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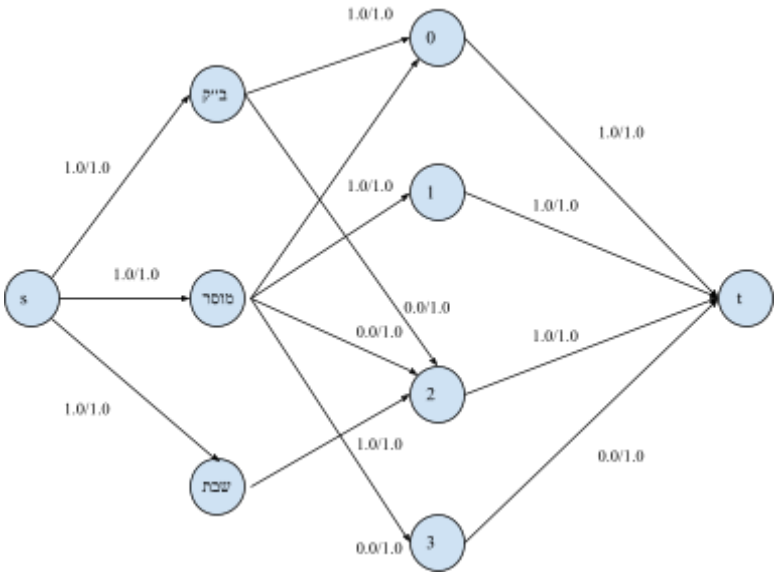
Problem Model

The *requestedHelp* map provides us with the number of requests made by talmidim per Torah topic. Each topic can be represented as a node in a network-flow graph, with edges from a *source* node that have weight capacities equal to the number of talmidim who requested help for the given topic. Moreover, the *rebbeim* can also be represented as nodes in this graph, with edges connecting them to the aforementioned topic nodes for those topics which they are capable of assisting in.[1] These edges all have capacities of 1, since only one talmid can meet with each *rebbe* for help. To round out the graph such that Ford-Fulkerson would be able to run on it, a final *sink* node is added to create single edges for each of the *rebbeim* also with capacities of 1. By finding the *maximum-flow* in this graph, all components of the problem can be solved. So long as all edges from *source* to the given topics are saturated and the *equilibrium constraint* is maintained, there exists a possible schedule. Analyzing this flow provides the information needed to map which *rebbe* is needed to help with which topic for the day.

Justification

This problem reduces to *maximum-flow* as it is almost a *maximum bipartite matching* problem, the only difference being that a single topic may be taught by multiple *rabbeim*.[2] Even though this may seem to negate this problem's *matching property*, it does not, since the model above is logically equivalent to one in which every request for a topic is represented as an individual node with an edge of capacity 1 from the *source* and 1 to the *rebbeim* capable of assisting with it. Accordingly, this problem model is justified by the same proof which showed that *maximum-flow* can be used to solve the *maximum bipartite matching* problem. See slide deck "*Applications of Network Flow: Bipartite Matching, Baseball Elimination, & Hall's Theorem*," slides 11 through 19.

Corresponding Network-Flow Graph



[1] Meaning, those *HelpTopics* listed under a given *Rebbe*'s *_helpTopics* list.
[2] The property that a *rebbe* will not teach more than one topic is maintained.