Market and Non-market Variance Risk in Individual Stock Returns

Sungjune Pyun*

National University of Singapore

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ABSTRACT

This paper shows that the price and quantity of variance risk of individual stock returns jointly explain the short-term variation of individual stock returns. The variance process of an individual stock inherits the factor structure of the stock returns. Therefore, the price of variance risk measures the priced second-moment risk of the stock, which includes variance shocks both due to market and non-market factors. Then, following the beta representation, the quantity of risk suggests how the stock's risk premium should be connected to the price of risk. Empirically, while stocks with a high negative quantity of variance risk tend to have higher subsequent returns, the spread largely depends on the size and sign of the price of variance risk. For stocks with a high negative price of variance risk, the spread between stocks with a negative and positive quantity of variance risk is 1.12% per month, which is -0.18% for stocks whose price of risk is small. This paper suggests that non-market variance risk is at least as important as variance risk of the market factor.

JEL classification: G11, G12, G15 and G17.

Keywords: Option-implied Variance, Variance Risk, Variance Risk Premium, Beta Representation

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I. Introduction

Variance risk is one essential component that determines asset prices. For example, Bollerslev, Tauchen, and Zhou (2009) find a strong predictive power of the variance risk premium for short-term market returns. Ang, Hodrick, Xing, and Zhang (2006) study how the exposure to aggregate variance risk explains cross-sectional variations among individual stocks. Variance risk might affect investors' portfolio decisions when investors make inter-temporal portfolio decisions since a positive variance shocks may decrease the investment opportunity set. High variance risk may also affect ambiguity averse investors as it means higher uncertainty about volatility.

It is generally acknowledged that the market factor is not the only factor that is cross-sectionally priced among individual stocks. In this regard, while Ang, Hodrick, Xing, and Zhang (2006) and many other studies[1] mainly focus on the role of market variance risk in explaining the variation of individual stock returns, it is yet unclear whether market variance is the only variance that should be priced among individual stocks. This paper uses individual stock returns and options data to provide a unique framework that captures the risk premium of individual stocks due to both the market and non-market variance shocks.

In this paper, I investigate the joint role of the price and quantity of the variance risk of individual stock returns in explaining the short-term variation of its future returns. I find that the price and quantity of variance risk has an interactive role. Therefore, the variance process of individual stock returns embeds information about the underlying factor structure and reflects changes in the discount rate. The price of variance risk of the individual stock is a linear combination of the price of relevant second-moment risk. Then, the quantity of variance risk measures how the risk premium of the stock should be related to its price of variance risk. Empirically, forming a trading strategy based on the combination of the price and quantity of variance risk generates a monthly risk-adjusted return that is at least 0.42% per month.

[1]See also, Adrian and Rosenberg (2008), Chang, Christoffersen, and Jacobs (2013), Conrad, Dittmar, and Ghysels (2013), and Cremers, Halling, and Weinbaum (2015) among others

Focusing on variance risk of individual stock returns is extremely useful for both empirical and logical reasons. First of all, using the price of variance risk is tractable. Variance risk is a unique type of risk in that both the price of risk and the underlying risk factor is observable. The price of variance risk can approximately be estimated as the difference between option-implied variance and historical realized variance of the underlying asset. The underlying factor, i.e., unexpected changes in variance, is also identifiable. Therefore, both the price and quantity of variance risk are essentially estimable as long as the underlying asset has tradable options. Then, from the 'beta representation,' we know that if the expected return of an asset is related to a particular price of risk, it should be related to the asset's exposure to that underlying risk. Therefore, combining the price and quantity of variance risk allows us to estimate an important fraction of the risk premium.

Second, the variance process of individual stocks contains information about the latent factor structure. For example, in a market model, the variance of any positive market beta stock will increase when market variance increases. When there is a negative relationship between market returns and market variance shocks, a high market beta stock will be exposed more to market variance risk. A similar argument follows in a multi-factor structure as the variance of factors varies over time[2]. The variance of an individual stock will increase when there is a positive factor variance shock only if the stock has exposure to that factor.

Finally, the combination of the price and quantity of risk has a direct interpretation as a risk premium. In particular, this combination measures the one-month risk premium of the stock that is due to variance shocks. The level of the variance of a stock is a measure of risk of the stock. A higher stock variance implies more risk for the stock investors. In particular, if the stock price sharply reacts to changes in the level of variance, the stock level variance is likely to contain some key information about the exposure to an unknown discount rate shock. Hence, variance risk at the stock level is likely to contain some key information about the stock risk premium.

[2] These may include liquidity shocks, downside risk or any other shock whose level of risk is time-varying. For example, Daniel and Moskowitz (2016) predict momentum crashes.

We know from the portfolio theory that the idiosyncratic component of the variance of individual stock returns is unpriced since it is diversifiable. However, the existence of an unpriced risk in a stock would at most have a small influence in this framework. First, the price of variance risk would not be affected, as the price of unpriced risk is zero. Second, the quantity of variance risk would also be unaffected if the size of unpriced risk does not vary over time[3]. Third, even when the unpriced volatility is time-varying, it is unclear whether *idiosyncratic volatility risk* can be diversified, especially, when idiosyncratic volatilities of different stocks co-move over time. For example, Herskovic, Kelly, Lustig, and Nieuwerburgh (2016) show that the co-movement of idiosyncratic volatility affects the cross-section of individual stock returns. They suggest that co-movements of idiosyncratic volatility may proxy for labor income risk of households.

Empirically, this paper finds that stocks with a negative price of variance risk tend to have higher subsequent returns. This finding is consistent with Bekaert and Hoerova (2014) who find a positive relationship between the difference of implied variance and realized variance and future stock returns. Although statistically insignificant, this paper also confirms that stocks with a high negative quantity of variance risk tend to have higher subsequent returns. The main question of this paper is whether there is an interactive relation between the price and quantity of risk. This paper finds that the price of variance risk matters only for stocks that have a negative exposure to its variance risk. Also, the risk exposure matters only if a stock has a high negative price of variance risk. During 1996 – 2015, for stocks that have options traded, the overall spread between stocks with a negative and positive price of variance risk is -0.55% per month. However, the sign and the size of the spread depend largely on how stock returns react to variance shocks. This spread is negative (-0.74%) if stocks have a negative variance risk exposure, but is positive (0.56%) if the variance risk exposure is positive. Similarly, the spread between the high minus low portfolio of the quantity-sorted stocks is -0.02%. The spread is much bigger for stocks with a high negative price of risk (-1.12%) than for stocks with a relatively small price of risk (0.08%). Finally, trading on the price and quantity of variance

[3]Time-varying idiosyncratic volatility would create a bias towards zero in the risk exposure

risk simultaneously yields as much as 0.89% per month. The risk-adjusted returns are 1.12% per month.

This paper provides three pieces of evidence that suggest the exposure to aggregate market variance risk (Ang, Hodrick, Xing, and Zhang 2006) has little role in explaining the interactive relation. First, I control for the aggregate variance risk exposure by controlling for market variance risk. The interactive relation remains both economically and statistically significant even when the risk exposure is estimated after controlling. Second, I construct a measure of the price and quantity of non-market variance risk using a market model. The interactive relation does not disappear even when the stocks are double sorted by the price and quantity of non-market variance risk. Finally, I control for the FVIX factor of Ang, Hodrick, Xing, and Zhang (2006). The variance risk factor return constructed using an interactive relation remains statistically significant even after controlling for the FVIX factor.

The price and quantity of non-market variance risk may also be important when investors are under-diversified. While the classical finance theory suggests that households should be entirely diversified, numerous studies suggest that households are actually severely under-diversified (e.g., Blume and Friend (1985)). Several motivations have been proposed. Under-diversion in households portfolio can come from investors' preference for lottery-like payoffs (Barberis and Huang 2008, Mitton and Vorkink 2007), lack of education (Campbell 2006, Calvet, Campbell, and Sodini 2007), wealth constraints (Liu 2014) or familiarity to certain firms or countries (Ivkovic and Weisbenner 2005, Van Nieuwerburgh and Veldkamp 2010). In particular, in the context of familiarity, Uppal and Wang (2003) show that cross-sectional difference of ambiguity on individual firms can be one motive for under-diversification. Therefore, for these under-diversified investors, the variance of non-market factors may also influence investors' portfolio decisions.

This article is a companion to Pyun (2017), who studies the interactive relation between the price and quantity of variance risk for predictability of aggregate stock returns. He shows that market returns are predictable in a statistically and economically significant manner when using a combination of the price and quantity of market variance risk. This paper is also closely related to three other streams of literature. First, this paper is connected to studies that analyze the role of variance risk in explaining the cross-section of stock returns. Bali and Hovakimian (2009) and Han and Zhou (2012) report a negative relationship between the price of variance risk and the subsequent stock returns. Han and Zhou (2012) argue that the price of risk is a proxy for the exposure to market variance risk, while Bali and Hovakimian (2009) claim that it proxies for jump risk. Ang, Hodrick, Xing, and Zhang (2006), Chang, Christoffersen, and Jacobs (2013), and Cremers, Halling, and Weinbaum (2015) also study how variance risk is priced in the cross-section, but their focus is on market variance risk. This paper focuses on both market and non-market variance risk. Furthermore, to my knowledge, this paper is also the first paper that studies the interactive effect between the price and quantity of variance risk for individual equities.

Second, previous studies including Goyal and Santa-Clara (2003), Fu (2009), Chen and Petkova (2012), Stambaugh, Yu, and Yuan (2015), and Hou and Loh (2016), among others, argue that idiosyncratic volatility is priced in stock returns. However, these articles mainly focus on the level of volatility rather than volatility risk. One exception is Herskovic, Kelly, Lustig, and Nieuwerburgh (2016), but they mainly focus on variance risk that is purely idiosyncratic.

Third, this paper also relates to studies using option implied information to explain the cross-section of stock returns. Buss and Vilkov (2012) and Chang, Christoffersen, Jacobs, and Vainberg (2012) study the role of option-implied forward-looking betas, and Buss, Schoenleber, and Vilkov (2016) argue that correlation risk explains systematic diversification benefits.

The remainder of the paper is organized in the following way. Section 2 introduces a simple model. Section 3 describes the estimation methodology and the data used in this article. Section 4 provides the empirical result. Section 5 concludes.

II. The Model

The expected return of any asset, under the beta representation, can be expressed as a linear function of multiple betas. The betas are explanatory variables that are specific to the asset in the cross-sectional relation and measures what extent the factors influence the risk premium of the assets. Factors are proxies for the marginal utility of consumption growth. Then, the representation tells us that the common price of the risk factor should be the slope that connects the betas to the expected return of the assets. Empirically, to understand how stock returns are priced, we commonly sort stocks by the betas and evaluate whether there is a positive/negative risk premium. This single sorting methodology is useful if we entirely understand the factor structure as well as the price of risk factors. However, entirely understanding the cross-section of expected returns can be challenging, especially in the short-run, when we do not entirely understand how the price of these proxies vary over time.

Recent studies suggest that variance risk is an important determinant of stock risk premium. For example, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with a negative exposure to market variance risk tend to outperform those stocks with a positive exposure. Chang, Christoffersen, and Jacobs (2013) find weaker explanatory power if the betas are estimated over a longer horizon, which suggests that the variance betas tend to be highly time-varying. Furthermore, Bollerslev, Tauchen, and Zhou (2009) find that the price of market variance risk, measured as the difference between VIX^2 and the realized variance of the market index, predicts future market returns and is highly time-varying.

A highly time-varying price of market variance risk suggests that there are periods when market variance risk tends to be more important. There are also times when market variance risk should be essentially unpriced. In the context of market return predictability, Pyun (2017) finds that the relation between the price of market risk and market variance risk largely depends on prices react to changes in variance. In other words, when the variance beta is small, the price of market variance risk and the price of market risk are rather unrelated.

Therefore, to understand how the risk premium differs across stocks at a given time, focusing only on the factor betas may be very different from evaluating the cross-sectional differences in the stock risk premium using two dimensions simultaneously– both the price and quantity of risk. However, looking at two dimensions at the same time is often very challenging since the time-varying price of a risk factor is often unobserved. This paper provides a unique case where we can observe both the price and quantity of risk that potentially affects individual stock returns. While the price of risk is difficult to observe ex-ante, Carr and Wu (2009) show that the price of variance risk of a liquid asset can be estimated using option prices and intra-day trading data. This paper shows that the price and quantity of variance risk of individual stock returns jointly determine part of the stock risk premium. In fact, this combination prices all variance risk that the stock is exposed to – both due to market variance risk and others – and regardless of the particular factor structure.

Individual stock returns are closely tied to the variance of its returns for various reasons. First of all, the variance of its stock returns is a natural measure of risk of the stock. A higher stock return variance means more risk for investors or a higher discount rate for the stock. Therefore, all else being equal, when there is a positive variance shock, the stock price must decrease simultaneously. As a result, stock returns and variance innovations should exhibit a negative relation. This explanation is reminiscent of volatility feedback (Pindyck 1984, French, Schwert, and Stambaugh 1987) and is clearly observed at the aggregate stock level (Bollerslev, Litvinova, and Tauchen 2006).

However, unlike in the case of the well-diversified market portfolio, the sign and the magnitude of the relation between individual stocks returns and variance shocks can also vary due to other reasons. If a stock has substantial unpriced idiosyncratic risk, stock prices will react less to changes in its variance shock. Price shocks and variance shocks of an individual stock can also be positively related. For example, firms may possess one or multiple valuable real options. Barinov (2013) studies the role of growth options in variance risk and value premium. Growth options will naturally grow as variance increases, as will increase firm value. Limited liability will also create an option-like payoff in equity returns. The existence of growth options and limited liability would particularly matter for smaller firms that has a higher likelihood of default, and will reduce the magnitude of the negative exposure to its variance risk.

Hence, when the stock price reacts more sharply to small variance shocks, the risk premium of those stocks would also strongly depend on the size of the price of variance risk. A high negative price of variance risk would directly correspond to a high positive stock risk premium. On the other hand, variance risk should be priced differently for stocks whose variance risk exposure is small or positive. The risk premium of these stocks is determined independently from what is implied by the price of its variance risk.

The following simple model describes the relationship between the expected return of a stock and the price of its variance risk. Suppose that the stock i ($S_{i,t}$)'s price process can be decomposed as the sum of the stock-specific systematic ($dY_{i,t}$) and idiosyncratic ($dW_{i,t}^{idio}$) risk:

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + b_i dY_{i,t} + \sigma_{idio} dW_{i,t}^{idio}.$$

$dY_{i,t}$, the systematic risk component can, for example, be linear functions of multiple factors and their corresponding slopes, the stock-factor specific betas. If stock i follows a N-factor structure, $dY_{i,t}$ can be represented as $\sum_{n=1}^{N} \beta_{i,n} dF_{n,t}$ for N-independent factors $dF_1, dF_2, \ldots, dF_N$[4]. The second component, $dW_{i,t}^{idio}$, represents unpriced idiosyncratic risk. For now, I assume a constant volatility for this component, but I will later relax the assumptions and discuss the implications of time-varying idiosyncratic volatility.

Let each of the risk factors $dF_{n,t}$ further follow a stochastic process that is correlated with the corresponding factor's variance process. That is, let

$$\begin{aligned} dF_{n,t} &= \mu_{n,t} dt + \sqrt{V_{n,t}}(\rho_n dW_{n,t}^{v} + \sqrt{1-\rho_n^2} dW_{n,t}^{o}) \\ dV_{n,t} &= \theta_n dt + \sigma_{nv} dW_{n,t}^{v}, \end{aligned}$$

where $V_{n,t}$ is the variance of factor n, and $dW_{n,t}^{v}$ and $dW_{n,t}^{o}$ are independent Brownian Motions. The correlation between factor returns and factor variance (ρ_n) is determined exogeneously. If $\rho_n < 0$, an increase in factor variance leads to an instantaneous negative shock in factor returns.

In this stochastic process, variations in each risk factor $F_{n,t}$ come from two different sources. First, they can be related to their second-order variance shocks. As the variance of factor returns is a measure of the risk of the factor, changes in factor variance is directly related to the price of the factor risk[5]. If there is a change in the factor risk premium, the factor returns should react immediately. Therefore, the variance shock related channel is directly related to discount rate shocks and should be substantial in explaining the factor risk premium. Second, there are other sources of factor shocks, which does not lead to changes in the variance of the

[4]The simplest form is to assume a single factor market model

[5]This can be observed from the equation $\mathrm{Cov}(-SDF, F) = \rho_{\mathrm{SDF},F}\sigma_{\mathrm{SDF}}\sigma_F$

factor. I refer all other risk that does not involve a time-varying level of the second moment as orthogonal shocks. Orthogonal shock also includes all cash-flow shocks, as they are one-time price shocks that do not affect the discount rate or the variance.

In this model, the factor structure of the underlying stock returns determines the relation between stock returns and its variance shocks. For example, the negative relation between returns and variance shocks may be stronger for stocks with a higher beta, large stocks, or stocks without relatively valuable growth options. According to the proposed model, factors that represent these firm characteristics may have stronger leverage effect than others. Therefore, either the firm characteristic or the risk structure of underlying stock returns determines the variance process and the underlying stock process simultaneously, which in the end affects the size of the leverage effect.

Solving the above three equations together and expressing stock returns as a function of factor variances yields[6]

$$\frac{dS_{i,t}}{S_{i,t}} = \{\cdot\}dt + b_i \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{n,t}}}{\sigma_{n,v}})dV_{i,t} + \sum_n B_{i,n} dW^o_{i,t} + dW^{idio}_{i,t},$$

where $B_{n,i} = b_i\sqrt{V_{n,t}(1-\rho_n^2}$ and $\{\cdot\}$ is used for some function that is can be solved explicitly, but is necessary to derive the final formula we need.

The connection between the expected stock returns and the price of variance risk of factor n $dV_{n,t}$ follows the argument of Pyun (2017). If we let the discounted marginal utility as Λ_t, the stochastic discount factor (SDF) in continuous time is $\frac{d\Lambda_t}{\Lambda_t}$. Thus, the expected return of a stock can be expressed as

$$\text{Cov}_t(-\frac{d\Lambda_t}{\Lambda_t}, \frac{dS_{i,t}}{S_{i,t}}) = b_i \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{nt}}}{\sigma_{nv}})\text{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}) + \sum_n B_{n,i}\text{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}).$$

[6]plug-in the third equation into $dW^v_{n,t}$ in the second equation, and put $dF_{n,t}$ into the first equation

Notice that $V_{n,t}$ is the variance of factor n (dF_n), and also that $b_i\beta_{n,i}\rho_n\frac{\sqrt{V_{n,t}}}{\sigma_{n,v}}$ is the beta we get when we regress individual stock returns on the variance of the n^{th} factor $dV_{n,t}$. The relation follows from

$$
\begin{aligned}
Cov_t(\frac{dS_{i,t}}{S_{i,t}}, dV_{n,t})/Var(dV_{n,t}) &= Cov_t(b_i\sum_n \beta_{n,i}dF_{n,t}, dV_{n,t})/Var(dV_{n,t}) \\
&= b_i\beta_{n,i}\rho_n\sqrt{V_{n,t}}\sigma_{n,v}/\sigma_{n,v}^2 \\
&= b_i\beta_{n,i}\rho_n\sqrt{V_{n,t}}/\sigma_{n,v}
\end{aligned}
$$

The above equations show that the expected return of stock i can be represented as the sum of the products of the price of variance risk of the factors that the stock is exposed to, and the stock's exposure to the variance shock of the particular factor. The first key result of this paper is represented as follows:

Result 1. Assume stock i follows a N-factor structure with factors $\{F_1, \ldots, F_N\}$ and betas $\{\beta_{i,1}, \ldots, \beta_{i,N}\}$ that correspond to the factors. Let $\lambda_{n,i,t}$ be the price of variance risk of the n^{th} systematic risk factor, $\lambda_{o,i,t}$ the linear combination of the price of risk due to orthogonal shocks[7], and $\beta_{n,i,t}$ the slope of a hypothetical regression of stock i's return on the variance of latent factor n (dF_n). Then, we have

$$E_t[R_{i,t+1}] = \sum \beta_{n,i,t}\lambda_{n,i,t} + \lambda_{o,i,t}, \tag{1}$$

Equation (1) tells us that the expected excess return of a stock can be represented as a linear combination of the product of the price and quantity of variance risk of the latent factors. Estimating these components is not straightforward since a direct estimation of the above equation would require us to assume a certain factor structure and compute the price of variance risk of each of these factors separately. In fact, it is even difficult to determine the factors that should be used.

[7]Here, $\lambda_{n,i,t} = \mathrm{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$ and $\lambda_{o,i,t} = \sum_n B_{n,i}\mathrm{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$

However, a simple transformation shows that the first component of Equation 1, the risk premium of a stock due to variance shocks of factors, can be estimated quite easily by just computing the price and quantity of variance risk of each individual stock. The second key result of this paper (proof in Appendix) is given as below.

Result 2.. Let the price of variance risk of stock i to be $\lambda_{v,i,t}$, and the stock i's exposure to its variance process be $\beta_{v,i,t}$, respectively. Assuming that idiosyncratic volatility stays constant, we have the following alternative representation of the Result 1, where the price of orthogonal risk $\lambda_{o,i,t}$ remains same as in Result 1.

$$E[R_{i,t}] = \beta_{v,i,t}\lambda_{v,i,t} + \lambda_{o,i,t} \tag{2}$$

The representation (2) shows that the product of the price and quantity of variance risk of each stock ($\beta_{v,i,t}\lambda_{v,i,t}$) exactly measures the risk premium of the stock that is implied by the second moment shocks – including market and non-market variance shocks– the stock is exposed to. This representation, in addition to the traditional factor models, is useful for several reasons. First, both the price and quantity of variance risk of individual stock returns are directly measurable. As mentioned, Carr and Wu (2009) show that the payoff of a variance swap is equivalent to the price of variance risk and can be represented as the difference between the risk-neutral expectation of variance and the real-world expectation of variance. At the individual stock level, the price of variance risk can also be estimated using individual stock option prices and high-frequency intraday trading data. The quantity of risk is estimable as well since the first-order difference of the option-implied variance is observable. We can measure the quantity of risk using a regression of stock returns on changes in its variance.

Second, the price-quantity combination of the individual stock is essentially a combination of the price of variance risk of all systematic risk proxies. The exact slope in the representation is determined by the factor variance risk exposure of the stock, but whether or not the variance risk of a particular factor dF is priced in a stock is determined by the factor structure of the underlying return process. In other words, the variance process inherits the underlying factor structure, and the factor structure determines whether the second moment risk of a factor, potentially related to marginal utility proxies, should be priced.

So far, the model assumes that unpriced idiosyncratic variance remains constant over time. One may wonder how the time-varying idiosyncratic volatility affects the relation between the expected stock returns and the price of variance risk. A recent study by Herskovic, Kelly, Lustig, and Nieuwerburgh (2016) suggest that idiosyncratic volatility is time-varying and their co-movement across stocks is priced. They argue that idiosyncratic volatility may affect heterogeneous agents, as idiosyncratic volatility shock may affect income risk faced by households.

The implications of a time-varying idiosyncratic variance are straightforward. If idiosyncratic variance risk has a common component that is priced as in Herskovic, Kelly, Lustig, and Nieuwerburgh (2016), the idiosyncratic risk of these firms should be treated as if there is a latent factor which has a variance that tracks the co-movement. A problem will arise if there is unpriced time-varying idiosyncratic volatility. A time-varying idiosyncratic variance means $\beta_{v,i,t}$ would be biased downwards, as it would create an error-in-variables problem in the estimation process. The quantity of variance risk of individual stock returns will have a downward bias, in which the magnitude depends on the level of the variance of idiosyncratic variance.

One limitation of this approach is that the risk premium due to the variance unrelated market shocks, namely the price of orthogonal risk $\lambda_{o,i,t}$, cannot be estimated in this framework. As mentioned, this type of risk includes systematic shocks that do not lead to changes in the level of variance. A macroeconomic shock that does not change the level of risk or uncertainty, measured by the variance, is rare, but one can theoretically always orthogonalize the variance shock from the first order shock.

Several implications of the main result are worth mentioning. The risk premium of a stock is likely to be higher for stocks whose price of variance risk is more negative. Intuitively, this may seem to be because when the price of variance risk of a stock is highly negative, a variance shock of the stock is more likely when marginal utility is high. When investors dislike an increase in the variance of the portfolio, the risk premium of the stock must be high. What the model of this paper suggest is a little different. The price of variance risk is important partly because stock returns are related to its variance shocks. The price of variance risk is a good measure of the risk premium of the stock only when returns are highly related to variance shocks.

Second, one may wonder why individual stocks have its price of risk. In this framework, the price of variance risk are different across individual stocks since individual stocks have its own factor structure (i.e., uniquely exposed to different risk factors). For example, when the price of market variance risk is high, the price of variance risk is high for stocks that have high exposure to market factor. Therefore, the price of variance risk differs across stocks not because they have a unique price of risk, but due to the factor exposures (betas) being different.

This paper suggests that the price and quantity of risk have a joint role in determining the short-term risk premium of individual stocks. The interactive effect implied by the model suggests that for stocks that are not exposed to its variance risk, the price of risk would have a smaller influence to the stock's risk premium. While the role of the exposure to variance risk and the price of variance risk have previously been studied separtely, the interactive relation in explaining the time-variation of individual stock returns has been commonly ignored for several reasons. First, the focus of the variance risk premium has been different from the interpretation as a price of risk. For various assets, this difference is often regarded as a measure of time-varying risk aversion. Second, the variance risk exposure is time varying, and is difficult to estimate.

The following sections empirically study the interaction between the price and the quantity of variance risk. I show the role of market variance risk and investigate the possibility that non-market variance risk is priced in individual stock returns. I also evaluate a possible trading strategy using the interactive relation between the price and quantity of risk.

III. Data and Methodology

The sample of this paper consists of stocks from NYSE, AMEX, and NASDAQ that have options actively traded in the market between 1996 and 2015. The study period is restricted due to data availability in both option pricing data and intra-day high-frequency trading data. The option-implied volatility is obtained from OptionMetrics, high-frequency intraday data from Trade and Quote, and individual stock returns from the Center for Research in Security Prices (CRSP). Among options that expire in the following month, several other filters are applied to the data to minimize possible data errors. First, following previous studies such as Goyal and Saretto (2009), options in which the ask price is lower than the bid price, options whose bid price is equal to zero, and options whose price violates arbitrage bounds are eliminated. Second, options with zero open interest are also deleted. Third, options that have moneyness smaller than 0.95 or higher than 1.05 are removed. Among these options, I choose one call and one put option that is closest to ATM and compute the simple average of the option-implied volatility between the call and the put. The implied variance (IV) of an individual stock is the square of the average. When IV is missing, I let the value to remain unchanged from the previous day.

The option-implied variances are then matched to CRSP using both CUSIP and tickers. Using daily observations of stock returns and IV, I use two different specifications to estimate a stocks exposure to its variance risk. First, similar to Ang, Hodrick, Xing, and Zhang (2006), in the main analysis, I use a two-factor structure

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\, R_{m,t} + \beta_{v,i}\, [IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}$$

where R_m are the excess market return, and R_i is the excess stock return. Here, $\hat{\beta}_{v,i}$ is estimated every month using the past twelve months of data. To be included in the sample, I require at least two months of observations of both returns and variance innovations to be available.

For each stock, the price of variance risk is computed as the difference between historical realized variance (RV) and IV. For each month, the price of variance risk ($\lambda_{v,i,t}$) is computed as the difference between RV and IV and each of them are averaged over the most recent week. This follows from the fact that

$$Cov_t(-SDF_{t+1}, RV_{i,t+1}) \approx E_t[RV_{i,t+1}] - E_t^Q[RV_{i,t+1}],$$

and option-implied variance proxies for the risk-neutral expectation of variance. The RV is estimated as the sum of 75-minute squared returns over the past month. To deal with possible microstructure noise, I use the average over five subsamples (Hansen and Lunde 2006). RV computed over 75-minutes follows earlier works (e.g. Bollerslev, Li, and Todorov (2016)) on intra-day individual stock returns. The frequency is lower than what is typically used for the market index but reflects the high possibility of microstructure noise at the individual stock level. However, the main results are unaffected by the choice of the sampling interval. Furthermore, while realize variance estimated in this manner may be inaccurate at the daily basis as there only 6 observations per day, they are only used after summing up to the monthly level.

It is still possible that the price of variance risk estimated in this manner is biased, as there may be a timing mismatch between the two variance components. While the RV reflects recent realizations, the IV reflects the future expectation of those variations. As a standard alternative, for the market index, a variance forecast model based on the intra-day square of high-frequency returns is used. Although they are more likely to be plausible at the index level, we need to be more cautious applying this alternative methodology to individul stocks. Above all, the sample period of this data is restricted to 20 years due to data availability. A time-varying out-of-sample variance forecast model over 240 observations would contain massive estimation error. In addition, the RV estimates of individual stocks are more subject to microstructure noise. A forecast model based on an already noisy estimate should be problematic. Finally, as Bekaert and Hoerova (2014) argue, relying on a particular variance forecast model can be controversial as the empirical implications may depend on the particular model used. Notably, Goyal and Saretto (2009) show how the difference between RV and IV, as defined in this paper, affects the cross-sectional average of option returns.

Table I summarizes the mean, median and standard deviations of the sample considered in this paper. There is a total of 245,807 stock-month during the sample period. While the entire CRSP sample has an average of 4,948 stocks in a given month during 1996/01-2015/12, on average there are 1,329 stocks that have options traded actively in the market. Out of them, 1,130 stocks have more than 44 observations over the past year. There are 396 stocks at the end of 1996 and 1,659 stocks at the end of the sample. The median of the IV is 0.011, which is equivalent to 36.3% in annual standard deviations, while the RV has a median of 0.007, equivalent to 29.0% in annual standard deviations. The mean of IV is 0.018, and the mean of RV is 0.022, which suggests that there are some outliers with unreasonably large RVs. These outliers are not necessarily a concern for the purpose of this paper, as they are not influential when forming portfolios based on the rankings. The table also suggests that the price of variance is mostly negative. The difference between the RV and IV tends to be negative on average, but a sizable proportion is still positive (23.8% in the panel), which is consistent with other previous research (e.g., Goyal and Saretto). Although I believe that a substantial proportion of stocks with a positive price of risk estimates is due to the estimation error in RV, especially for small stocks, there may also be some downward bias in IV due to using only ATM options.

The variance betas, or the variance exposures are, on average, negative, suggesting that for an average stock, the leverage effect or volatility feedback tends to dominate the possible influence of growth or real options. However, for a notable number of stocks (22.7%), returns are positively related to contemporaneously variance movements, which is partly due to growth or real options. Compared to the entire CRSP database, the chosen sample has similar average market beta. Also, stocks in the sample tend to be larger than the CRSP average. This is natural as the sample systematically excludes options on these stocks that are less likely to be traded.

There is possibility that the price and quantity of variance risk are strictly related. A high negative price of variance risk means that the variance process is either extremely volatile or highly correlated with the latent marginal utility process of investors. The stock prices of these stocks are more likely react sharply to changes in variance. As a result, these stocks may have both a negative price and quantity of variance risk. However, if we compute the average of the estimated cross-sectional correlation coefficient between the price and quantity of variance risk, it is -0.095 (not reported in the table) which indicates that there are not heavily related.

IV. Empirical Results

The 'beta representation' of expected returns suggests that if the expected return is represented as a linear function of the price of risk, the slope or the weights should be the betas or the exposure to the underlying risk factor. The model of the previous section also implies this. If stock risk premium is connected to a price of variance risk, the slope should be the stock return's beta on the variance shock of the underlying factor. This linear combination can be measured alternatively using the price of its own variance risk and the stock return's exposure to own variance shocks.

Empirically, when the price of variance risk is highly negative, we expect the quantity of risk to affect future stock returns more than stocks with a less negative price of variance risk. Analogously, the price of variance risk should matter most when it is highly exposed to its variance shocks. If the variance risk exposure is positive, either due to valuable growth options or due to limited liability constraint, those a high negative price of variance risk may even mean lower subsequent returns. In simple terms, the price and quantity of risk would have an interactive relation. This section provides empirical evidence suggesting that both the price and quantity are important in explaining the time-variation of individual stock returns. To test the hypothesis, I first sort the stocks by the price or quantity of variance risk and evaluate the performance of single-sorted quartile portfolios separately. Then, the performance of price-quantity double-sorted portfolios is provided. Finally, I evaluate the role of aggregate variance risk exposure.

1. The Interactive Relation between the Price and Quantity

Before evaluating the performance of the price-quantity double-sorted portfolios, I first evaluate the performance sorted by each of these variables. Table II summarizes the performance of single-sorted portfolios. To do so, I first estimate the price and quantity of variance risk for each stock. Then, the stocks are sorted either by the price or the quantity of variance risk. The stocks are divided into quartiles based on the estimates of the sorting variables, and simple averages of the sorting variable, value-weighted returns, risk-adjusted returns, the market beta as well as the average market capitalization are computed and reported. Risk-adjusted returns are controlled for the size, value (Fama and French 1993), and momentum (Carhart 1997) and denoted by α_4). Both returns and risk-adjusted returns are evaluated over the subsequent month after formation.

Panel A summarizes the performances of the price of variance risk sorted portfolios. This panel is a slightly modified version of two previous studies including Bali and Hovakimian (2009) and Han and Zhou (2012). These studies investigate how the difference between the option-implied variance and the realized variance affects future stock returns, but does not interpret this measure as being the price of variance risk of individual stock returns. For example, Han and Zhou (2012) argue that the difference proxies for aggregate variance risk exposures and report a negative relationship between the price of variance risk and subsequent returns. Bali and Hovakimian (2009) interpret this spread as a proxy for jump risk of the stock. The first two columns of Panel A essentially replicate these studies and confirms that stocks with a high negative price of variance risk tend to have higher subsequent returns. The next two columns summarize the average contemporaneous values of the price and quantity of variance risk. The last few columns provide some summary statistics. Stocks with a high negative price of variance risk tend to be smaller, have higher market betas, and have a smaller exposure to variance risk.

Previous studies also suggest that market variance risk is priced in the cross-section of stock returns[8]. These studies focus on the exposure to market variance fluctuations. Here, the focus is the variance risk of individual stock returns. Since the variance of a typical stock return is likely to depend on market variance movements, the market variance beta and the individual stock variance beta may be closely related.

Panel B shows the performances of quantity (variance risk exposure) - sorted portfolios. The first two rows summarize the returns and risk-adjusted returns of the portfolios. There is little difference in performance between stocks with a high risk exposure and the ones with a low risk exposure, especially in excess returns. For risk-adjusted returns, while being statistically

[8]See, for example, Coval and Shumway (2001), Ang, Hodrick, Xing, and Zhang (2006), Bali and Hovakimian (2009), Cremers, Halling, and Weinbaum (2015) and Chang, Christoffersen, and Jacobs (2013).

insignificant, stocks with high negative risk exposure slightly outperform stocks with small or positive variance risk exposure. This result is similar to the result of Chang, Christoffersen, and Jacobs (2013) that finds a weak relation between the market variance risk exposure if more than a single month of data is used for estimation. The average stock with a high negative exposure to variance risk tends to be bigger and have a slightly lower market beta.

The focus of this paper is on whether there is an interactive relation between the price and quantity of variance risk. If there is an interactive effect, the long-short portfolio of a single sort will be stronger or weaker depending on the level of the other sort. In particular, the long-short portfolio returns of the risk exposure should be much stronger and bigger when the price of risk is highly negative. The long-short portfolio returns concerning the price of risk may be significant only for stocks whose variance risk exposure is highly negative. This possibility is evaluated by analyzing the performance of the double-sorted portfolios.

Panel A of Table III summarizes the performance of the price-quantity double-sorted portfolios. All stocks in the sample are sorted independently by the price and quantity of variance risk and divided into four groups. That is, a total of 16 portfolios are formed based on the estimates of the price and quantity of variance risk. The value-weighted returns along with the four-factor returns of the subsequent month are evaluated. The table summarizes the time-series average of returns and its t-statistics computing using Newey-West methodology. Each row represents different levels of the variance exposure, and the columns represent different levels of the price of variance risk. For both the price and quantity of variance risk, high means highly negative and low means only slightly negative and does include positive.

The price of variance risk has the biggest impact on subsequent returns for stocks that are negatively exposed to variance risk. The spread between stocks with a small or positive price of variance risk and a negative price of risk is -0.74% in monthly excess returns and -1.29% in risk-adjusted returns. Although being statistically insignificant, the spread for stocks with a positive variance beta switches its sign. The difference in the spread of the price of risk sorted portfolios is 0.56% in excess returns and 0.52% in risk-adjusted returns, respectively. Analogously, the spread between stocks with a high negative exposure to its variance risk and with small or positive exposure to variance risk is -1.12% in excess returns and -1.70% in risk-adjusted returns when the price of variance risk is highly negative. When the price is small or close to zero, the spread also switches its sign but is essentially close to zero, with 0.18% in excess returns and 0.10% in risk-adjusted returns. However, notably, the difference between the two spreads is highly statistically significant at 1% level for both the excess returns and risk-adjusted returns. To summarize, the variance risk exposure mostly affects stock returns whose price of variance risk is highly negative. The price of risk matters most for stocks with a negative exposure to its variance risk. In other words, the interactive effect of the price and quantity is clearly observable.

Panel B summarizes the performance of the double-sorted portfolios using equally-weighted returns. Overall, the spread decreases somewhat by equally weighting the stocks within each portfolio. This is in part because firm size is related to the sorting variables. Growth options are more likely going to be important for small stocks, and limited liability constraints are more likely to be binding. Also, statistically, small firms tend to have a more negative price of risk. Compared to the value-weighted returns, the spread between low and high exposure stocks decreases to 0.38% in monthly excess returns for stocks with a high negative price of risk. The spread still changes its sign when the price of risk is small or positive to 0.22% in excess returns. For risk-adjusted returns, the spread is -0.46% and 0.08%, respectively. Similarly, the spread between stocks with a high negative price of variance risk and small price of variance risk is -0.25% in excess returns and -0.38% in risk-adjusted returns. The spread changes its sign for stocks with a positive variance risk exposure. While most of the spreads are statistically insignificant, Except for the risk-adjusted spread for high price quantity of variance risk stocks, the difference between the two groups (difference-in-difference) is still statistically significant with 0.59% per month in excess returns. Also, after controlling for the common risk factors, the difference is 0.54% in monthly risk-adjusted returns.

2. Relation to the Aggregate Variance Risk Exposure

The exposure to market variance risk closely relates to the exposure to own variance risk as individual stocks returns are exposed to market risk. One possibility is that only market variance risk affects the risk premium of the stock, and the variance betas are simply a correlated measure of the market beta. The betas may be related since the variance process of high beta stocks is more likely to be heavily exposed to market variance risk. If for some reason market variance risk is only important for a certain subsample, the interactive relation between the price and quantity of variance risk presented in previous tables may simply be a result of it. This part of the paper investigates this possibility by studying the relationship between the aggregate variance risk exposure (Ang, Hodrick, Xing, and Zhang 2006) and the exposure to own variance risk. In several different ways, I show that individual variance risk that is unrelated to market variance risk is still important and is what essentially drives the interactive relation between the price and quantity of variance risk.

First, instead of using a two-factor model, a three-factor model including market excess returns, changes in VIX^2, and changes in the implied variance of individual stocks is used to estimate the quantity of variance risk. The regression is given as,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\, R_{m,t} + \beta'_{x,i}\, [VIX_t^2 - VIX_{t-1}^2] + \beta'_{v,i}\, [IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}.$$

This regression is designed so that a stock's own variance risk exposure captures the risk exposure that is not driven by market variance shocks. To distinguish the variance risk exposure after controlling for market variance risk ($\beta'_{v,i}$) from the one before, I use a prime ($'$) whenever a three-factor model is used to control for market variance risk. I call this the market variance risk controlled variance beta.

Table IV summarizes the performance of double-sorted portfolios when the market variance risk controlled beta is used. The spread between high and low variance risk exposure decreases for stocks with a high price of variance risk, but if we focus on the other dimension, for high market variance controlled beta stocks, the spread created by the price of variance risk is higher than before. Overall, the interactive relation is still present and statistically significant, which suggests that aggregate variance risk is not the key component that affects the relationship.

Second, using a market model, I decompose both the price and quantity of variance risk into two components – one that represents those driven by market variance risk and the other that represent those from non-market variance risk. The second part, the component due to non-market variance risk, measures how the risk premium of a stock relates to variance of possible factors or systematic shocks excluding the market factor. The decomposition works in the following manner:

Assuming a market model, we have

$$R_{i,t} = a_i + \beta_{m,i} R_{m,t} + \epsilon_{i,t},$$

where returns are excess returns. If individual stock return $R_{i,t}$ follows a N-factor structure ($N > 1$), $a_i + \epsilon_{i,t}$ would capture the sum of the variations due to changes in factors other than the market factor and, also possibly, idiosyncratic shock. The market model also implies a one-factor structure for the variance process.

$$\Delta Var_t(R_{i,t+1}) = \beta_{m,i}^2 \Delta Var_t(R_{m,t+1}) + \Delta Var_t(\epsilon_{i,t}).$$

Similarly, $\Delta Var_t(\epsilon_{i,t})$ would be associated with all non-market variance shocks that affects the variance process of the stock. Thus, the variance risk of a stock i can be represented as the sum of variance risk due to changes in market variance and due to non-market variance shocks.

If we replace the changes in actual variance with option-implied variance, we can estimate the slope of the following regression to obtain the non-market related variance shock of stock i ($\hat{e}_{it}$). Also, the square of the market beta ($\beta_{m,i}^2$), or the variance process of stock i's exposure to market variance risk, can be accomplished by estimating the slope of

$$\Delta IV_{i,t} = \beta_{vx,i} \Delta VIX_t^2 + e_{it}. \tag{3}$$

We can run the above regression using, for example, one year of data, as in other regressions of this paper. The implicit assumption is that $\hat{\beta}_{m,i}^2 = \hat{\beta}_{vx,i}$ would hold, approximately.

Then, non-market variance risk of stock i is $\hat{e}_{i,t}$, and the exposure to non-market variance risk ($\beta_{e,i}$) can be estimated as the slope of a regression of individual stock returns on non-market variance risk.

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta_{e,i}\ \hat{e}_{i,t} + \epsilon_{i,t} \tag{4}$$

The slope measures how a stock return reacts to changes in variance but that is unrelated to the market variance shocks.

Also, the price of variance risk of individual stock returns can then be represented as

$$\begin{aligned}\mathrm{Cov}_t(-SDF_{t+1}, \Delta Var_t(R_{i,t+1})) = \beta_{vx,i}\mathrm{Cov}_t(-SDF_{t+1}, \Delta \mathrm{Var}_t(R_{m,t+1})) \\ + \mathrm{Cov}_t(-SDF_{t+1}, e_{i,t+1}),\end{aligned}$$

where SDF is the stochastic discount factor. The price of variance risk due to non-market factors can be written as,

$$\widehat{Cov}_t(-SDF_{t+1}, \Delta e_{i,t+1}) = (RV_{i,t} - IV_{i,t}) - \hat{\beta}_{vx,i}(RV_{m,t} - VIX_t^2),$$

which is equivalent to writing as

$$\hat{\lambda}_{e,i,t} = \hat{\lambda}_{v,i,t} - \hat{\beta}_{vx,i,t}\hat{\lambda}_{x,t}, \tag{5}$$

where $\lambda_{x,t}$ is the price of market variance risk. I call $\lambda_{e,i,t}$ as the price of non-market variance risk in the sense that it is driven by second moment of factors other than the market factor. This price of risk is computed by subtracting the price of market variance risk from the price of variance risk of individual stock returns with the relevant scaler. The goal is to evaluate whether the interactive relation is mainly driven either by the price or by quantity that is related to market variance risk.

Table V shows the performance of the price and quantity of non-market variance risk double-sorted portfolios. This table shows that the high minus low spread for the price-sorted portfolios is still bigger for stocks that have a more negative non-market variance risk exposure. Also, the spread for the quantity-sorted portfolios is bigger for stocks whose price of non-market variance risk is more negative. Comparing it to the earlier table where stocks are double-sorted by the price and quantity of total variance risk, the difference in the spread is slightly smaller with 0.97% in excess returns and 0.92% in risk-adjusted returns. However, the difference in spreads is still statistically significant, which suggest that the interactive relation is mainly driven by non-market variance risk.

In short, I find no evidence that market variance risk is the only variance factor that affects individual stock returns. Furthermore, the result of this section suggests that even the non-market factor based variance risk is priced and determines the short-term variation of individual stock returns.

3. The Price-Quantity Combination and Possible Trading Strategies

Earlier tables clearly confirm the interactive and multiplicative relation between the price and quantity of variance risk. Furthermore, the model as well as the 'beta representation' directly implies that the product of the price and the quantity should represent a fraction of the risk premium of the stock. Therefore, a natural next step is to see whether the product directly explains future individual stock returns.

However, multiplying two noisy estimates can create an empirical problem, especially when the estimation error switches the sign of the estimates. As the model suggests, the risk premium implied in individual stock returns is high when both the price of variance risk and the variance risk exposure are negative. The empirical issue is that several small stocks tend to be illiquid and is more likely to be subject to microstructure noise. Therefore, the RV of these stocks will be over-estimated, which means that the price of variance risk is likely to be estimated positively. At the same time, small stocks are more likely to have a positive variance beta. While these stocks have smaller price of variance risk, they may look as if they are a high risk premium stock if their variance risk exposure is positive.

This is shown in Table VI. The price and quantity of variance risk is estimated as in the same manner as in previous tables and the returns of the subsequent month is evaluated. Panel

A shows the result for the portfolios sorted by the product of the price and quantity. The Panel suggests that stocks with a high variance risk premium (i.e., high positive value of the product) tend to have higher returns. While the difference in the 5-1 spread is not statistically significant, the spread has the correct sign with 0.26% in excess returns, and 0.53% in risk-adjusted returns.

However, as hypothesized, this spread largely depends on whether a stock has a positive or a negatively estimated price of variance risk. Panel A-1 summarizes the result when only stocks with a negative price of variance risk are used in the sample, while Panel A-2 shows the performance of stocks with a positive price of variance risk. The results in Panel A-1 confirms that for stocks with a negative price of variance risk, the product of price and quantity of risk is priced and affects future stock returns. The 5-1 spreads are statistically significant with 0.58% in excess returns and 0.60% in risk-adjusted returns.

Of course, it is possible that stocks with positively estimated price of variance risk truly have a positive price for their variance risk. That is, there may be types of variance shocks that is positively related to the marginal utility of investors. If this is the case, stocks with a high price-quantity combination should embed a higher risk premium and should have higher subsequent returns regardless of the sign of the price of risk. Panel A-2 suggest that this is not necessarily true. Stocks with a more negative variance risk implied premium (i.e., price $\times$ quantity), tend to outperform stocks with a positive risk premium. The sign of the spread is against the hypthesis that there is a good variance, in which a substantial proportion of stocks is exposed to. Overall, the results suggest that possible noise in RV, for small stocks, makes it difficult to regard the price-quantity product as a measure of the risk premium implied in individual stocks. Finally, when both market variance risk and non-market variance risk is priced, we expect that the risk premium due to these two sources of risk to be priced separately. Since we can estimate the price of market variance risk as well as the price of non-market variance risk, we can express the expected return of the stock as a linear combination of the two prices of risk. The market variance beta ($\beta_{x,i}$) is from a three-factor model, and the non-market beta is estimated from Equation 4 is then used as the slope in this combination.

Panel B of Table VI summarizes the performance of the portfolios of the risk premium implied in stocks assuming a two-factor structure. The entire sample is used regardless of the sign of the price of risk, and the results are stronger when only the sample with a negative price of risk is used. Compared to Panel A, dividing the variance risk into market variance based and non-market variance based risk increases the spread to 0.42% in excess returns and 0.63% in risk-adjusted returns. They are both statistically significant. These results confirm that non-market variance risk is important that affects the cross-section of individual stock returns.

Despite the strong evidence of Table III, which shows that there is an interactive effect between the price and quantity of variance risk, building a tradable strategy is complicated and partially misleading since the returns given in the table are essentially sums of two long-short positions. To obtain the profits provided in the main table, one needs to buy a combination of stocks with high negative price/ high negative quantity and stocks with low price/low quantity and sell stocks with a combination of low price/high quantity stocks and high price/low quantity stocks. The presented profit could be misleading because this is different from the returns of a single long-short portfolio. As most of the abnormal returns arise from the portfolio with both high negative price and quantity of variance risk, a simpler trading strategy is implementable by buying stocks that have both high price and quantity of risk and selling those that have either/both low price and quantity of variance risk.

In forming feasible trading strategies, I consider the following seven described as follows:

Strategy 1	Trade only on stocks with a high negative risk exposure.
	Buy Q1/Q1[9] and sell Q4/Q1 of Table III.
Strategy 2	Trade only on stocks with a high negative price.
	Buy Q1/Q1 and sell Q1/Q4 in Table III.
Strategy 3	Trade on the price-quantity combination. Panel A of Table VI
Strategy 4	Trade on the price-quantity combination and market variance risk.
	Panel B of Table VI
Strategy 5	Trade on the price-quantity combination, but only when price negative
	($\lambda_{v,i,t} < 0$). Panel A-1 of Table VI
Strategy 6	Buy Q1/Q1 and sell a combination of Q4/Q1 and Q1/Q4 in Table III.
Strategy 7	Buy Q1/Q1 and Q4/Q4 and sell Q4/Q1 and Q1/Q4 of Table III.

Table VII summarizes the performance of the seven trading strategies. The proposed trading strategies are all positive and both economically and statistically significant except for the excess returns in trading strategy 3, where the strategy is based on the product of the price and quantity. The trading strategies generate a profit of 0.42%-1.12% in excess returns, and 0.63%-1.70% in risk-adjusted returns. Also, most importantly, a substantial proportion of the profits of the trading strategy comes from the long side (stocks with both high price/quantity combination), hence the trading cost is not high.

One of the weaknesses of this approach is that the price and quantity of variance risk can be computed only for stocks that have options traded and is liquid enough to be able to compute the realized variance. A natural next step is to see whether we can build factor on variance risk using factor mimicking portfolios. Building factors are useful since it allows us to understand how market and non-market variance risk is priced across the entire stock universe.

I consider the last three out of seven trading strategies. I select a subset because the abnormal performance of the trading strategies are not mutually exclusive. Especially, strategy 6 is a combination of strategy 1 and 2. Strategy 5 is a subset of strategy 3, which performs better due to possible estimation error. Strategy 4 gives higher returns than strategy 3, but strategy 4 comes from two different sources of risk – market and non-market variance risk. In the next section, I study the relation between the factors generated using individual stock options and the one generated from the market variance risk exposure.

Factors are directly built from the trading strategies. Table VIII shows the performance of the decile portfolios sorted by the factor betas. Note that this is formed from the entire stock universe, which includes all stocks (excluding penny stocks) traded in NYSE, NASDAQ, and AMEX. For all three factors considered, the table suggests that stocks that are highly exposed to this variance factor tend to have higher subsequent returns. The spread is 0.79% - 1.19% in monthly excess returns and 0.96-1.42% in risk-adjusted returns, and they are all statistically significant.

4. Time-variation of the Variance Risk Factor

Table III -Table VIII overall suggest that non-market variance risk is priced among individual stocks, and the risk premium is unrelated to common risk factors (value, size, and momentum). In fact, the spread and its significance sometimes even increases after controlling for the risk factor. This subsection analyzes how the variance risk factor common risk factors as well as other well-known anomalies that seem directly connected to variance risk.

Table Table IX directly reports the time series regressions of monthly variance risk premium on risk factors:

$$\lambda_{VF,k,t} = \alpha_k + \sum_{j=1}^{J} \beta_{k,j} F_{j,t} + \epsilon_{k,t}$$

where $F_{j,t}$ are returns to factor-mimicking portfolios in month t. The dependent variable is the variance risk factor constructed using mimicking portfolios of trading strategies 5-7. Panel A-C each represent the regression result for trading strategies 5-7, respectively.

The first rows of each Panel provide the regression result when controlling for market, value, size, and momentum factors. Then, I build factor-mimicking portfolios using popular anomalies including idiosyncratic volatility, FVIX (Ang, Hodrick, Xing, and Zhang 2006), coskewness (Kraus and Litzenberger 1976). As the table indicates, the regression intercepts are both economically and statistically significant even after controlling for various risk factors. Also, while the variance risk factor is negative related to the common risk factors, it is positively related to idiosyncratic volatility and coskewness. The relation to the market variance factor is extremely weak.

V. Conclusion

This paper investigates the interactive role of the price and quantity of variance risk. The findings suggest that these two dimensions are interrelated. When stocks are sorted by the quantity of variance risk of its returns, those stocks that are more exposed to its variance risk tend to have higher subsequent returns. However, this effect is substantially stronger for stocks whose price of variance risk is also highly negative. For stocks with a small or positive price, the risk exposure does not matter. Similarly, the price of variance risk matters most for stocks that have high negative exposure to variance risk. Conclusively, the price of individual stock variance risk and the quantity of variance risk have an interactive relation.

This paper finds that the interactive relations are mainly driven by variance risk of factors other than the market factor. The interactive relation exists even when market variance risk is controlled or when stocks are sorted by the price and quantity of non-market variance risk. The results suggest that variance risk of factors other than the market factor does affect individual stock returns. The fact that both the price of variance risk as well as the underlying factor (i.e., variance shocks) is observable makes the new way of representing the risk premium of an asset, as suggested in this paper, a useful way of explaining the time-variation of individual stock returns.

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A. Appendix

We have assumed that the price process of a stock i (S_{it}) be the sum of the stock-specific systematic risk and unpriced idiosyncratic risk.

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + b_i dY_{i,t} + \sigma_{idio} dW_{i,t}^{idio},$$

where $dY_{i,t} = \sum_n \beta_{i,n} dF_n$. and

$$\begin{aligned} dF_{n,t} &= \mu_{n,t} dt + \sqrt{V_{nt}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o) \\ dV_{n,t} &= \theta_n dt + \sigma_{nv} dW_{n,t}^v, \end{aligned}$$

We transform the factor returns and variances as two stochastic processes for stock i, Y_i and the variance of Y_i (Called $\bar{V}_i$). Then,

$$dY_{i,t} = b_i \sum_n \beta_{i,n} \mu_{n,t} dt + b_i \sum_n \beta_{i,n} (\sqrt{V_{nt}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o))$$

Let $\bar{\mu}_{i,t} = b_i \sum_n \beta_{i,n} \mu_{n,t}$, and $\bar{V}_{i,t} = \sum_n b_i^2 \beta_{i,n}^2 V_{nt}$. Then, $d\bar{V}_{i,t} = \sum_n b_i^2 \beta_{i,n}^2 dV_{nt}$. Let $\bar{\theta}_{i,t} = b_i^2 \sum_n \beta_{i,n}^2 \theta_n$ and $\bar{\sigma}_i^2 = \sum_n (b_i^2 \beta_{i,n}^2 \sigma_{n,v})^2$. Then, the above equations can be rewritten as,

$$\begin{aligned} dY_{i,t} &= \bar{\mu}_{i,t} dt + \sqrt{\bar{V}_{i,t}} \bar{\rho}_i dW_{i,t}^v + \sqrt{1-\bar{\rho}_i^2} dW_{i,t}^o \\ d\bar{V}_{i,t} &= \bar{\theta}_i dt + \bar{\sigma}_i dW_{i,t}^v, \end{aligned}$$

where $\bar{\rho}_i$ measures the correlation between $dY_{i,t}$ and $d\bar{V}_{i,t}$. Following the same logic as in the main text, we can show that

$$E_t[R_{i,t+1}] = \beta_{Y,i,t} \lambda_{Y,i,t} + \lambda_{o,t},$$

where $\lambda_{Y,i,t}$ is the price of systematic variance risk of stock i, $\lambda_{Y,i,t}$ is the price of risk due to orthogonal shocks, and $\beta_{Y,i,t}$ is the slope of a hypothetical regression of stock i's return on the variance of latent systematic risk dY_i since

$$\begin{aligned} Cov(\frac{dS_{it}}{S_{it}}, dV_{i,t})/Var(dV_{i,t}) &= Cov(b_i dY_{it}, dV_{i,t})/Var(dV_{i,t}) \\ &= b_i \rho_i \sqrt{V_{it}} \sigma_{vi}/\sigma_{vi}^2 \\ &= b_i \rho_i \sqrt{V_{it}}/\sigma_{vi} \end{aligned}$$

This equation is equivalent to the representation of Result 2 since

$$\begin{aligned} \lambda_{v,i,t} \times \beta_{v,i,t} &= Cov(\frac{dS_{it}}{S_{it}}, b_i^2 dV_{i,t})/Var(b_i^2 dV_{i,t}) \times b_i^2 \lambda_{Y,i,t} \\ &= Cov(\frac{dS_{it}}{S_{it}}, dV_{i,t})/Var(dV_{i,t}) \times \lambda_{Y,i,t} \\ &= \beta_{Y,i,t} \lambda_{Y,i,t} \end{aligned}$$

B. Tables

Table I
Summary Statistics

This table provides the summary statistics for several variables of interest. Implied variance (IV) is the average option-implied variance of at-the-money call and put options. Historical realized variance (RV) is the variance of daily stock returns over the past month. The variance risk exposure (β_{vi}) is the slope of the regression,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i} \left[IV_{i,t} - IV_{i,t-1} \right] + \epsilon_{i,t}$$

where $R_{i,t}$ is the return of stock i over the past month, and $R_{m,t}$ is the excess value-weighted excess market return. The mean, median, and standard deviations for the sample is computed and the table compares them with those from the entire CRSP database.

	Entire CRSP	Sample				
	Mean	Mean	Median	St Dev	% Negative	% Positive
IV_i	-	0.018	0.011	0.024	-	100
RV_i	-	0.022	0.007	0.061	-	100
$RV_i - IV_i$	-	-0.001	-0.003	0.016	0.762	0.238
$\hat{\beta}_{v,i}$	-	-1.599	-1.249	1.784	0.773	0.227
Market Beta	1.097	1.161	1.103	0.484	0.002	0.998
Market Cap	2.721B	12.613B	3.1489B	33.32B	-	100
# of Stocks	4,948	1,130	1,242	334	-	-
# of Stock-Month		245,807				

Table II
The Characteristics and Performance of Single Variable Sorted Portfolios

This table summarizes the performance and characteristics of the price and quantity of variance risk sorted portfolios. Excess returns, the risk adjusted (value, size, momentum) returns of the subsequent month, the contemporaneous values of the price and quantity of variance risk, firm size, and market beta of each of the portfolios is summarized in the table.

A. Portfolios Sorted by the Price of Variance Risk ($\hat{\lambda}_{i,v}$)

	Returns	4-Factor α_4	$\hat{\lambda}_{v,i}$	$\hat{\beta}_{v,i}$	Size	Market Beta
Quartile 1	0.98	0.73	−0.014	−1.797	4.232B	1.401
	(1.71)	(3.12)				
Quartile 2	0.72	0.24	−0.005	−2.610	12.821B	1.089
	(2.10)	(2.66)				
Quartile 3	0.61	0.04	0.000	−2.969	21.121B	0.924
	(1.97)	(0.43)				
Quartile 4	0.43	−0.21	0.014	−2.694	12.386B	1.051
	(1.15)	(−2.14)				
Q4-Q1	−0.55*	−0.94***	0.028	−0.897	8.148B	−0.349
	(−1.70)	(−3.71)				

B. Portfolios Sorted by the Variance Risk Exposure ($\hat{\beta}_{i,v}$)

	Returns	4-Factor α_4	$\hat{\lambda}_{v,i}$	$\hat{\beta}_{v,i}$	Size	Market Beta
Quartile 1	0.72	0.19	0.001	−4.711	24.662B	0.932
	(2.52)	(2.90)				
Quartile 2	0.52	0.01	0.001	−2.216	12.801B	1.146
	(1.35)	(0.17)				
Quartile 3	0.81	0.14	0.000	−0.919	7.619B	1.176
	(2.10)	(1.07)				
Quartile 4	0.70	0.02	0.000	0.504	5.558B	1.028
	(2.15)	(0.22)				
Q4-Q1	−0.02	−0.17	0.000	5.215	−19.104B	0.096
	(−0.12)	(−1.24)				

Table III
The Performance of Price-Quantity of Variance Risk Double Sorted Portfolios

This table summarizes the value-weighted (Panel A) and equally-weighted (Panel B) returns of the quartile portfolios sorted by the price and quantity of individual stock variance risk. Excess returns and risk adjusted (value, size, momentum) returns of the subsequent month are provided. Portfolios are first constructed after sorting the stocks with respect to the price and then by the risk exposure.

A. Value-weighted Returns

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	1.21	1.05	0.74	0.51	1.28	0.76	0.09	−0.65	−1.12**	−1.70***
		(2.16)	(3.54)	(1.19)	(1.94)	(2.17)	(2.38)	(0.16)	(−1.86)	(−2.41)	(−3.71)
	Q2	0.84	0.41	0.52	−0.02	0.68	0.23	0.76	0.14	−0.08	−0.27
		(2.51)	(3.09)	(1.29)	(−0.13)	(1.60)	(0.98)	(2.49)	(0.65)	(−0.31)	(−0.95)
$\hat{\lambda}_{v,i}$	Q3	0.66	0.41	0.52	−0.02	0.68	0.23	0.76	0.14	−0.08	−0.27
		(2.25)	(3.09)	(1.29)	(−0.13)	(1.60)	(0.98)	(2.49)	(0.65)	(−0.31)	(−0.95)
	Q4 (Positive)	0.47	−0.24	0.33	−0.16	0.69	0.02	0.65	−0.13	0.18	0.10
		(1.36)	(−1.62)	(0.77)	(−0.82)	(1.20)	(0.10)	(1.54)	(−0.52)	(0.65)	(0.38)
	Q4-Q1	−0.74*	−1.29***	−0.41	−0.68**	−0.59	−0.74**	0.56	0.52	1.30***	1.80***
		(−1.92)	(−3.79)	(−1.09)	(−2.16)	(−1.28)	(−2.12)	(1.59)	(1.32)	(2.93)	(3.76)

B. Equally-weighted Returns

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	0.93	0.37	1.04	0.61	0.89	0.33	0.55	−0.09	−0.38	−0.46**
		(1.94)	(1.93)	(1.79)	(2.72)	(1.59)	(1.37)	(1.01)	(−0.52)	(−1.38)	(−2.13)
	Q2	0.93	0.33	0.71	0.10	0.68	0.05	0.87	0.27	−0.05	−0.06
		(2.87)	(3.34)	(1.84)	(0.98)	(1.68)	(0.36)	(2.44)	(2.02)	(−0.36)	(−0.45)
$\hat{\lambda}_{v,i}$	Q3	0.64	0.33	0.71	0.10	0.68	0.05	0.87	0.27	−0.05	−0.06
		(2.16)	(3.34)	(1.84)	(0.98)	(1.68)	(0.36)	(2.44)	(2.02)	(−0.36)	(−0.45)
	Q4 (Positive)	0.69	0.00	0.61	−0.02	0.50	−0.19	0.90	0.08	0.22	0.08
		(1.90)	(−0.03)	(1.37)	(−0.10)	(1.02)	(−1.13)	(2.01)	(0.36)	(0.89)	(0.34)
	Q4-Q1	−0.25	−0.38*	−0.44	−0.63***	−0.39	−0.52*	0.35	0.17	0.59*	0.54*
		(−0.95)	(−1.66)	(−1.58)	(−3.06)	(−1.32)	(−1.95)	(1.14)	(0.65)	(1.87)	(1.78)

Table IV
Price-Quantity of Variance Risk Double Sorted Portfolios Controlling for Market Variance Risk

This table summarizes the value-weighted returns of the price and quantity of variance risk double-sorted portfolios where the variance risk exposure is estimated using a three factor model

$$R_{i,t} = \alpha' + \beta'_{m,i} R_{m,t} + \beta'_{v,i}(IV_{i,t} - IV_{i,t-1}) + \beta'_{x,i}(VIX_t^2 - VIX_{t-1}^2) + \epsilon_t$$

where $R_{m,t}$ is the market excess return, IV is the option-implied variance of the individual stock, and VIX is the volatility index.

		Variance Risk Exposure ($\hat{\beta}'_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	1.28	1.07	0.71	0.63	1.10	0.48	0.36	−0.31	−0.92**	−1.38***
		(2.35)	(3.56)	(1.19)	(2.46)	(1.82)	(1.56)	(0.63)	(−0.96)	(−2.06)	(−3.21)
	Q2	0.81	0.38	0.61	0.02	0.69	0.26	0.76	0.12	−0.05	−0.26
		(2.38)	(2.94)	(1.65)	(0.13)	(1.69)	(1.27)	(2.39)	(0.54)	(−0.19)	(−0.93)
$\hat{\lambda}_{v,i}$	Q3	0.63	0.38	0.61	0.02	0.69	0.26	0.76	0.12	−0.05	−0.26
		(2.16)	(2.94)	(1.65)	(0.13)	(1.69)	(1.27)	(2.39)	(0.54)	(−0.19)	(−0.93)
	Q4 (Positive)	0.44	−0.22	0.35	−0.14	0.61	−0.06	0.78	−0.04	0.33	0.18
		(1.27)	(−1.59)	(0.82)	(−0.68)	(1.23)	(−0.28)	(1.78)	(−0.16)	(1.13)	(0.62)
	Q4-Q1	−0.84**	−1.29***	−0.36	−0.77**	−0.49	−0.55	0.42	0.27	1.26***	1.56***
		(−2.22)	(−3.72)	(−1.00)	(−2.41)	(−1.05)	(−1.59)	(1.18)	(0.73)	(2.85)	(3.30)

Table V
Price- Quantity of Non-market Variance Risk Double Sorted Portfolios

This table summarizes the value-weighted returns of the price and quantity of non-market variance risk double-sorted portfolios. The price of non-market variance risk ($\lambda_{o,t}$) is estimated as

$$\lambda_{o,t} = \lambda_{v,i,t} - \hat{\beta}_{vx,i}\lambda_{x,t}$$

, and the quantity ($\beta_{i,o}$) is estimated from

$$R_{i,t} = \alpha + \beta_{m,i}R_{m,t} + \beta'_{e,i}\hat{e}_t + \epsilon_t$$

where

$$IV_{i,t} - IV_{i,t} = \beta_{vx,0} + \beta_{vx,i}(VIX_t^2 - VIX_{t-1}^2) + e_t.$$

, $\lambda_{x,t}$ is the price of market variance risk, $IV_{i,t}$ is the option implied variance of stock i, VIX is the square of the market volatility index.

		Non-market Variance Risk Exposure ($\hat{\beta}_{e,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	1.08	0.82	0.70	0.47	0.93	0.53	0.46	−0.15	−0.62	−0.97**
		(2.04)	(2.85)	(1.11)	(1.82)	(1.63)	(2.16)	(0.95)	(−0.42)	(−1.47)	(−2.12)
	Q2	0.98	0.46	0.30	−0.08	0.78	0.36	0.66	0.05	−0.31	−0.41
		(2.95)	(2.96)	(0.77)	(−0.56)	(1.92)	(1.64)	(2.16)	(0.24)	(−1.23)	(−1.61)
$\hat{\lambda}_{e,i}$	Q3	0.67	0.46	0.30	−0.08	0.78	0.36	0.66	0.05	−0.31	−0.41
		(2.10)	(2.96)	(0.77)	(−0.56)	(1.92)	(1.64)	(2.16)	(0.24)	(−1.23)	(−1.61)
	Q4 (Positive)	0.53	−0.05	0.22	−0.23	0.48	−0.02	0.88	−0.09	0.35	−0.04
		(1.35)	(−0.29)	(0.45)	(−1.30)	(0.93)	(−0.11)	(2.20)	(−0.45)	(1.26)	(−0.18)
	Q4-Q1	−0.55	−0.86***	−0.48	−0.71**	−0.44	−0.55*	0.43	0.06	0.97**	0.92**
		(−1.53)	(−2.78)	(−1.09)	(−2.17)	(−1.03)	(−1.79)	(1.47)	(0.16)	(2.28)	(1.97)

Table VI
Portfolios Sorted by the Price-Quantity Combination

This table summarizes the value-weighted returns of the single-sorted portfolios. Stocks in the sample are sorted by the combination of the price and quantity of variance risk. Panel A summarizes the result when stocks are sorted by the product. Panel A-1 shows the performance for the sub-sample of stocks whose price of variance risk is estimated to be negative. Panel A-2 is for the sub-sample with a positive price. Finally, Panel B summarizes the result when stocks are sorted by the sum of the price-quantity combination of the market and non-market variance risk.

A. Entire Sample Sorted by $\hat{\beta}_{i,v} \times \hat{\lambda}_{i,v}$

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Ret	0.54	0.47	0.81	0.70	0.80	0.26
	(1.53)	(1.47)	(2.73)	(2.11)	(2.02)	(1.44)
α_4	−0.07	−0.16	0.21	0.13	0.46	0.53***
	(−0.94)	(−1.81)	(1.95)	(1.20)	(3.95)	(3.59)

A-1. Subsample ($\hat{\lambda}_{v,i} < 0$)

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Ret	0.27	0.73	0.70	0.70	0.84	0.58**
	(0.82)	(2.65)	(2.05)	(1.91)	(1.98)	(2.10)
α_4	−0.14	0.12	0.15	0.23	0.47	0.60***
	(−1.04)	(0.96)	(1.21)	(1.60)	(2.95)	(3.12)

A-2. Subsample ($\hat{\lambda}_{v,i} > 0$)

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Ret	0.54	0.68	0.58	0.73	0.26	−0.28
	(1.44)	(1.69)	(1.69)	(2.26)	(0.74)	(−1.17)
α_4	0.00	−0.06	−0.02	−0.08	−0.44	−0.44**
	(0.03)	(−0.41)	(−0.10)	(−0.69)	(−2.39)	(−1.99)

B. Entire Sample Sorted by $\hat{\beta}_{e,i}\hat{\lambda}_{e,i} + \hat{\beta}'_{x,i}\hat{\lambda}_x$

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Ret	0.51	0.61	0.58	0.66	0.93	0.42**
	(1.29)	(1.73)	(1.99)	(2.15)	(2.43)	(2.37)
α_4	−0.09	0.07	0.03	0.11	0.54	0.63***
	(−0.87)	(0.70)	(0.24)	(1.06)	(4.06)	(3.46)

Table VII
Trading Strategy

This table summarizes the performance of possible trading strategies. Strategy 1 consists of buying stocks with both a high negative price and quantity of variance risk (Table 3), and selling stocks with low price of risk. Strategy 2 consists of buying stocks with both a high negative price and quantity of variance risk (Table 3), and selling stocks with low quantity of risk. Strategy 3 comes from Table 6-A and consists of a long/short strategy based on the product of the price and quantity. Strategy 4 comes from Table 6-B and consists of a long/short strategy based on the combination of non-market and market variance risk premium. Strategy 5 comes from Table 6-A-1 and is the strategy based on the product of price and quantity of variance risk, but excludes those stocks with a positive price of risk. Strategy 6, from Table 3, consists of buying stocks that have both high or low price and quantity of risk (i.e., Q1/Q1 or Q4/Q4) and selling stocks that has either a low price or quantity of risk (i.e., Q1/Q4 or Q4/Q1). Finally, strategy 7, also from Table 3 is built by buying stocks with both high (Q1/Q1) price and quantity of risk and selling stocks that has either a low price or quantity of risk (i.e., Q1/Q4 or Q4/Q1 in Table 3).

	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6	Strategy 7
Returns	0.74*	1.12**	0.26	0.42**	0.58**	0.57**	0.89**
	(1.92)	(2.41)	(1.44)	(2.37)	(2.10)	(2.38)	(2.39)
CAPM α_1	0.77**	1.21***	0.36**	0.52***	0.57**	0.59***	0.95***
	(2.25)	(2.72)	(2.18)	(3.13)	(2.51)	(2.58)	(2.77)
α_3	1.16***	1.63***	0.47***	0.60***	0.62***	0.65***	1.00***
	(3.38)	(3.65)	(2.94)	(3.16)	(2.99)	(2.81)	(2.92)
α_4	1.29***	1.70***	0.53***	0.63***	0.60***	0.74***	1.12***
	(3.79)	(3.71)	(3.59)	(3.46)	(3.12)	(3.11)	(3.09)

Table VIII
Performance of Portfolios Sorted by the Variance Risk Factor Betasβ_{FVR}

This table summarizes the value-weighted returns of the portfolios formed by sorting the stocks of the entire CRSP universe by its exposure to the variance risk factor. The factors are formed as the mimicking portfolios generated from strategies 5-7 as in Table 7. The factor betas are the regression coefficients of

$$R_{i,t} = \beta_{0f} + \beta_{m,i} R_{m,t} + \beta_{FVR(j),i} F_{j,t} + \epsilon_t$$

where F_j is one of the three variance risk factors.

A. Factor Built on Strategy 5

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	10-1
Return	0.19	0.2	0.57	0.61	0.55	0.89	0.69	0.71	0.87	1.37	1.19**
	(0.39)	(0.49)	(1.69)	(1.90)	(1.65)	(2.78)	(2.06)	(1.86)	(1.84)	(2.21)	(2.04)
α_4	−0.33	−0.22	0.08	−0.06	−0.05	0.1	−0.03	−0.13	0.08	0.63	0.96**
	(−1.62)	(−1.80)	(0.80)	(−0.64)	(−0.50)	(0.94)	(−0.21)	(−1.19)	(0.52)	(2.18)	(2.35)

B. Factor Built on Strategy 6

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	10-1
Return	0.25	0.31	0.67	0.68	0.62	0.59	0.61	0.74	0.84	1.04	0.79**
	(0.38)	(0.71)	(2.01)	(2.24)	(2.08)	(1.92)	(1.72)	(1.96)	(2.17)	(1.93)	(2.38)
α_4	−0.63	−0.29	0.07	0.09	−0.05	−0.07	0.01	0.17	0.29	0.38	1.01***
	(−1.96)	(−1.88)	(0.53)	(0.99)	(−0.61)	(−0.87)	(0.12)	(1.59)	(2.10)	(1.82)	(2.72)

C. Factor Built on Strategy 7

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	10-1
Return	0.1	0.43	0.49	0.78	0.54	0.74	0.69	0.74	0.83	0.89	0.80**
	(0.15)	(0.99)	(1.49)	(2.83)	(1.66)	(2.26)	(2.22)	(2.04)	(1.90)	(1.63)	(2.12)
α_4	−0.99	−0.2	−0.18	0.14	−0.05	0.07	0.16	0.01	0.36	0.43	1.42***
	(−3.15)	(−1.20)	(−1.95)	(1.22)	(−0.52)	(0.74)	(1.76)	(0.10)	(1.90)	(1.81)	(3.12)

Table IX
Time Variation of Variance Risk Factors

This table summarizes the results of time series regressions of the variance risk factors on other common risk factors such as the market, value (HML), size (SMB), past returns (UMD), market variance risk (FVIX), coskewness (CSK), idiosyncratic volatility (IVOL) factors. The variance risk factor of Panel A, B, and C is built from trading strategies 5,6, and 7 in Table 7, respectively.

A. Factor Built on Strategy 5

Regression	Intercept	Mktrf	HML	SMB	UMD	IVOL	CSK	FVIX	Adj-R^2
Reg (1-1)	0.177***	2.505	−8.118***	−1.131	−5.672***				0.204
	(3.25)	(1.63)	(−3.41)	(−0.61)	(−4.46)				
Reg (1-2)	0.134**	1.870				0.116***			0.121
	(2.41)	(1.18)				(2.68)			
Reg (1-3)	0.134**	4.638***					0.147		0.083
	(2.24)	(2.97)					(1.52)		
Reg (1-4)	0.115**	5.256***						−0.050	0.066
	(1.99)	(3.21)						(−0.59)	
Reg (1-5)	0.190***	0.587	−6.056***	−3.966	−4.105***	−0.036	0.105**	0.110	0.234
	(3.50)	(0.39)	(−2.69)	(−1.56)	(−3.30)	(−0.60)	(2.21)	(1.22)	

B. Factor Built on Strategy 6

Regression	Intercept	Mktrf	HML	SMB	UMD	IVOL	CSK	FVIX	Adj-R^2
Reg (2-1)	0.125**	1.559	−3.195	−0.741	−0.801				0.013
	(2.49)	(1.12)	(−1.47)	(−0.39)	(−0.63)				
Reg (2-2)	0.115**	1.129				0.033			0.013
	(2.34)	(0.89)				(1.01)			
Reg (2-3)	0.111**	2.029					0.008		0.006
	(2.24)	(1.50)					(0.09)		
Reg (2-4)	0.109**	2.138						−0.025	0.007
	(2.15)	(1.64)						(−0.36)	
Reg (2-5)	0.126**	1.052	−2.495	−1.592	−0.324	−0.027	0.034	0.010	0.005
	(2.56)	(0.80)	(−1.10)	(−0.65)	(−0.22)	(−0.36)	(0.76)	(0.10)	

C. Factor Built on Strategy 7

Regression	Intercept	Mktrf	HML	SMB	UMD	IVOL	CSK	FVIX	Adj-R^2
Reg (3-1)	0.208***	3.331	−7.771**	−1.757	−3.531*				0.072
	(2.92)	(1.54)	(−2.00)	(−0.60)	(−1.89)				
Reg (3-2)	0.178**	2.083				0.108*			0.060
	(2.36)	(0.96)				(1.92)			
Reg (3-3)	0.188**	4.407**					0.213*		0.053
	(2.43)	(2.11)					(1.75)		
Reg (3-4)	0.158**	5.449**						−0.117	0.040
	(1.98)	(2.36)						(−0.99)	
Reg (3-5)	0.227***	1.228	−5.704	−3.831	−1.341	−0.139	0.115*	0.218*	0.111
	(3.27)	(0.59)	(−1.63)	(−0.96)	(−0.81)	(−1.31)	(1.65)	(1.81)	