Assignment 7

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A 1. The role matrices for the scenario are shown in Figure 2, 3 and 4. The visualization of the simulation is shown in Figure 1.

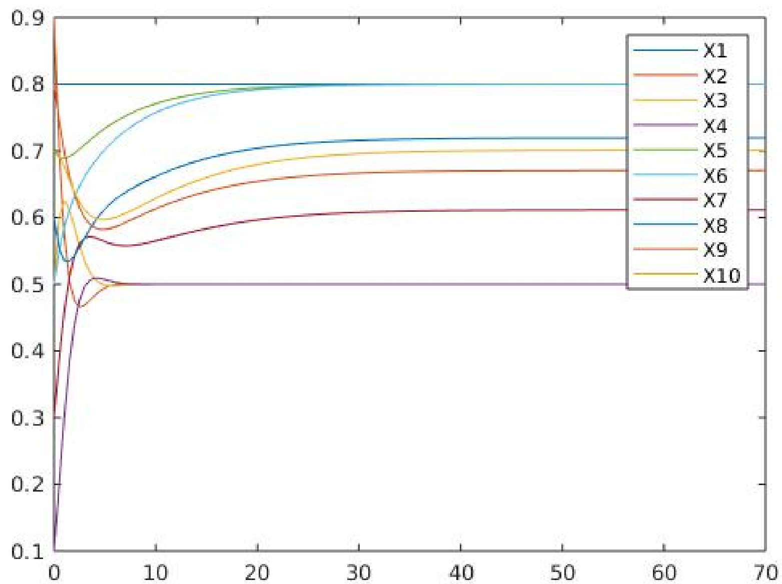


Figure 1: Visualization of scenario described in assignment 7.

2. See table below. The model has an accuracy of 10^{-3} on average.

m_b	**1**	**2**	**3**
X_1	1		
X_2	4		
X_3	2		
X_4	3		
X_5	1	6	
X_6	5		
X_7	3	9	10
X_8	6	7	
X_9	3	8	
X_{10}	8	9	

(a) Base connectivity matrix m_b.

m_{cwv}	**1**	**2**	**3**
X_1	1		
X_2	1		
X_3	1		
X_4	1		
X_5	0.8	0.7	
X_6	1		
X_7	0.8	0.6	0.6
X_8	0.8	0.6	
X_9	0.2	0.7	
X_{10}	0.5	0.3	

(b) Connection weight matrix m_{cwv}.

Figure 2: Connectivity role matrices.

m_{cfw}	**ssum**	**id**
X_1		1
X_2		1
X_3		1
X_4		1
X_5	1	
X_6		1
X_7	1	
X_8	1	
X_9	1	
X_{10}	1	

(a) Combination function weights matrix m_{cfw}.

m_{cfp}	**ssum**		**id**	
X_1				
X_2				
X_3				
X_4				
X_5	1.5			
X_6				
X_7	2			
X_8	1.4			
X_9	0.9			
X_{10}	0.8			

(b) Combination function parameters matrix m_{cfpv}.

Figure 3: Aggregation role matrices.

State X_i	7	3	4	9	5
Time point t	4.6	1.3	5.8	4.9	1
$X_i(t)$	0.56374	0.59715	0.5031	0.58906	0.69182
aggimpact$X_i(t)$	0.56256	0.57229	0.5022	0.58483	0.69634
aggimpact$X_i(t) - X_i(t)$	-0.00118	-0.02486	-0.0009	-0.00423	0.00452
	8	2	4	5	7
	1.4	3.4	5.3	5.3	5.3
	0.54526	0.4825	0.50295	0.7351	0.56284
	0.54869	0.4837	0.50312	0.7421	0.59179
	0.00343	0.0012	-0.0098	0.007	0.02895

3. There only occur errors in the states with more than one incoming connection (states that have the scaled sum as combination function). However, these errors are approximately 10^{-4} and thus negligible.

m_s	
X_1	0.5
X_2	0.5
X_3	0.5
X_4	0.5
X_5	0.5
X_6	0.5
X_7	0.5
X_8	0.5
X_9	0.5
X_{10}	0.5

(a) Speed parameters matrix m_s.

i_v	
X_1	0.8
X_2	0.9
X_3	0.5
X_4	0.1
X_5	0.7
X_6	0.5
X_7	0.3
X_8	0.6
X_9	0.8
X_{10}	0.7

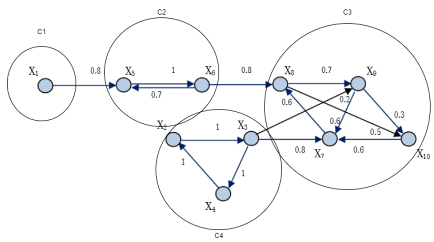
(b) Initial values matrices. i_v.

Figure 4: Miscellaneous matrices.

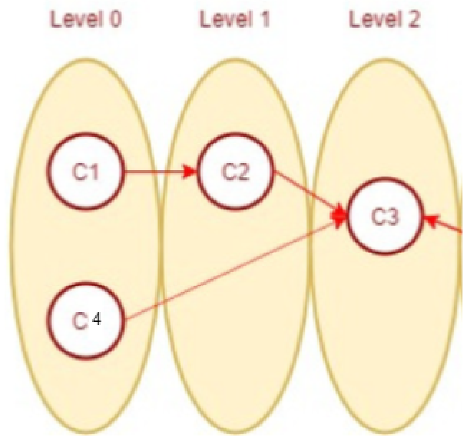
State X_i	1	2	3	4	5
Time point t	300	300	300	300	300
$X_i(t)$	0.8	0.5	0.5	0.5	0.8
aggimpact$X_i(t)$	0.8	0.5	0.5	0.5	0.8
aggimpact$X_i(t) - X_i(t)$	0	0	0	0	0
	6	7	8	9	10
	300	300	300	300	300
	0.8	0.61142	0.71918	0.67047	0.70091
	0.8	0.61142	0.71918	0.67047	0.70091
	0	0	0	0	0

B 4. See Figure 5.

5.
- Network N is normalised because for each state Y it holds that $Cy(\omega_{x_1,Y}, ..., \omega_{x_k,y}) = 1$ where $X_1, ..., X_k$ are the states from which Y gets its incoming connections.
- Its combination functions are scalar-free because for all $c(\alpha V_1, ..., \alpha V_k) = \alpha c(V_1, ..., V_k)$
- Its combination functions are strictly monotonic because for all $\alpha.0$ and all $V_1, ..., V_k$ it holds $Ui \leq Vi$ for all i, and $Uj < Vj$ for at least one $j \rightarrow c(U1, ..., Uk) < c(V1, ..., Vk)$.
- In level 0 there is C1 and C4. The values V of all states in N belonging to C4 (X_2, X_3, X_4) have the same equilibrium value = 0.5. C1 only contains 1 state without any incoming connection, and its equilibrium value is indeed between 0.1 and 0.9 (theorem 5.ii). Specifically, the equilibrium value of $X1$ is 0.8 because it is the only state in N belonging to C1 and has the initial value of 0.8, thus its equilibrium value is also 0.8 (theorem 5.iii).



(a) Strongly connected components



(b) Stratified condensation graph for the scenario.

Figure 5: Network Connectivity Characteristics Based on Strongly Connected Components.

In level 1, X_5 and X_6, forming $C2$, have only one incoming connection from $C1$. Thus, X_5 and X_6 have the same equilibrium value as X_1, that is, 0.8.
In level 2, $C3$ has three incoming connections from both $C2$ and $C5$. Their equilibrium values are not equal. Therefore, the above theorems do not imply that the equilibrium values of the components of $C3$ are the same. It is observed in 6 that they are indeed different.

- See Figure 6. All the equilibrium points of all states became 0.8 after the initial value of X_4 became 1, instead of 0.1. This is because the compound C_5 equilibrium point became 1 as their components are strongly connected. Applying Theorem 6a, it is observed how with the equilibrium values of all states in every strongly connected component of level 0 in $SC(N)$ are equal to 0.8, then the equilibrium state values of all states in N are equal to 0.8.
- See Figure 7. States X_5 to X_{10} form the compounds from level 2 onwards. The equilibrium values of the compounds in level 0 have not changed, thus, the equilibrium value has not changed- it is still 0.8. The only difference is that all of these states have an initial value of 0 instead of their respective former values.

6. See figure 7.
7. The above outcomes will be the same when instead of the normalised scaled sum a normalized Euclidean combination function of higher order

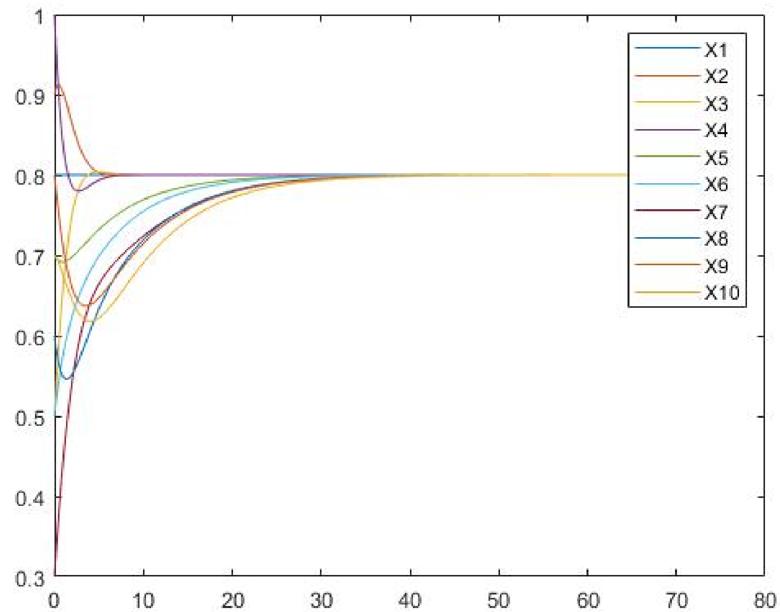


Figure 6: Scenario with initial value of $X_4 = 1$.

n.1 is used. This is because Euclidean combination functions also fulfill the three requirements mentioned in question 6.

C 9. This system has infinitely which are, according to WIMS (with parameters r_i):

- $x = 0.2058823529411764\ r_1 - 0.04092071611253196r2$
- $x_1 = 0.7999999999999999$
- $x_{10} = 1.372549019607843\ r1 - 0.2148337595907928r2$
- $x_2 = 0.9999999999999999r_2$
- $x_3 = r_2$
- $x4 = 0.9999999999999999r_2$
- $x_5 = 1.970588235294117r_1 - 0.391669711362806r_2$
- $x_6 = 1.970588235294117r_1 - 0.391669711362806r_2$
- $x_7 = 0.3273657289002557r_2 + 0.6862745098039213r_1$
- $x_8 = r_1$
- $x_9 = 0.9150326797385618r_1 - 0.02728047740835464r_2$

If we put the equations into matrix A the system will have a $det(A) = 0$ thus creating infinite many solutions.

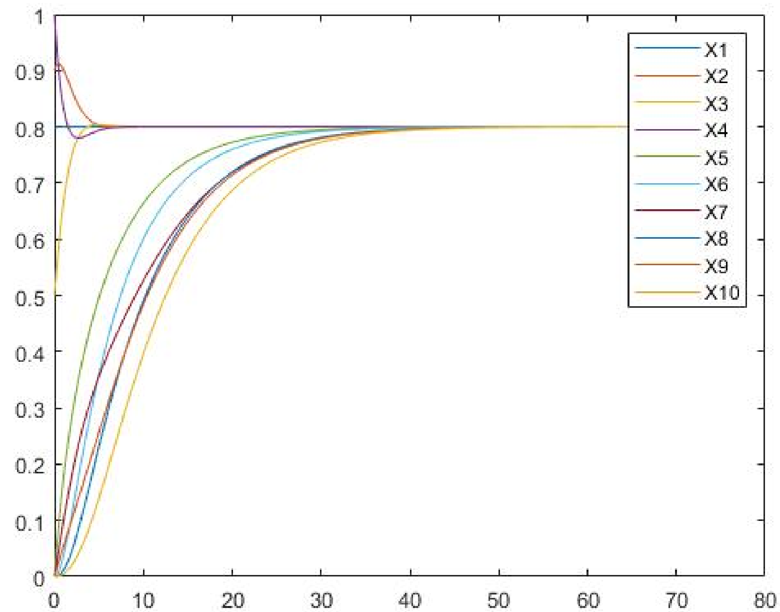


Figure 7: Scenario with initial values from $X_5 - X_{10} = 0$.

10. According to WIMS, this system now has a unique solution for X_2, X_3 and X_4 a unique solution, which is:
    - $x_2 = 0.4999999999999999$
    - $x_3 = 0.4999999999999999$
    - $x_4 = 0.4999999999999999$

    The NOME framework in MATLAB gave value 0.5 for X_2, X_3 and X_4 creating an error which is insignificant.

    The infinitely many solutions happened because in the fifth condensation component $C5$ it was specified before that all the states that formed it were dependent on each other. Moreover, as $C5$ is part of level 0, the values of the components remained unspecified. The specification of one single state in a strongly correlated network $X2 = 0.5$ made all the states that form part of the component also adopt the value of 0.5, as the formulas indicate: $X2 = x3 = x4$.
11. This system has infinitely many solutions, which are (with parameters ri):
    - $a = 0.9999999999999999r1$
    - $x_1 = 0.9999999999999997r1$
    - $x_{10} = 1.0r1$
    - $x_2 = 0.9999999999999999r1$

- $x_3 = 0.9999999999999999r1$
- $x_4 = 0.9999999999999999r1$
- $x_5 = 0.9999999999999997r1$
- $x_6 = 0.9999999999999997r1$
- $x_7 = 0.9999999999999999r1$
- $x_8 = 1.0r1$
- $x_9 = r1$

In this example, one more level is added before the current level 0. Thus, condensation components $C1$ and $C5$ become dependent in an unspecified state a, that is the new level 0. Because both levels are dependent on a single state, they acquire the value of such state. However, this value is unspecified in the equation. Therefore, their values are also unspecified.

D12. For the reification states X_{11} and X_{12} we find the values in the equilibrium to be 0.15726 and 0.2876 respectively. Using the formula in box 3.1 (book 2) we get for $X_{11} = \frac{0.32302 \cdot 0.34661}{1-0.4+(0.32302 \cdot 0.34661)} = 0.15725834826$ and for $X_{12} = \frac{0.8 \cdot 0.30278}{1-0.4+(0.8 \cdot 0.30278)} = 0.28760044833$, resulting in an error of 0.00000165174 for X_{11} and 0.0000004883 for X_{12}. This translates to an accuracy of 10^{-6} and 10^{-7} which is probably due to the rounding off of the values in Matlab.

13. The reification states X_{11} and X_{12} are introduced with (in this case) both a Hebian combination function. However, they only have an influence on the weights of the base network. Therefore the network will not be a normalized network anymore. The scalar-free and strictly monotonic characteristics will still hold, as described in B.

14. According to box 3.2 (book 2) the value of the reification states can be either 0, 1 or $|V_1 - V_2| = \tau$. We can see that for X_{11} we find indeed that the value is 1. However, for $t = 0.1 * 10,000 = 1,000$ we have still have not reached 1 for X_{12}. We therefore assume that $|V_1 - V_2| = \tau$ holds which is indeed true by $|0.95 - 0.8| = 0.15 = \tau$.

15. In this case the difference is in the reification states using a slhomo instead of Hebian combination functions, which again does only change the base network in being not normalized anymore. For the other two characteristics the same applies as described earlier in B.