Point Pattern Analysis: Basics

SERGIO REY

GPH 483/598
Geographic Information Analysis
School of Geographical Sciences
Arizona State University
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Outline

1. Point Pattern Analysis Objectives and Examples
   - Objectives
   - Definitions
   - Examples and Terminology
2. Centrography
   - Central Tendency
   - Dispersion and Orientation
   - Geometry

Point Pattern Analysis Objectives

Goals

- Pattern detection
- Assessing the presence of *clustering*
- Identification of individual *clusters*

General Approaches

- Estimate intensity of the process
- Formulating an idealized model and investigating deviations from expectations
- Formulating a stochastic model and fitting it to the data

Point Pattern Analysis Definitions

Spatial Point Pattern

A set of events, irregularly distributed within a region A and presumed to have been generated by some form of stochastic mechanism.

Representation

$\{Y(A), A \subset \Re\}$, where $Y(A)$ is the number of events occurring in area A.

Events, points, locations

Event an occurrence of interest

Point any location in study area

Event location a particular point where an event occurs

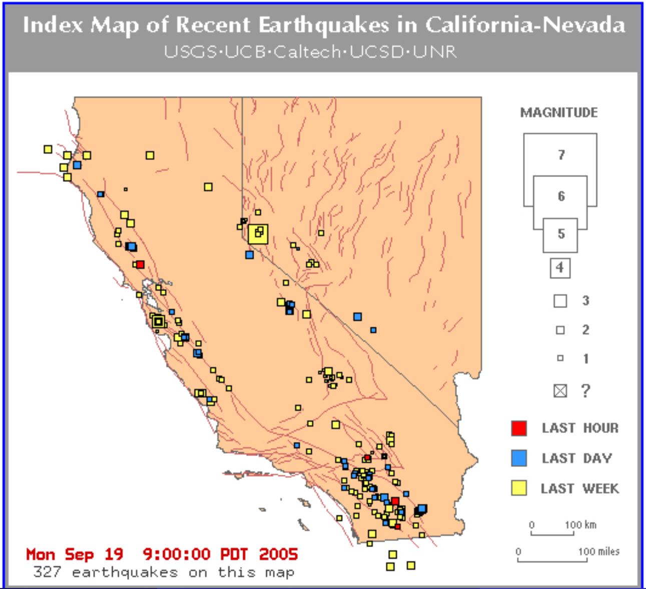
Point Pattern Analysis Definitions

Region: A

- Most often planar (two-dimensional Euclidean space)
- One dimensional applications also possible
- Three-dimensional increasingly popular (space + time)
- Point processes on networks (non-planar)

Space-Time Point Patterns

Space-Time Point Patterns



Point Patterns on Networks

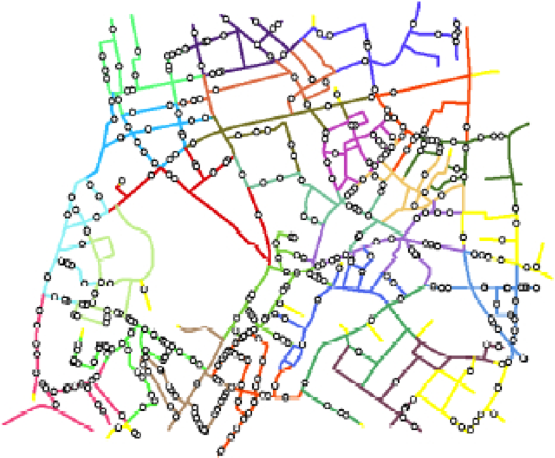


Figure 2: Retail stores assigned to the street network in Shibuya, Tokyo
(cells are indicated by different colors)

Point Patterns

Unmarked Point Patterns

- Only location is recorded
- Attribute is binary (presence, absence)

Marked Point Patterns

- Location is recorded
- Non-binary stochastic attribute
- e.g., sales at a retail store, dbh of tree

Realizations

Mapped Point Patterns

- *All* events are recorded and mapped
- Complete enumeration of events
- Full information on the realization from the process

Sampled Point Patterns

- *Sample* of events are recorded and mapped
- Complete enumeration of events impossible or intractable
- Partial information on the realization from the process
- Presence/“absence” data (ecology, forestry)

Points as Events

Mapped Patterns

not a sample

Selection Bias

- events are mapped
- but non-events are not

Research Questions

Location Only

are points randomly located or patterned

Location and Value

- marked point pattern
- is combination of location and value random or patterned

Both Cases

What is the Underlying Process?

Points on a Plane

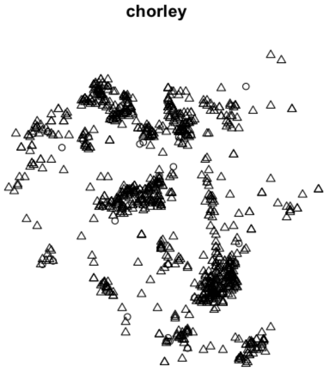
Classic Point Pattern Analysis

- points on an isotropic plane
- no effect of translation and rotation
- classic examples: tree seedlings, rocks, etc

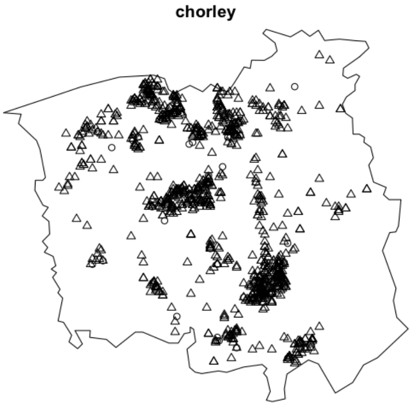
Distance

- no directional effects
- no translational effects
- straight line distance only

Events: Point Map



Points in Context



Intensity

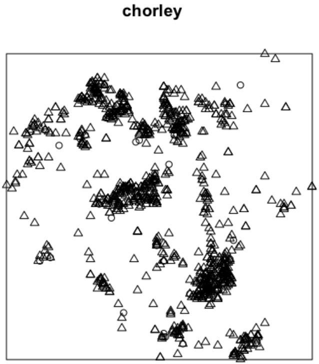
First Moment

- number of points N, area of study $|A|$
- intensity: $\lambda = N/|A|$
- area depends on bounds, often arbitrary

Artificial Boundaries

- bounding box (rectangle, square)
- other (city boundary)

Bounding Box



District Boundary

Convex Hull

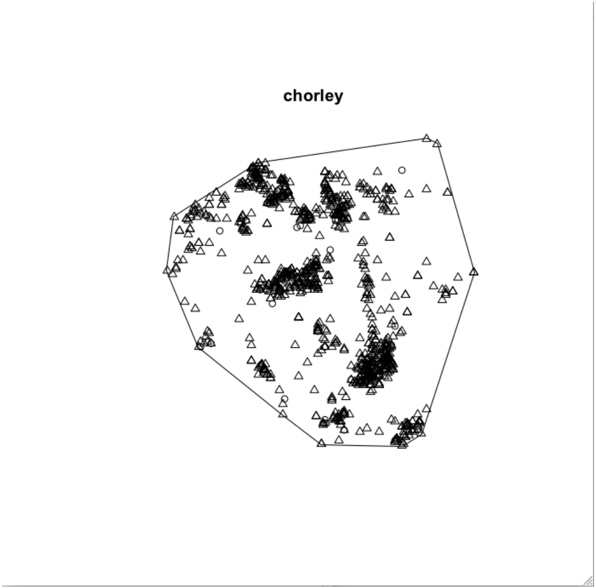
Tightest fit

various algorithms

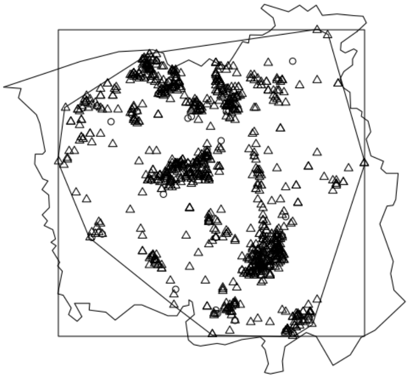
Rescaled Convex Hull (Ripley-Rasson)

- adjust to properly reflect spatial domain of point process
- use centroid of convex hull
- rescale by $1/[\sqrt{(1-m/N)}]$
- m: number of vertices of convex hull

Convex Hull



Multiple Boundaries



Intensity Estimates

	Area km^2	Intensity $cases/km^2$
District Boundary	315.155	3.29
Bounding Box	310.951	3.33
Convex Hull	229.421	4.52

N=1036

Points on a Network

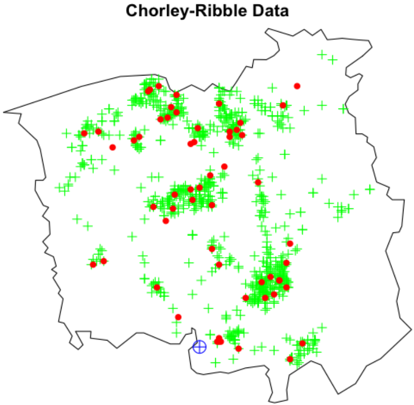
Realistic Location

- addresses
- remove impossible locations (lakes)

Network Distance

- shortest path along network
- Manhattan block distance
- distance vs. travel time or cost

Case-Control Design: Lancashire Cancer

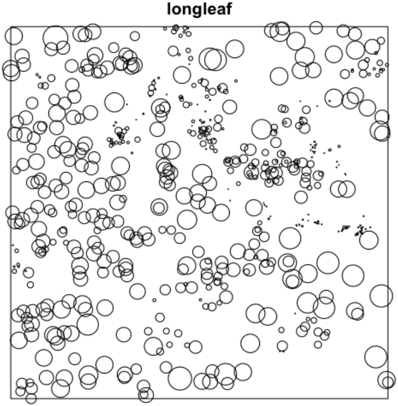


Marked Point Patterns

Both Location and Value

- Patterns in the Location
- Value Associated with Location

Marked Point Pattern: Longleaf Pine

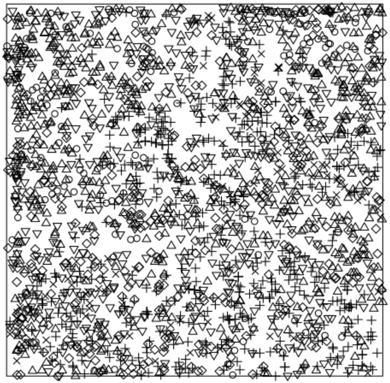


Multi-Type Patterns

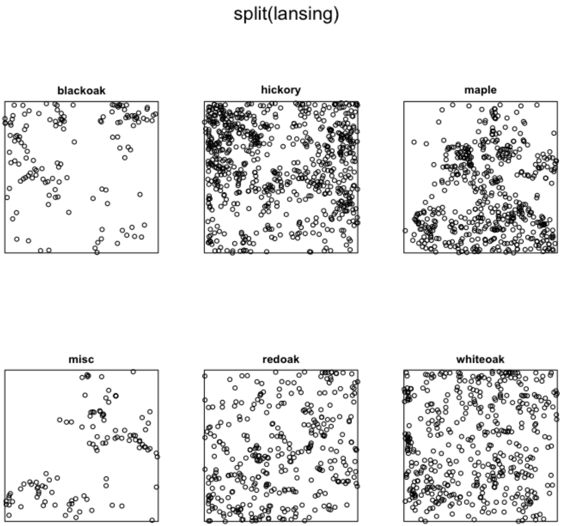
Multiple Categories

- Patterns in Single Category
- Association between Patterns in Other Categories
- Repulsion or Attraction

Multi-Type Pattern: Lansing Woods



Multi-Type Pattern: Lansing Woods



Areal Aggregation

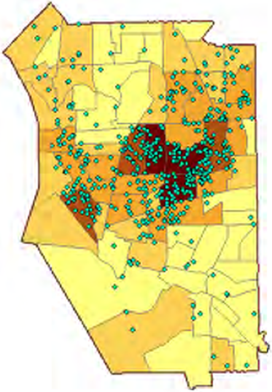
Event Counts

- points aggregated by areal unit
- spatially extensive variable

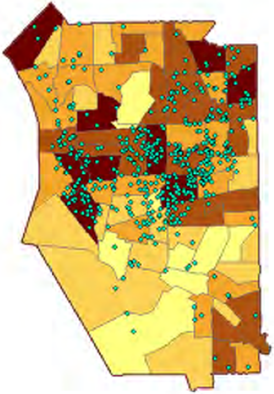
Rates

- events / population at risk
- non-homogeneous population at risk
- risk = probability of an event
- rate is an estimate of underlying risk

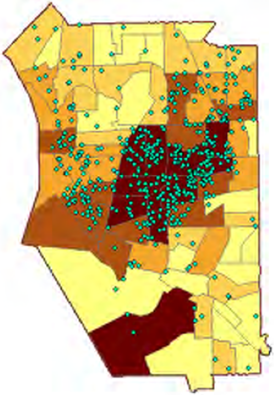
Homicide Counts by Census Tracts



Population Count by Census Tracts



Homicide Rate by Census Tracts



Central Tendency

Purpose

- Provide a “center point”
- Similar to first moment of a distribution
- “Representative point”

Measures

- Mean Center
- Weighted Mean Center
- Median Center
- Center of Minimum Distance

Example Data

i	x_i	y_i	w_i
1	20	40	10
2	30	60	20
3	34	52	10
4	40	40	20
5	44	42	10
6	48	62	80
7	50	10	10
8	60	50	90
9	90	90	100

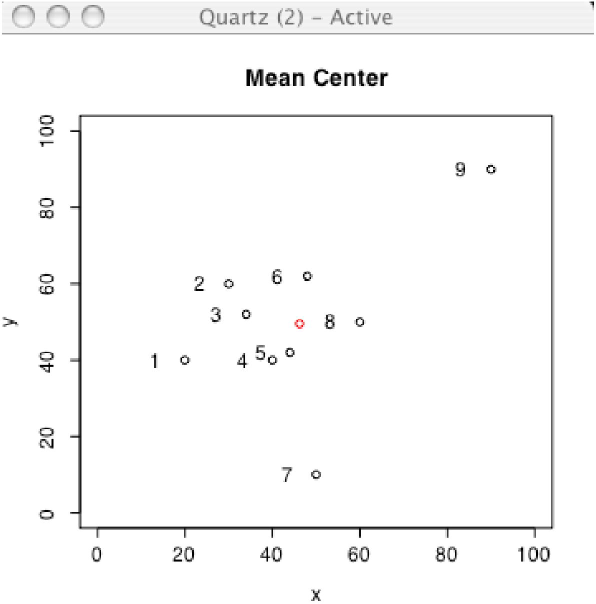
Mean Center

(x_m, y_m)

$$x_m = 1/n \sum_{i=1}^{n} x_i \tag{1}$$

$$y_m = 1/n \sum_{i=1}^{n} y_i \tag{2}$$

Mean Center



Quartz (2) – Active
Mean Center
9
2
6
3
8
1
4
5
7
y
x
0
20
40
60
80
100
0
20
40
60
80
100

Weighted Mean center

(x_m, y_m)

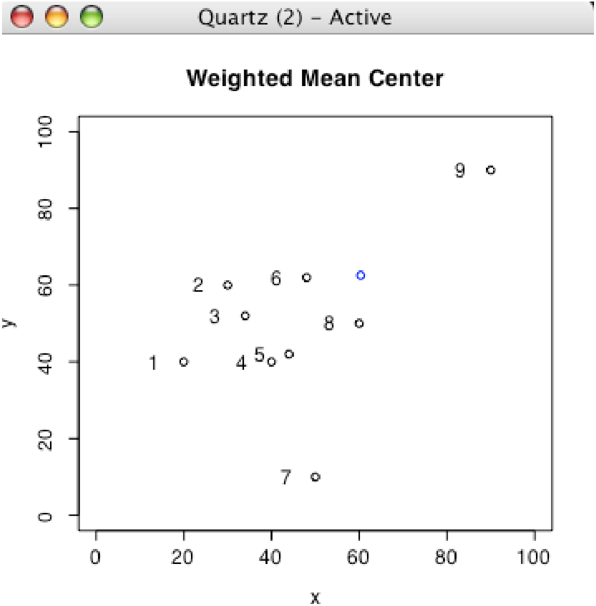
$$x_m = 1/n \sum_{i=1}^{n} x_i \frac{w_i}{\sum_{i=1}^{n} w_i} \quad (3)$$

$$y_m = 1/n \sum_{i=1}^{n} y_i \frac{w_i}{\sum_{i=1}^{n} w_i} \quad (4)$$

w_i weight

- Marked point patterns
- Continuous mark
- Not categorical mark

Weighted Mean Center



Median Center

Definition(s)

English Statistics: The intersection of two orthogonal axes, each which has an equal number of points on either side.

American: The center of minimum travel.

English Median Center

Manhattan Median

$$Min\ f(x_m, y_m) = \sum_{i=1}^{n} |x_i - x_m| + |y_i - y_m| \quad (5)$$

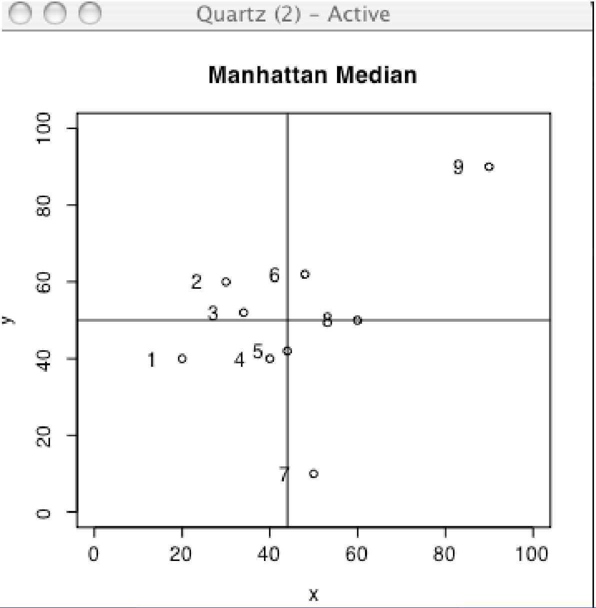
Advantages

- Can be found very quickly
- No calculations are typically required (other than intersection)

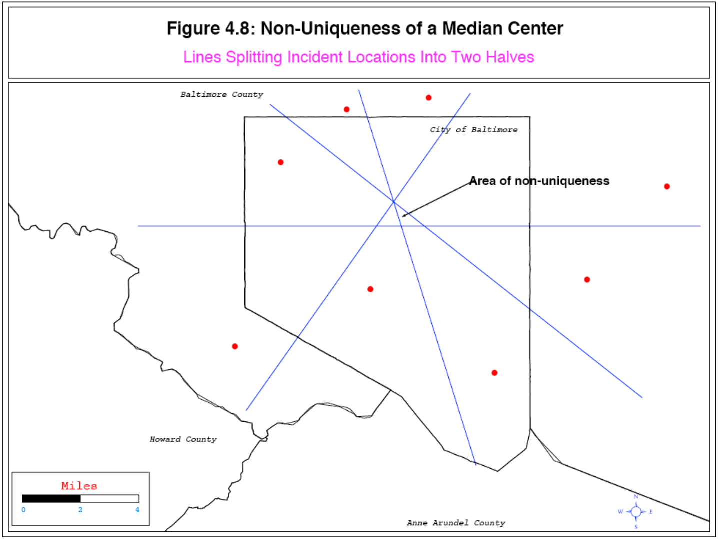
Disadvantage

- Never unique with even n
- Always unique with odd n
- Not unique under axis rotation

Manhattan Median



Non-Uniqueness



Center of Minimum Travel

Euclidean Median

The location from which the sum of the Euclidean distances to all points in a distribution is a minimum.

Euclidean Median

$$Min\ f(x_m, y_m) = \sum_{i=1}^{n} \sqrt{(x_i - x_m)^2 + (y_i - y_m)^2} \tag{6}$$

Weighted Euclidean Median

$$Min\ f(x_m, y_m) = \sum_{i=1}^{n} \frac{w_i}{\sum_{i=1}^{n} w_i} \sqrt{(x_i - x_m)^2 + (y_i - y_m)^2} \tag{7}$$

Euclidean Median

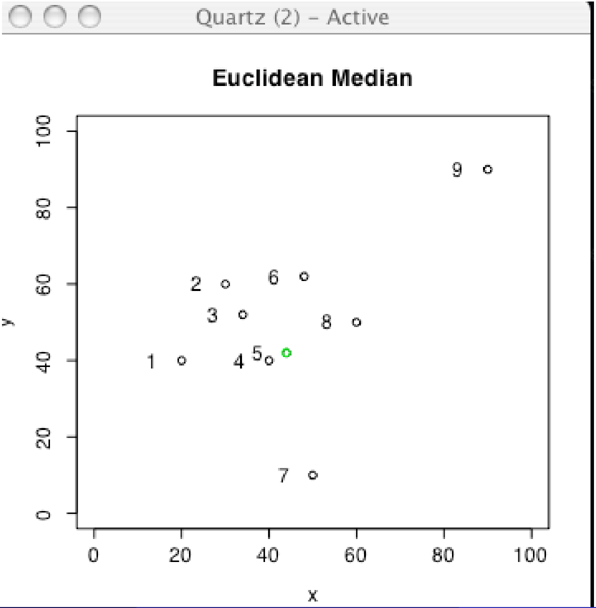
Weber Problem

Find the optimal location for a factory: one that minimizes transport costs between sources of raw materials and delivery to the market.

Solutions

- No closed form solution
- First iterative alogrithm: Kuhn and Kuenne (1962)
- Important for more general location allocation problems

Euclidean Median



Dispersion and Orientation

Measures

- Standard Distance
- Major/minor axes
- Standard Deviational Ellipse

Standard Distance

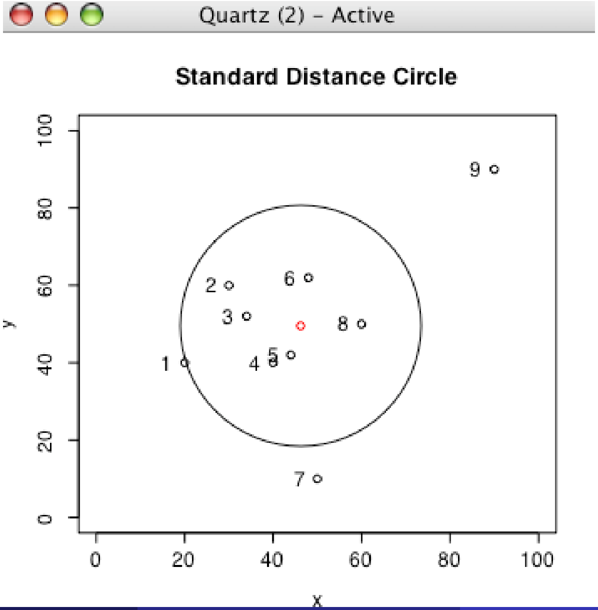
Euclidean Based

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x_m)^2}{n} + \frac{\sum_{i=1}^{n}(y_i - y_m)^2}{n}} \tag{8}$$

Uses

- Similar to standard deviation
- Combine with Mean Center for “outlier detection”
- Sensitive to extreme values

Standard Distance Circle



Standard Deviational Ellipse

Relative to Standard Distance

- Measures dispersion
- Sensitive to *shape* of distribution
- Measures dispersion in two dimensions

Components

- Angle of rotation
- Dispersion along major axis
- Dispersion along minor axis

Standard Deviational Ellipse

Major, minor axes

- Major axis defines the direction of maximum spread in the distribution
- Minor axis is orthogonal to major axis
- Minor axis defines the direction of minimum spread

Steps

1. Determine rotation angle of Y-axis
2. Calculate standard deviations for transposed axes
3. Determine length of axes
4. Determine area of the ellipse

Rotation Angle Θ

$$\Theta = ARCTAN\left\{\left(\sum_i (x_i - \bar{x})^2 - \sum_i (y_i - \bar{y})^2\right) + \left[\left(\sum_i (x_i - \bar{x})^2 - \sum_i (y_i - \bar{y})^2\right)^2 + 4\left(\sum_i (x_i - \bar{x})(y_i - \bar{y})\right)^2\right]^{1/2}\right\} / 2\sum_i (x_i - \bar{x})(y_i - \bar{y})$$

Standard Deviations On Transposed Axes

S_x

$$S_x = \sqrt{2\frac{(\sum_{i=1}^{n}(x_i - \bar{x})Cos(\Theta) - \sum_{i=1}^{n}(y_i - \bar{y})Sin(\Theta))^2}{n-2}} \quad (9)$$

S_y

$$S_y = \sqrt{2\frac{(\sum_{i=1}^{n}(x_i - \bar{x})Sin(\Theta) - \sum_{i=1}^{n}(y_i - \bar{y})Cos(\Theta))^2}{n-2}} \quad (10)$$

Ellipse Axes

Lengths

$$L_x = 2S_x \tag{11}$$

$$L_y = 2S_y \tag{12}$$

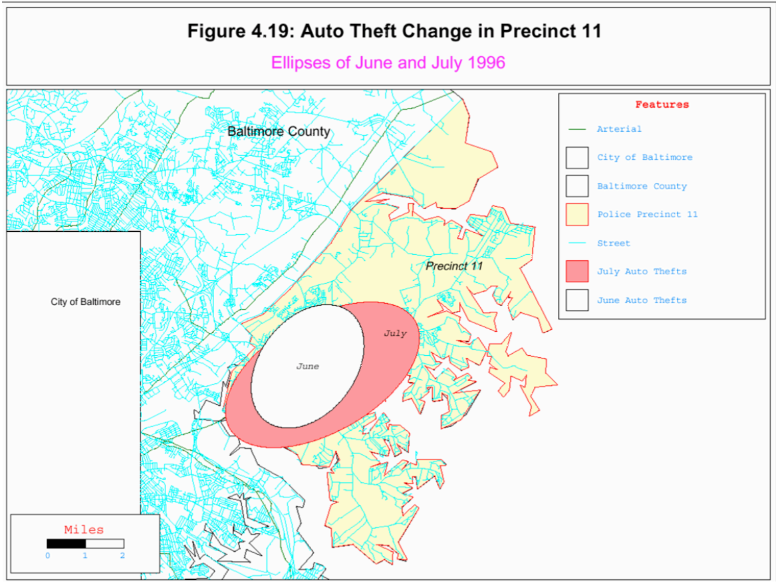
Mid Point

Mean Center of Point Pattern (x_m, y_m)

Area

$$A = \pi S_x S_y \tag{13}$$

Standard Deviational Ellipse

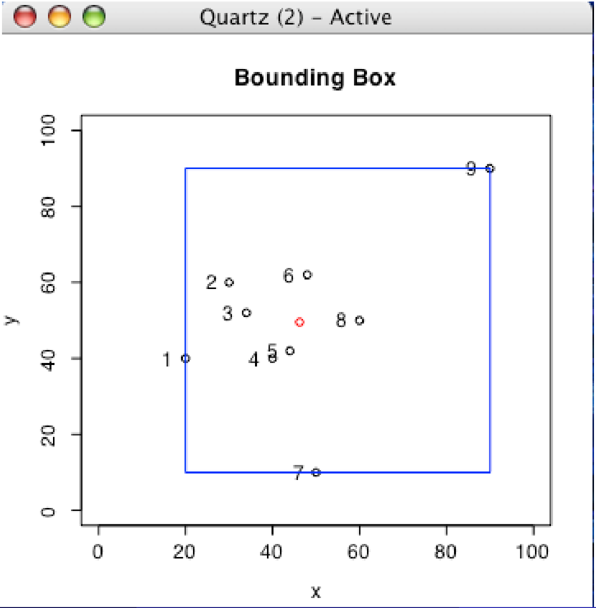


Shape Analysis of Point Patterns

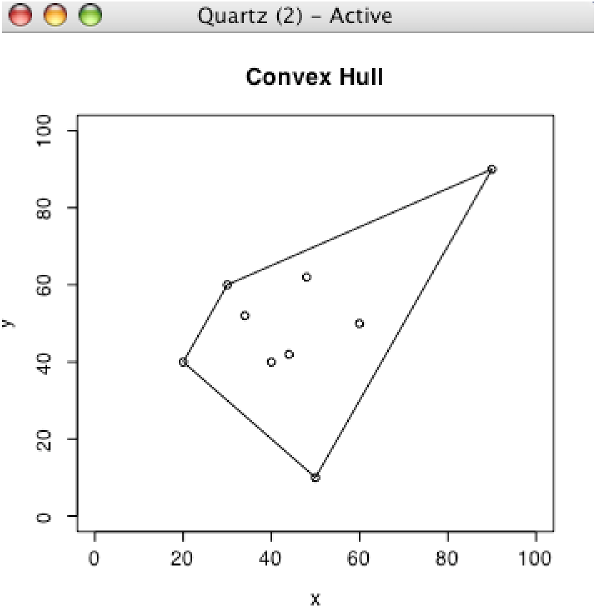
Geometry

- Bounding Box
- Convex Hulls

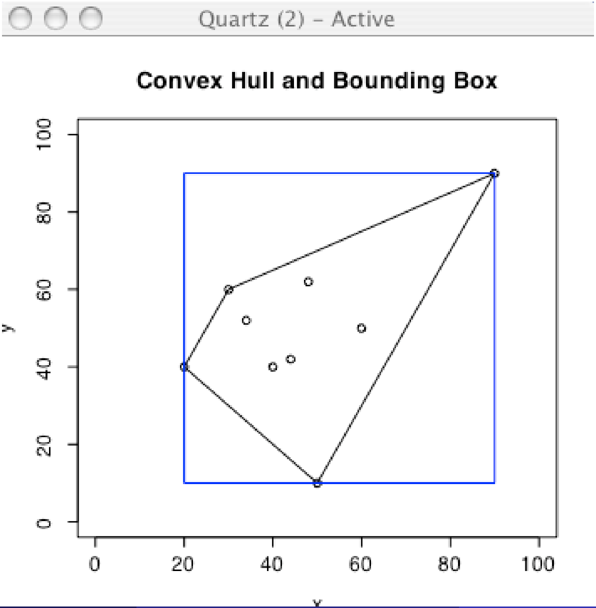
Bounding Box



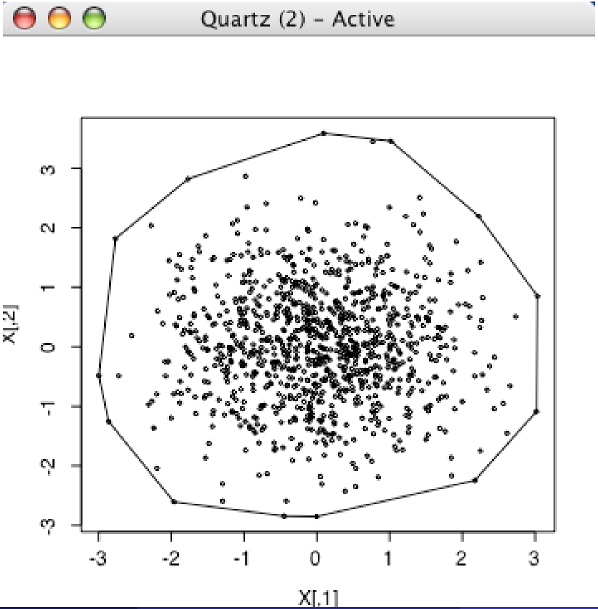
Convex Hull



Convex Hull and Bounding Box



Convex Hull (Large n)



Nested Convex Hulls