Exp-4

Laws of Radiation

Reiley-Jeans law

density of states is defined in phase space

① ν ; frequency

② $V_x V_p / h^3 \rightarrow \dfrac{8\pi \nu^2 d\nu}{c^3} V_0$

↑ ↳ vol. in momentum space.

volume in position space

↑
standing waves in a shapeless body.

③ Energy $\frac{1}{2} kT \times 2$

(4) total energy $KT \times G(\nu)\, d\nu = N(\nu)$

avg. " $\bar{E}_\nu \times G(\nu)\, d\nu = dU(\nu)$

energy density

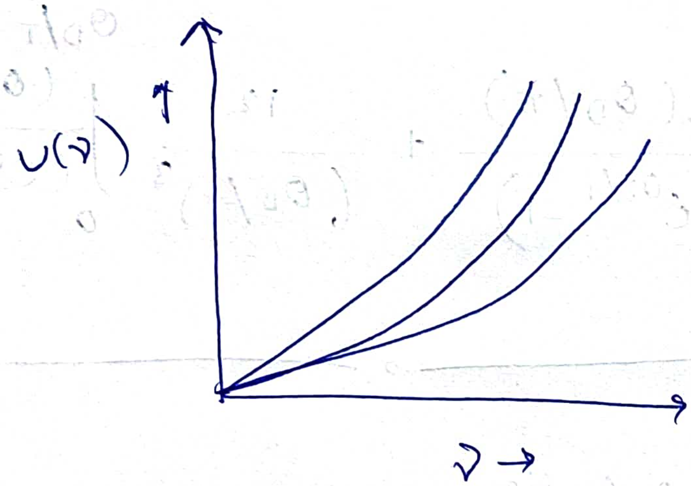
$$\frac{\bar{E}_\nu \times G(\nu)\, d\nu}{V_0} = \frac{dU(\nu)}{V_0}$$

$$= \frac{8\pi\nu^2 d\nu}{c^3} \cdot KT$$

Rayleigh - Jeans

$$U(\nu)\, d\nu = \frac{8\pi K_B}{c^3} T \nu^2 d\nu$$

ν range $10^{10} - 10^{30}$



Planck

$$U(\nu) = \frac{h\nu}{(e^{h\nu/KT} - 1)} \frac{8\pi\nu^2 d\nu}{c^3} = \frac{\bar{E}_\nu G(\nu) d\nu}{V_0}$$

To Do →

(1) Plot density of states for R-J and Planck

$$G(\nu)\, d\nu = \frac{8\pi\nu^2 d\nu}{c^3} V_0$$

(1') l_0 is length

$$G(\nu)\, d\nu = \frac{8\pi\nu^2 d\nu}{c^3} l_0^3$$

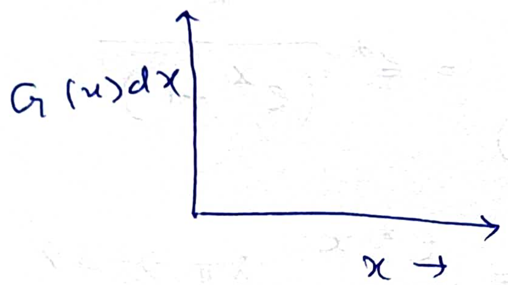
(ii) $\nu_0 \lambda_0 = c \Rightarrow \nu_0 = \frac{c}{\lambda_0} = \frac{c}{2l_0}$, $\lambda_0 = 2l_0$ (say)

$$G(\nu)\,d\nu = \pi \frac{\nu^2\, d\nu}{\nu_0^3} = \pi \left(\frac{\nu}{\nu_0}\right)^2 d\left(\frac{\nu}{\nu_0}\right)$$

$$G(x) = \pi x^2$$

$$G(x)\,dx = \pi x^2\, dx$$

$$x = \frac{\nu}{\nu_0}$$



(2) $$u(\nu)\,d\nu = kT \frac{8\pi\, \nu^2 d\nu}{c^3} = e^* \frac{8\pi\, \nu^2 d\nu}{c^3}$$

$$e^* = kT$$

$$e^* = p^* c \Rightarrow p^* = \frac{e^*}{c}$$

$$\Rightarrow l^* = h/p^*$$

$$= hc/\epsilon^*$$

$$x = \frac{e}{e^*} = \frac{h\nu}{e^*} \qquad \nu^* = e^*/h$$

$$u(\nu)\,d\nu = \frac{8\pi}{c^3} e^* \frac{1}{h^3} e^2\, de$$

$$= \frac{8\pi}{h^3 c^3} \epsilon^* \epsilon^{*3} x^2\, dx$$

$$u(\nu)\,d\nu = 8\pi \frac{\epsilon^*}{l^{*3}} x^2\, dx$$

$$G(x)\,dx = \pi x^2\, dx = \pi f_{RJ}(x)\,dx$$

input ν ; calculate $x = h\nu / \epsilon^*$

calculate f_{RJ} $\quad x = 0$ to 12

" $U(x)$ $\quad T = 1000K, 1200K$, 1800K

transform $U(x) \rightarrow U(\nu)$

Planck

$$\bar{\epsilon} = \frac{\epsilon}{(e^{\epsilon/\epsilon^*} - 1)} = \epsilon^* \frac{x}{(e^x - 1)}$$

$$U(\nu) d\nu = \frac{\epsilon^* x}{(e^x - 1)} \frac{8\pi \nu^2 d\nu}{c^3}$$

$$= \frac{\epsilon^* x}{(e^x - 1)} \frac{8\pi \epsilon^2 d\epsilon}{h^3 c^3}$$

$$= \left(\frac{\epsilon^* x}{e^x - 1}\right) \frac{8\pi x^2 dx}{h^3 c^3} \epsilon^{*3}$$

$$= \frac{8\pi \epsilon^*}{l^*} \frac{x^3 dx}{(e^x - 1)}$$

$$= \frac{8\pi \epsilon^*}{l^*} f_p(x)$$

$$f_{RJ}(x) = x^2$$

$$f_p(x) = \frac{x^3}{(e^x - 1)}$$