. These results are in line with recent papers by Laibson and Mollerstrom (2010), Mian and Sufi (2011), Chen, Michaux, and Roussanov (2020) have shown a stronger tendency of consumption to be driven by fluctuating asset valuations over this period. Our new finding is simply that asset returns have effects on consumption growth at long horizons than documented previously, particularly in the latter period.

We demonstrate the changing relation between asset returns and consumption growth by computing the predictive correlation

$$\mathrm{Corr}\left(R_t, \Delta c_{t+k}\right)$$

for different horizons k and using returns R_t on different wealth proxies as defined in Appendix B . We analyze horizons from one to 12 quarters and compute the correlations separately for the period before 1998 and starting in 1998.

The results, shown in Figure 6, suggest a major shift in the predictive relationship, particularly at the longer horizon. Prior to 1998, the predictive relationship between asset returns and future consumption growth was relatively weak. Statistical significance, which is indicated by the estimated correlation exceeding the corresponding dashed line, is observed in some cases, but mainly at short horizons. In the sample starting in 1998, correlations are in many cases twice as large, if not more, and highly significant even at multi-year lags. The biggest shift is in the response to housing returns, to which consumption responded little before 1998 and very significantly after.

These results are not meant to be definitive, and a more careful analysis would likely require a further enhancement of the theoretical framework that we have adopted here.

IV. Conclusion

While the exogenous consumption process examined by Bansal and Yaron (2004) is highly successful in replicating key moments of asset returns, its assumption of independent shocks is inconsistent both with macroeconomic theory and with consumption data. In particular, the model does not account for the relationship between shocks to current consumption growth and expected future consumption growth, which we term consumption shock persistence (CGP). In theory, this relationship may be positive or negative, depending on whether permanent or transient shocks to income or productivity are more prevalent. The model also does not account for the negative correlation between shocks to consumption growth and consumption volatility, which likely arises from the precautionary savings motive.

Because of these assumptions, the model cannot match several well-documented features of financial markets. The correlation between stocks and bonds is highly time-varying in the data and appears to vary with the level of stock market volatility. These effects are absent in the model of Bansal and Yaron, which features a constant stock/bond correlation.

We propose a model that allows for a significantly more realistic dependence structure. Shocks to current and expected future consumption growth are stochastically correlated, which we view as a reduced form approach to modeling the relative importance of transitory and permanent shocks. Shocks to current consumption and consumption growth are negatively correlated at a fixed value, which maintains parsimony and reflects the likely importance of the precautionary savings motive.

The model implies that the correlation between stock and bond returns is decreasing in CGP. So is the stock market leverage effect. Empirically, we see that consumption growth tends to become more serially correlated during periods of more negative stock/bond correlations. This result provides evidence of time variation in CGP, and it also links CGP to correlations that are readily estimable from high-frequency asset price data. We also see strong evidence that the SB correlation is positively related to the stock market leverage effect, which is implied by our model and new to this paper.

Our model also predicts the negative relation between stock market volatility and the SB correlation that has been observed in prior studies, such as Connolly, Stivers, and Sun (2005) or Baele, Bekaert, and Inghelbrecht (2010). This is because high consumption persistence makes cash flows and discount rates negatively correlated, which amplifies the effects of these shocks. Empirically, we find strong evidence for this relation.

We also find evidence of a time-varying relation between current uncertainty and future consumption growth. Nakamura, Sergeyev, and Steinsson (2017) show that this relation is generally negative, particularly during economic contractions. We confirm our model's implication that the correlation should be more negative when CGP is high or equivalently when the SB correlation.

Finally, the model implies that the slope coefficient of the predictive relationship between current bond yields and future stock returns also varies as a function of CGP. Using our CGP proxies, we confirm this prediction in the data. Stock returns are strongly related to lagged bond yields, but only in environments where the SB correlation. We also show that the source of this predictability is the real yield rather than the inflation component.

Combined with the observation that consumption growth persistence increased markedly since 1998, our model provides a new explanation for the dramatic downward shift in the stock/bond correlation around that time. Additional evidence suggests that the increase in persistence may be the result of a greater role for asset valuations in driving long-run consumption growth rates. While a more detailed analysis of changes in the response of consumption to asset values is beyond the scope of this study, we believe that it is an intriguing result that merits further attention.

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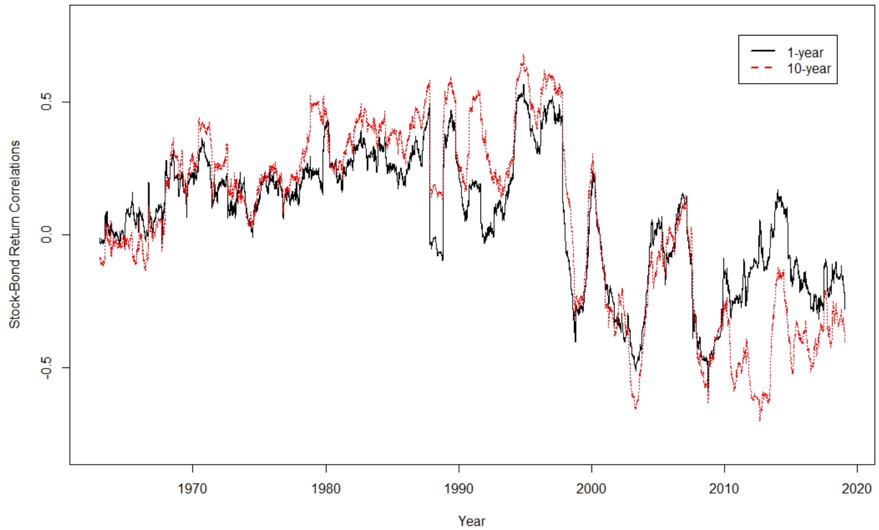
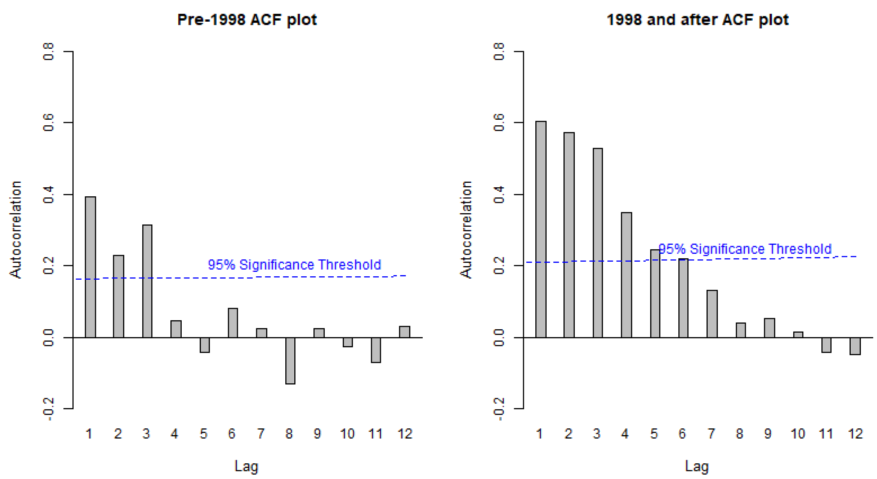


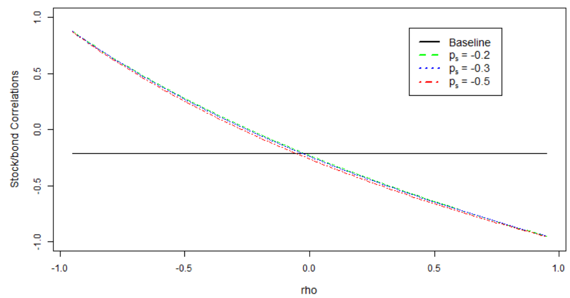
(a) Time-series of SB correlation



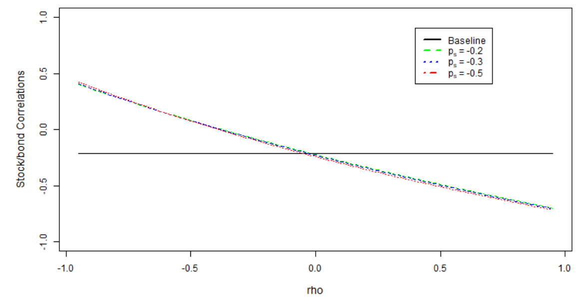
(b) Autocorrelation plot of consumption growth

Figure 1. Pre-1998 vs. Post-1998 Comparison

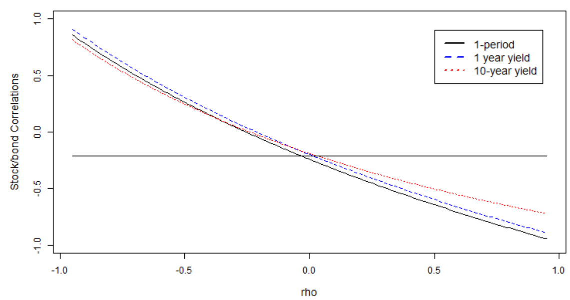
This figure shows the time-variation in the stock/bond return (SB) correlation (Panel a) and the autocorrelogram (Panel b) for the pre-1998 and post-1998 periods separately. The SB correlation is estimated using the first difference in daily one-year and ten-year bond yields over a rolling window of twelve months. Consumption is real personal consumption expenditure on non-durables and services as defined in the data appendix.



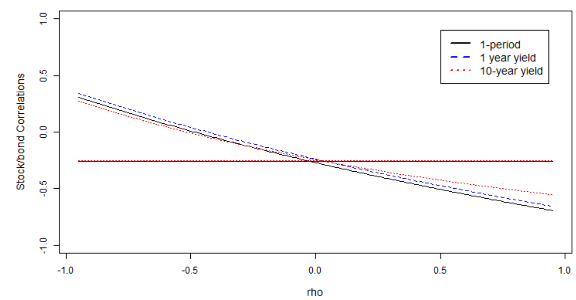
(a) Consumption-only (stock/bond) across ϱ_{ps}



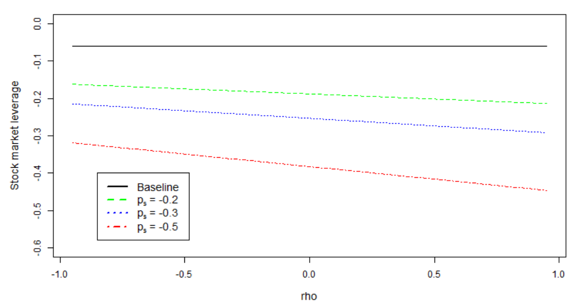
(b) Full model (stock/bond) across ϱ_{ps}



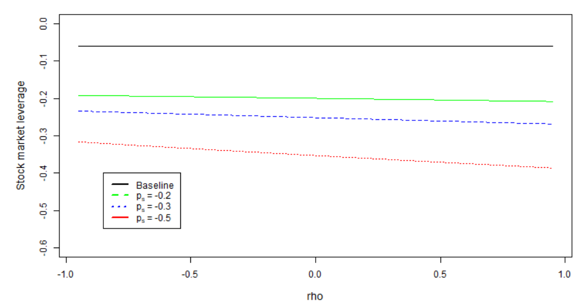
(c) Consumption-only (stock/bond) across maturity



(d) Full model (stock/bond) across maturity



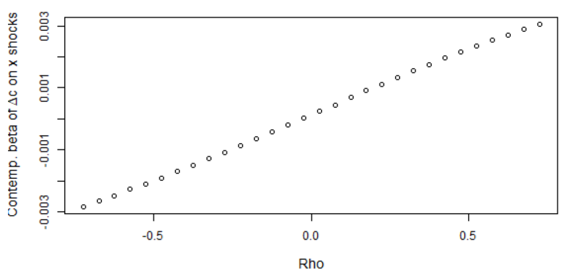
(e) Consumption-only (leverage correlation)



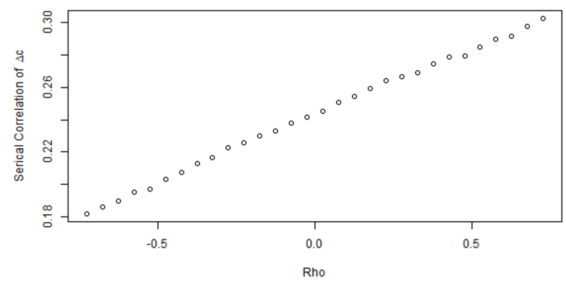
(f) Full model (leverage correlation)

Figure 2. Consumption Persistence and Model-based Correlations

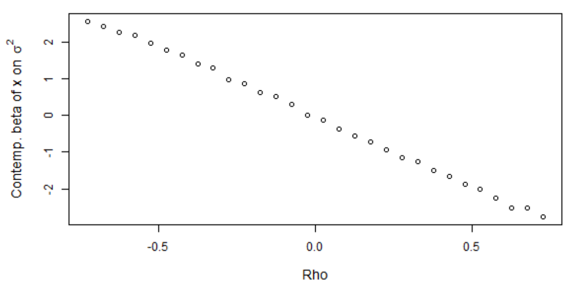
This figure shows the relationships between CGP and the stock/bond return correlations under the consumption and dividend dynamics provided in the main text.



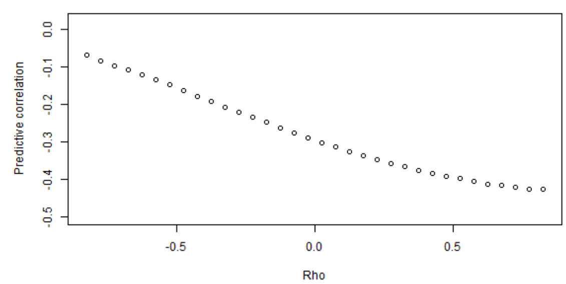
(a) Regression of Δc_t on x_t shocks



(b) Regression of Δc_{t+1} on Δc_t



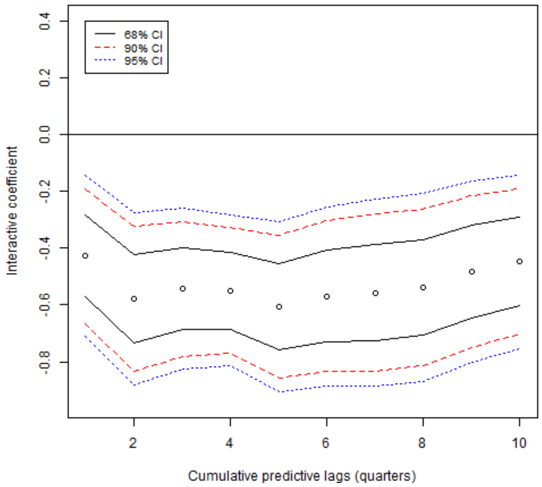
(c) Regression of x_t on σ_t^2



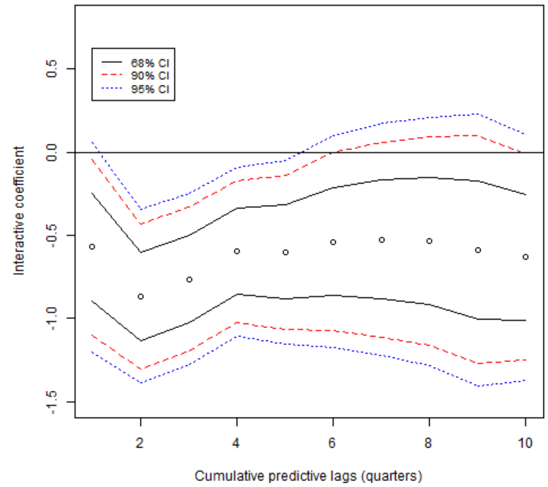
(d) Correlation between MRP_t and y_t

Figure 3. Simulation-based Regression Betas Conditional on Consumption Growth Leverage

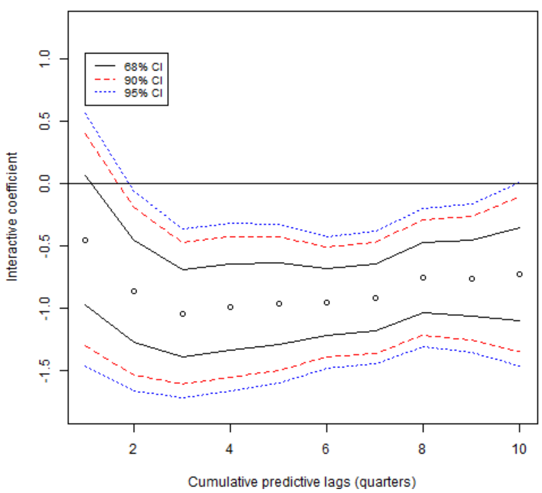
This figure describes the relationship between the slope coefficients of various simple linear regressions for different values of CGP. MRP_t and y_t denotes the market risk premium and one-period bond yields at time t



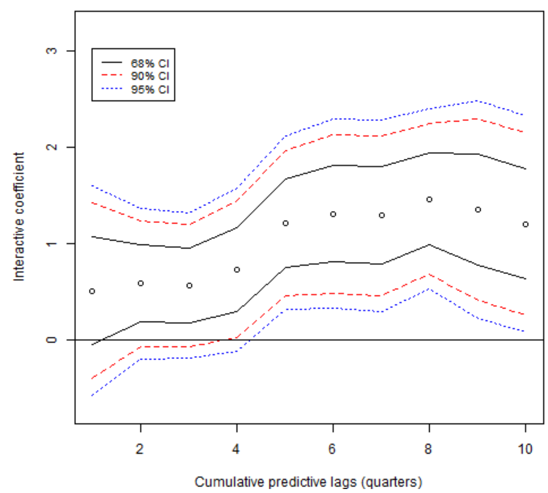
(a) Stock/bond correlations



(b) Stock/bond correlations (Shorter Sample)



(c) Stock/real bond correlation

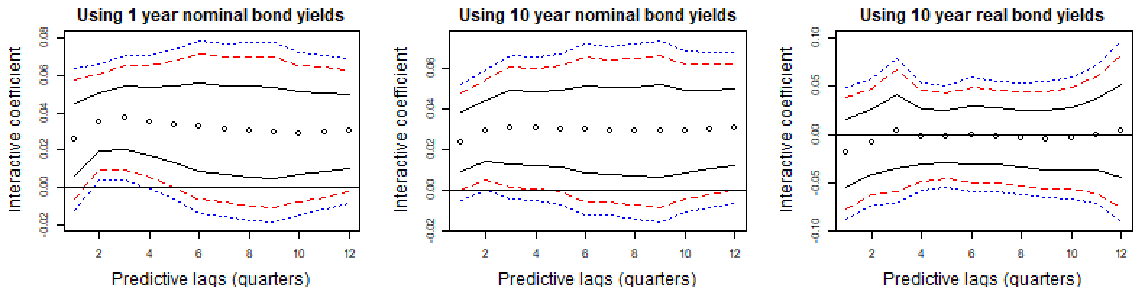


(d) Stock/inflation correlation

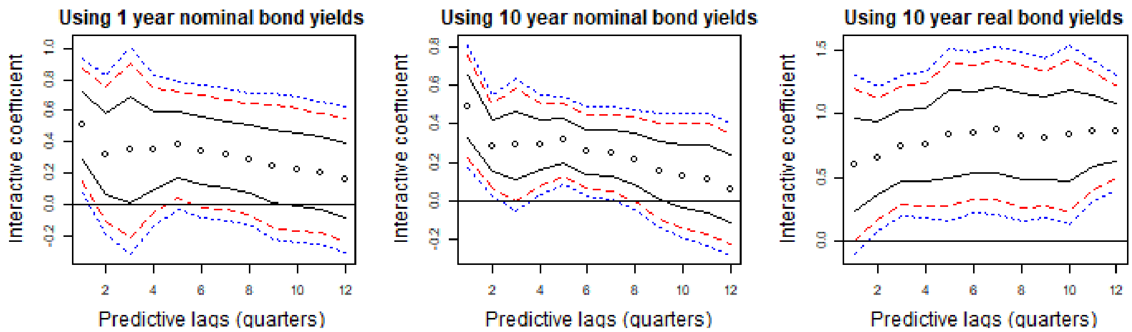
Figure 4. Interactive Beta of Consumption Growth Regressions For Multiple Lags
This figure plots the slope estimates ($\hat{\alpha}_{3,k}$) of the interactive regressions

$$\sum_{k=1}^{K} \Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K} \Delta c_t + \alpha_{2,K} \hat{\rho}_{SB,t} + \alpha_{3,K} \hat{\rho}_{SB,t} \times \Delta c_t + \epsilon_{t+K},$$

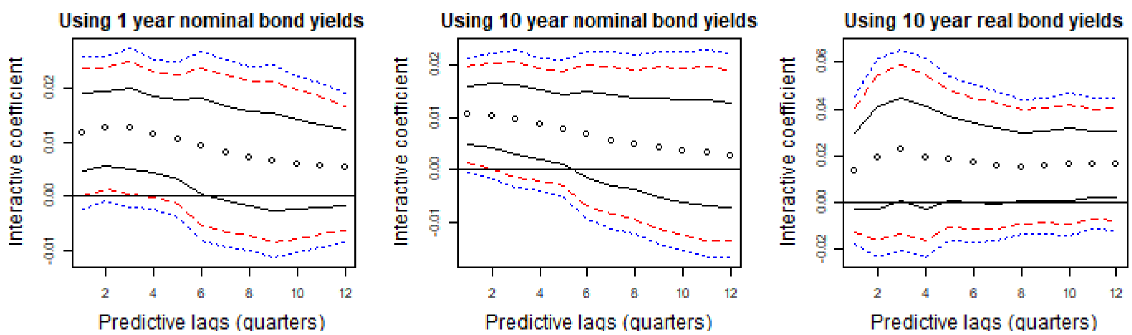
for different values of the interval (K), where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation. The lines show the 68%, 90%, and 95% confidence intervals computed using Newey-West standard errors using 12 lags, respectively.



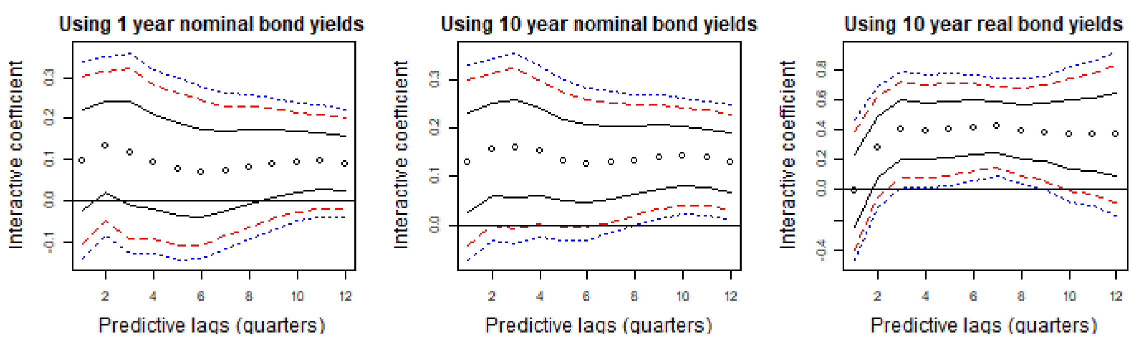
(a) Uncertainty = Macro Uncertainty



(b) Uncertainty = Monetary Policy Uncertainty



(c) Uncertainty = Consumption growth variance



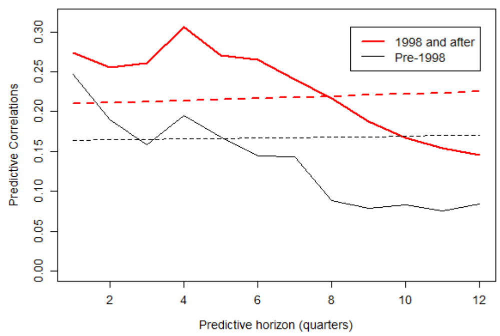
(d) Uncertainty = Stock market variance

Figure 5. Interactive Beta of Consumption Growth Regressions For Multiple Lags

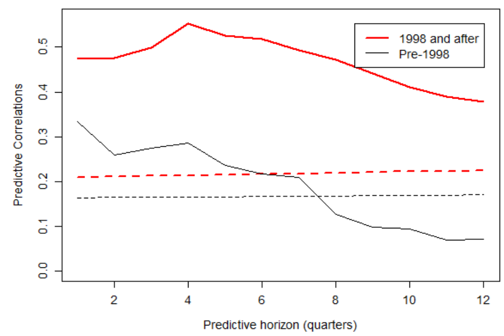
This figure plots the slope estimates ($\hat{\alpha}_{3,k}$) of the interactive regressions

$$\frac{1}{K}\sum_{k=1}^{K}\Delta c_{t+k} = \alpha_{0,K} + \alpha_{1,K}\hat{\rho}_{SB,t} + \alpha_{2,K}UNC_t + \alpha_{3,K}\hat{\rho}_{SB,t} \times UNC_t + \epsilon_{t+K},$$

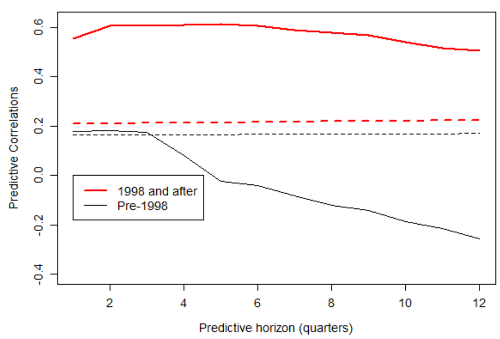
for different values of the interval (K), where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation, UNC$_t$ is one of the uncertainty measures as described in the main text. The lines show the 68%, 90%, and 95% confidence intervals computed using Newey-West standard errors using 12 lags, respectively.



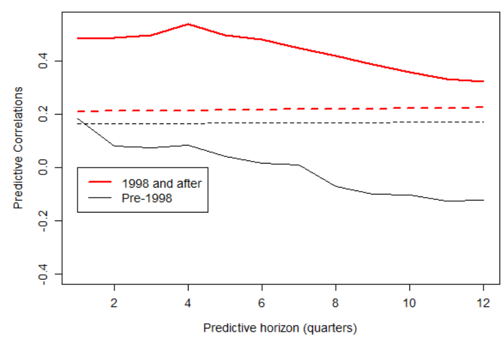
(a) Market returns



(b) Asset growth



(c) Housing Price Index returns



(d) Net worth growth

Figure 6. Consumption Growth Response to Asset Price Shocks : Pre-1998 vs. Post-1998 Comparison

This figure computes the predictive correlation Cor($\Delta c_{t+k}, R_{.,t}$ for $k = 1, \ldots, 12$, where $R_{.,t}$ is the market returns (Panel a), asset growth (Panel b), Housing Pricing Index returns (Panel c), and Net worth grown (Panel d) as defined in the data appendix.

Table I
Model Calibration

This table summarizes the parameters that describe the representative investor's preference and the dynamics of consumption and dividend growth, volatility, and covariance processes used as the main specification as well as asset pricing moments implied by these parameters. Panel A shows the values of the parameters, and Panel B shows the moments obtained via simulating the dynamics. y denotes bond yields, $R_{TW/m}$ is the return of the wealth (consumption only) or the market portfolio (full model), σ is the volatility of the wealth/market portfolio, ρ_{SB} denotes the stock/bond return correlation, ρ_{SV} is the correlation between stock returns and stock market variance shocks. Values in Panel B are scaled to the annual level.

Panel A. Parameters

Parameters		Parameters	
Preference Parameters		Consumption Parameters	
γ	10	μ	0.0015
ψ	1.5	p_x	0.943
β	0.9983	ϕ_x	0.038
Correlation Parameters		Variance Parameters	
ω_0	0	s_0	4.87×10^{7}
ω_1	0.93	s_1	0.992
σ_p	0.9×10^{-3}	σ_v	3.20×10^{-4}
Dividend Parameters		Precautionary Savings	
μ_d	0.0015	ϱ_{ps}	−0.3
ϕ_d	2.5		
φ_{cd}	3.50		
φ_d	4.50		

Panel B. Simulated Moments

		Model			Data	
	Maturity	Baseline Model	Consumption Only	Full Model	Real	Nominal
$E[R_{TW/m}]$		3.34%	3.45%	5.98%	6.64%	10.44%
y	1Y	2.35%	2.30%	2.30%	1.35%	5.15%
	10Y	2.40%	2.30%	2.30%	2.38%	6.18%
$\sigma_{TW/m}$		2.76%	2.82%	16.07%	15.25%	15.26%
$SD(\Delta y)$	1Y	1.84%	1.90%	1.91%	1.35%	1.67%
	10Y	0.70%	0.77%	0.78%	0.91%	1.13%
ρ_{SB}	1Y	−0.212	−0.197	−0.182	−0.197	−0.149
	10Y	−0.205	−0.189	−0.183	−0.171	−0.131
ρ_{SV}		−0.059	−0.233	−0.246	−0.244	−0.244

Table II
Relationships Between Simulated Values

This table summarizes the correlation between macroeconomic and asset pricing variables based on the simulation for the types of model considered. Panel A shows the relationship between Δc_{t+1}, the shocks to x_{t+1}, σ_{t+1}, and the first-order difference in one-year bond yields ($y_{1,t+1}$) and ten-year bond yields ($y_{10,t+1}$), returns of the total wealth/market portfolio ($R_{TW/m}$), and the first-order difference in the variance of the wealth/market portfolio ($\sigma_{TW/m,t+1}$). Panel B shows the relationship between CGP (ρ_t) and the model-based one-year and ten-year stock/bond correlations ($\rho_{SB,1}, \rho_{SB,10}$) or the correlation between stock returns and stock market variance shocks (ρ_{SV}). The simulations are based on 1,000,000 observations, where the first 100,000 are dropped when calculating the correlations.

Panel A. Relationships between simulated variables

	Model	$\Delta y_{1,t+1}$	$\Delta y_{10,t+1}$	$R_{TW/m,t+1}$	$\Delta\sigma^2_{TW/m,t+1}$
Δc_{t+1}	Baseline	-0.019	-0.017	0.976	0.001
	Consumption	0.027	0.148	0.977	-0.175
	Full Model	0.027	0.148	0.613	-0.229
$x_{t+1} - E_t[x_{t+1}]$	Baseline	0.975	0.834	0.205	0.001
	Consumption	0.934	0.767	0.202	-0.001
	Full Model	0.934	0.767	0.194	-0.001
$\sigma^2_{t+1} - E_t[\sigma^2_{t+1}]$	Baseline	-0.145	-0.529	-0.058	0.997
	Consumption	-0.149	-0.544	-0.349	0.591
	Full Model	-0.149	-0.544	-0.270	0.769
$p_{t+1} - E_t[p_{t+1}]$	Baseline	-	-	-	-
	Consumption	-0.275	-0.300	-0.028	0.787
	Full Model	-0.275	-0.300	-0.029	0.622

Panel B. Relationships between correlations

	Model	$\rho_{SB,1}$	$\rho_{SB,10}$	ρ_{SV}
ρ_t	Baseline	0.000	0.000	0.000
	Consumption	-0.990	-0.982	-0.938
	Full Model	-0.996	-0.991	-0.999

Table III
Consumption Growth Persistence and SB Correlation

This table summarizes the slopes and the Newey-West adjusted (12 lags) t-statistics of quarterly regressions that examine the relationship between CGP and SB correlations. Panel A and B summarize the results of

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{SB,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{SB,t} + \epsilon_{1,t+k},$$

for $k = 1, 2, 3, 4$ where $\hat{\rho}_{SB,t}$ is the stock/bond return correlation estimated using either one-year or ten-year Treasury yields. Panel C shows the results of the contemporaneous regression

$$\Delta c_t = \alpha'_0 + \alpha'_1 \Delta y_t + \alpha'_2 \hat{\rho}_{SB,t} \times \Delta y_{1,t} + \alpha'_3 \hat{\rho}_{SB,t} + \epsilon_{2,t},$$

where y_t is the nominal one-year bond yield.

Panel A. Serial correlation of consumption growth

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.494	0.370	0.420	0.186
	(7.39)	(4.30)	(6.54)	(2.22)
Adj-R^2	0.219	0.134	0.173	0.030

Panel B. Serial correlation of consumption growth and SB correlation

Dep. Var.:	Δc_{t+1}		Δc_{t+2}		Δc_{t+3}		Δc_{t+4}	
Maturity:	1Y	10Y	1Y	10Y	1Y	10Y	1Y	10Y
Δc_t	0.480	0.482	0.367	0.371	0.397	0.406	0.150	0.164
	(10.80)	(10.48)	(7.34)	(6.78)	(8.03)	(7.62)	(2.47)	(2.79)
$\Delta c_t \times \hat{\rho}_{SB,t}$	−0.400	−0.312	−0.690	−0.520	−0.372	−0.294	−0.475	−0.164
	(−2.79)	(−2.17)	(−3.76)	(−2.87)	(−1.92)	(−1.94)	(−2.39)	(−3.38)
R_t	0.004	0.003	0.005	0.004	0.005	0.003	0.005	0.004
	(3.38)	(3.46)	(4.39)	(4.00)	(3.19)	(2.59)	(2.96)	(2.88)
Adj-R^2	0.242	0.243	0.173	0.171	0.202	0.194	0.069	0.068

Panel C. Consumption growth and changes in nominal yields

Dep. Var.:	Δc_t		
SB Cor:		1Y	10Y
Δy_t	0.096	0.165	0.164
	(3.80)	(4.10)	(4.13)
$\Delta y_t \times \hat{\rho}_{SB,t}$		−0.365	−0.282
		(−3.08)	(−2.18)
R_t		0.003	0.002
		(1.49)	(1.79)
Adj-R^2	0.042	0.134	0.102

Table IV
Consumption Growth Persistence and Real SB Correlation

This table summarizes the slopes and the Newey-West adjusted (12 lags) t-statistics of quarterly regressions that examine the relationship between CGP and SB correlations computed using 10-year real yields. Panel A and C shows the results of the regression

$$\Delta c_{t+k} = \alpha_0 + \alpha_1 \Delta c_t + \alpha_2 \hat{\rho}_{.,t} \times \Delta c_t + \alpha_3 \hat{\rho}_{.,t} + \epsilon_{1,t+k},$$

for $k = 1, 2, 3, 4$, where $\hat{\rho}_{.,t}$ is either the stock/bond return correlation computed using 10-year real yields or the correlation between stock returns and the 10-year breakeven inflation rate ($\hat{\rho}_{S\pi}$). Panel B shows the results of the contemporaneous regression

$$\Delta c_t = \alpha'_0 + \alpha'_1 \Delta r_t + \alpha'_2 \hat{\rho}_{SR,t} \times \Delta r_t + \alpha'_3 \hat{\rho}_{SR,t} + \epsilon_{2,t},$$

where $\hat{\rho}_{SR,t}$ is the stock/bond return correlation computed using ten-year real yield from TIPS.

Panel A. Consumption growth persistence and Real stock/bond correlation

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.451	0.240	0.218	0.125
	(2.85)	(1.69)	(3.55)	(1.68)
$\Delta c_t \times \hat{\rho}_{SR,t}$	−0.451	−1.138	−1.394	−0.895
	(−0.87)	(−2.42)	(−4.46)	(−2.29)
$\hat{\rho}_{SR,t}$	0.005	0.007	0.008	0.007
	(3.49)	(5.14)	(3.08)	(2.99)
Adj-R^2	0.232	0.346	0.424	0.186

Panel B. Consumption growth and changes in real yields

Dep. Var.:	Δc_t	
Δr_t	−0.035	−0.149
	(−0.58)	(−2.06)
$\Delta r_t \times \hat{\rho}_{SR,t}$		−0.779
		(−3.58)
$\hat{\rho}_{SR,t}$		0.003
		(1.14)
Adj-R^2	0.014	0.087

Panel C. Consumption growth persistence and stock/inflation correlation

Dep. Var.:	Δc_{t+1}	Δc_{t+2}	Δc_{t+3}	Δc_{t+4}
Δc_t	0.684	0.742	0.804	0.738
	(3.40)	(4.20)	(3.12)	(4.29)
$\Delta c_t \times \hat{\rho}_{S\pi,t}$	0.412	0.739	0.721	1.193
	(0.92)	(1.73)	(0.91)	(1.85)
$\hat{\rho}_{S\pi,t}$	0.002	−0.002	−0.004	−0.006
	(0.53)	(−0.88)	(−1.00)	(−1.89)
Adj-R^2	0.231	0.275	0.338	0.153

Table V
Stock/Bond Return Correlations and the Stock Market Leverage Effect

This table summarizes the slopes and Newey-West adjusted standard errors (12 lags) of the contemporaneous regressions of stock/bond correlations (SB Cor) on the correlation between stock returns and stock market variance innovations (SV Cor), where the variance innovation is defined as daily changes in VXO^2. One month of daily data is used to estimate the correlations, and end of the month observation is used for VXO_t^2 .

	SB Correlation					
Bond maturity:	1Y		10Y		10Y Real	
$\text{VXO}_t^2/10000$		−1.485		−0.755		−1.041
		(−2.99)		(−0.98)		(−2.79)
$\hat{\rho}_{SV,t}$	1.033	0.982	1.602	1.576	0.775	0.979
	(5.25)	(5.32)	(5.11)	(5.07)	(2.09)	(2.11)
Adj-R^2	0.315	0.378	0.355	0.362	0.067	0.169

Table VI
Stock Market Volatility Estimation

The table summarizes the parameters estimated for the extended two-factor EGARCH model of Brandt and Jones (2006) with the stock/bond return correlations. The model is given as:

$$\ln h_t - \ln h_{t-1} = \kappa_h(\ln q_{t-1} - \ln h_{t-1}) + \phi_h X_{t-1} + \delta_h \frac{R_{t-1}}{h_{t-1}}$$

$$\ln q_t - \ln q_{t-1} = \kappa_h(\theta - \ln q_{t-1}) + \phi_q X_{t-1} + \delta_q \frac{R_{t-1}}{h_{t-1}} + \delta_{qc}\hat{\rho}_{SB,t-1}\frac{R_{t-1}}{h_{t-1}}$$

$$X_t = \left(\left| \frac{R_{t-1}}{h_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) / \sqrt{1 - \frac{2}{\pi}},$$

where $R_t \sim \text{N}(0, h_t^2)$, and where δ_{qc} is set to 0 in the baseline case. The stock/bond return correlation is estimated using one-year nominal, ten-year nominal, and ten-year real yields using a rolling window of 365 calendar days. Asymptotic t-statistics are reported in parentheses.

	κ_h	ϕ_h	δ_h	κ_q	θ	ϕ_q	δ_q	δ_{qc}
Baseline	0.100	0.042	−0.073	0.004	−4.423	0.022	−0.009	
	(12.09)	(22.81)	(−33.49)	(6.97)	(−71.93)	(15.96)	(−5.40)	
SB Cor	0.104	0.042	−0.074	0.004	−4.461	0.019	−0.013	0.028
(1Y yield)	(12.42)	(23.20)	(−33.29)	(7.48)	(−81.43)	(15.41)	(−7.60)	(6.92)
SB Cor	0.108	0.042	−0.075	0.005	−4.458	0.020	−0.012	0.018
(10Y Yield)	(12.43)	(22.99)	(−33.40)	(7.61)	(−82.25)	(15.87)	(−7.49)	(6.67)
SB Cor	0.095	0.024	−0.119	0.010	−4.537	0.029	0.008	0.073
(Real)	(8.32)	(5.19)	(−20.98)	(6.13)	(−79.10)	(10.47)	(1.07)	(3.29)

Table VII
Stock Market Variance Predictability

This table summarizes the relationship between stock/bond correlations and the market variance. In all regressions, the dependent variable is the realized variance of stock returns (RV_{t+1}) estimated using the sum of daily squared returns in the following month. Independent variables include measures of macro uncertainty of Jurado, Ludvigson, and Ng (2015), the monetary policy uncertainty of Baker, Bloom, and Davis (2016), and the consumption growth volatility of Schorfheide, Song, and Yaron (2018). Uncertainty measures are interacted with daily stock/bond return correlations.

Panel A. Uncertainty is Macro Uncertainty

	Dependent Variable: RV_{t+1}								
Bond Maturity:	1Y			10Y			10Y Real		
MU_t	0.016	0.008	0.024	0.016	0.008	0.024	0.055	0.024	0.014
	(2.60)	(2.77)	(3.75)	(2.44)	(2.66)	(3.72)	(3.09)	(2.59)	(1.05)
$MU_t \times \hat{\rho}_{SB,t}$	−0.005	−0.003	−0.084	−0.003	−0.002	−0.070	−0.006	−0.004	−0.165
	(−2.54)	(−2.69)	(−2.85)	(−2.45)	(−2.67)	(−2.91)	(−1.89)	(−1.60)	(−1.69)
RV_t		0.441			0.457			0.527	
		(3.12)			(3.15)			(8.12)	
$\hat{\rho}_{SB,t}$			0.074			0.061			0.147
			(2.77)			(2.87)			(1.67)
Adj-R^2	0.153	0.301	0.282	0.128	0.305	0.271	0.400	0.564	0.462

Panel B. Uncertainty is Monetary Policy Uncertainty

	Dependent Variable: RV_{t+1}								
Bond Maturity:	1Y			10Y			10Y Real		
MPU_t	−0.008	−0.069	−0.114	0.099	−0.201	0.019	0.022	−0.113	−0.248
	(−0.14)	(−1.28)	(−1.26)	(1.36)	(−2.48)	(0.35)	(0.19)	(−1.09)	(−1.19)
$MPU_t \times \hat{\rho}_{SB,t}$	−0.619	−0.477	−1.350	−0.218	−0.021	−0.561	−0.910	−0.844	−2.174
	(−2.42)	(−2.41)	(−2.13)	(−2.11)	(−0.35)	(−1.63)	(−1.82)	(−1.78)	(−1.82)
RV_t		0.323			0.366			0.417	
		(2.46)			(2.55)			(2.46)	
$\hat{\rho}_{SB,t}$			0.009			0.004			0.019
			(1.67)			(1.26)			(1.65)
Adj-R^2	0.128	0.213	0.161	0.072	0.186	0.083	0.135	0.284	0.160

Panel C. Uncertainty is consumption growth volatility

	Dependent Variable: RV_{t+1}								
Bond Maturity:	1Y			10Y			10Y Real		
$H_{x,t}$	0.004	0.002	0.005	0.004	0.002	0.005	0.013	0.003	0.003
	(2.79)	(2.49)	(2.44)	(2.48)	(2.24)	(2.05)	(2.40)	(1.77)	(0.75)
$H_{x,t} \times \hat{\rho}_{SB,t}$	−0.006	−0.003	−0.016	−0.003	−0.001	−0.010	−0.005	−0.004	−0.062
	(−2.40)	(−2.40)	(−1.59)	(−2.08)	(−2.11)	(−1.30)	(−1.26)	(−1.24)	(−1.99)
RV_t		0.441			0.475			0.630	
		(3.05)			(3.08)			(7.53)	
$\hat{\rho}_{SB,t}$			0.011			0.008			0.062
			(1.26)			(1.08)			(1.95)
Adj-R^2	0.157	0.313	0.181	0.103	0.303	0.120	0.236	0.549	0.342

Table VIII
Predictability of Consumption Growth (II)

This table summarizes the results of the predictive regression

$$\Delta c_{t+1} = \beta_0 + \beta_1 UNC_t + \beta_2 \hat{\rho}_{SB,t} \times UNC_t + \beta_3 \hat{\rho}_{SB,t} + \beta_4 \Delta c_t + \epsilon_{t+1},$$

where$\hat{\rho}_{SB,t}$ is the stock/bond correlation estimated in various ways, the proxies of uncertainty (UNC) are defined as in previous tables with the addition of the long-run stock market variance (q) estimate from the baseline model presented in Table VI.

Panel A. One-year stock/bond return correlation

	Dependent Variable: Δc_{t+1}											
	UNC = MU			UNC = MPU			UNC= H_x			UNC= q		
UNC_t	−0.025	−0.016	−0.028	0.016	0.034	0.020	−0.005	−0.002	−0.006	0.071	−0.020	−0.068
	(−5.51)	(−4.16)	(−6.16)	(0.20)	(0.65)	(0.48)	(−2.19)	(−2.25)	(−3.31)	(−1.71)	(−0.73)	(−1.51)
$\text{UNC}_t \times \hat{\rho}_{SB,t}$	0.005	0.004	0.026	0.361	0.299	0.506	0.005	0.003	0.011	0.071	0.056	0.098
	(2.91)	(3.13)	(1.35)	(1.76)	(2.49)	(2.38)	(2.50)	(2.40)	(1.68)	(1.36)	(1.85)	(0.81)
$\hat{\rho}_{SB,t}$			−0.024			−0.001			−0.007			−0.001
			(−0.99)			(−0.51)			(0.99)			(−0.23)
Δc_t		0.342			0.406			0.413			0.419	
		(5.46)			(4.86)			(7.55)			(8.06)	
Adj-R^2	0.205	0.287	0.208	0.093	0.252	0.103	0.135	0.274	0.139	0.102	0.242	0.099

Panel B. Ten-year stock/bond return correlation

	Dependent Variable: Δc_{t+1}											
	UNC = MU			UNC = MPU			UNC= H_x			UNC= q		
UNC_t	−0.260	−0.017	−0.028	0.051	0.059	0.094	−0.005	−0.003	−0.006	−0.073	−0.024	−0.068
	(−5.68)	(−4.59)	(−6.94)	(0.70)	(1.04)	(1.29)	(−2.63)	(−2.18)	(−2.98)	(−1.89)	(−0.95)	(−1.57)
$\text{UNC}_t \times \hat{\rho}_{SB,t}$	0.004	0.003	0.023	0.316	0.241	0.487	0.003	0.002	0.011	0.060	0.045	0.129
	(3.12)	(3.37)	(1.68)	(2.19)	(2.83)	(2.94)	(2.64)	(2.68)	(1.89)	(1.81)	(2.26)	(1.27)
$\hat{\rho}_{SB,t}$			−0.018			−0.002			−0.007			−0.003
			(−1.30)			(−1.19)			(−1.27)			(−0.68)
Δc_t		0.329			0.406			0.413			0.419	
		(5.41)			(4.39)			(7.38)			(8.11)	
Adj-R^2	0.223	0.295	0.228	0.124	0.252	0.125	0.133	0.274	0.144	0.113	0.247	0.114

Panel C. Real stock/bond return correlation

	Dependent Variable: Δc_{t+1}											
	UNC = MU			UNC = MPU			UNC= H_x			UNC= q		
UNC_t	−0.029	−0.017	−0.036	0.084	0.015	0.082	−0.005	−0.001	−0.004	−0.082	−0.027	−0.102
	(−3.58)	(−3.08)	(−4.01)	(0.10)	(0.36)	(0.76)	(−1.05)	(−0.39)	(−0.55)	(−2.39)	(−0.64)	(−3.12)
$\text{UNC}_t \times \hat{\rho}_{SB,t}$	0.007	0.006	−0.019	0.613	0.344	0.599	0.007	0.005	0.013	0.111	0.098	−0.003
	(1.96)	(2.80)	(−0.54)	(3.85)	(2.52)	(1.66)	(2.50)	(2.57)	(0.83)	(1.97)	(2.62)	(−0.01)
$\hat{\rho}_{SB,t}$			0.024			0.000			−0.007			0.006
			(0.69)			(0.03)			(−0.40)			(0.43)
Δc_t		0.365			0.513			0.497			0.394	
		(3.36)			(3.70)			(3.03)			(2.85)	
Adj-R^2	0.251	0.303	0.243	0.098	0.256	0.084	0.296	0.432	0.282	0.215	0.256	0.208

Table IX
Market Return Predictability

This table summarizes the results of the regression

$$R^e_{S,t+1} = \beta_0 + \beta_1 y_t + \beta_2 y_t \times \hat{\rho}_{SB,t} + \epsilon_{t+1},$$

where R^e_S is the leading value-weighted market excess return over one-month, three-month, six-month, or twelve-month, and y_t is either the one-year nominal ($y_{1,t}$), ten-year nominal ($y_{10,t}$), or the ten-year real yield ($r_{10,t}$). $\hat{\rho}_{SB,t}$ is the estimated correlation between stock and bond returns The t-statistics are adjusted for heteroscedasticity and autocorrelation using Newey-West standard errors with a lag of 12.

Panel A. SB correlation using 1-year yields

	One-month		Three-month		Six-month		Twelve-month	
$y_{1,t}$	−0.100	−0.295	−0.239	−0.818	−0.408	−1.512	−0.648	−2.252
	(−1.88)	(−3.45)	(−1.89)	(−3.95)	(−1.57)	(−3.89)	(−1.35)	(−3.06)
$y_{1,t} \times \hat{\rho}_{SB,t}$		0.616		1.857		3.539		5.144
		(2.63)		(3.11)		(3.20)		(2.46)
Adj-R^2	0.005	0.016	0.009	0.047	0.014	0.082	0.018	0.092

Panel B. SB correlation using 10-year yields

	One-month		Three-month		Six-month		Twelve-month	
$y_{10,t}$	−0.088	−0.429	−0.201	−1.204	−0.339	−2.175	−0.621	−2.976
	(−1.64)	(−3.63)	(−1.56)	(−4.24)	(−1.28)	(−3.99)	(−1.29)	(−3.14)
$y_{10,t} \times \hat{\rho}_{SB,t}$		0.571		1.655		3.016		4.152
		(3.36)		(3.65)		(3.48)		(2.85)
Adj-R^2	0.003	0.022	0.006	0.054	0.009	0.089	0.008	0.086

Panel C. SB correlation using 10-year real yields

	One-month		Three-month		Six-month		Twelve-month	
$r_{10,t}$	−0.835	−0.254	−2.341	−0.462	−3.749	−0.367	−6.026	−2.131
	(−2.48)	(−0.82)	(−2.88)	(−0.69)	(−2.70)	(−4.69)	(−2.41)	(−0.81)
$r_{10,t} \times \hat{\rho}_{SB,t}$		4.624		15.180		27.328		31.477
		(2.68)		(3.73)		(3.23)		(2.23)
Adj-R^2	0.027	0.075	0.076	0.242	0.092	0.346	0.126	0.303

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A. Technical appendix

1. The wealth-consumption ratio

Following the Campbell-Shiller decomposition, the returns to total wealth portfolio can be represented by

$$R_{TW,t+1} = \kappa_0 + \Delta c_{t+1} + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) + A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t).$$

The intertemporal marginal rate of substitution (IMRS) is

$$\begin{aligned} m_{t+1} = \theta \log \beta - \gamma \Delta c_{t+1} + (\theta - 1)\big[&\kappa_0 + A_0(\kappa_1 - 1) + A_1(\kappa_1 x_{t+1} - x_t) \\ &+ A_2(\kappa_1 \sigma_{t+1}^2 - \sigma_t^2) + A_3(\kappa_1 p_{t+1} - p_t)\big]. \end{aligned}$$

The unexpected component of the IMRS is represented by

$$m_{t+1} - \mathrm{E}_t[m_{t+1}] = \lambda_c \sigma_t \epsilon_{c,t+1} + \lambda_x \sigma_t \epsilon_{x,t+1} + \lambda_v \sigma_t \epsilon_{v,t+1} + \lambda_\delta \sigma_t \epsilon_{p,t+1},$$

where $\lambda_c = -\gamma$, $\lambda_x = (\theta - 1)\kappa_1 A_1 \phi_x$, $\lambda_v = (\theta - 1)\kappa_1 A_2 \sigma_v$, and $\lambda_\delta = (\theta - 1)\kappa_1 A_3 \sigma_p$.

We solve for A_0, A_1, A_2, and A_3 using equation using the Euler equation $\mathrm{E}_t[m_{t+1} + R_{TW,t+1}] + \mathrm{Var}_t[m_{t+1} + R_{TW,t+1}] = 0$. For A_1, we collect all terms associated with x_t:

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 p_1}.$$

Collecting the terms from the Euler equation that are functions of σ_t^2 and p_t, it can be seen that A_2 and A_3 must jointly satisfy the conditions

$$2A_2(\kappa_1 s_1 - 1) + \theta\big((A_1 \kappa_1 \varphi_x)^2 + (A_2 \kappa_1 \sigma_v)^2 + (A_3 \kappa_1 \sigma_p)^2 + (1 - \frac{1}{\psi})^2\big) + 2(1 - \gamma)\kappa_1 A_2 \sigma_v \varrho_{ps} = 0$$

$$A_3 = A_{30} + A_{32} A_2,$$

where $A_{30} = \frac{(1-\gamma)\kappa_1 A_1 \varphi_x}{1 - \kappa_1 \omega_1} < 0$ and $A_{32} = \frac{\theta \varrho_{ps} \kappa_1^2 A_1 \varphi_x \sigma_v}{1 - \kappa_1 \omega_1} > 0$.

A_2 can be obtained by solving a quadratic equation after plugging the second equation into the first. It can also be shown that $A_2 < 0$ when $\gamma > 1$ and $\psi > 1$ by evaluating the characteristics of the quadratic equation. We obtain two values for A_2. We choose the value that is closer to the baseline model. The second value generates unrealistic moments of asset returns. The negative sign of A_2 also implies $A_3 < 0$.

Finally, A_0 satisfies $A_0 = \frac{1}{1-\kappa_1}\left[\log\beta + \kappa_0 + (1-\frac{1}{\psi})\mu + k_1(A_2 s_0 + A_3\omega_0)\right]$.

2. The price-dividend ratio

Similar to the wealth-consumption ratio, market returns can be expressed as

$$R_{m,t+1} = \kappa_0 + \Delta d_{t+1} + A_{m,0}(\kappa_1 - 1) + A_{m,1}(\kappa_1 x_{t+1} - x_t) + A_{m,2}(\kappa_1\sigma^2_{t+1} - \sigma^2_t) + A_{m,3}(\kappa_1 p_{t+1} - p_t).$$

We again solve for the coefficients using the Euler equation $E_t[m_{t+1}+R_{m,t+1}]+0.5Var_t[m_{t+1}+R_{m,t+1}] = 0$. Collecting the terms associated with x_t, σ^2_t, and p_t, we can solve for $A_{m,0}$, $A_{m,1}$, $A_{m,2}$, and $A_{m,3}$. First, we have

$$A_{m,1} = \frac{\phi_d - \frac{1}{\psi}}{1 - \kappa_1 p_1}.$$

As in the wealth-consumption ratio, $A_{m,2}$, and $A_{m,3}$ must jointly satisfy the conditions

$$\begin{aligned}
&2A_{m,2}(\kappa_1 s_1 - 1) + 2(\theta-1)(\kappa_1 s_1 - 1)A_2 + 2(\varphi_{cd} + \lambda_c)(\kappa_1 A_{m,2}\sigma_v + \lambda_v)\varrho_{ps} \\
&\qquad + (\kappa_1 A_{m,1}\varphi_x + \lambda_x)^2 + (\kappa_1 A_{m,2}\sigma_v + \lambda_v)^2 + (\kappa_1 A_{m,3}\sigma_p + \lambda_\delta)^2 + (\varphi_{cd} + \lambda_c)^2 + \varphi_d^2 = 0 \\
&A_{m,3} = A_{m,30} + A_{m,32}A_{m,2},
\end{aligned}$$

where $A_{m,30} = \frac{1}{1-\kappa_1\omega_1}\big((\varphi_{cd}+\lambda_c)(\kappa_1 A_{m,1}\varphi_x+\lambda_x)+(\theta-1)(\kappa_1\omega_1-1)A_3+\lambda_v(\kappa_1 A_{m,1}\varphi_x+\lambda_x)\varrho_{ps}\big)$ and $A_{m,32} = \frac{1}{1-\kappa_1\omega_1}\kappa_1\sigma_v(\kappa_1 A_{m,1}\varphi_x + \lambda_x)\varrho_{ps}$. Evaluating the characteristics of the quadratic function, similar to the earlier case, $A_{m,2} < 0$ when $\gamma > \varphi_{cd} > 1$, which is consistent with a general long-run risk specification. Also, one can show that $A_{m,30} <$ and $A_{m,32} > 0$, under the condition of $\gamma > \phi_d$ and $\varphi_{cd} > 1$, which implies $A_{m,3} < 0$.

Finally, $A_{m,0}$ satisfies

$$A_{m,0} = \frac{1}{1-\kappa_1}\big(\theta \log\beta + \theta\kappa_0 + (1-\gamma)\mu$$

$$+ \kappa_1 s_0(A_2(\theta-1) + A_{m,2}) + \kappa_1\omega_0(A_3(\theta-1) + A_{m,3}) + (\theta-1)(\kappa-1)A_0)\big).$$

3. Bond yields

Denote the state vector as

$$\Sigma_t = \begin{bmatrix} \Delta C_t & x_t & \sigma_t^2 & p_t \end{bmatrix}'$$

We can write the conditional mean as

$$\mathrm{E}_t\left[\Sigma_{t+1}\right] = K_0 + K\Sigma_t,$$

where

$$K_0 = \begin{bmatrix} \mu & 0 & s_0 & \omega_0 \end{bmatrix}'$$

and

$$K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & s_1 & 0 \\ 0 & 0 & 0 & \omega_1 \end{bmatrix}$$

The conditional covariance matrix is

$$\mathrm{Cov}_t\left(\Sigma_{t+1}, \Sigma_{t+1}'\right) = \begin{bmatrix} \sigma_t^2 & \phi_x p_t & \varrho_{ps}\sigma_v\sigma_t^2 & 0 \\ \phi_x p_t & \phi_x^2\sigma_t^2 & \sigma_v\varrho_{ps}p_t & 0 \\ \varrho_{ps}\sigma_v\sigma_t^2 & \sigma_v\varrho_{ps}p_t & \sigma_v^2\sigma_t^2 & 0 \\ 0 & 0 & 0 & \sigma_p^2\sigma_t^2 \end{bmatrix} = \Omega_1\sigma_t^2 + \Omega_2 p_t,$$

where

$$\Omega_1 = \begin{bmatrix} 1 & 0 & \varrho_{ps}\sigma_v & 0 \\ 0 & \phi_x^2 & 0 & 0 \\ \varrho_{ps}\sigma_v & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_p^2 \end{bmatrix} \quad \text{and} \quad \Omega_2 = \begin{bmatrix} 0 & \phi_x & 0 & 0 \\ \phi_x & 0 & \varrho_{ps}\sigma_v & 0 \\ 0 & \varrho_{ps}\sigma_v & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In vector notation, we can write the log pricing kernel as

$$m_{t+1} = m_0 + M_1'\Sigma_{t+1} - M_2'\Sigma_t$$

with

$$m_0 = \theta \log \beta + (\theta - 1)\left(\kappa_0 + A_0(\kappa_1 - 1)\right),$$

$$M_1 = \begin{bmatrix} -\gamma & (\theta-1)\kappa_1 A_1 & (\theta-1)\kappa_1 A_2 & (\theta-1)\kappa_1 A_3 \end{bmatrix}'$$

and

$$M_2 = \begin{bmatrix} 0 & (\theta-1)A_1 & (\theta-1)A_2 & (\theta-1)A_3 \end{bmatrix}'$$

where A_1, A_2, and A_3 are defined earlier.

The log price of a riskless one-period bond is derived by

$$\begin{aligned} B_1(t) =& \mathrm{E}_t\left[m_{t+1}\right] + 0.5\mathrm{Var}_t\left(m_{t+1}\right) \\ =& m_0 + M_1'K_0 + \left(M_1'K - M_2'\right)\Sigma_t + 0.5M_1'\mathrm{Cov}_t\left(\Sigma_{t+1}, \Sigma_{t+1}'\right)M_1 \\ =& m_0 + M_1'K_0 + \left(M_1'K - M_2'\right)\Sigma_t + +0.5M_1'\Omega_1 M_1\sigma_t^2 + 0.5M_1'\Omega_2 M_1 p_t \\ =& m_0 + M_1'K_0 + \left(M_1'K - M_2'\right)\Sigma_t + 0.5\Psi'\Sigma_t \\ =& m_0 + M_1'K_0 + \left(M_1'K - M_2' + 0.5\Psi'\right)\Sigma_t, \end{aligned}$$

where

$$\Psi' = \begin{bmatrix} 0 & 0 & M_1'\Omega_1 M_1 & M_1'\Omega_2 M_1 \end{bmatrix}'.$$

Therefore, the yield of a one-period bond is represented by

$$y_t = Y_0 + Y\Sigma_t,$$

where

$$Y_0 = -m_0 - M_1' K_0$$

and

$$Y = -M_1' K + M_2' - 0.5\Psi'$$

It can be shown that for

$$Y = \begin{bmatrix} 0 & Y_x & Y_v & Y_p \end{bmatrix}'$$

that $Y_x > 0$, $Y_{\sigma^2}, Y_p < 0$.

Now suppose that the n-period bond has a log price

$$B_n(t) = D_{n,0} + D_n' \Sigma_t$$

Then the $n+1$-period bond has a price that is equal to the conditional expectation of

$$\mathrm{E}_t\left[m_{t+1} + B_n(t+1)\right] + 0.5\mathrm{Var}_t\left(m_{t+1} + B_n(t+1)\right),$$

where

$$m_{t+1} + B_n(t+1) = m_0 + D_{n,0} + (M_1 + D_n)'\Sigma_{t+1} - M_2'\Sigma_t.$$

The log price of the bond can be solved as

$$\begin{aligned} B_{n+1}(t) =& m_0 + D_{n,0} + (M_1 + D_n)'(K_0 + K\Sigma_t) - M_2'\Sigma_t + 0.5(M_1 + D_n)'\mathrm{Cov}_t\left(\Sigma_{t+1}, \Sigma_{t+1}'\right)(M_1 + D_n) \\ =& m_0 + D_{n,0} + (M_1 + D_n)'K_0 + \left((M_1 + D_n)'K - M_2'\right)\Sigma_t + 0.5\Psi_n'\Sigma_t, \end{aligned}$$

where

$$\Psi_n = \begin{bmatrix} 0 & 0 & (M_1 + D_n)'\Omega_1(M_1 + D_n) & (M_1 + D_n)'\Omega_2(M_1 + D_n) \end{bmatrix}'$$

The log of $n+1-$period bond price is therefore

$$B(n+1,t) = D_{n+1,0} + D'_{n+1}\Sigma_t,$$

where

$$D_{n+1,0} = m_0 + D_{n,0} + (M_1 + D_n)'K_0$$

and

$$D_{n+1} = K'(M_1 + D_n) - M_2 + \frac{1}{2}\Psi_n.$$

The $n+1-$period yield is derived as

$$y_{n+1,t} = Y_{n+1,0} + Y_{n+1}\Sigma_t,$$

where $Y_{n+1,0} = -D_{n+1,0}$ and $Y_{n+1} = -D_{t+1}$. Y_{n+1} can further be represented as

$$Y_{n+1} = \begin{bmatrix} 0 & Y_{n+1,x} & Y_{n+1,v} & Y_{n+1,p} \end{bmatrix}'$$

4. The stock/bond return correlation

The stock/bond return correlation is the negative of the correlation between stock return and changes in bond yield. The unexpected return of the total wealth portfolio and the market return are derived using the Campbell-Shiller decomposition:

$$R_{TW,t+1} - E_t[R_{TW,t+1}] = \kappa_1\phi_x A_1\sigma_t\epsilon_{x,t+1} + \kappa_1\sigma_v A_2\sigma_t\epsilon_{v,t+1} + \kappa_1\sigma_p A_3\epsilon_{p,t+1} + \sigma_t\epsilon_{c,t+1}$$

$$R_{m,t+1} - E_t[R_{m,t+1}] = \kappa_1\phi_x A_{m,1}\sigma_t\epsilon_{x,t+1} + \kappa_1\sigma_v A_{m,2}\sigma_t\epsilon_{v,t+1} + \kappa_1\sigma_p A_{m,3}\epsilon_{p,t+1} + \varphi_{cd}\sigma_t\epsilon_{c,t+1} + \varphi_d\sigma_t\epsilon_{d,t+1}$$

We represent the above relationship by:

$$S_{j,x}\sigma_t\epsilon_{x,t+1} + S_{j,v}\sigma_t\epsilon_{v,t+1} + S_{j,p}\sigma_t\epsilon_{p,t+1} + S_{j,c}\sigma_t\epsilon_{c,t+1} + S_{j,d}\sigma_t\epsilon_{d,t+1},$$

where j is either TW for the wealth portfolio or m for the market portfolio. From the above equation, we can derive the stock/bond return correlation by taking the negative of conditional correlation between wealth portfolio/market returns and bond yields.

The conditional covariance between a n-period bond yield and stock returns can be expressed as

$$\mathrm{Cov}_t(R_{j,t+1}, y_{n,t+1}) = (Y_{n,x}S_{j,x}\varphi_x + Y_{n,v}S_{j,v}\sigma_v + Y_{n,p}S_{j,p}\sigma_p + Y_{n,v}S_{j,c}\sigma_v\varrho_{ps})\,\sigma_t^2$$
$$+ ((Y_{n,x}\varphi_x S_{j,v} + Y_{n,v}S_{j,x}\sigma_v)\varrho_{ps} + Y_{n,x}S_{j,c}\varphi_x)\,p_t.$$

The conditional variance of the bond yield is

$$\mathrm{Var}_t\,(y_{n,t+1}) = (Y_n'\Omega_1 Y_n + Y_n'\Omega_2 Y_n \rho_t)\,\sigma_t^2.$$

Similarly, the conditional variance of the wealth portfolio/market returns is

$$\mathrm{Var}_t\,(R_{j,t+1}) = \sigma_{j,t}^2 = (V_{j,v} + V_{j,p}\rho_t)\sigma_t^2$$

for $j = \{TW, m\}$, where $V_{j,v} = S_{j,x}^2 + S_{j,v}^2 + S_{j,p}^2 + S_{j,c}^2 + S_{j,d}^2 + 2S_{j,c}S_{j,v}\varrho_{ps}$ and $V_{j,p} = 2S_{j,x}S_{j,v}\varrho_{ps} + 2S_{j,c}S_{j,x}$.

5. The stock market leverage effect

The leverage correlation is the conditional covariance between the returns and variance shocks of the wealth portfolio divided by the conditional standard deviations of each. The covariance can be represented by

$$\mathrm{Cov}_t(R_{j,t+1}, \sigma_{j,t+1}^2) = [(S_v + S_c\varrho_{ps})V_{j,v}\sigma_v + S_p V_{j,p}\sigma_p + S_x V_{j,v}\sigma_v\varrho_{ps}\rho_t]\,\sigma_t^2,$$

for $j = \{TW, m\}$. Dividing the above by the variance of variance shocks yields the stock market leverage effect. The variance of the market variance shocks is

$$\big((V_v\sigma_v)^2 + (V_p\sigma_p)^2\big)\sigma_t^2.$$

6. The market risk premium

The risk premium of the wealth/market portfolio can be expressed as

$$\mathrm{Cov}_t(-m_{t+1}, R_{j,t+1}) = \big(- \lambda_c(S_{j,c} + S_{j,v}\varrho_{ps}) - \lambda_x S_{j,x} - \lambda_v S_{j,v} - \lambda_\delta S_{j,p} - S_{j,c}\lambda_v\varrho_{ps}\big)\sigma_t^2$$
$$+ (-\lambda_x S_{j,v}\varrho_{ps} - \lambda_v S_{j,x}\varrho_{ps} - \lambda_c S_{j,x} - \lambda_x S_{j,c})p_t$$

for $j = \{TW, m\}$.

B. Data

Quarterly consumption data is obtained from the national income and product accounts (NIPA) provided by the Bureau of Economic Analysis. We measure consumption at the quarterly frequency as the sum of the real personal consumption expenditure on non-durables and services. We take the quantity index of NIPA Table 2.3.3 and divide it by the total population obtained from NIPA Table 7.1. Consumption growth is defined as the first log difference and is computed from 1962 to 2019.

Bond yields are obtained from the website of the Federal Reserve Bank of St. Louis and are available from 1962 to 2019. We use the one-year yield, ten-year yield, and ten-years TIPS data is used compute the ten-year real yields. Excess market returns and total market returns are from Ken French's data library.

In Section II.2, the averages of real variables are computed by subtracting the average changes in consumer price index, obtained from the Bureau of Labor Statistics, over the entire calibration period. To compute the standard deviation of real bond yields and the stock/bond return correlation, we make several assumptions. We assume that the relative fraction of the shocks to inflation to the nominal yields remains constant over the entire sample period. We also assume that inflation follows a unit root process. That is, changes in expected inflation equals unexpected prices changes in the previous period. This assumption implies that nominal dividend is represented as the sum of real dividends and shocks to expected inflation (π).

$$d^n_{t+1} = d_{t+1} + \pi_{t+1},$$

where d^n_{t+1} is the nominal dividend at time $t+1$. Finally, a zero nominal-real correlation is assumed.

We first calculate the variance ratio (VR) of the inflation shocks (e.g., Duffee 2018a) using the TIPS data between 2003 – 2019. The variance of the real yields is computed by multiplying the variance of nominal yields by $(1 - VR)$. Similarly, the real SB covariance $Cov(r_t, R^r_{m,t})$ is computed as

$$Cov(\Delta r_t, R^r_{m,t}) = Cov(\Delta y_t - \Delta\pi_t, R_{m,t} - \Delta\pi_t) = Cov(\Delta y_t, R_{m,t}) - Var(\Delta\pi_t),$$

where y_t is the nominal bond yield, r_t is the real bond yield, $R_{m,t}$ is the nominal stock return, and the real return of a stock investment is $R^r_{m,t}$. The variance of the inflation shock $Var(\pi_t)$ is computed as $VR \times Var(y_t)$. The variance of real stock returns is $Var(R^r_{m,t}) = Var(R_{m,t}) - Var(\Delta\pi_t)$, which is quantitatively similar to the variance of nominal stock returns.

We measure macroeconomic uncertainty in three different ways. First, we use the 12-month macro uncertainty measure from Jurado, Ludvigson, and Ng (2015), which is obtained from Sydney Ludvigson's website and is available from 1961 to 2019. These data are available on a monthly basis, and we convert to quarterly by choosing the last value of each quarter. Second, we use the monetary policy uncertainty from Baker, Bloom, and Davis (2016). This uncertainty index is estimated using textual analysis of newspaper articles and is substantially different from those estimated from macroeconomic aggregates. The data covers the period from 1985 to 2019 and can be downloaded at the authors' Economic Policy Uncertainty website. Third, we use the volatility estimate of expected consumption growth estimated using the long-run risk model of Schorfheide, Song, and Yaron (2018). That series, which was provided by the authors of the paper, is available from 1962 to 2014.

We also use several different measures of stock market volatility. The first is a monthly measure, the so-called "realized variance" computed as the squared daily excess market returns. The second volatility measure is the VXO index of the Chicago Board Options Exchange (CBOE). VXO is the predecessor of the VIX and measures the implied volatility of options on the S&P 100 Index (as opposed to the VIX, which is the model-free implied volatility of S&P 500 Index options). We choose it because it is available going back to 1986, while the VIX starts in 1990. Finally, we estimate measure of equity market volatility using the two-factor EGARCH model of Brandt and Jones (2006), which is closely related to the model of Engle and Lee (1999). Specifically, we use the long-run factor from the most general specification of Brandt and Jones, which we fit using daily market returns from 1950 to 2019. By focusing on the long-run factor, we are excluding volatility fluctuations with very low persistence, which we believe are less relevant for explaining macroeconomic dynamics at horizons of one quarter or more.

We also use several measures of wealth. The value weighted stock market index is from French data library, the value of assets is from Lettau and Ludvigson (2001), the Housing Price Index is the All-Transactions Housing Price Index of the U.S. Federal Housing Finance Agency, and the net worth is the net worth of Households and Nonprofit Organizations is from the Federal Reserve of St. Louis.