

Drill

$Women = \{A, B, C\}$ $Men = \{1, 2, 3\}$	Day 1	Day 2	Day 3	Day 4	Day 5
A	1, 3	1	1, 2	2	2
B	2	2, 3	3	1, 3	1
C					3

Proofs

1. In any execution of the algorithm, if a woman receives a proposal on day_i , then she receives some proposal on every subsequent day until the algorithm terminates.
 - a. Axiom: Each morning, every man proposes to his most highly ranked woman who has not yet rejected him.
 - b. Implication: Once a woman receives a proposal from a man, he will continue to propose to her (as she is *his most highly ranked woman*) until either she rejects him or the algorithm terminates.
 - i. Explanation: The only way for a woman to not receive a proposal from the man who has previously proposed to her is if some other man proposes to her, until the algorithm terminates.
 - c. Thus, she receives *some* proposal on every subsequent day until the algorithm terminates.
 - d. QED
2. In any execution of the algorithm, if a woman receives no proposal on day_i , then she receives no proposal on any previous day_j , $1 \leq j < i$.
 - a. Premise: A woman receives a proposal on day_j , and does not receive a proposal on day_i , where $1 \leq j < i$.
 - b. Contradiction: As defined by Proof 1, this cannot be. Once a woman receives any proposal she will always be guaranteed at least one until the termination of the algorithm.
 - c. Thus, if a woman does not receive a proposal on day_i , then she must have not received a proposal on day_j , where $1 \leq j < i$.
 - d. QED
3. In any execution of the algorithm, there is at least one woman who only receives a single proposal.
 - a. Axiom: The algorithm runs until no offers are rejected: at that point, every man and woman are paired.
 - b. Inference: From Proof 1 and Proof 2 it can be inferred that so long as the algorithm is running, one of the women received multiple proposals.

- i. Explanation: Proof 1 states that once a woman gets a proposal she is guaranteed at least some proposal in all coming days, and Proof 2 states that if on any day a woman does not have a proposal then she never received a proposal prior to that day. Thus, on each day of the algorithm's execution, one of the women must be receiving multiple proposals and rejecting all but one of them such that at least one other woman has not yet received any proposal.
- c. Implication: One woman does not receive a proposal until the final day of the algorithm's execution, whereupon she must accept it and the algorithm terminates.
- d. QED