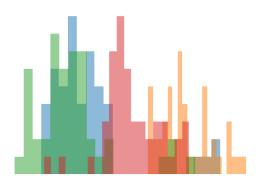
# Agent Based Modelling: Measuring Flow of Different Traffic Policies on Intersections

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# Abstract

An agent-based model is developed to analyse the differences between certain kind of traffic intersections. The environment (intersection) and the cars (agents) are introduced in the first chapter. The different rule sets of the intersections and the details of the agents are explained in chapter two. Chapter 4 contains the results of some experiments and the validation of the model. Throughout the report we show that intersections without traffic lights have higher throughput than intersection with traffic lights.

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# Introduction

Traffic plays a major role in our everyday lives. It enables us to transport freely, commute to our jobs while it also plays a key role in the logistics chain. With the growth of world economies over the past decades, the demand on our infrastructure has drastically increased. This has led to more and more traffic jams, which has negative effects on our economy, environment and public health [14][2][13].

Testing different traffic control policies in real life requires a lot of resources. Therefore, modelling traffic on computers can be a very useful tool to give better predictions of the effects of different traffic control policies on traffic flow. In this report we describe a model used for modelling traffic, investigating the influence of different type of rule sets on a intersection.

The model description follows the ODD +D (Overview, Design concepts, Details, Decision) protocol [4][12].

### 0.1 Theoretical background

Traffic consists of heterogeneous entities interacting in a non-linear way on a network of roads. Traffic therefore can be seen as a complex system. Modelling traffic has been done in a variety of ways. For example, traffic on larger roads such as highways can modelled using mathematical flow models, resembling gas flux, using differential equations [9]. Mathemetical models can be very useful in modelling a network of roads, for they very efficiently compute traffic flow on roads between intersections. However, these models cannot capture the complex dynamics in traffic that result from interactions between vehicles, such as lane changing, individual driving behaviour and different destinations of vehicles.

Cellular automata (CA) have also been repeatedly used to model traffic starting with the model of Nagel and Schreckenberg in 1992. Cellular automata efficiently use computational resources and can explain complex phenomena in traffic such as the formation of traffic jams [6]. Although efforts have been made to formulate CA models that can model a heterogeneous population of vehicles, it is shown that CA models are not well suited to model inter vehicle interactions, or complex road networks [10].

Agent based models can also by very useful in modelling traffic. Complex interactions between vehicles determine the behaviour of traffic as a whole, which can accurately be represented in an agent-based model (ABM). Behaviour of vehicles in traffic is greatly influenced by factors of the environment in which the vehicles move [16]. For example turns in the road, presence of other drives or extreme weather conditions. Position of vehicles relative to each other is also very important in determining their behaviour.

Lastly, vehicles in traffic are non-homogeneous entities, different drivers may have different driving strategies, and different kinds of vehicles may exhibit different driving behaviour [11]. All these properties of traffic are hard to simulate in CAs or mathematical models making ABMs very suited for research in traffic [1]. For these reasons current project applies agent-based modelling to traffic. We based our model on the widely used and validated intellegent driver model[15]. This model describes distance dependent acceleration and deceleration of cars in traffic using a set of ODEs.

As one can imagine a hybrid multi-level approach is considered the holy grail in traffic modelling, for they can both model complex interactions between vehicles, and integrate these in a larger network modelled by linked PDE's or CA's [5]. However these kind of models are computationally very costly and require resources unavailable for current project. Therefore we choose to focus on modelling behaviour of heterogeneous vehicles at a crossing using different traffic control policies. Using agent-based models to simulate driving behaviour at a sequence of intersections, it has already been shown that extremely cautious driving techniques can result in a chaotic traffic pattern, possibly causing traffic jams [17]. Another research by Doniec et al. 2008 showed that a well validated agent-based model incorporating norm violating and anticipatory behaviour of drivers. This project aims to research the results of driver-driver interactions on traffic flow at different types of intersections.

# 1 Model overview

## 1.1 Purpose

The purpose of our research is to determine the effect of different type of traffic policies on a particular intersection, which is illustrated in figure 1.1. The created model should give us information about which policy yields the highest throughput and thus the lowest amount of waiting time for each car. As mentioned earlier, it is highly advantageous to lower this waiting time for both the environment and personal health [3].

The created model can be used to guide policy makers on different traffic policies and their effect on the throughput of intersections.

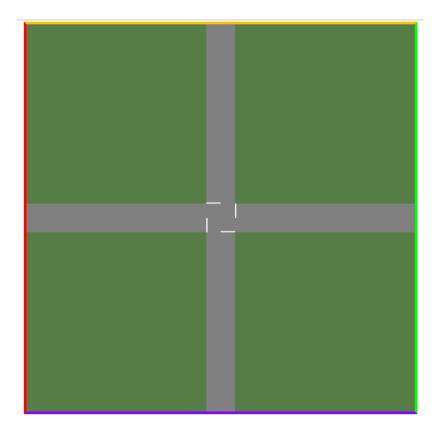


Figure 1.1: Empty environment

### 1.2 Entities, state variables, and scales

#### 1.2.1 Environment

The environment (figure 1.1) consists of two perpendicular roads, that intersect each other in the centre of the environment. The intersection is modelled on a grid of 216 x 216 cells, in which one cell represents a space of 0.5 by 0.5 metre. This means our simulated environment represents a square of  $108 \times 108$  metre. In our environment we have used discrete time in which each time step represents one second.

Furthermore, we have chosen to label the direction parameters in our model by cardinal directions. We have visualized these directions in our environment by colours: east for green, north for yellow, west for red and south for purple. The cars are coloured by their goal direction.

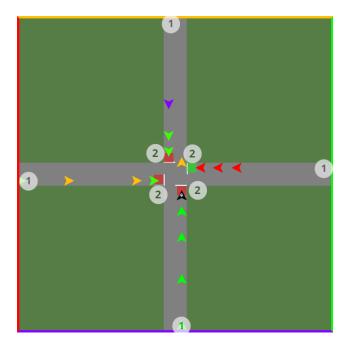


Figure 1.2: Stop light policy in action

#### Road

Each road is bi-directional and consists of two lanes; one lane to go straight ahead and one lane for the opposite direction. Each lane is 8 cells wide, thus making a road 16 cells wide. This boils down to an intersection of  $16 \times 16$  cells.

The beginning of each road has a starting location ((marked by the "1" in figure 1.2)) where the cars are spawned. Each road has its own spawning probability for its cars, which is the probability of a car spawning on the road during a time step.

Additionally, each road contains the probabilities which will determine the goal direction of the cars. This probability of a car coming from direction i, going to direction j is denoted by  $P_i^j$ . We have fixed these probabilities like so:

- $P_{north}^{west} = P_{west}^{south} = P_{east}^{east} = P_{east}^{north} = 0.33$
- $P_{north}^{east} = P_{west}^{north} = P_{south}^{west} = P_{east}^{south} = 0.33$
- $\bullet \ \ P_{north}^{south} = P_{west}^{east} = P_{south}^{north} = P_{east}^{west} = 0.33$
- $P_{north}^{north} = P_{west}^{west} = P_{south}^{south} = P_{east}^{east} = 0.01$

In other words, this means a car has the same probability of turning right (point 1), turning left (point 2) and moving straight ahead (point 3). However, there also is a small chance of a car making a u-turn (point 4).

Finally, each road has a maximum speed. The velocity of the cars will be around this speed limit. They can however be higher due to the variance added to the speed by a Gaussian distribution.

Symbol	Description	Value
$P_i^j$	Probability of car from direction i going to direction j	[0,1]
$F_{bmw}$	Fraction of cars being BMW	[0, 1]
$V_{max}^{vertical}$	The max velocity of the vertical road	[5, 15]
$V_{max}^{horizontal}$	The max velocity of the horizontal road	[5, 15]

Table 1.1: Environment parameters

#### Stop place

All lanes that are directed towards the intersection contain a stop place (marked by the "2" in figure 1.2). The stop place either contains a stop line or a traffic light, based on the current traffic policy of the environment. When the cars approach the stop line they always need to stop, even if the other roads are clear. A green traffic light means the car approaching can proceed without stopping.

#### Car

The agents in our model are cars. All cars have the same measurements, 4 x 8 cells, thus representing a car of 2 by 4 metres.

A car has multiple parameters defining its movement, such as its velocity (its current speed) and an acceleration (determines how fast it can accelerate). Cars also have a deceleration speed, which determines how fast it loses speed, and a minimum distance parameter which is the minimum distance it should keep between itself and the car in front of it. Using the Intelligent Driver Model [8] these parameters together determine the position of a car each time step, keeping in mind that a car can only be positioned on the roads.

A car contains three direction parameters:

- 1. Initial direction, determined by the road on which it spawned.
- 2. Current direction
- 3. Next direction, the direction a car is going to.

Additionally, each car has a Better Move out of my Way factor (BMW factor), which represents how antisocial a driver is. This factor adds in noise to our model, which represents human behaviour not always being rational. The higher the BBMW value, the more antisocial the driver and the more likely he will take precedence on the intersection over others, when no one has clear right of way.

An overview of the various environment parameters in our model can be found in 1.1.

### 1.3 Process overview and scheduling

In our environment time is discretized. For every time step, the velocity of all cars are updated simultaneously. It will slow down when it is approaching the intersection or another car, or speed up when there is space in front of the car while not exceeding the maximum road speed.

If a car is at the intersection, it will check if it has precedence over other cars based on the current traffic policy. The rules may not always be followed when a car has a high BMW factor. On the intersection a car may also change its current direction, based on its goal direction.

Every time step each road may spawn a car based on its car spawn probability.

# 2 Design Concepts

### 2.1 Theoretical and Empirical Background

The different traffic policies we have used concerning our research are restricted to four different traffic rule sets. We have chosen these specific policies since they represent type of intersections that are frequently seen on the western roads.

- 1. The **4-way stop**<sup>1</sup> is a traffic rule set which is derived from the traffic rules typically used to regulate intersections in the United states, which relies on the first-come, first-served principle. It consists of the following rules:
  - a) The car that first reaches its stop line has priority over cars that arrive later at their stop line.
  - b) If two vehicles reach the stop line at the same time, the vehicle on the right is given priority.
  - c) If multiple vehicles reach the stop line at the same time, the drivers informally sign to each other which car can take priority.
- 2. Right priority<sup>2</sup> is a traffic rule set which is seen in the European continent:
  - a) All cars must give priority to cars to their right.
    - Thereafter, if the car is turning right or moving straight ahead it can proceed.
    - If the car instead is going left it must give priority to traffic from the opposite direction (unless they're turning left as well).

<sup>&</sup>lt;sup>1</sup>See e.g. Negotiating Intersections.

<sup>&</sup>lt;sup>2</sup>See e.g. part 4 of Road Rules.

- b) When the intersection is congested the drivers circulate priority to the right one vehicle.
- 3. **Dumb** traffic lights. Another way to regulate intersections is by using traffic lights. At this type of intersection, car drivers must obey the traffic lights, meaning cars can only cross the intersection if the traffic lights allow them to by sending out a green light. Each road has its own traffic light, that are all sending out a green light one after another. In our model, we have modelled this scenario by giving the four different traffic lights an equal amount of "green time".
- 4. **Smart** traffic lights. This rule set is an addition to the previous scenario, but now we take into account the density of each road. We connect the density to a probability of a road getting a green light. Thus giving more dense roads a higher amount of green time compared to less dense roads. In real life this rule set is very common where the density can be measured by a simple button or by detection loops.

#### 2.1.1 Agents

The goal of the cars is to get from point a to point b. While doing so it must move. This happens by determining the position of a car by following the Intelligent Driver Model (IDM). The IDM ensures the cars have realistic movement, containing acceleration and deceleration. Each car is adjusting its speed to the car in front of it, which prevents collisions. For example, a car will slow down if the car in front of it has a lower speed.

The intelligent driver model is implemented using the following ordinary differential equations:

$$\dot{x}_{\alpha} = \frac{\mathrm{d}x_{\alpha}}{\mathrm{d}t} = v_{\alpha}$$

$$\dot{v}_{\alpha} = \frac{\mathrm{d}v_{\alpha}}{\mathrm{d}t} = a \left( 1 - \left( \frac{v_{\alpha}}{v_{0}} \right)^{\delta} - \left( \frac{s^{*}(v_{\alpha}, \Delta v_{\alpha})}{s_{\alpha}} \right)^{2} \right)$$
with  $s^{*}(v_{\alpha}, \Delta v_{\alpha}) = s_{0} + v_{\alpha}T + \frac{v_{\alpha}\Delta v_{\alpha}}{2\sqrt{a}b}$ 

The different symbols, their description and the values we have assigned to it can be found in table 2.1.

Symbol	Description	Value
$v_0$	Desired velocity	[5, 25]
$v_A$	Velocity of car $\alpha$ at time $t$	[0,25]
$v_0$	Minimum distance between two cars	2
Τ	Minimum possible time between two cars	1.5
$\alpha$	Maximum vehicle acceleration	[0.6, 2.0]
b	Breaking deceleration	[1, 3]
$x_{\alpha}$	Postion of car $\alpha$ at time $t$	[0, 216]
$\delta$	Acceleration component	4

Table 2.1: Intelligent Driver Model parameter description

To make sure the equations fit our model some small adaptations have been made. These adaptations have been made in the  $\dot{v}_{\alpha}$  equation, which can be divided into two equations.

First we have  $a(1-(\frac{v_{\alpha}}{v_0})^{\delta})$ , which makes sure the velocity of the car changes in order to get it to the desired velocity. Here the desired velocity could be the speed limit of the road. However to make it more realistic the cars will desire different velocities around the speed limit.

Secondly you have the part of the equation that takes care of interactions with other objects on the road, namely:

$$-a\left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2$$

In the original equations these objects are the other vehicles on the road. However in this model the stop sign and traffic lights are also counted as vehicles in this part of the equation, with the change that they always have a velocity of 0. This way cars also decelerate naturally for other objects than cars.

The last adaptation that was made has to do with the fact that the is designed for continuous time. Thus we had to convert the results of the ODE into discrete values by rounding numbers to the nearest integer.

The Intelligent Driver Model is chosen, because it is a state of the art model used in traffic modelling. Thus it is used by many researchers and referenced in many scientific papers.

# 2.2 Individual Decision-Making

Every time step the cars make a decision to adjust their position based on the earlier described IDM. The other part of the decision making takes places at the moment that a car is at the stop place. At this moment a car has to decide whether it has to wait or it can make its turn. This decision is influenced by the the current traffic policy of the environment. For the decision making there is a distinction between the stop line and traffic light rulesets. For the traffic light policies, a car always obeys the traffic light, meaning that it will only cross the intersection if the light is green.

The decision making for the stop line policies is more complex. Firstly, a car has to determine whether it can cross the intersection based on either the 4-way stop principle or the right priority ruleset. However, the cars also take into account whether the parts of the intersection they will cross during their turn are free. Thus meaning multiple cars can cross the intersection as long as they do not overlap.

### 2.3 Individual Sensing

For the different types of the traffic policies we have researched the cars must sense different variables.

Each car can sense the frame of the roads and their lanes, thus keeping the car aligned to the centre of the right lane. Other state variables that each car must sense are the presence of a car directly in front of it as well as the stop line or the traffic light at the end of its road.

The main part of sensing takes place when a car is standing still at the stop line or traffic light:

- For the 4-way stop and right priority policies the car that is standing still at the stop line must also know if any other cars are present at the other stop lines, if any.
- For the traffic light policies the car must know the state of the traffic light, being either green (go) or red (stop).

Globally, a car making a turn also needs to sense the free parts of the intersection.

## 2.4 Heterogeneity

While cars have the exact same shape, their velocity, maximum acceleration, deceleration and BMW factor vary per car. Besides this, cars have different knowledge of the cars surrounding them. While driving, a car will only see the car directly in front of it. If a car is standing still at a stop line, it has knowledge over other cars that are also in front of the intersection and pending to make a turn.

The goal direction of a car is also heterogeneous, and is determined by the probability described in section 1.2.1.

# 2.5 Stochasticity

We have added stochasticity into multiple parameters of our model, which we will cover in this section.

Each road has a spawn probability, which represents the probability that a car will spawn on the road each timestep. The road also contains a goal direction probability which denotes the probability of a car going into a certain direction.

The desired velocity of a car, maximum acceleration and the deceleration speed are drawn from a normal distribution, while the minimum value of those is bounded by a positive constant to ensure that the agent will be able to move forward. This lower bound is justified since otherwise the model would be filled with cars which are standing still. The BMW factor is sampled from a beta distribution with  $\alpha=2$  and  $\beta=5$ , so that the mean of the BMW factor is around 0.3. We have chosen a beta distribution, because it is bounded on the interval [0,1], which are the desired bounds of the BMW factor.

Finally there is stochasticity in the 'smart' traffic lights. First, the number of cars on each road are counted, which we will call  $w_1$ ,  $w_2$ ,  $w_1$  and  $w_4$ . Next we take the sum  $s := w_1 + w_2 + w_3 + w_4$  and find four intervals in [0, s], i.e.  $[0, w_1)$ ,  $[w_1, w_1 + w_2)$ ,  $[w_1+w_2, w_1+w_2+w_3)$  and  $[w_1+w_2+w_3, s]$ . Lastly we generate a uniform random number p between 0 and s and find the interval in which p lies. As the roads are represented by the intervals, we now find which road get green light. This way the road with the most cars is more likely to get a green light.

#### 2.6 Observation

The model collects different kinds of data, which are the throughput, the mean crossover time and the average speed. The throughput is measured by the amount of cars that reached their destination divided by the amount of time steps. The mean crossover time is the average time it takes a car to reach its destination. The average speed is equal to the mean velocity of the active agents. All the data is collected during the execution of the model.

# 3 Details

## 3.1 Implementation Details

The model was created using the Mesa library of Python 3. Mesa is a framework for agent-based modelling. The visualisation is created using Mesa's visualisation module. The code of the model can be found in this GitHub respository (link).

#### 3.2 Initialisation

The model is always initialised as an empty intersection. The values of variables are arbitrarily chosen and may be changed by the user of the model. Thus the rule set of the intersection and the parameters may change during runs, but only if chosen to do so by the user.

### 3.3 Input data

The model does not use input data from external sources to represent processes that change over time.

# 4 Results

### 4.1 Comparing intersections

To compare the three intersection types we analysed the traffic flow, average crossover time and average speed of 3000 simulation runs. To sample input parameters we used Saltelli sampling over all parameters (as described in table 1.1) except BMW factor. For every output variable a non-parametric ANOVA (Kruskal-Wallis) was executed. When significant differences were found we performed a Nemenyi post-hoc to locate the differences. Distributions are shown in figure 4.1. Traffic flow, crossover time and average speed distributions of the 4-way and right-priority intersection types where all very similar (p > .05). For traffic flow and crossover time there was a clear distinction between intersections with smart traffic lights, dumb traffic lights and the intersections without a traffic lights (p < .001). Figure 4.1 shows a heatmap of all p-values of differences between distributions.

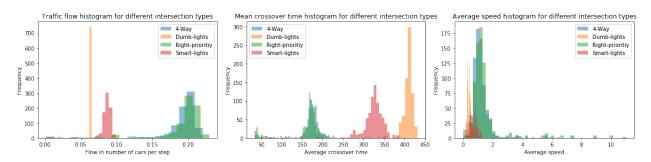
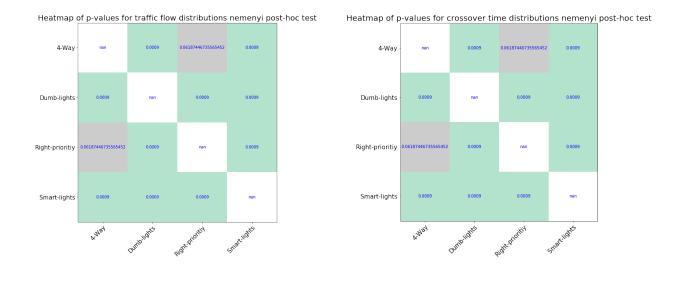


Figure 4.1: Histograms of distributions of different output variables for different intersection types.



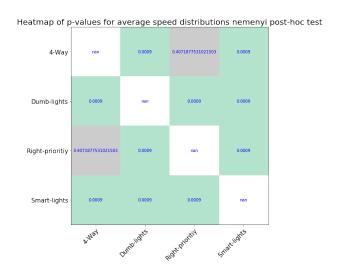


Figure 4.2: Heat maps visualising the p-values of resulting form Nemenyi post-hoc tests.

#### 4.2 Validation

To validate our basic implementation of the intelligent driver model we adapted the model so there were only cars driving from north to south and vice versa. Speed and spawning probability were varied over 2800 parameter combinations. Spawning probability was varied from 0 to 1 in 100 steps and speed was varied from 3 to 30 in 28 discrete steps. We ran the simulations for either 100 steps(flow vs density) or 1000 steps. Subsequently we plotted the relations between flow and density, speed and density and speed and flow. We compared these relations with graphs adopted from Immers and Logghe, 2002 [7]. Due to difference in environmental parameters, of the data that resulted in

the graphs from Immers and Logghe and our simulations, we choose not to quantify the differences.

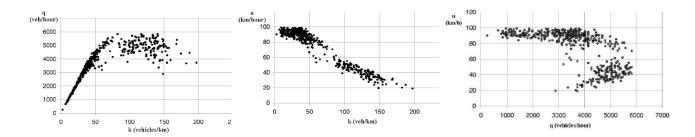


Figure 4.3: Graphs retrieved from Immers and Logghe, 2002 showing typical relations between different measures of traffic data

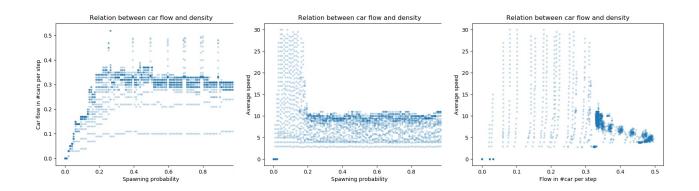


Figure 4.4: Results of 2800 simulations of straight driving cars. Input parameters were varied; speed between 3 and 30 (grid places per step) and spawning probability from 0 to 1.

## 4.3 Sensitivity analysis

A Sobol method for global sensitivity analysis was performed to quantify effect of different input parameters. We use Saltelli sampling to generate a parameter space consisting of 5000 parameter sets. We only performed a global sensitivity analysis on the effect of input parameters on traffic flow. We choose not to perform a analysis on the effect on average speed and mean crossover time for two reasons: time it takes to run the simulation for 5000 steps and more importantly the possible correlation between output parameters. Every simulation was run for 1000 steps. First and total order sensitivity indexes that had a significant effect on output were intersection type and spawning

probability. The effect of intersection type was also significantly larger than the effect of spawning probability. No significant second order indices were found.

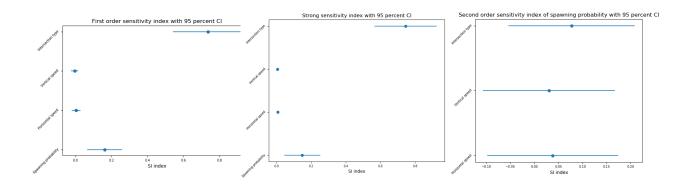


Figure 4.5: Results of 2800 simulations of straight driving cars. Input parameters were varied; speed between 3 and 30 (grid places per step) and spawning probability from 0 to 1.

# 5 Discussion

The experiments with the different intersection rule sets have shown that the traffic lights have a negative influence on all our output variables. This can partly be explained by the fact that our model does not take safety into account. Although traffic lights do not prevent cars to enter the intersection while they are not supposed to, they might increase the safety of the intersection. The cars currently perfectly follow the rule set of the intersection, were it not that cars with high BMW value take priority while they should not. In real life it is expected that when there is a high concentration of cars with high BMW value that the intersection without traffic lights become a mess and the output variables become worse. This is something we also would like to factor within the model.

Also, we created our model using both discrete time and discrete space. The latter proved to have been the wrong choice since later in this project we decided to base the positioning of the car on the intelligent Driver Model, which is based on a continuous space model. Therefore we had to convert the result of the IDE into discrete space values which has a negative effect on the accuracy of the positioning. Another limiting factor of our simulations was the scale. We modelled an intersection with single lane roads of 56 meters, and the maximum of cars waiting in front of the intersection was 10. Therefore interactions between agents and environment might not have emerged in our

simulations because of the limited time agents where simulated for. This also contributed to the difference between our model behaviour and traffic flow data we adopted form Immers and Logghe. In our simulations car densities reach a maximum quite soon, which influences the behaviour of the system as ha whole. To better validate our model in the future, we would like to use data from a crossway or road better comparable with the environment of our simulations.

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