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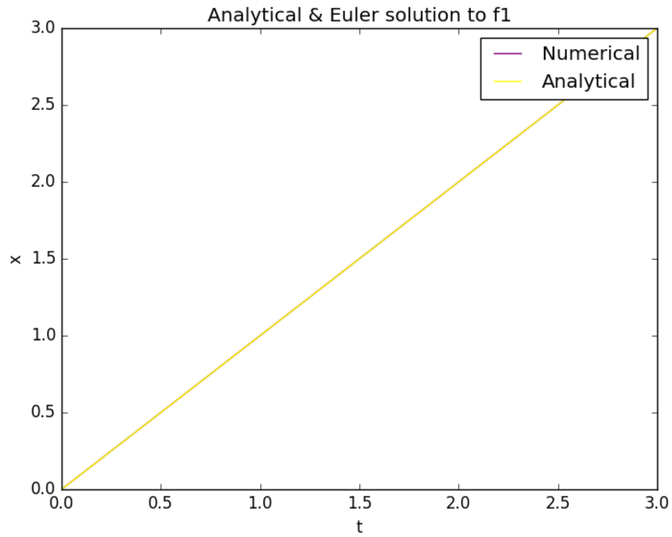
1. (a) Separable: a differential equation where the variables can be separated (x's on one side, y's on the other)
   (b) Linear/non-linear: a linear equation is an equation where the differentials are linear. Non-linear is an equation where the differentials are in non-linear (e.g with powers)
   (c) First-order/Second order: this refers to the order of the differentials. So an equation with $\frac{dx}{dt}$ is first order, but $\frac{d^2x}{dt^2}$ is second order

2. (a) Explicit/implicit solutions: Explicit solution is when the function is defined as a function over some vars: $x(t) = 2t$, implicit is when you can distract the the solution. E.g. for $x^2 + y(x)^2 = r^2$
   (b) Fixed point(s): a fixed point is a value for x such that x does not change anymore.
   (c) Locally/globally stable fixed points; unstable fixed points:
      i. locally stable: iff x(t) = F (where 6= 0) moves towards F whenever it is (very) close to F already (so 'small enough' but still non-zero).
      ii. It is called 'globally stable' if x(t) will always converge to F no matter the initial condition, so for any value of .
      iii. A fixed point is called 'unstable' if x(t) moves away from the fixed point, no matter how close x(t) is to F – with the single exception of exactly x(t) = F of course.

3. *See euler.py*
   (a) $\frac{dx}{dt} = 1$, with $x(0) = 0$
   This can be solved via:

$$
\begin{aligned}
dx &= dt \cdot 1 \\
\int dx &= \int dt \\
x &= t + C \\
\\
0 &= 0 + C \\
C &= 0 \\
\\
x &= t
\end{aligned} \tag{1}
$$

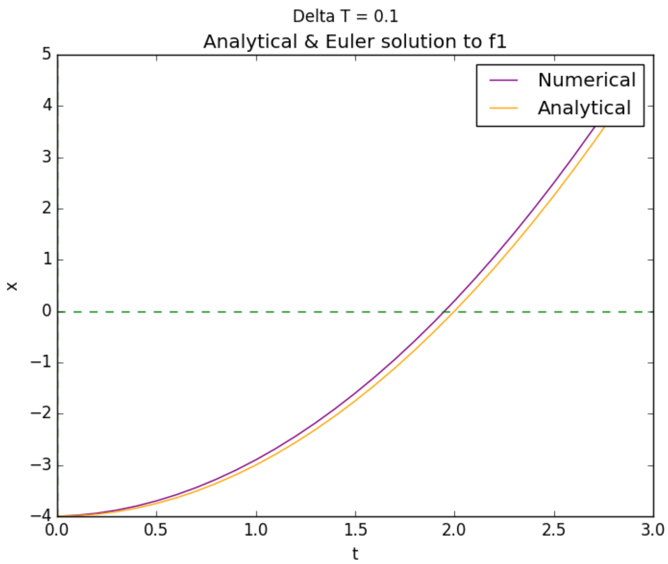
   Which is linear line, therefore the Δt does not matter. The analytical and numerical solution will overlap:



(b) $\frac{dx}{dt} = 2t$, with $x(0) = -4$
Can be solved analytical:

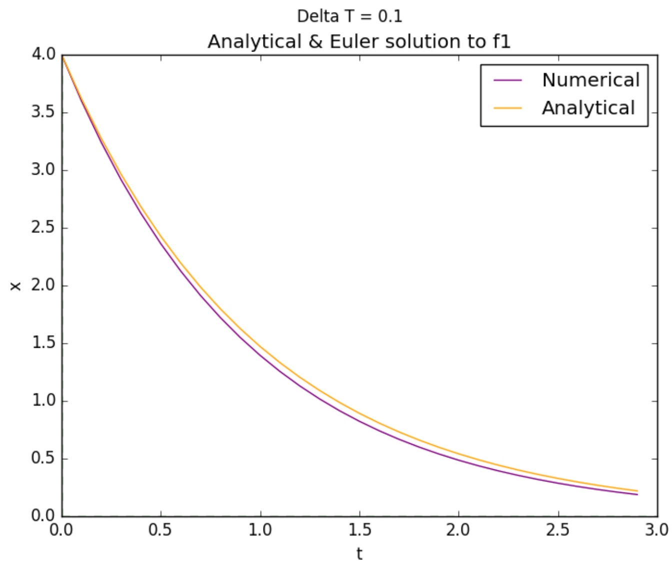
$$\begin{aligned} dx &= 2t \cdot dt \\ \int dx &= \int 2t \cdot dt \\ x &= t^2 + C \\ \\ -4 &= 0^2 + C \\ C &= -4 \\ \\ x &= t^2 - 4 \end{aligned} \tag{2}$$

Because this function has an exponent, we choose $\Delta t = 0.1$:



(c) $\frac{dx}{dt} = x$, with $x(0) = 4$
Can be solved analytical via:

$$\begin{aligned}
\frac{dx}{-x} &= dt \\
\int \frac{dx}{-x} &= \int dt \\
-1 \cdot ln(x) &= t + C \\
x &= e^{-t-C} \\
&= e^{-t} \cdot C \\
\\
4 &= e^{-0} \cdot C \\
C &= 4 \\
\\
x &= e^{-t} \cdot 4
\end{aligned} \tag{3}$$



4. (a) G is the rate at which the molecules are generated. -K is the rate at which the molecules should be decreased.

   (b)

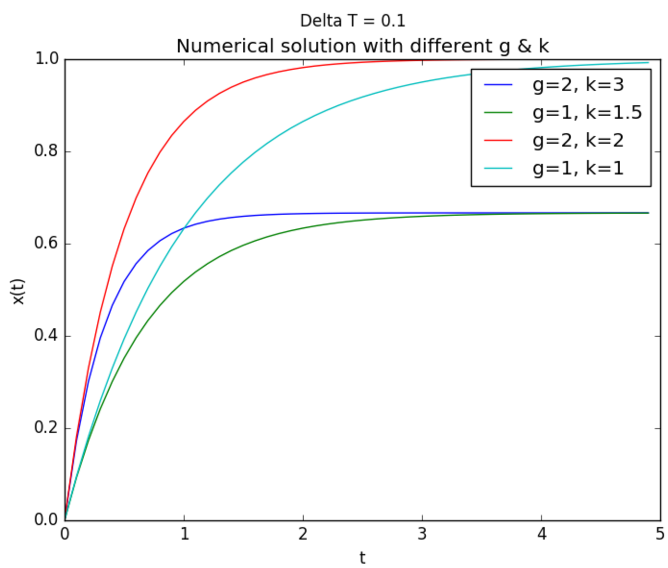
$$
\begin{aligned}
\frac{dx}{dt} &= g - k \cdot x \\
\frac{dx}{g - k \cdot x} &= dt \\
\int \frac{dx}{g - k \cdot x} &= \int dt \\
-\frac{1}{k} \cdot ln(g - k \cdot x) &= t + C \\
ln(g - k \cdot x) &= -k(t + C) \\
g - k \cdot x &= e^{-k \cdot t - k \cdot C} \\
&= e^{-k \cdot t} \cdot C \\
x &= \frac{e^{-kt} \cdot C - g}{-k} \\
&= -\frac{1}{k} \cdot C \cdot e^{-kt} + \frac{g}{k} \\
x &= C \cdot e^{-kt} + \frac{g}{k}
\end{aligned}
\tag{4}
$$

   (c) C is a number which adjusts the integral to be correct, so the derivative of this integral is the original formula.

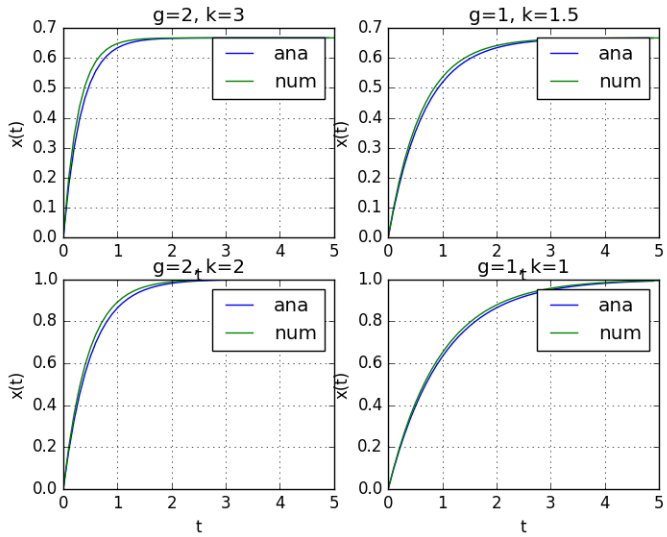
(d)

$$
\begin{aligned}
0 &= C \cdot e^{-k\cdot 0} + \frac{g}{k} \\
&= C + \frac{g}{k} \\
C &= -\frac{g}{k}
\end{aligned} \tag{5}
$$

(e) G seems to determine the height (x(t)) of the fixed part. K seems to influence the amount of t it takes to reach a stable state.



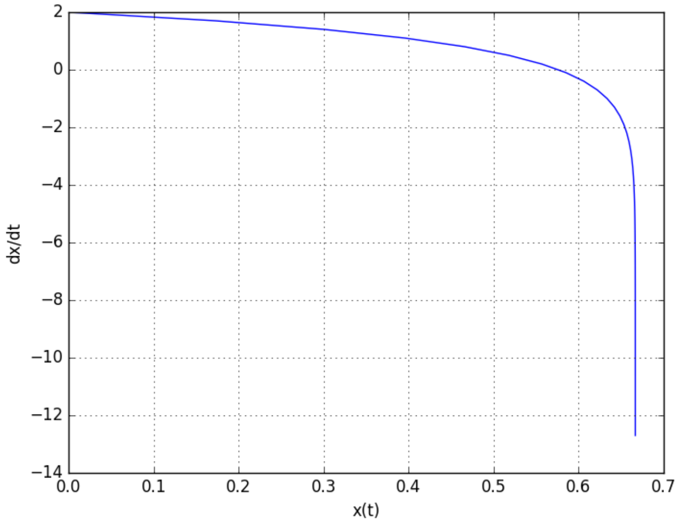
(f) All $\Delta t = 0.01$



(g) The fixed(s) point can be found by equating the first derivative to zero:

$$\begin{aligned} \frac{dx}{dt} &= g - k \cdot x = 0 \\ -k \cdot x &= -g \\ x &= \frac{g}{k} \end{aligned} \tag{6}$$

(h) Seems globally stable since it is converging to F



(i)

(j) In real life there are much more variables which influence the production, so this is a mean field approximation model since it tries to recreate the production but leaves out a few variables.

5. (a) Since we are only looking at reproduction and death rates, the growth of the population can be captured in the given formula where rx is the reproduction of the population and $k * x$ is the deaths in the population.

(b)

$$\begin{aligned}
\frac{dx}{dt} &= r \cdot x - k \cdot x \\
&= x(r-k) \\
\int \frac{1}{x(r-k)} \cdot dx &= \int dt \\
\frac{1}{r-k} \cdot ln(x) &= t + C \\
ln(x) &= (r-k) \cdot (t+C) \\
&= rt - kt + rC - kC \\
x &= e^{t(r-k)} \cdot C
\end{aligned} \tag{7}$$

(c) Fixed point are for $x = 0$ and $r - k = 0$

(d)

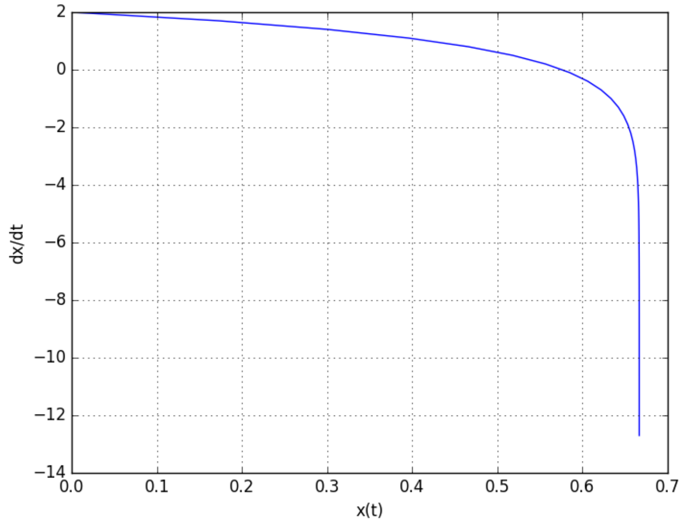
(e)

(f) When x increases linear, the population will increase quadratic.

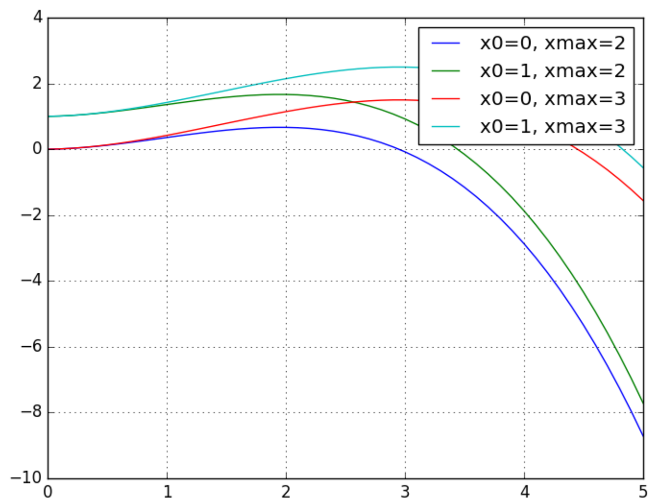
(g)

$$\begin{aligned}
\frac{dx}{dt} &= r \cdot x \cdot x - k \cdot x \\
&= x(rx - k) \\
\frac{1}{x(rx-k)} \cdot dx &= dt \\
\int \frac{1}{x(rx-k)} \cdot dx &= \int dt \\
\frac{1}{k} \cdot ln(\frac{rx-k}{x}) &= t + C \\
ln(\frac{rx-k}{x}) &= k(t + C) \\
\frac{rx-k}{x} &= e^{kt} \cdot C \\
rx - k &= C \cdot e^{kt} + k \\
rx - k - C \cdot e^{kt} &= k \\
x(r - C \cdot e^{kt}) &= k \\
x &= \frac{k}{r - C \cdot e^{kt}} \\
\\
x(0) &= \frac{k}{r - C \cdot 1} = x_0 \\
x_0(r - C) &= k \\
x_0 \cdot r - x_0 \cdot C &= k \\
-x_0 \cdot C &= k - x_0 \cdot r \\
C &= -\frac{k}{x_0} + r \\
\\
x &= \frac{k}{r - (-\frac{k}{x_0} + r) \cdot e^{kt}}
\end{aligned} \tag{8}$$

(h) .



(i) x_{max} will be the maximum the function can reach



(j)

(k)

$$
\begin{aligned}
0 &= x(1 - x/x_{max}) \\
x &= 0 \\
\\
1 - x &= 0 \\
x &= 1
\end{aligned} \tag{9}
$$

Fixed points for $x = 0$ and $x = 1$.
Second derivative proves the fixed points are both stable

$$
\begin{aligned}
\frac{d}{dx}(x(1 - x/x_{max}) &= 1 \cdot (1 - x/x_{max}) + (-\frac{1}{x_{max}})x \\
&= 1 - \frac{2x}{x_{max}}
\end{aligned} \tag{10}
$$

(l)

(m) $\frac{dx}{dt} = x(1 - x/x_{max}) - r$ The rabbits die at rate r, so $r \cdot x$ is the this rate applied to the population. Though only a r will be added since this is the derivative.

(n)

(o)