

# Spatial Data

SERGIO REY

**GPH 483/598**

**Geographic Information Analysis**

School of Geographical Sciences and Urban Planning

Arizona State University

Fall 2010

## 1 Spatial Data

- Types
- Spatial Effects

## 2 Spatial Data Analysis

- Spatial Process
- Point Pattern Analysis
- Lattice
- Geostatistical
- Network Data

## 3 Transformations of Spatial Data

- Area to Point
- Point to Area
- Change of Support Problem

## 1 Spatial Data

- Types
- Spatial Effects

## 2 Spatial Data Analysis

- Spatial Process
- Point Pattern Analysis
- Lattice
- Geostatistical
- Network Data

## 3 Transformations of Spatial Data

- Area to Point
- Point to Area
- Change of Support Problem

## 1 Spatial Data

- Types
- Spatial Effects

## 2 Spatial Data Analysis

- Spatial Process
- Point Pattern Analysis
- Lattice
- Geostatistical
- Network Data

## 3 Transformations of Spatial Data

- Area to Point
- Point to Area
- Change of Support Problem

- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

# Spatial Data is Special

*Spatial data comes in many varieties and it is not easy to arrive at a system of classification that is simultaneously exclusive, exhaustive, imaginative, and satisfying.*

– G. Upton & B. Fingleton

# Types of Spatial Data

## Events

addresses of crimes

## Continuous **surfaces**

air quality, rainfall

## Discrete spatial **objects**

county income

# What is special about spatial data?

## Location, Location, Location

where matters

## Dependence is the rule, not the exception

- spatial interaction, contagion
- spatial externalities
- spillovers, copy-catting

## Spatial Scale

Inference can change with scale



# Nature of Spatial Data

## Georeferenced

attribute data together with location

## Geocoding

- associate observations with location
- point: latitude-longitude (GPS)
- areal unit: spatial reference

# Geocoding on-line



## Converting Addresses to/from Latitude/Longitude in One Step

[Stephen P. Morse](#), San Francisco

[Batch Mode](#) [Deg/Min/Sec to Decimal](#) [Frequently Asked Questions](#) [My Other Webpages](#)

address	975 S. Forest Mall	latitude	
city	Tempe	longitude	
state	AZ	above values must be in decimal with minus signs for south and west	
zip	85281		
country	United States		
<a href="#">Determine Latitude/Longitude</a>		<a href="#">Determine Address</a>	

# Where is the office?

## Converting Addresses to/from Latitude/Longitude in One Step

[Stephen P. Morse](#), San Francisco

Batch Mode

Deg/Min/Sec to Decimal

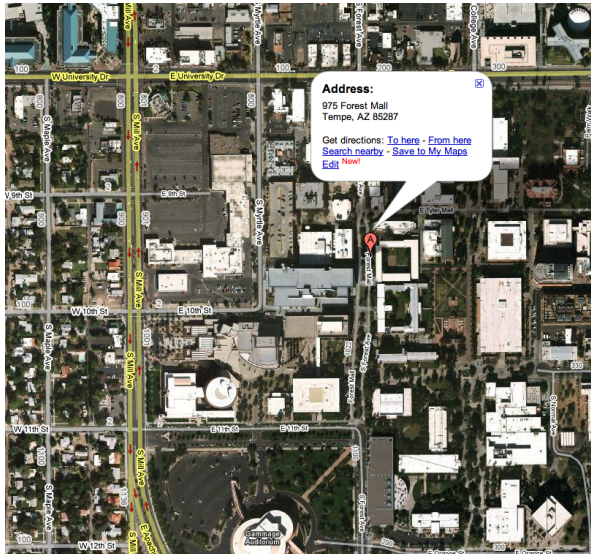
Frequently Asked Questions

My Other Webpages

address	<input type="text" value="975 S. Forest Mall"/>	latitude	<input type="text"/>
city	<input type="text" value="Tempe"/>	longitude	<input type="text"/>
state	<input type="text" value="AZ"/>	<i>above values must be in decimal with minus signs for south and west</i>	
zip	<input type="text" value="85281"/>		
country	<input type="text" value="United States"/>		
<input type="button" value="Determine Latitude/Longitude"/>		<input type="button" value="Determine Address"/>	

from <a href="#">google</a>	latitude	longitude
decimal	33.4197396	-111.936517
deg-min-sec	33° 25' 11.0626"	-111° 56' 11.4612"

# Geocoding: google link



## Converting Addresses to/from Latitude/Longitude in One Step

[Stephen P. Morse](#), San Francisco

Batch Mode

Deg/Min/Sec to Decimal

Frequently Asked Questions

My Other Webpages

address <input type="text"/>	latitude <input type="text" value="33.4197396"/>
city <input type="text"/>	longitude <input type="text" value="-111.936517"/>
state <input type="text"/>	<i>above values must be in decimal with minus signs for south and west</i>
zip <input type="text"/>	
country <input type="text" value="United States"/>	
<input type="button" value="Determine Latitude/Longitude"/>	<input type="button" value="Determine Address"/>

## Converting Addresses to/from Latitude/Longitude in One Step

[Stephen P. Morse](#), San Francisco

[Batch Mode](#) [Deg/Min/Sec to Decimal](#) [Frequently Asked Questions](#) [My Other Webpages](#)

address <input type="text"/>	latitude <input type="text" value="33.4197396"/>
city <input type="text"/>	longitude <input type="text" value="-111.936517"/>
state <input type="text"/>	<i>above values must be in decimal with minus signs for south and west</i>
zip <input type="text"/>	<input type="button" value="Determine Address"/>
country <input type="text" value="United States"/>	
<input type="button" value="Determine Latitude/Longitude"/>	

Virtual Earth  
Street, Tempe, Arizona 85281, United States



998 Myrtle Ave S, Tempe, AZ 85281

## Location as a **Given**

- in most spatial data analyses no choice in location
- no sampling in the usual sense
- data = attributes augmented with **locational** information

## 1 Spatial Data

- Types
- Spatial Effects

## 2 Spatial Data Analysis

- Spatial Process
- Point Pattern Analysis
- Lattice
- Geostatistical
- Network Data

## 3 Transformations of Spatial Data

- Area to Point
- Point to Area
- Change of Support Problem



## The Trilogy

- Spatial Dependence
- Spatial Heterogeneity
- Spatial Scale

# First Law of Geography

## Waldo Tobler

- "everything depends on everything else, but closer things more so"
- Structure of spatial dependence
- Distance Decay
- Closeness = Similarity

## Spatial Instability

- Process varies in some way over spatial units
- Multiple forms
  - Discrete = regimes
  - Continuous = expansion method, GWR
- Trade-off
  - spatial homogeneity = stationary process
  - uniqueness = extreme heterogeneity

## Mismatch

- Spatial scale of the process
- Spatial scale of our measurement

## Issues

- points too far apart = miss small distance variation
- area aggregates cannot provide information on individual behavior
- ecological fallacy

# Modifiable Areal Unit Problem (MAUP)

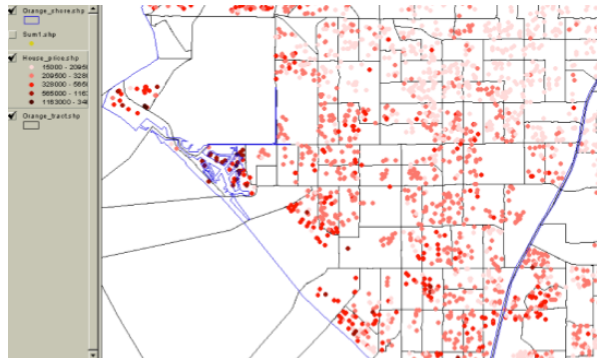
## Aggregation Problem

- special case of ecological fallacy
- spatial heterogeneity
- a million spatial autocorrelation coefficients

## Zonation Problem

- size
- arrangement

# Spatial Heterogeneity: Housing Prices



## Dependence, Heterogeneity and Scale

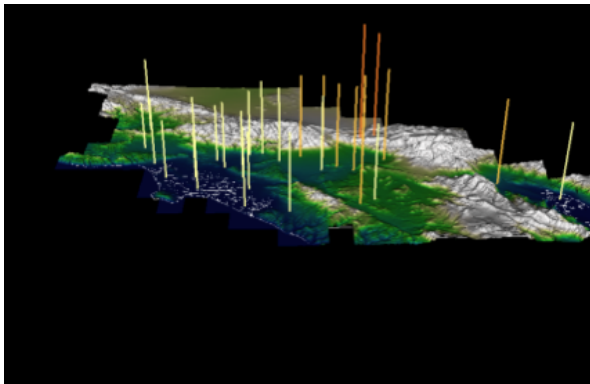
- not necessarily orthogonal
- distinguishing between dependence and heterogeneity can be challenging

## Location as an Experimental Design Problem

- Spatial sampling = where to collect the data
  - which villages to survey
  - where to locate air quality monitoring stations



# Spatial Sampling



- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - **Spatial Process**
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

## Spatial Random Field

- a mathematical construct to capture randomness of values distributed over space
- $\{Z(s) : s \in D\}$ 
  - $s \in R^d$  : location (e.g., lat-lon)
  - $D \in R^d$  : index set = possible locations
  - $Z(s)$  : random variable at location  $s$

# Types of Spatial Analysis

## Point Pattern Analysis

spatial distribution of events

## Geostatistical Analysis

surface modeling

## Lattice Data Analysis

spatial patterns of attributes observed for spatial objects

## 1 Spatial Data

- Types
- Spatial Effects

## 2 Spatial Data Analysis

- Spatial Process
- **Point Pattern Analysis**
- Lattice
- Geostatistical
- Network Data

## 3 Transformations of Spatial Data

- Area to Point
- Point to Area
- Change of Support Problem

# Point Pattern Analysis

## Data

- mapped pattern = all the events
- not a sample in the usual sense

## Spatial Process

- observations as a realization of a random point process
- points occur in space according to a mathematical model

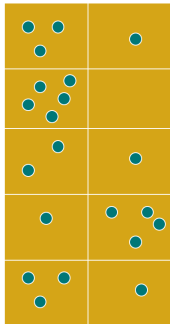
## Spatial Domain: $D$

- Domain is *random*
- *Number* of points is *random*
- *Location* of points is *random*

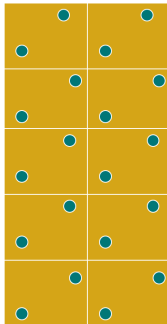
## Focus: Properties of $D$

- Number of points observed
- Pattern of the point locations

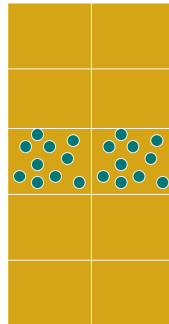
# Point Patterns



RANDOM



UNIFORM



CLUSTERED



# Point Patterns

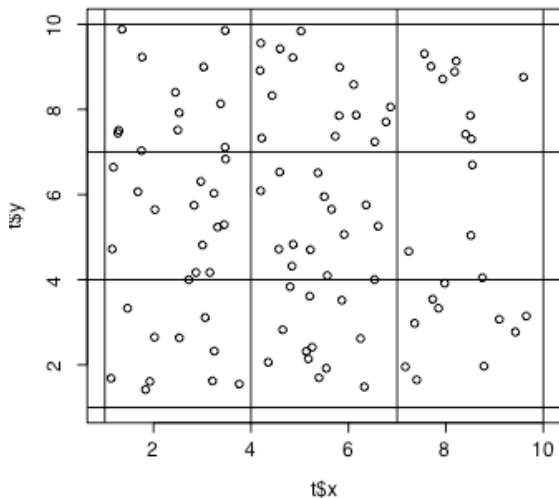
## Unmarked Point Pattern

- Only location is recorded
- No other attribute information

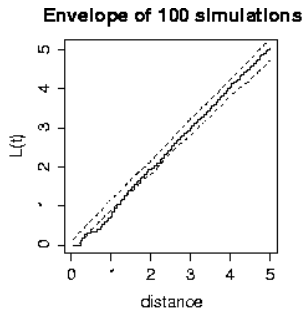
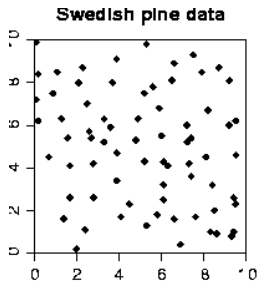
## Marked Point Patterns

- Location is recorded
- Stochastic attributes also recorded
- e.g., sales at location, dbh of tree

# Point Pattern Analysis: Quadrat Methods



# Point Pattern Analysis: Distance Based Methods



# Points on Networks



**Figure 2: Retail stores assigned to the street network in Shibuya, Tokyo**  
(cells are indicated by different colors)

# Burkitt's Lymphoma Data: Time Series of Point Data



- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - **Lattice**
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels



## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Spatial Domain: $D$

- Discrete and fixed
- Locations nonrandom
- Locations countable

## Examples of lattice data

- Attributes collected by ZIP code
- census tract
- remotely sensed data reported by pixels

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites



# Lattice Data: Indexing

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

# Lattice Data: Indexing

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

# Lattice Data: Indexing

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

# Lattice Data: Indexing

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

## Site

- Each location is now an area or *site*
- One observation on  $Z$  for each site
- Need a spatial index:  $Z(s_i)$

## $Z(s_i)$

- $s_i$  is a representative location within the site
- e.g., centroid, largest city
- Allows for measuring distances between sites

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
  - Aggregated: event counts (number of crimes per tract)
  - Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus



# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: Aggregation and Coverage

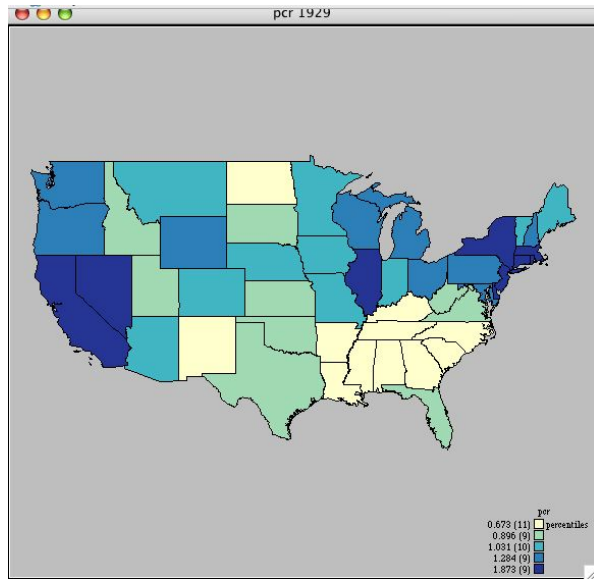
## Sites are areal units

- Attribute is typically aggregated or averaged
- Aggregated: event counts (number of crimes per tract)
- Averaged: per capita income by state

## Coverage

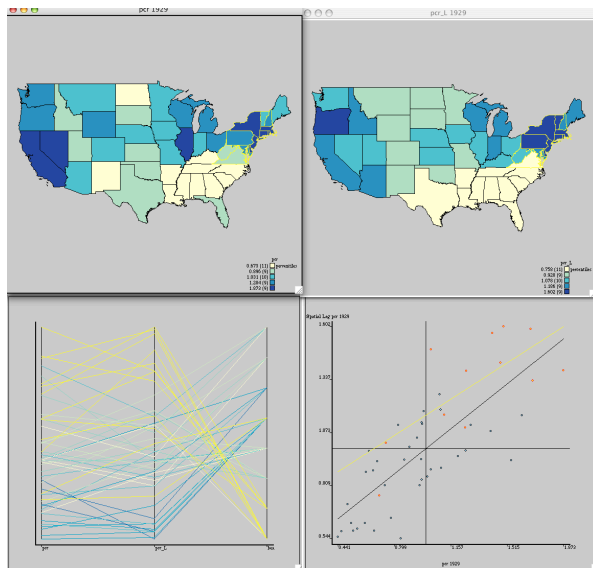
- Lattice data is usually exhaustive in coverage
- e.g., U.S. states, census tracts in San Diego
- Prediction or interpolation not meaningful
- Explaining attribute variation across sites is the focus

# Lattice Data: State Per Capita Incomes





# Lattice Data: Spatial Autocorrelation



- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - **Geostatistical**
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as "spatial data with continuous variation."
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.



## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

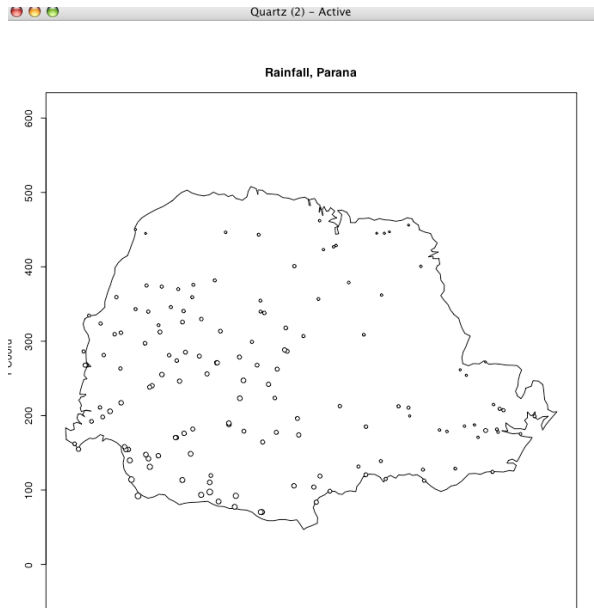
## Spatial Domain: $D$

- A continuous and fixed set.
- Meaning  $Z(s)$  can be observed everywhere within  $D$ .
- Between any two sample locations  $s_i$  and  $s_j$  you can theoretically place an infinite number of other samples.
- By fixed: the points in  $D$  are non-stochastic

## Continuous Variation

- Because of the continuity of  $D$
- Geostatistical data is referred to as “spatial data with continuous variation.”
- Continuity is associated with  $D$ .
- Attribute  $Z$  may, or may not, be continuous.

# Geostatistical Data: Rainfall in Parana State Brazil



## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging



## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

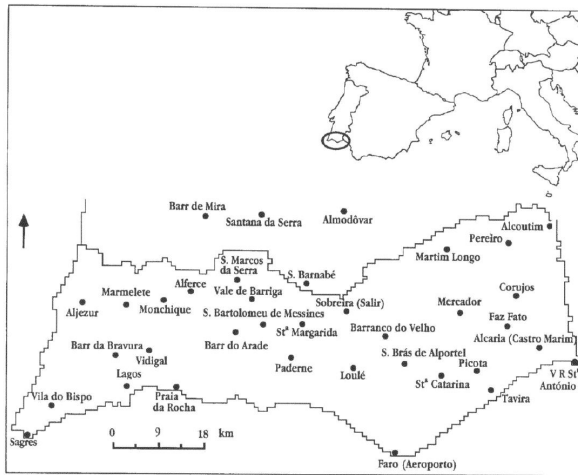
## Continuous variation

- Potentially measurable anywhere in  $D$
- Impossible to sample  $D$  exhaustively

## Reconstruction of the surface from observed sites

- Tessellation based methods
- Interpolation
- Kriging

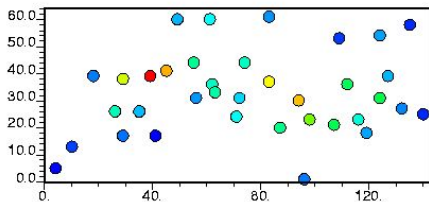
# Surface Reconstruction: Example<sup>1</sup>



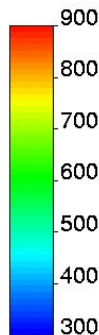
<sup>1</sup>From Goovaerts, P. (1999) "Performance comparison of geostatistical algorithms for incorporating elevation into the mapping of precipitation". *Geocomputation '99*.

# Surface Reconstruction: Tessellation Based Method

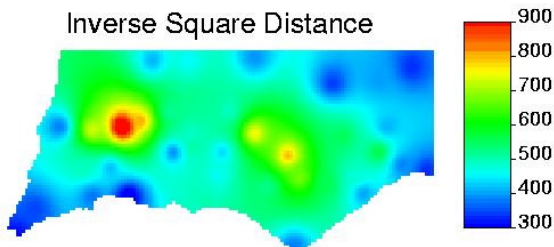
Annual rainfall data (mm)



Thiessen Polygons

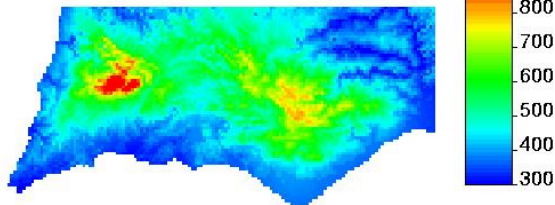


# Surface Reconstruction: Spatial Interpolation



# Surface Reconstruction: Kriging

Simple Kriging with Local Means





- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - **Network Data**
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

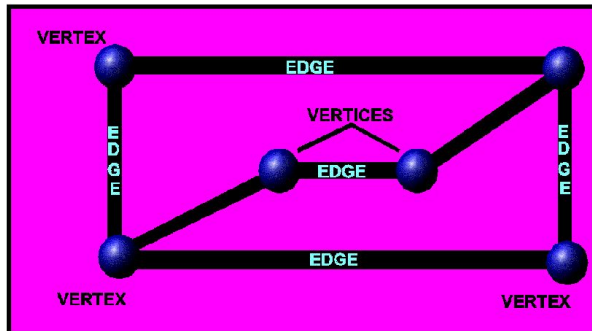
- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.

## Networks

- A network is a system of linear features connected at intersections and interchanges.
- These intersections and interchanges are called nodes.
- The linear feature connecting any given pair of nodes is called an arc.
- Formally, a network is defined as a directed graph  $G = (N, A)$  consisting of an indexed set of nodes  $N$  with  $n = |N|$  and a spanning set of directed arcs  $A$  with  $m = |A|$ , where  $n$  is the number of nodes and  $m$  is the number of arcs.
- Each arc on a network is represented as an ordered pair of nodes, in the form from node  $i$  to node  $j$ , denoted by  $(i, j)$ .
- In the GIS literature, a network arc is often called a network link.



# Network Data: Graph Theory



## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

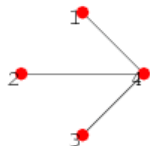
## In this course

- We will not be analyzing network data per se
- We will be drawing on graph theory to help in ESDA

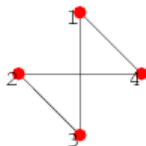
## Uses of graph theory in ESDA

- Properties of adjacency matrices
- Clustering and regionalization algorithms

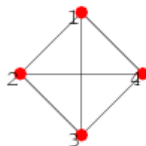
# Network Data: Adjacency Matrices



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$



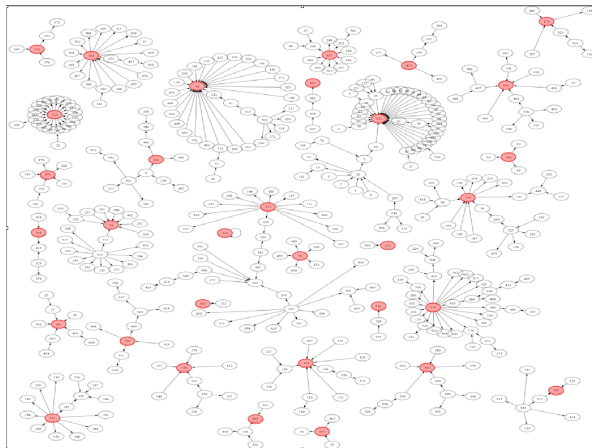
$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

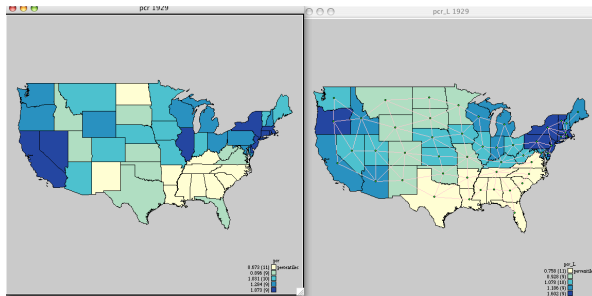


# Network Data: Clustering Visualization



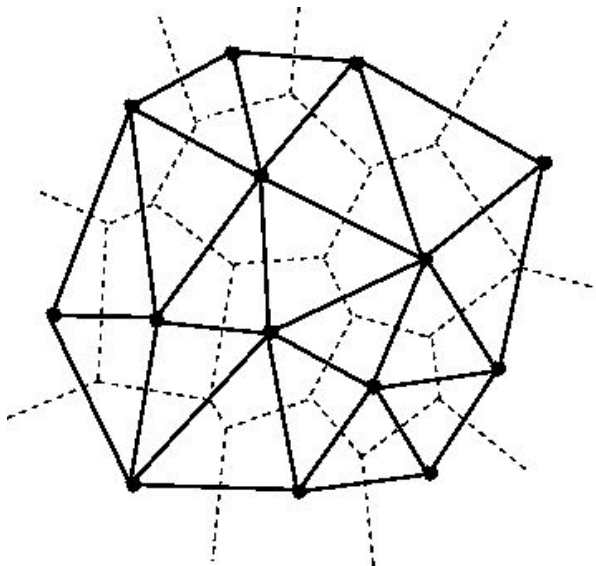
- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - **Area to Point**
  - Point to Area
  - Change of Support Problem

# Area to Point Transformation: Centroids



- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - **Point to Area**
  - Change of Support Problem

# Point to Area Transformation: Thiessen Polygons



- 1 Spatial Data
  - Types
  - Spatial Effects
- 2 Spatial Data Analysis
  - Spatial Process
  - Point Pattern Analysis
  - Lattice
  - Geostatistical
  - Network Data
- 3 Transformations of Spatial Data
  - Area to Point
  - Point to Area
  - Change of Support Problem

# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas

# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas



# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas

# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas

# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas

# Change of Support Problem

## Transformation from one spatial framework to another

- Point data to area data
- Regional data: areal interpolation

## Scaling

- Upscaling: points to areas or, areas to larger areas
- Downscaling: larger areas to smaller composite areas

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points  $(s(1), \dots, s(n))$  are the supports
- Prediction is for the area  $(A)$ ,  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the area
  - Ignores the difference in variance:  $V[Y(A)] < V[\hat{Y}]$
  - Sample size may vary over spatial supports  $(s(i))$
  - Covariate information is not used

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points  $(s(1), \dots, s(n))$  are the supports
- Prediction is for the area  $(A)$ ,  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the area
  - Ignores the difference in variances:  $V[Y(A)] < V(\hat{Y})$
  - Sample size may vary over the area (e.g.,  $Y(i)$  is a point sample,  $Y(A)$  is an area sample)

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points  $(s(1), \dots, s(n))$  are the supports
- Prediction is for the area  $(A)$ ,  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems

- Ignores spatial correlation in  $Y(i)$  over the area
- Ignores the difference in variances  $Var(Y(A)) \neq Var(Y(i))$
- Ignores the difference in support area

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points  $(s(1), \dots, s(n))$  are the supports
- Prediction is for the area  $(A)$ ,  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems

•  $\hat{Y}$  ignores spatial correlation in  $Y(i)$  over the area of interest

•  $\hat{Y}$  ignores the difference in variances  $\text{Var}(Y(A)) \neq \text{Var}(Y(i))$

•  $\hat{Y}$  ignores the difference in supports  $A \neq i$



# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better

# Change of Support Problem: Point to Area

## Example

- Estimate soil contamination for a census tract given point samples in the tract.
- Sample points ( $s(1), \dots, s(n)$ ) are the supports
- Prediction is for the area ( $A$ ),  $Y(A)$

## Estimation

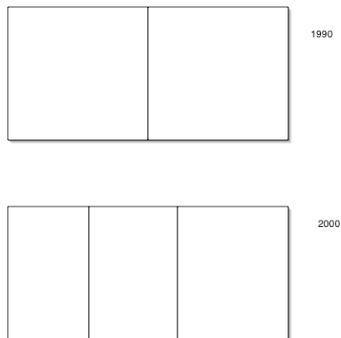
- Assume a constant mean (contamination), then  $E[Y(i)] = \mu \forall i$
- Simple estimator:  $\hat{Y} = (1/n) \sum_{i=1}^n y(i)$
- Problems
  - Ignores spatial correlation in  $Y(i)$  over the sites
  - Ignores the difference in variances:  $V(Y(A)) < V(\hat{Y})$
  - May significantly over or under estimate  $Y(A)$
  - Geostatistical methods are better



# Change of Support Problem: Areal Interpolation

## Example

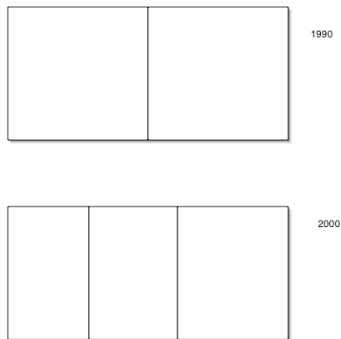
- Census tracts from 2000 and 1990
- Due to population growth, some 1990 tracts may have been split



# Change of Support Problem: Areal Interpolation

## Example

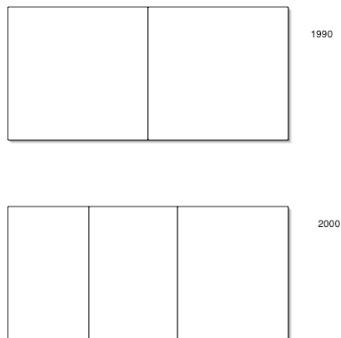
- Census tracts from 2000 and 1990
- Due to population growth, some 1990 tracts may have been split



# Change of Support Problem: Areal Interpolation

## Example

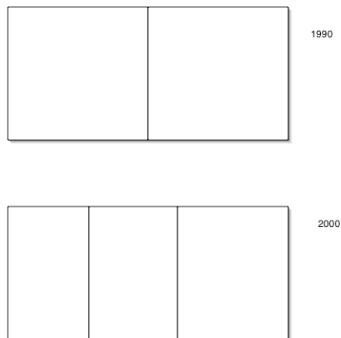
- Census tracts from 2000 and 1990
- Due to population growth, some 1990 tracts may have been split



# Change of Support Problem: Areal Interpolation

## Example

- Census tracts from 2000 and 1990
- Due to population growth, some 1990 tracts may have been split



# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate



# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate

# Change of Support Problem: Areal Interpolation

## Methods

- Cartographic based
- Statistical based

## Cartographic

- Polygon overlap
- Adjacency information to smooth

## Statistical

- Fit models to surface data on ancillary variables
- Use models to interpolate