Assignment 7 Analysis of a Network's Emerging Behaviour Based on its Structure

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Assignment Outcomes

After doing this assignment it is expected that you will understand the following:

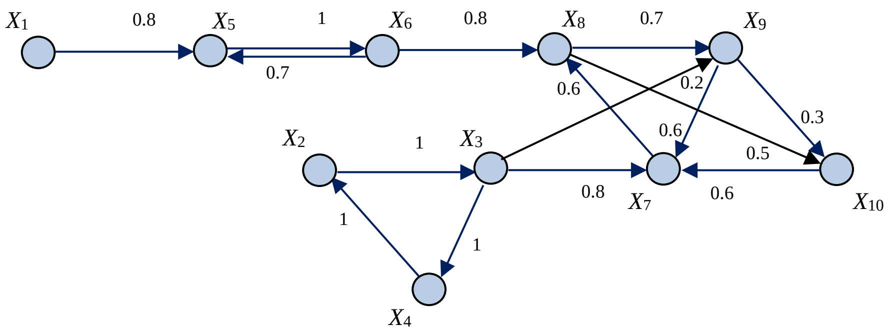
- Stationary points and equilibria of a network model, and the equations for them
- How to determine relations for the values in equilibria of a network model by mathematical analysis
- How to use such mathematical analysis for verification of a network model by comparison to simulation values

References

For this analysis refer to Book 1, Chapter 12 and Book 2, Chapter 1, Section 1.6, Chapter 2, Section 2.5, and Chapter 12.

Example Network Model

In this assignment for an example network, stationary points are analysed, and relations for the final equilibrium values are predicted. The example network used in A., B., and C. is depicted here:



As combination functions normalised scaled sum functions are chosen (for the states X_1, X_2, X_3, X_4, X_6 with one incoming connection you may as well choose for the identity combination function, as this works the same). All speed factors are set at 0.5. The connection weights are indicated as labels to the arrows in the above graph; note that there is also a not depicted connection from X_1 to itself with weight 1. The following (normalising) scaling factors λ are used:

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
λ	1	1	1	1	1.5	1	2	1.4	0.9	0.8

Use the following initial values

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
initial values	0.8	0.9	0.5	0.1	0.7	0.5	0.3	0.6	0.8	0.7

A. Checking stationary point and equilibrium equations (Ch 12, Section 12.2, p. 327)

1. First specify the role matrices for this example network model and run a simulation.

2. Determine from the simulation a number of temporary stationary points for well-chosen time points $t \leq 10$ for the different states and check with what accuracy the stationary point equations hold for them. Show this in a table as shown below with 5 rows for

    - The chosen state X_i and time point t
    - The state value $X_i(t)$ at time t
    - The value of the aggregated impact $\mathbf{aggimpact}_{X_i}(t)$ for X_i at time t
    - The difference between $\mathbf{aggimpact}_{X_i}(t)$ and $X_i(t)$

State X_i										
Time point t										
$X_i(t)$										
$\mathbf{aggimpact}_{X_i}(t)$										
$\mathbf{aggimpact}_{X_i}(t)$ - $X_i(t)$										

    Did the model generate correct values? With what accuracy (= what is the maximal deviation)?

3. Determine from the simulation for the final equilibrium for time points $t \geq 300$ for different states their equilibrium value and check with what accuracy the equilibrium equations hold for them; again create such a table, now using time point 300 all the time. Did the model generate correct values? With what accuracy?

B. Analysis based on the network's strongly connected components

For this part refer to Book 2, Chapter 12, and to Theorems 5 and 6 of Section 12.6 of this chapter.

4. Indicate in the graph the strongly connected components C_1, C_2, …, and depict the network's stratified condensation graph (similar to Figs. 12.5 and 12.7 in Book 2, Chapter 12, Section 12.4).
5. In Theorems 5 and 6 three conditions are mentioned in the first sentence. Explain why for this example these conditions are fulfilled, so that these theorems are applicable here (Hint: refer to Chapter 12, Table 12.2, and for normalisation to Definition 6 and subsequent text). Apply Theorem 5 for the level 0 components and Theorem 6 for the components of level > 0 and by this explain the final equilibrium values in the simulation from the strongly connected components and how they relate in the stratified condensation graph. With what accuracy?
6. Now set the initial value of X_4 at 1 (instead of 0.1), and explain what happens. Why can this pattern be predicted based on the strongly connected components and the above theorems?
7. Set all initial values for X_5 to X_{10} at 0, and leave the other initial values the same. What happens? Can you explain this by the above theorems?
8. Do you expect that the above outcomes will be the same when instead of the normalised scaled sum a normalised Euclidean combination function of higher order $\boldsymbol{n}$>1 is used? Explain that by the above theorems.

C. Symbolic solving of the equilibrium equations (Ch 12, Section 12.4, p. 334)

In Book 1, Chapter 12, Section 12.4 (p. 334) the equilibrium equations for a network using scaled sum combination functions are described (here for k = 10 states):

$$\lambda_1 X_1 = \omega_{1,1} X_1 + \ ... + \omega_{10,1} X_{10}$$

$$.....$$

$$\lambda_{10} X_{10} = \omega_{1,10} X_1 + \ ... + \omega_{10,10} X_{10}$$

In a normalised case as for the example here, for each i it holds that λ_i is the sum of the weights of the incoming connections:

$$\lambda_i = \ \omega_{1,i} + \ ... + \omega_{10,i}$$

Note that this is the sum of the values in the i^{th} row of matrix **mb**, but also indicated above. For identity combination functions with single incoming weight 1 it works the same as for a scaled sum function with scaling factor 1.

For state X_1 the (constant) value is 0.8. Based on this, the equilibrium equations for the considered example network are

$$\begin{aligned}
&X_1 = 0.8\\
&X_2 = X_4\\
&X_3 = X_2\\
&X_4 = X_3\\
&1.5X_5 = 0.8X_1+0.7X_6\\
&X_6 = X_5\\
&2X_7 = 0.8X_3+0.6X_9+0.6X_{10}\\
&1.4X_8 = 0.8X_6+0.6X_7\\
&0.9X_9 = 0.2X_3+0.7X_8\\
&0.8X_{10} = 0.5X_8+0.3X_9
\end{aligned}$$

9. Show by copying the above set of equations and pasting them in the WIMS Linear Solver at https://wims.unice.fr/wims/wims.cgi that this does not give a single solution for this set of equations. Explain why this is.

10. However, show that by adding an extra equation

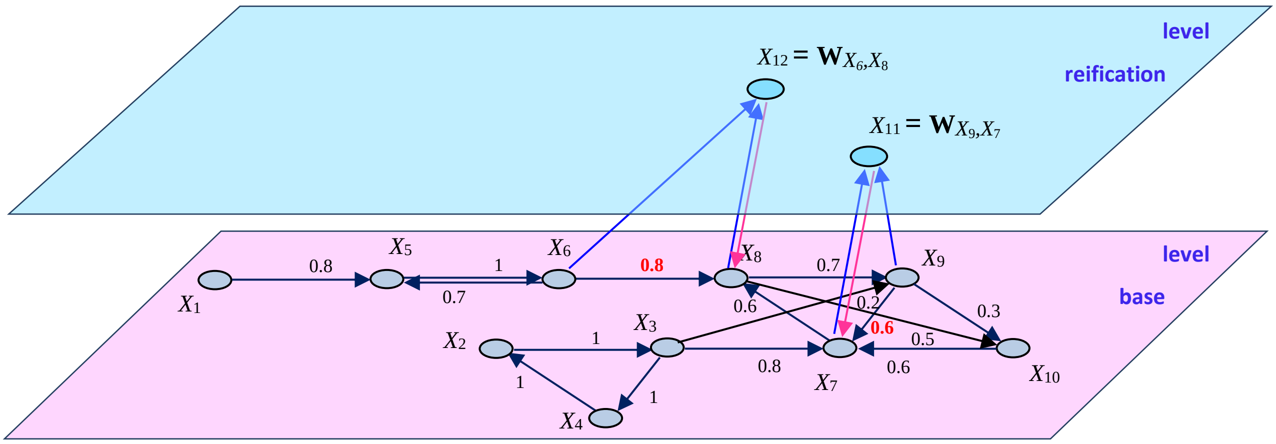
    $$X_2 = 0.5$$

    for the observed equilibrium value for the level 0 component for X_2, X_3, X_4, it does provide a single solution. Show in a table the equilibrium values from this WIMS solution and the observed equilibrium values in the simulation, and their differences. Did the model generate correct values? With what accuracy?

11. Now replace the equations $X_1 = 0.8$ and $X_2 = 0.5$ by $X_1 = a$ and $X_2 = a$ for one fixed parameter a and in the Linear Solver also insert this a in the slot for parameters. Report what happens and explain this from Theorems 5 and 6 considering the strongly connected components.

D. Analysis of an adaptive network model

Next, the above network model is made adaptive by adding two reification states $X_{11} = \mathbf{W}_{X_9,X_7}$ and $X_{12} = \mathbf{W}_{X_6,X_8}$ for the connections from X_9 to X_7 and from X_6 to X_8. See the picture below, and see Book 2, Chapter 3, Section 3.6.1 for more background. The red values for the two adaptive connections now are initial values. First, for the combination function of the two reification states the Hebbian learning function $\mathbf{hebb}_\mu(V_1, V_2, W)$ is used. In the Matlab file Assignment7adaptive_hebbian.m this model is already given with appropriate settings.



First, we do analysis of the reification level, based on what is shown in Box 3.1 of Book 2, Chapter 3, Section 3.6.1. Next, we check in how far the analysis of the base network based on its strongly connected components (as in B. above) according to Theorems 5 and 6 of Book 2, Chapter 12, Section 12.6 still can be applied now some connection weights are not constant.

12. With the given settings, run a simulation long enough that an equilibrium occurs. For the end point of this simulation, for both reification states, check whether the relation found in Box 3.1 holds for the simulation. With what accuracy?
13. For the base network, check again the three conditions that are mentioned in the first sentence of Theorems 5 and 6 of Book 2, Chapter 12, Section 12.6. Discuss for each of these three conditions for which parts of the base network they are fulfilled, and for which part they are not (Hint: refer to Chapter 12, Table 12.2, and for normalisation to Definition 6 and subsequent text). Discuss why for the parts where all three conditions are fulfilled, indeed the conclusions of these theorems hold for the equilibrium values shown in the new simulation, and with what accuracy. Discuss whether the conclusions of these theorems do or do not hold for the other part(s) of the base network.

As another variant, next the simple linear combination function $\mathbf{slhomo}_{\sigma,\tau}\mathbf{(}V_1, V_2, W\mathbf{)}$ for bonding by homophily is used for the two reification states. In the Matlab file Assignment7adaptive_homophily.m this model is already given with appropriate settings. First, we do analysis of it, based on what is shown in Box 3.2 of Book 2, Chapter 3, Section 3.6.1. Next, we check in how far the analysis of the base network based on its strongly connected components (as in B. above) according to Theorems 5 and 6 of Book 2, Chapter 12, Section 12.6 still can be applied now some connection weights are not constant.

14. With the given settings, run a simulation long enough that an equilibrium occurs. For the end point of this simulation, for both reification states, check whether the relation found in Box 3.2 holds for the simulation. With what accuracy?
15. For the base network, check again the three conditions that are mentioned in the first sentence of Theorems 5 and 6 of Book 2, Chapter 12, Section 12.6 for this case. Discuss for each of these three conditions for which parts of the base network they are fulfilled, and for which part they are not (Hint: refer to Chapter 12, Table 12.2, and for normalisation to Definition 6 and subsequent text). Discuss why for the parts where all three conditions are fulfilled, indeed the conclusions of these theorems hold for the equilibrium values shown in the new simulation, and with what accuracy. Discuss whether the conclusions of these theorems do or do not hold for the other parts of the base network.