1)(b) Let,

m_e = mass of electron

$-e$ = charge of electron

r_n = radius of nth Bohr orbit

$+e$ = charge on nucleus

v_n = linear velocity of electron in nth orbit

Z = number of electrons in an atom

n = principal quantum number

Now,

from Bohr's first postulates,

$$m_e r_n v_n = \frac{nh}{2\pi}$$

$$\Rightarrow m_e^2 r_n^2 v_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\therefore v_n^2 = \frac{n^2 h^2}{4\pi^2 m_e^2 r_n^2} \longrightarrow (1)$$

and, Coloumb's force F_e = Centripetal force F_{cp}

$$\therefore \frac{Ze^2}{4\pi\epsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}$$

$$\therefore v_n^2 = \frac{Ze^2}{4\pi\epsilon_0 r_n m_e} \longrightarrow (2)$$

From equations (1) and (2),

$$\frac{n^2 h^2}{4\pi^2 m_e^2 r_n^2} = \frac{Ze^2}{4\pi\epsilon_0 r_n m_e}$$

$$\therefore r_n = \frac{n^2 h^2 \epsilon_0}{\pi m_e Z e^2}$$

$$\therefore r_n = \frac{\epsilon_0 h^2}{\pi m_e Z e^2} n^2 \rightarrow (3)$$

This is the required expression for n^{th} orbit radius

Again the energy E of an electron in an orbit is the sum of kinetic and potential energies.
Using equation of centripetal force, we have,

$$m v^2 = \frac{Z e^2}{4 \pi \epsilon_0 r}$$

The kinetic energy of the electron is,

$$KE = \frac{1}{2} m v^2 = \frac{Z e^2}{8 \pi \epsilon_0 r} \rightarrow (4)$$

On substituting for r on equation (4) we get kinetic energy of the electron in the n^{th} orbit,

$$KE = \frac{m Z^2 e^4}{8 \epsilon_0^2 h^2} \left(\frac{1}{n^2}\right)$$

In terms of Rydberg constant R, its simplified form is,

$$KE = \frac{Rhc}{n^2}; \left[R = \frac{m e^4}{8 \epsilon_0^2 c h^3}\right]$$

The potential energy of the electron in an orbit of radius r due to the electrostatic attraction by the nucleus is given by,

$$PE = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(-e)}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Z e^2}{r}$$

In terms of Rydberg constant R, it's simplified form is,

$$PE = \frac{2Rhc}{n^2}$$

The total energy of electron is therefore,

$$E = KE + PE$$

$$= \frac{Ze^2}{8\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\epsilon_0 r}$$

$$= -\frac{Rhc}{n^2}$$

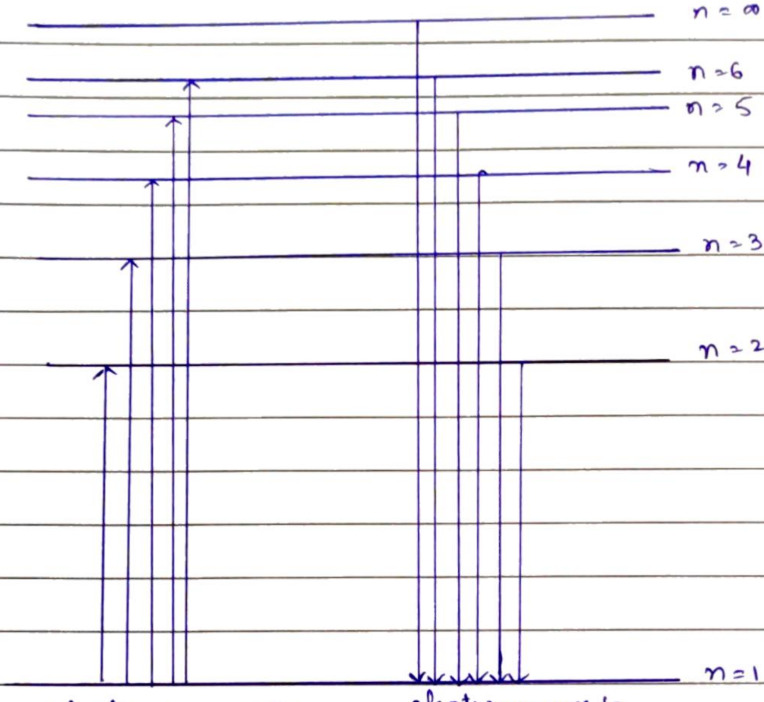
On substituting from equation (3) we have,

$$E = \frac{mZ^2e^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

where $n = 1, 2, 3 \ldots$

and for $Z = 1$ we have

$$E = \frac{me^4}{32\cancel{8}\epsilon_0^2\pi^2\hbar^2}\left(\frac{1}{n^2}\right)$$



we know the radii is given by the expression

$$r = \frac{n^2}{Z} a_0 \quad \text{where } a_0 \rightarrow \text{Bohr radius} = 5.292 \times 10^{-11}$$

for hydrogen Z = 1

$\therefore$ r to be equal to 1 mm

$$1 \times 10^{-3} = n^2 (5.292 \times 10^{-11})$$

$$\Rightarrow n^2 = \frac{10^{-3}}{5.292 \times 10^{-11}}$$

$$= 1.8 \times 10^{8}$$

$$= 2 \times 10^{8}$$

$$\therefore n \approx 1.4 \times 10^{4}$$

(d) The radius of the n^{th} Bohr orbit is,

$$r = \frac{\epsilon_0 h^2 n^2}{\pi m e^2} \quad \text{—(i)}$$

where ϵ_0 is the ~~parti~~ permitivity of free space, h is the Planck's constant, n is the principal quantum number, m is the mass of the electron and e is the charge of the electron.

The linear speed of the electron in this orbit is

$$v = \frac{e^2}{2\epsilon_0 n h} \quad \text{—— (ii)}$$

Since the angular speed $\omega = \frac{v}{r}$ then from eq (i) and (ii) the angular speed of the electron in the n^{th} Bohr orbit is,

$$\omega = \frac{v}{r} = \frac{e^2}{2\epsilon_0 n h} \cdot \frac{\pi m e^2}{\epsilon_0 h^2 n^2}$$

$$= \frac{\pi m e^4}{2\epsilon_0^2 h^3 n^3} \quad \text{—— (iii)}$$

From equation (iii) the frequency of ~~rot~~ revolution of electron,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \times \frac{\pi m e^4}{2\epsilon_0^2 h^3 n^2}$$

$$= \frac{m e^4}{4\epsilon_0^2 h^3 n^3}$$

$$\Rightarrow f_{orb} = \frac{m e^4}{32\pi^3 \epsilon_0^2 \hbar^3}\left(\frac{1}{n^3}\right)$$

(e) We have,

n^{th} energy level,

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right)$$

and $(n-1)^{th}$ energy level

$$E_{(n-1)} = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{1}{(n-1)^2}\right)$$

Now, the energy emitted by electron when making a transition from n^{th} to $(n-1)^{th}$ orbit is,

$$\Delta E = E_n - E_{(n-1)}$$

$$= \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$$= \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left\{\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right\}$$

$$= \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{2n-1}{n^2(n-1)^2}\right)$$

Now we know,

$$E = h\nu$$

$$\Rightarrow \nu = \frac{E}{h} \quad \text{and} \quad h = \hbar 2\pi$$

$\therefore$ frequency of radiation

$$f_{qn} = f_{n\to n-1} = \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{2n-1}{n^2(n-1)^2}\right)\frac{1}{\hbar 2\pi}$$

$$f_{qn} = \frac{me^4}{64\pi^3\epsilon_0^2\hbar^3}\left\{\frac{2n-1}{n^2(n-1)^2}\right\}$$

Now for very large n

$$\frac{2n-1}{n^2(n-1)^2}$$

$$= \frac{2-\frac{1}{n}}{n(n-1)^2} \quad \text{(deviding both sides by } n\text{)}$$

$$= \frac{2-\frac{1}{n}}{n^3\left(1-\frac{1}{n}\right)^2}$$

Now for very large n

$$\lim_{n\to\infty} \frac{2-\frac{1}{n}}{n^3\left(1-\frac{1}{n}\right)^2}$$

$$= \frac{2}{n^3} \quad \left[\text{as } \frac{1}{n} \to 0\right].$$

$$\therefore f_{q_n} = \frac{me^4}{32\pi^3\epsilon_0^2 h^3} \frac{1}{n^3},$$

which is equal to f_{eln}.