EsitmateSecretAlgorithms2

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Experimental Apparatus

To generate data from which a conclusion could be drawn regarding the order of growth of each given "SecretAlgorithm", I composed a client code which contained four methods in correspondence to the four algorithms. Each method runs a for loop, where some integer n is doubled until a certain higher quantity.¹ Iterating through this loop, the value n is used in a call to the BigOMeasurable "setup" method, which sets up the data structure behind each algorithm as a function of n. Thus, the algorithm will be positioned to run though data whose quantity is n. At this point the client code saves some double value as the starting time, provided by calls to "System.currentTimeMillis()" or "System.nanoTime()", and then executes the program by calling the BigOMeasurable "execute" method. Upon completion of each "SecretAlgorithm"'s execution another double value of time is saved, by calling the aforementioned System methods a second time, and the time elapsed is calculated by subtracting the starting time from this ending time. Once the data has been recorded for how much time the "SecretAlgorithm"s took to run on each data size, the logarithms (base 2) of both the data size n and corresponding times are calculated via a private method in my client code.³ These logarithms allow for the data to be plotted on a log-log scale, where a power law holds if they fall out on a straight line.⁴ The slope of this line is equivalent to b in the equation aN^b which is the curve that fits the original data. That value b is then the power of 2 for which the time increases as the data sizes double, and can be used to determine the "SecretAlgorithm"'s order of growth.6

¹ These higher quantities of *n* were determined based on step 4 on slide 22 from the "Analysis of Algorithms: Motivation, Techniques & Order-of-Growth" lecture. Each algorithm was run with doubled data sizes until enough data had been collected to estimate the order of growth.

² Whether time was measured in units of milliseconds or nanoseconds was dependent on the performance of the algorithms. The data from SecretAlgorithm1 and SecretAlgorithm3 was discernible from measurements of milliseconds, while SecretAlgorithm2 and SecretAlgorithm4 were faster and therefore required a more refined measurement of time.

³ The method generates a logarithm of base 2 of a given integer i by dividing the logarithm of base 10 of i by the logarithm of base 10 of 2. Both logarithms of base 10 are calculated by the "Math.log()" method.

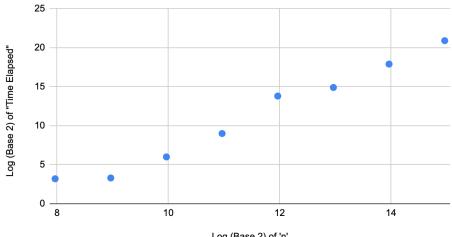
⁴ As per slide 23 from the "Analysis of Algorithms: Motivation, Techniques & Order-of-Growth" lecture.

⁵ Meaning the data sizes and elapsed times prior to the taking of their base 2 logarithms.

⁶ Hence, if *b* is 1 then as the data sizes double the time elapsed increases by 2, meaning that the algorithm is linear. If *b* is 2 then as the data sizes double the time elapsed increases by 4, meaning that the algorithm is quadratic. Similarly if *b* is 3 then as the data sizes double the time elapsed increases by 8, meaning that the algorithm is cubic.

SecretAlgorithm1





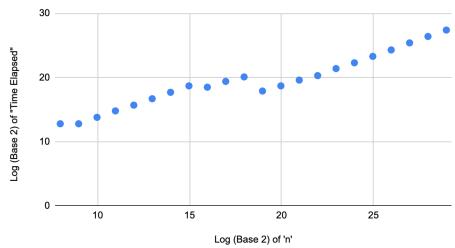
Log (Base 2) of 'n'

Data Size 'n'	Log (Base 2) of 'n'	f(n) / Time Elapsed (milliseconds)	Log (Base 2) of $f(n)$ / "Time Elapsed"
250	8.0	9	3.2
500	9.0	10	3.3
1000	10.0	66	6.0
2000	11.0	526	9.0
4000	12.0	14500	13.8
8000	13.0	30555	14.9
16000	14.0	240792	17.9
32000	15.0	1921785	20.9

The line of the log-log scale has a rough slope of 3, due to the fact that there are some imperfect points as the data was produced from a real experiment. Thus the value of b is 3, meaning that as the data size doubles the time required for the algorithm to process it increases by a ratio of 8. Accordingly it can be concluded that the order of growth of "SecretAlgorithm1" is cubic, $O(n^3)$.

SecretAlgorithm2





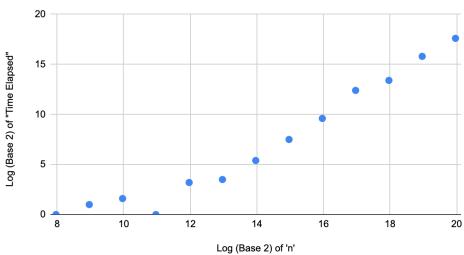
Data Size 'n'	Log (Base 2) of 'n'	f(n) / Time Elapsed (nanoseconds)	Log (Base 2) of $f(n)$ / "Time Elapsed"
250	8.0	7334	12.8
500	9.0	7072	12.8
1000	10.0	14698	13.8
2000	11.0	27673	14.8
4000	12.0	54083	15.7
8000	13.0	106973	16.7
16000	14.0	213192	17.7
32000	15.0	428249	18.7
64000	16.0	358814	18.5
128000	17.0	676526	19.4
256000	18.0	1113756	20.1
512000	19.0	245115	17.9
1024000	20.0	430445	18.7
2048000	21.0	791531	19.6
4096000	22.0	1327809	20.3
8192000	23.0	2682214	21.4
16384000	24.0	5208455	22.3
32768000	25.0	10252449	23.3
65536000	26.0	20658606	24.3
131072000	27.0	42814520	25.4

262144000	28.0	86774249	26.4
524288000	29.0	181548323	27.4

The line of the log-log scale has a rough slope of 1, due to the fact that there are some imperfect points as the data was produced from a real experiment. Thus the value of b is 1, meaning that as the data size doubles the time required for the algorithm to process it increases by a ratio of 2. Accordingly it can be concluded that the order of growth of "SecretAlgorithm2" is linear, O(n).

SecretAlgorithm3



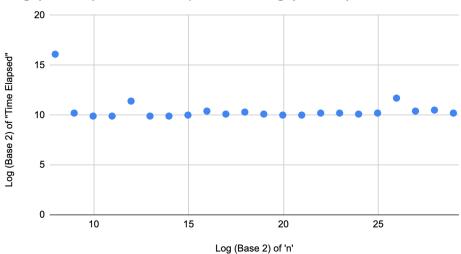


Data Size 'n'	Log (Base 2) of 'n'	f(n) / Time Elapsed (milliseconds)	Log (Base 2) of $f(n)$ / "Time Elapsed"
250	8.0	1	0.0
500	9.0	2	1.0
1000	10.0	3	1.6
2000	11.0	1	0.0
4000	12.0	9	3.2
8000	13.0	11	3.5
16000	14.0	42	5.4
32000	15.0	175	7.5
64000	16.0	784	9.6
128000	17.0	5299	12.4
256000	18.0	10555	13.4
512000	19.0	58004	15.8
1024000	20.0	198218	17.6

The line of the log-log scale has a rough slope of 2, due to the fact that there are some imperfect points as the data was produced from a real experiment. Thus the value of b is 2, meaning that as the data size doubles the time required for the algorithm to process it increases by a ratio of 4. Accordingly it can be concluded that the order of growth of "SecretAlgorithm3" is quadratic, $O(n^2)$.

SecretAlgorithm4





Data Size 'n'	Log (Base 2) of 'n'	f(n) / Time Elapsed (nanoseconds)	Log (Base 2) of $f(n)$ / "Time Elapsed"
250	8.0	71630	16.1
500	9.0	1179	10.2
1000	10.0	973	9.9
2000	11.0	951	9.9
4000	12.0	2649	11.4
8000	13.0	931	9.9
16000	14.0	978	9.9
32000	15.0	1021	10
64000	16.0	1378	10.4
128000	17.0	1125	10.1
256000	18.0	1241	10.3
512000	19.0	1073	10.1
1024000	20.0	1048	10
2048000	21.0	1057	10

22.0	1216	10.2
23.0	1166	10.2
24.0	1084	10.1
25.0	1180	10.2
26.0	3220	11.7
27.0	1323	10.4
28.0	1410	10.5
29.0	1176	10.2
	23.0 24.0 25.0 26.0 27.0 28.0	23.0 1166 24.0 1084 25.0 1180 26.0 3220 27.0 1323 28.0 1410

The line of the log-log scale has a rough slope of 0, due to the fact that there are some imperfect points as the data was produced from a real experiment. Thus the value of b is 0, meaning that as the data size doubles the time required for the algorithm to process it does not increase. Accordingly it can be concluded that the order of growth of "SecretAlgorithm4" is constant, O(1).