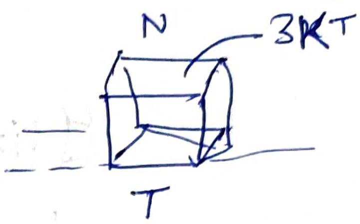
Specific Heat of Solid

Dulong and Petit's Law ↳

Vibrational Energy associated with an atom = 3KT

for N atom



$$E = 3NKT$$

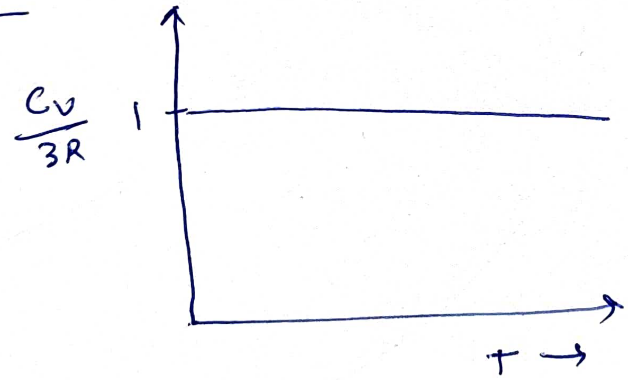
$$\Delta E = 3NK\,\Delta T$$

Specific heat $C_V = \left.\frac{\Delta E}{\Delta T}\right|_V = 3NK$

$$\therefore C_V = 3N_A K = 3R$$

↑ gas constant

~~plot~~ plot I



part b:- Planck's Law,

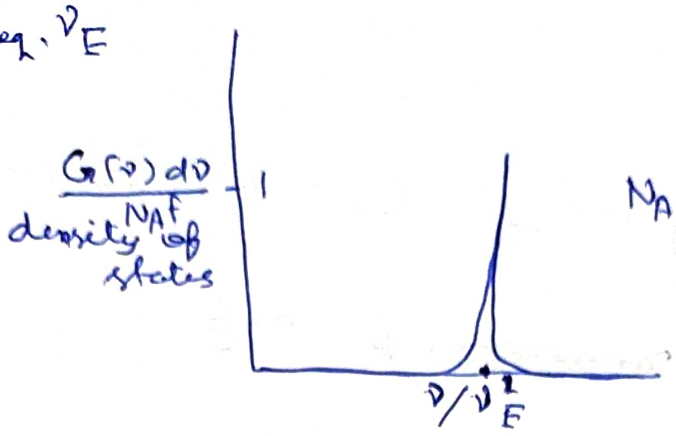
avg energy $E_\nu = h\nu/[e^{h\nu/KT} - 1]$

each atom has energy $h\nu$

Role of temp^r: if $T >>$ $E_\nu = h\nu/[1 + h\nu/KT - 1]$

$= KT$

Einstein freq. ν_E



* At a given freq. available energy

$$dE_\nu = G(\nu)\, d\nu \times \epsilon_\nu$$

$$E = \int_\nu dE_\nu = \int_\nu G(\nu)\, \epsilon_\nu \, d\nu$$

$$= \int_\nu N_A f \, \delta(\nu - \nu_E)\, \epsilon_\nu \, d\nu$$

$$= 3 N_A \frac{h\nu_E}{\left[e^{h\nu_E/KT} - 1\right]} \quad \text{(total energy)}$$

Now, specific heat,

$$y \quad C_V = \left(\frac{dE}{dT}\right)_V$$

$$\Rightarrow \frac{C_v}{3R} = \left(\frac{h\nu_E}{KT}\right)^2 \frac{e^{h\nu_E/KT}}{\left(e^{h\nu_E/KT} - 1\right)^2} \quad \text{—(1)}$$

Einstein temp^o. $\theta_E = \frac{h\nu_E}{K}$

take $x = T/\theta_E$

$$(1) \Rightarrow y = \left(\frac{1}{x}\right)^2 \frac{e^{1/x}}{\left(e^{1/x} - 1\right)^2}$$

Debye Law :-

phonons

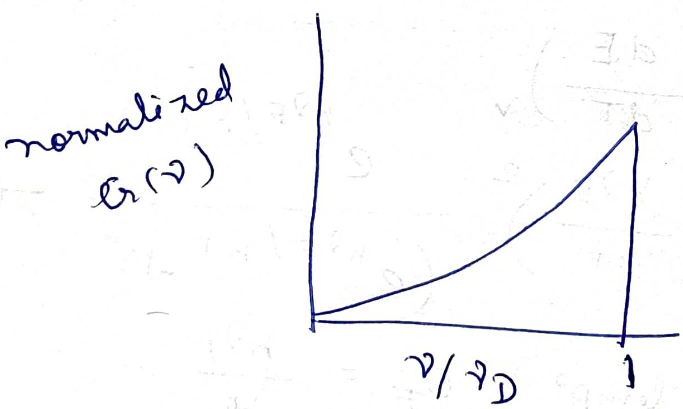
avg energy, $\epsilon_\nu = h\nu / [e^{h\nu/kT} - 1]$, BE statistics

momentum, $p = h\frac{\nu}{c}$

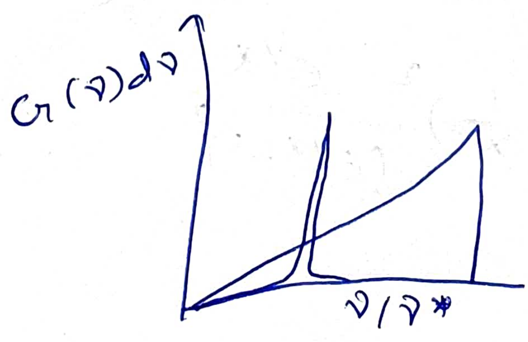
$\therefore$ density of states,

$$G(\nu)\,d\nu = V_0 \frac{4}{3}\pi \frac{\nu^2 d\nu}{c^3}$$

Cutoff freq / debye freq is introduced



Comparison of debye and Einstein freq.



Total degree of freedom

$$\int_0^{\nu_D} G(\nu)\, d\nu = N_A f$$

Energy,

$$E_D = \int_{\nu=0}^{\nu_D} G(\nu)\, d\nu\, \epsilon_\nu$$

$$= \frac{9 N_A}{\nu_D^3} \int_0^{\nu_D} \frac{h\nu^3}{(e^{h\nu/kT} - 1)}\, d\nu \quad \Bigg|\ \text{take } \frac{kT}{h} = \nu^*$$

Specific heat,

constant T

$$\frac{C_V}{3R} = -\frac{3(\Theta_D/T)}{(e^{\Theta_D/T} - 1)} + \frac{12}{(\Theta_D/T)^3} \int_0^{\Theta_D/T} \frac{(\Theta_D/T)^3\, d(\frac{\Theta_D}{T})}{(e^{\Theta_D/T})}$$

dummy T