

Geographic Information Analysis

Spatial Weights

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Outline

- Weights General Concepts
- Contiguity Weights
- Distance Based Weights
- General Weights
- Guidance

Weights General Concepts

Why Spatial Weights?

- Identification Problem
 - total number of interactions is $N(N-1)/2$
 - all possible unique pairs i-j
 - only N observations in a cross-section
- Incidental Parameter Problem
 - number of parameters increases in size sample, $O(N^2)$
 - more data not the solution

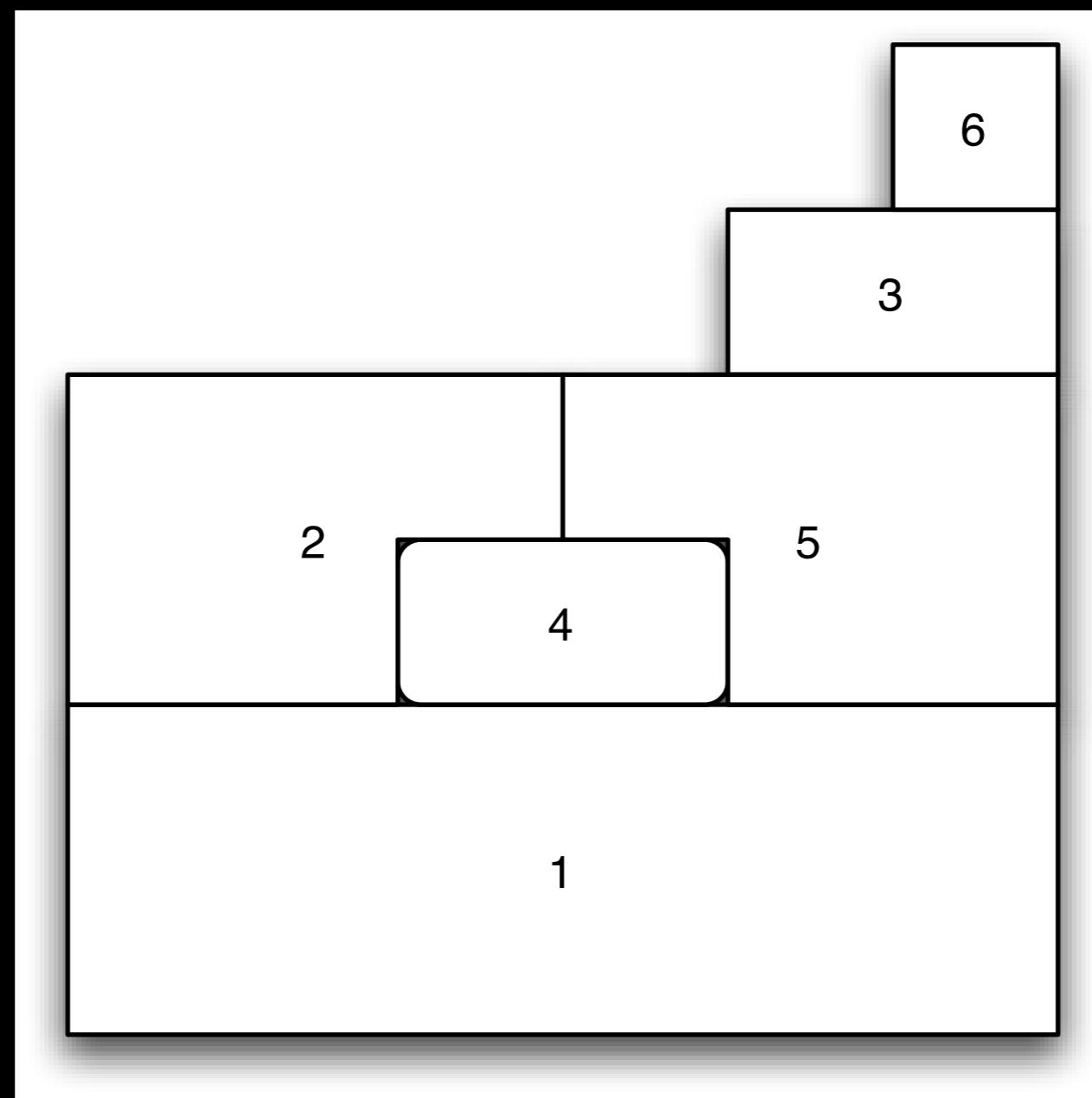
Problem

- Spatial Correlation
 - $C[y_i, y_j] \neq 0 \forall i \neq j$
 - $N(N-1)/2$ covariances
 - Only N observations

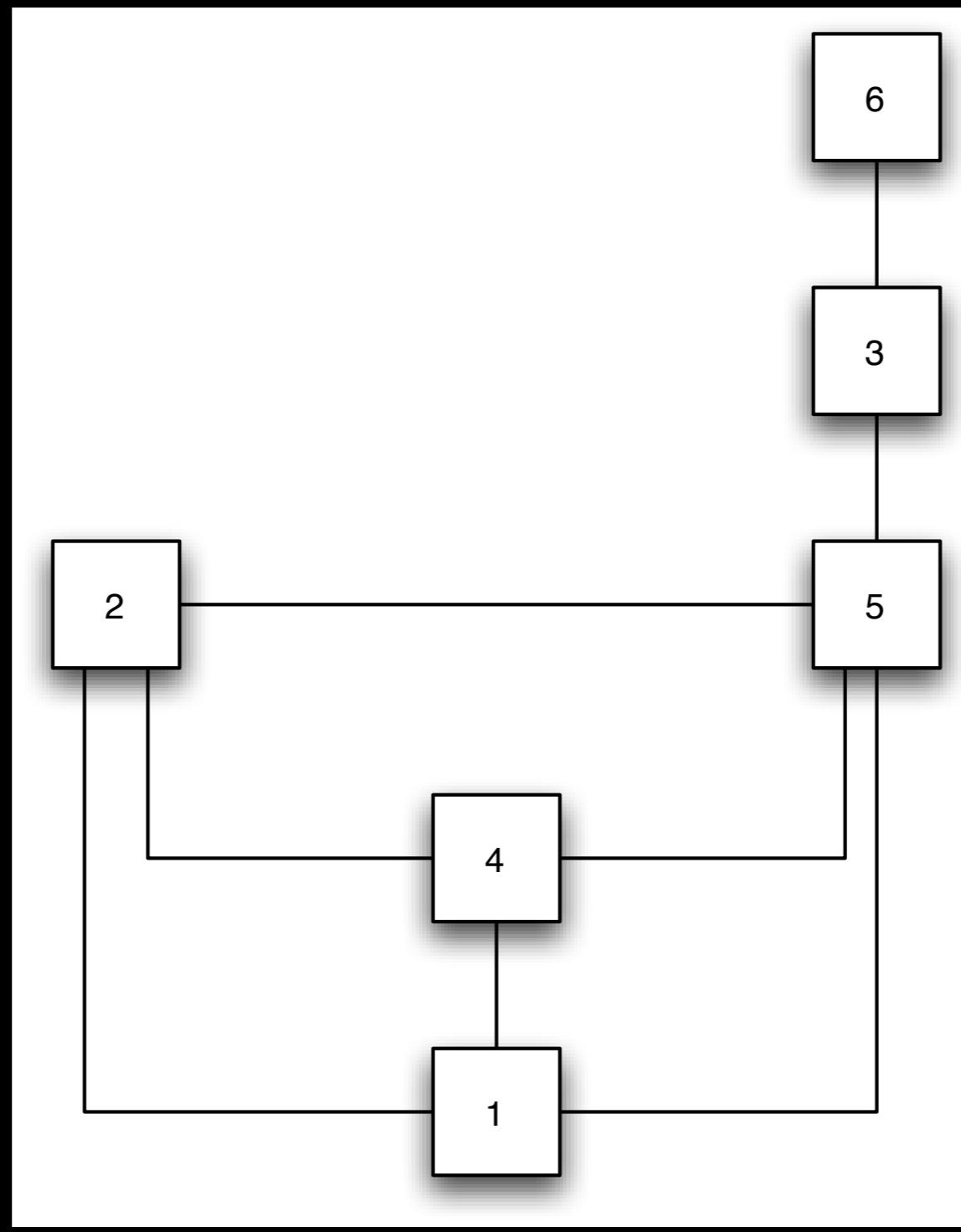
Solution

- Impose Structure of the Problem
 - set some interactions to zero
 - only “neighbors” interact directly
 - constrain the number of neighbors
- Assume a Single Parameter
 - spatial autocorrelation coefficient

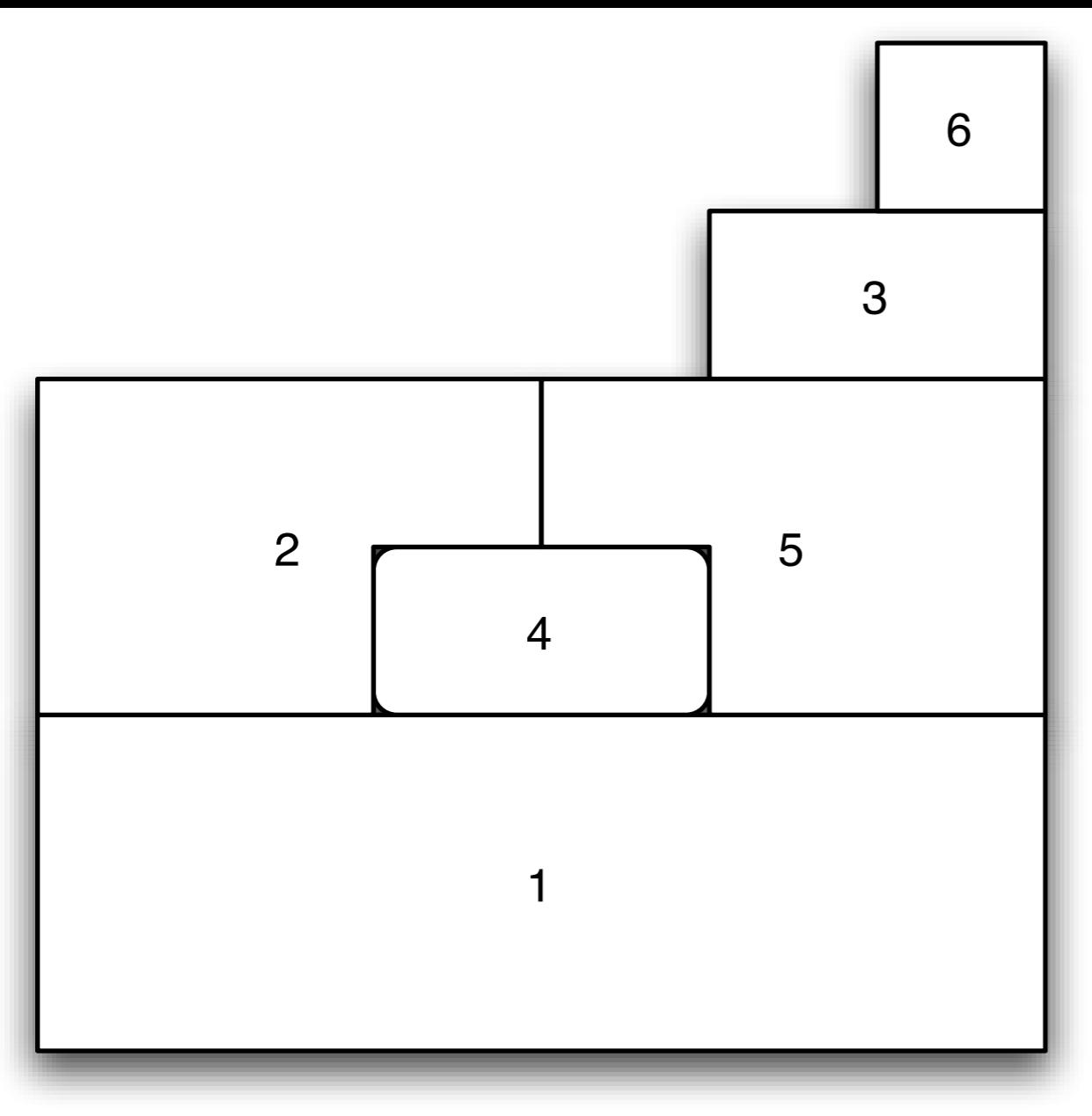
Irregular Lattice (Polygons)



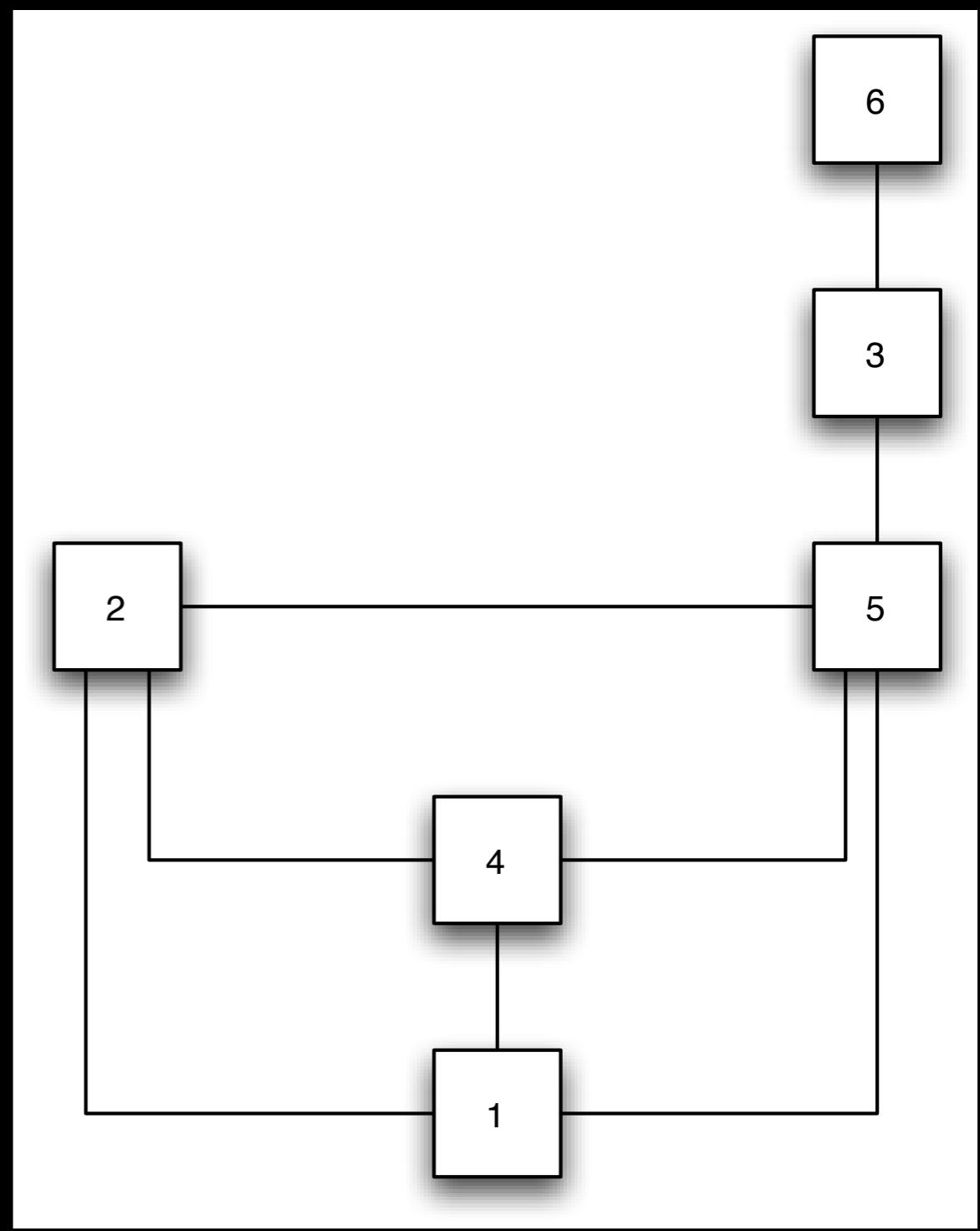
Neighbor Structure as a Graph



Map



Graph



Spatial Weights Matrix

- Definition
 - N by N positive matrix \mathbf{W} , elements w_{ij}
 - w_{ij} nonzero for neighbors, 0 otherwise
 - $w_{ii} = 0$, no self-neighbors

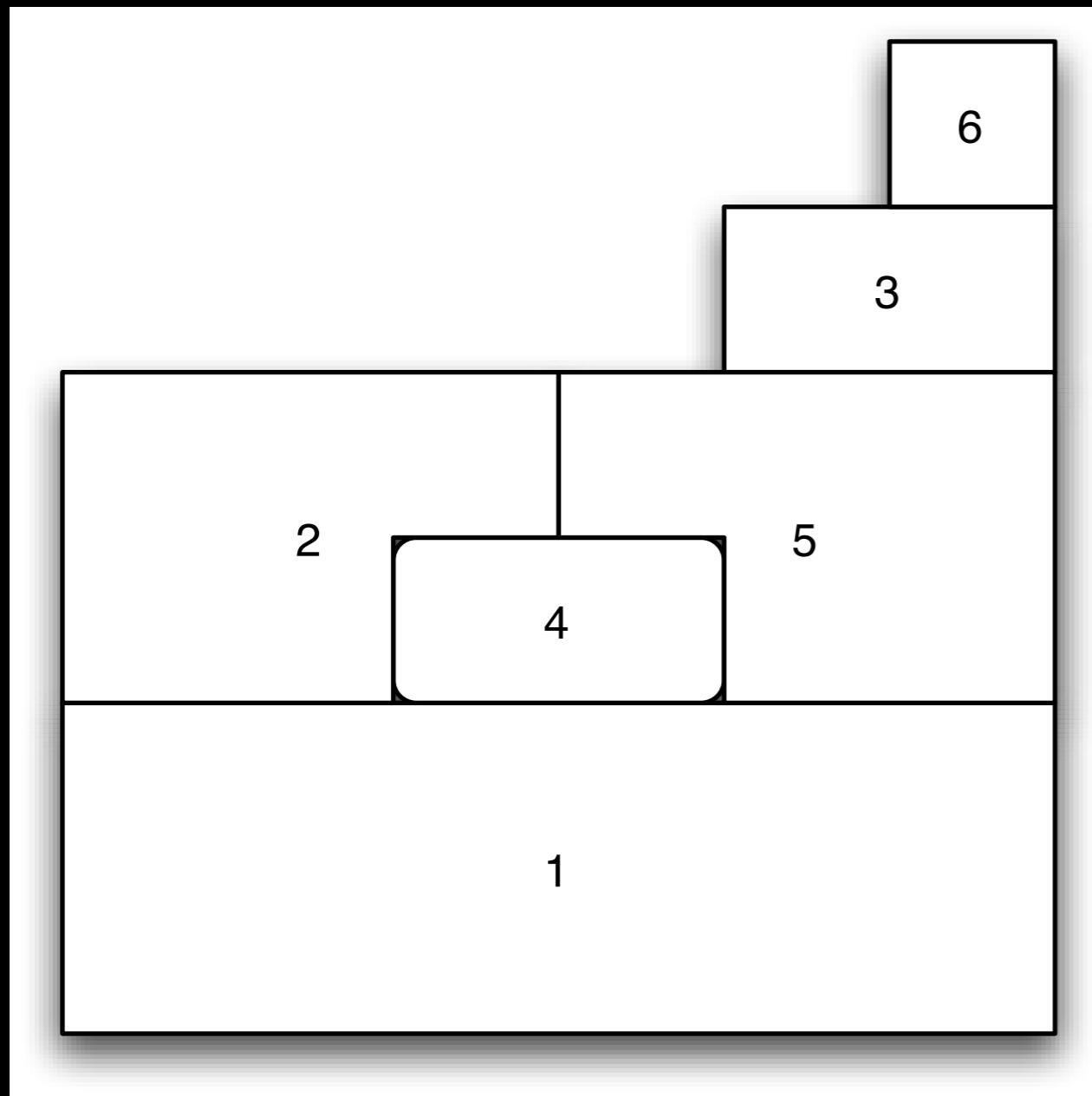
$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{bmatrix}$$

Binary Contiguity

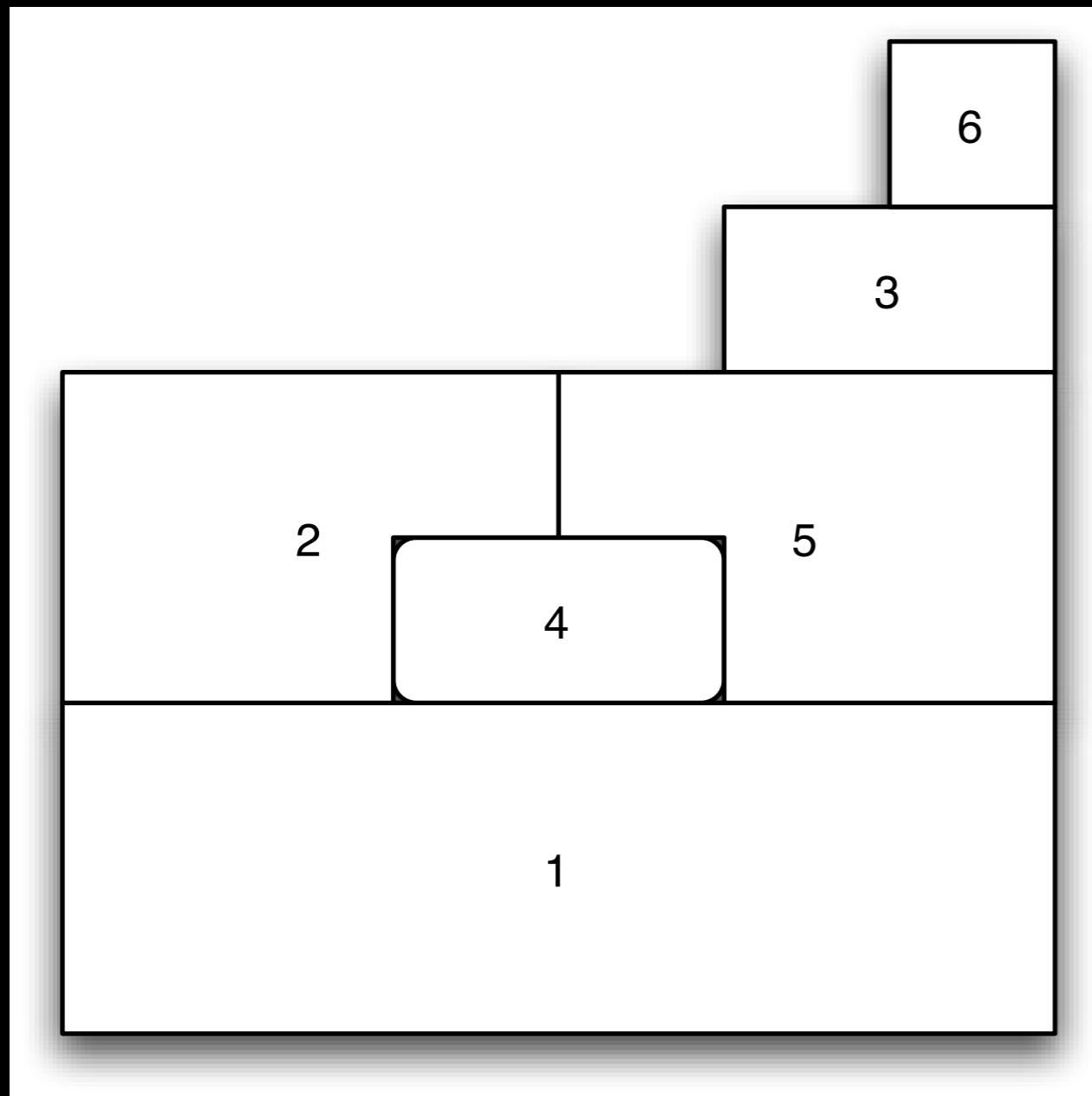
- Weights are 0 or 1
 - for i, j neighbors, $w_{ij} = 1$
- Example, using common boundary

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Example

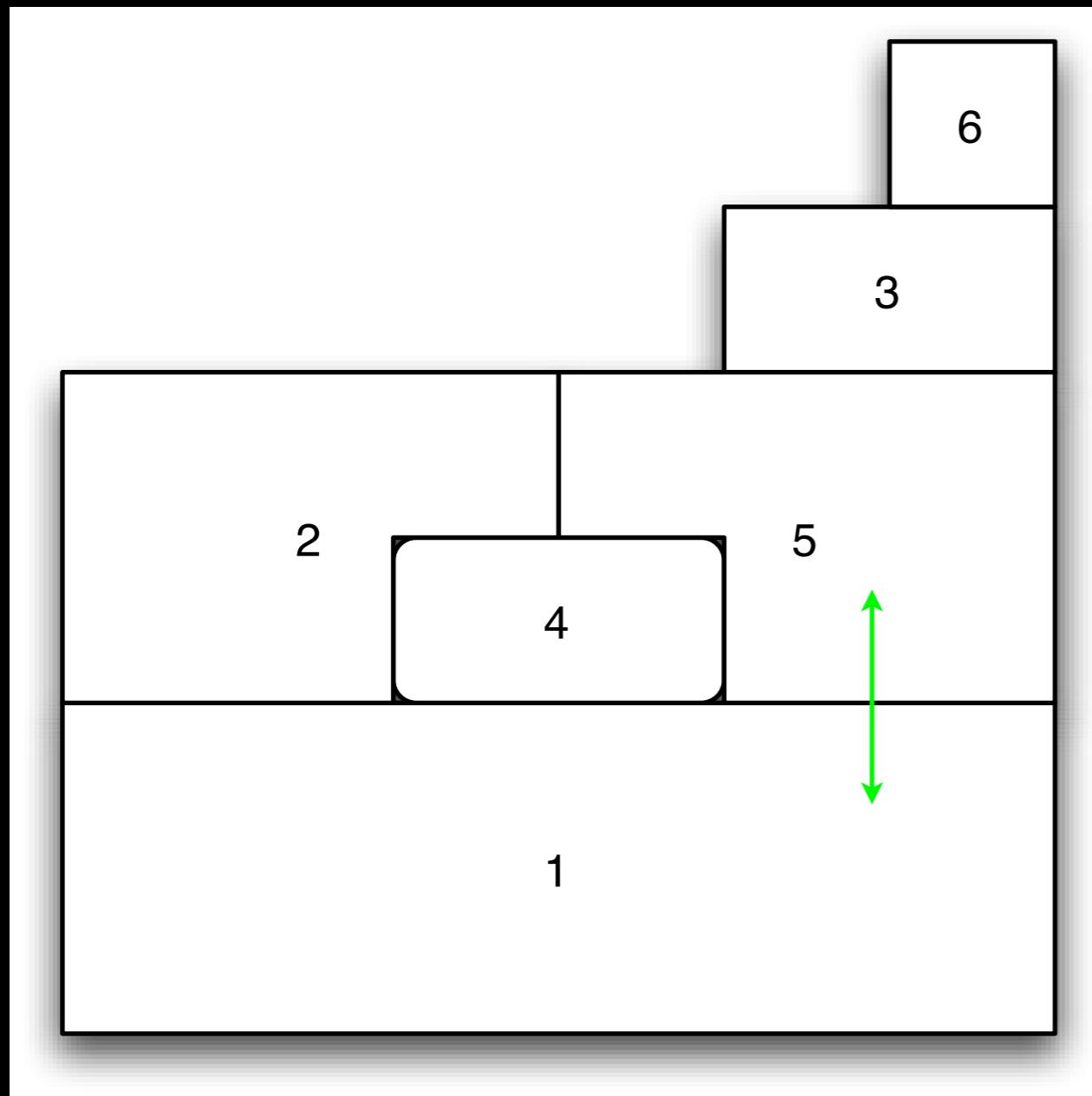


Example



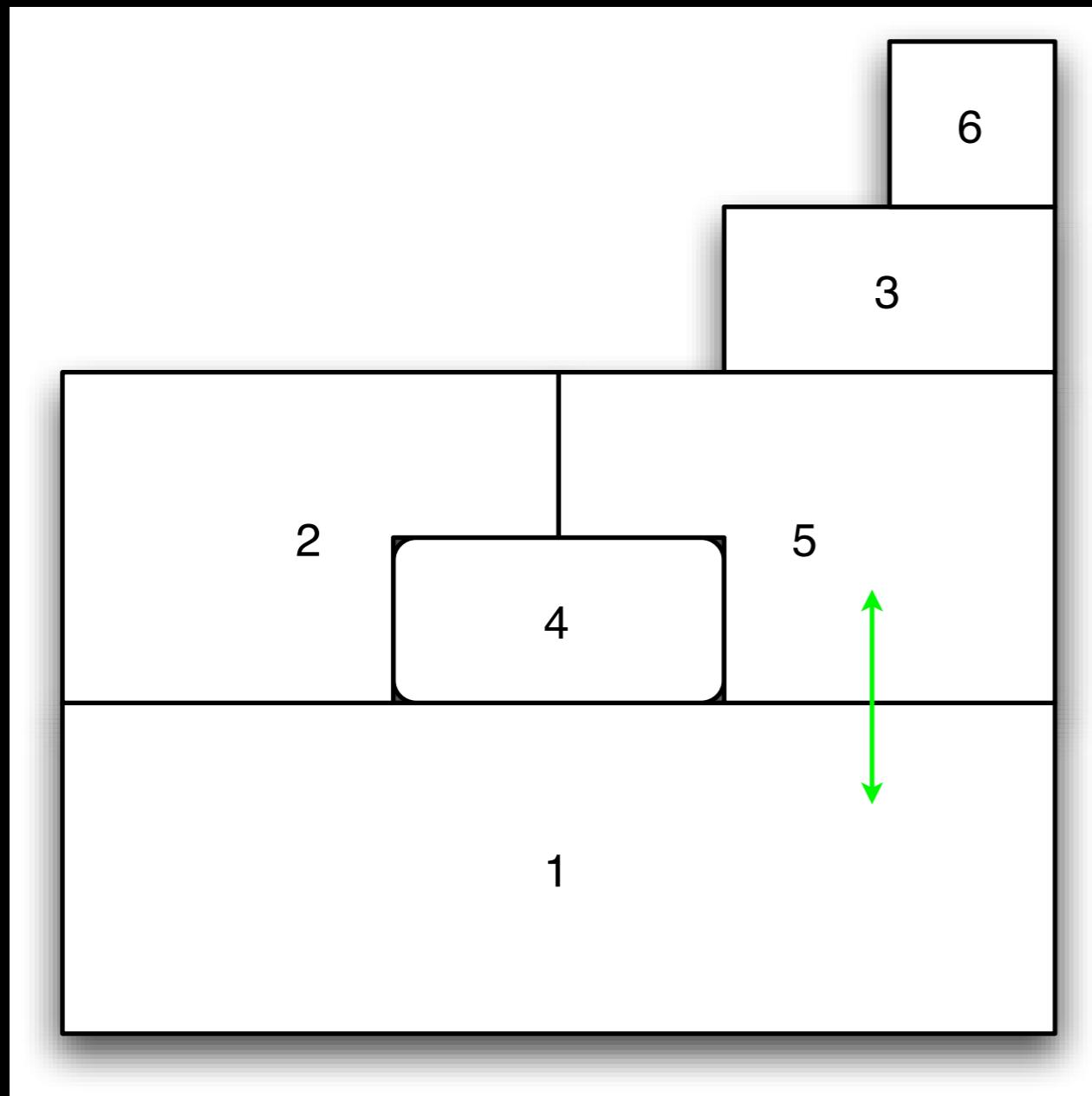
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Example



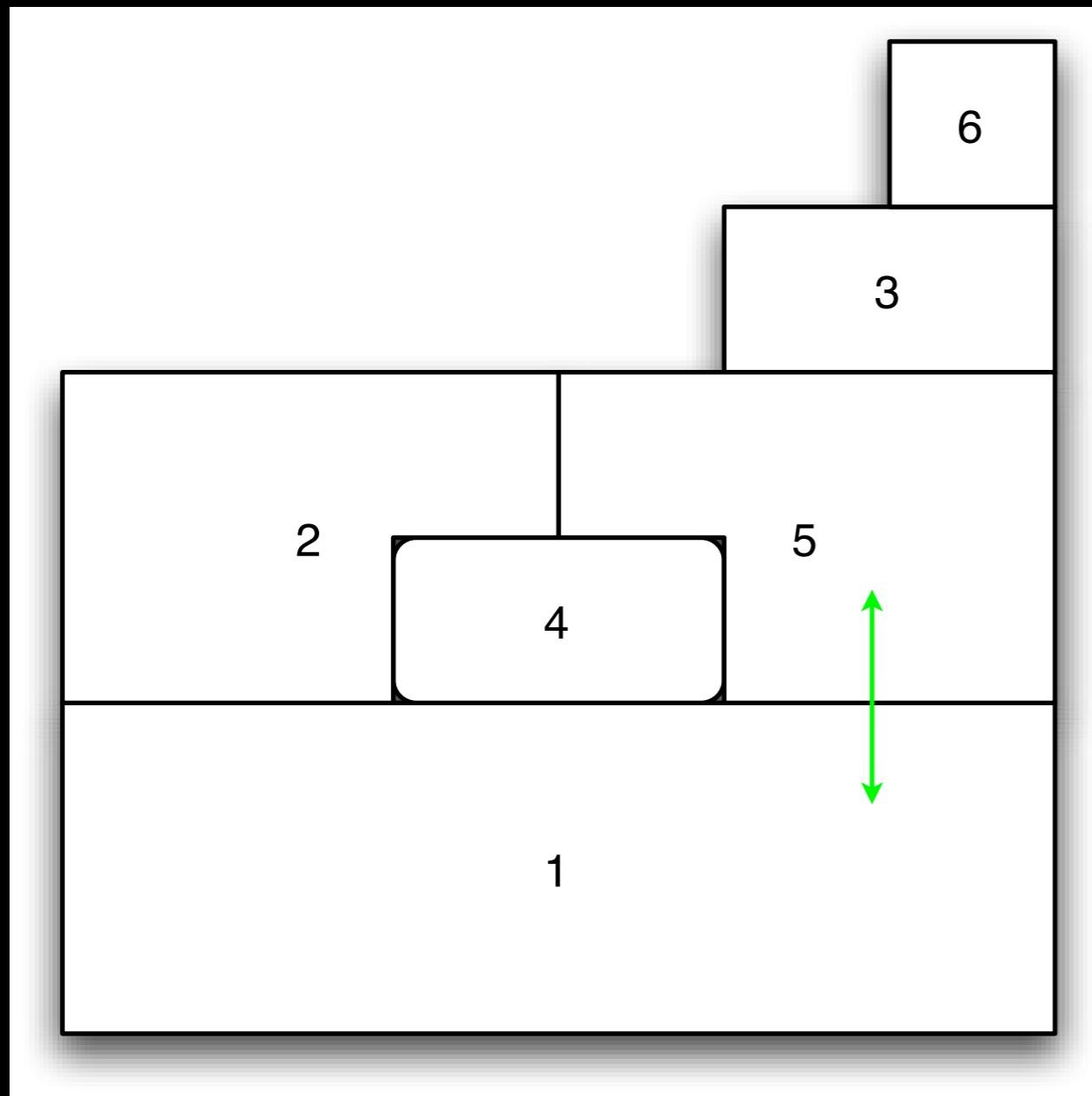
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How to Define W

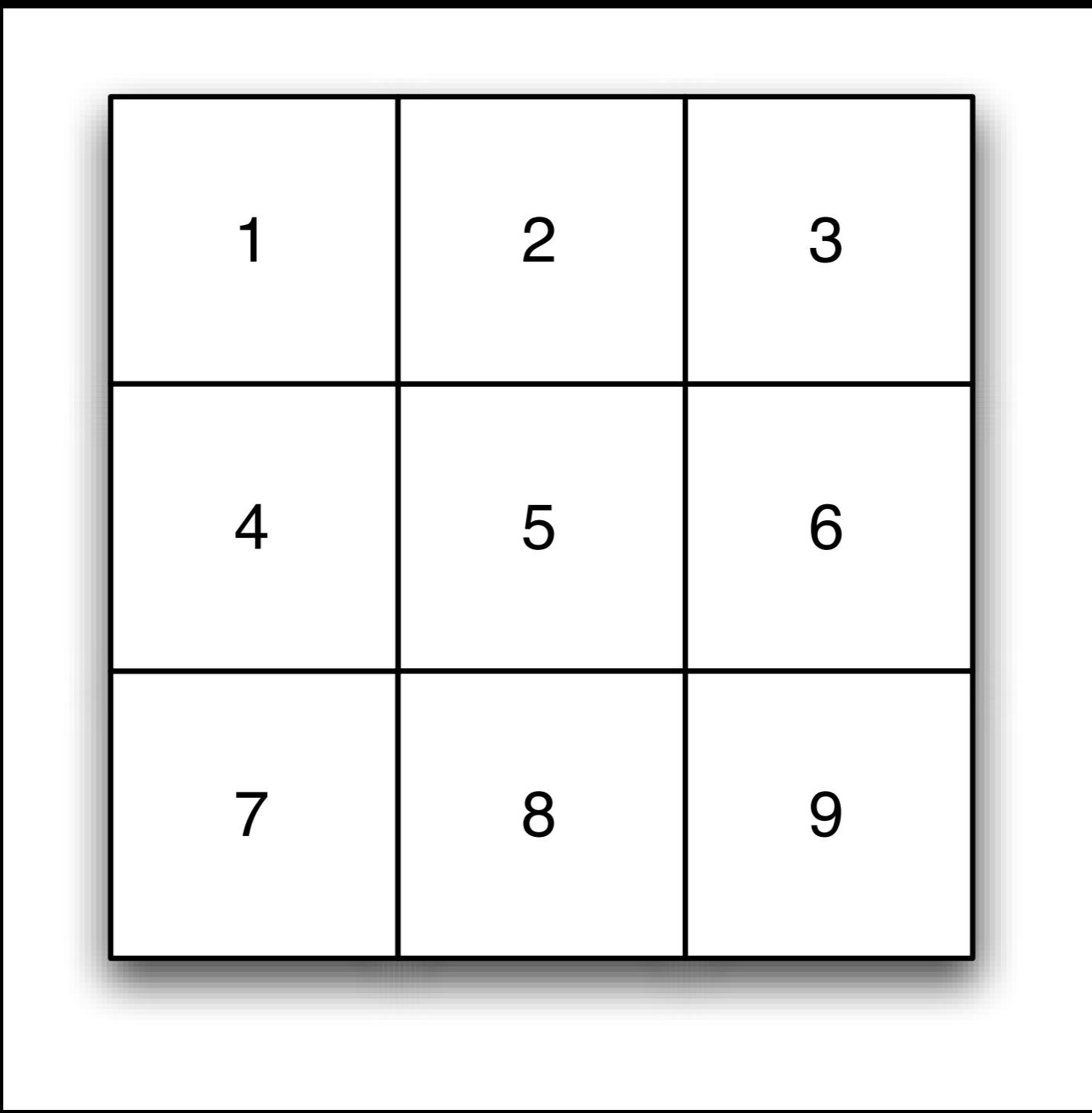
- Geographic Weights
 - contiguity
 - distance
 - general
 - graph-based weights
- Socio-Economic Weights

Contiguity

Contiguity Weights

- Contiguity
 - sharing a common boundary of non-zero length
- What is a Non-zero Boundary?
- Three Views of Contiguity
 - rook
 - bishop
 - queen

Example Layout: Regular Lattice N=9



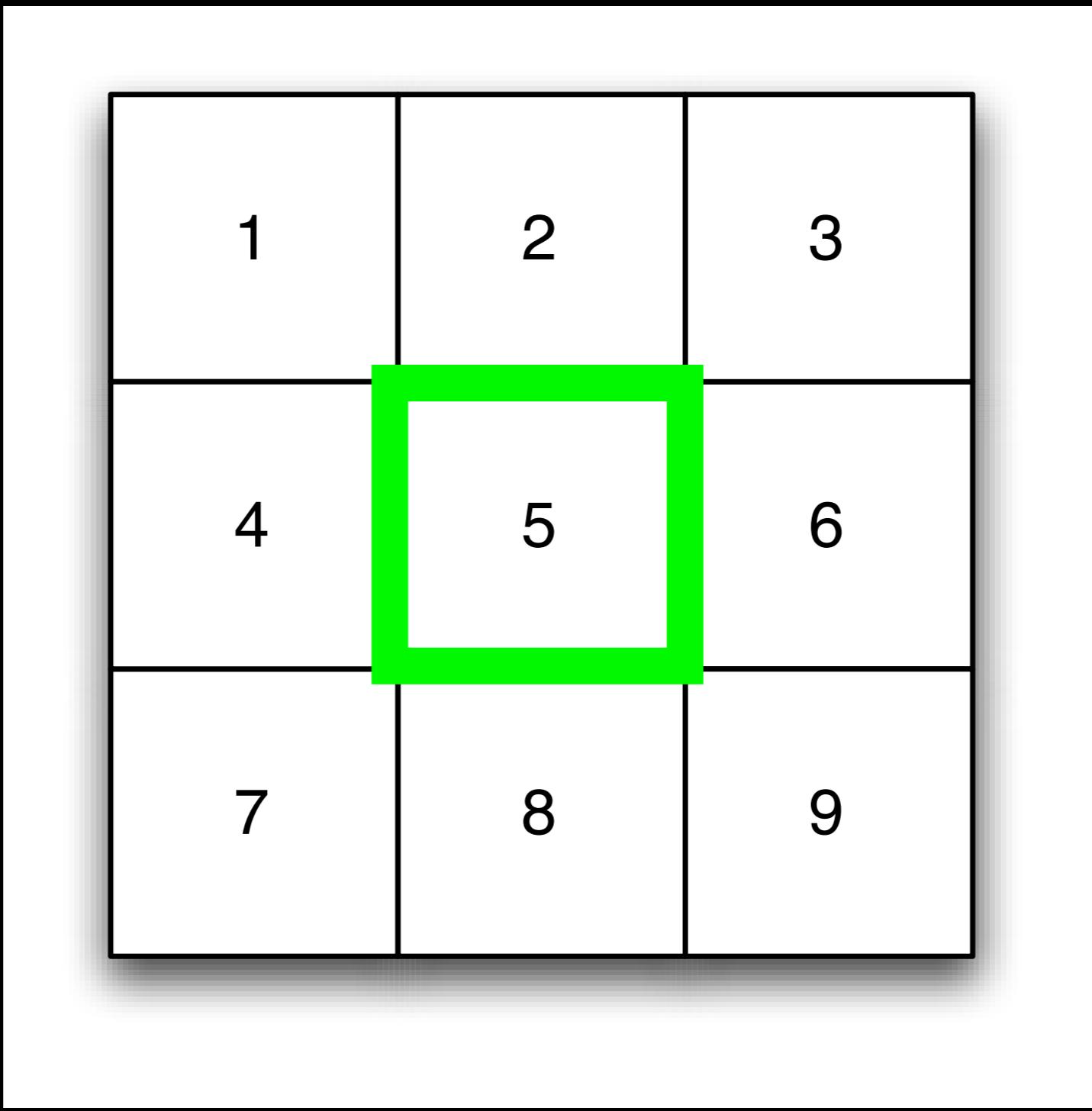
Example Layout: Regular Lattice N=9

Focus on
Central
Location

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

Example Layout: Regular Lattice N=9

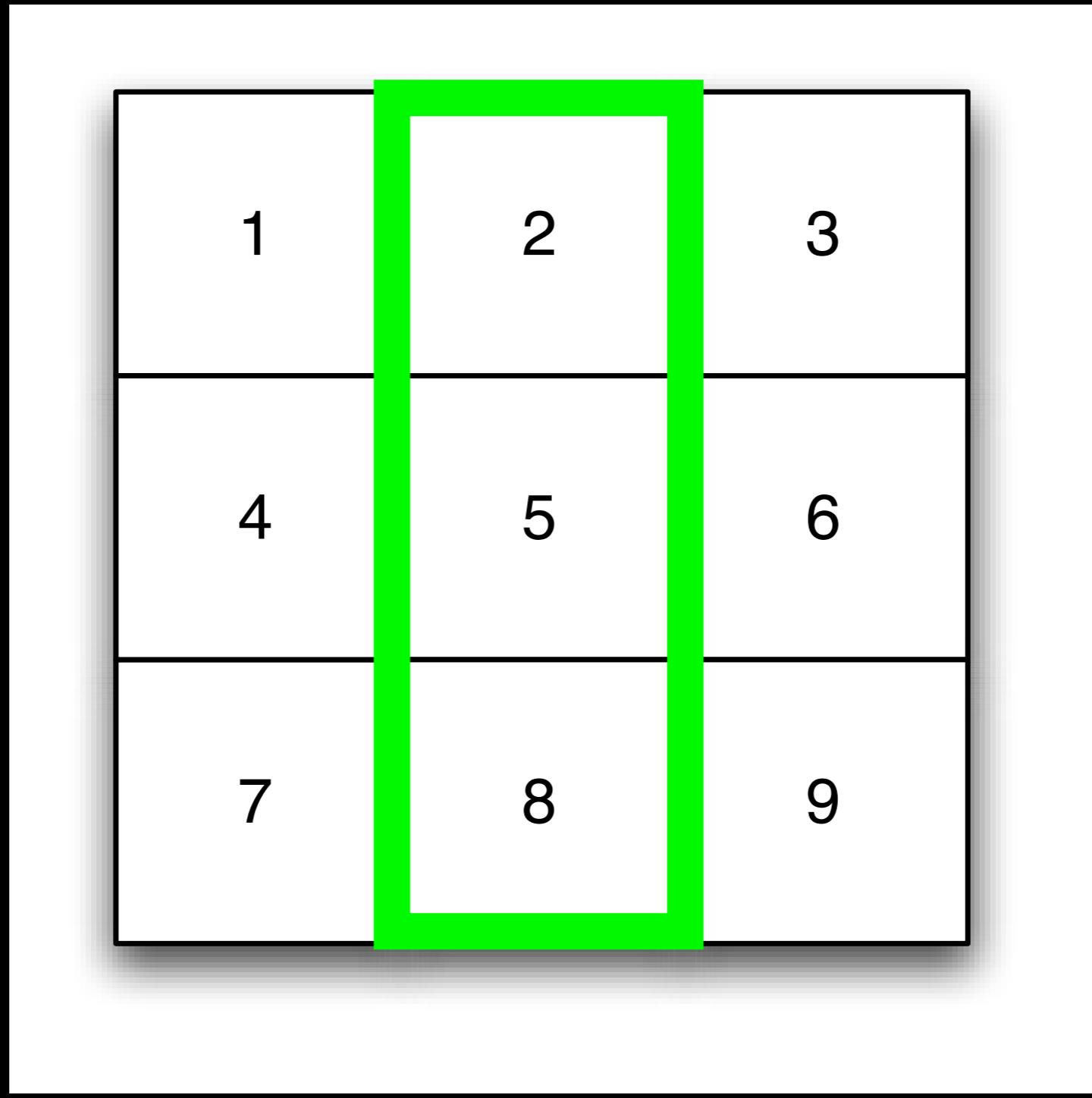
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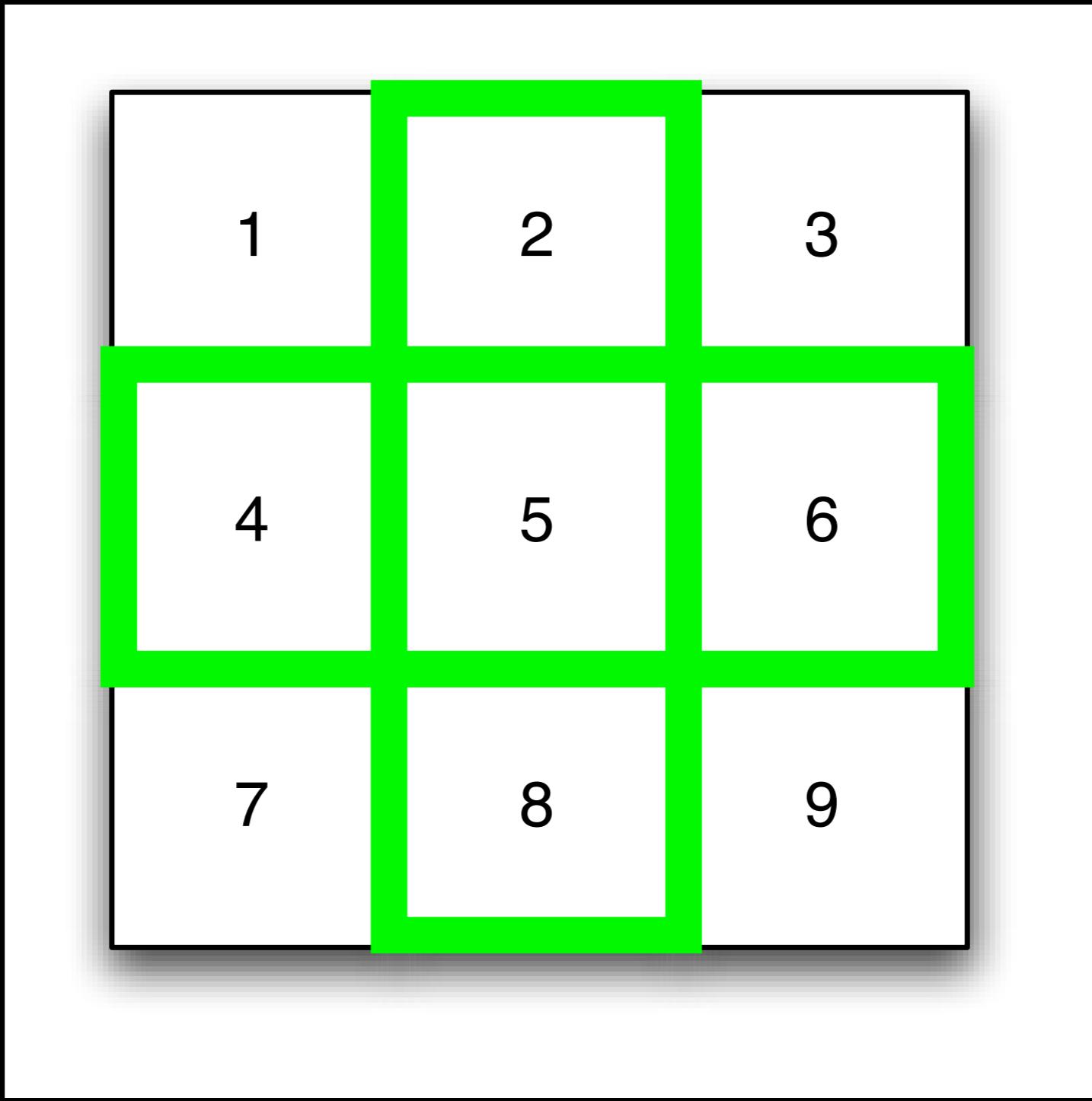
Rook Contiguity

| | | |
|---|---|---|
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| 4 | 5 | 6 |
| 7 | 8 | 9 |

Rook Contiguity



Rook Contiguity

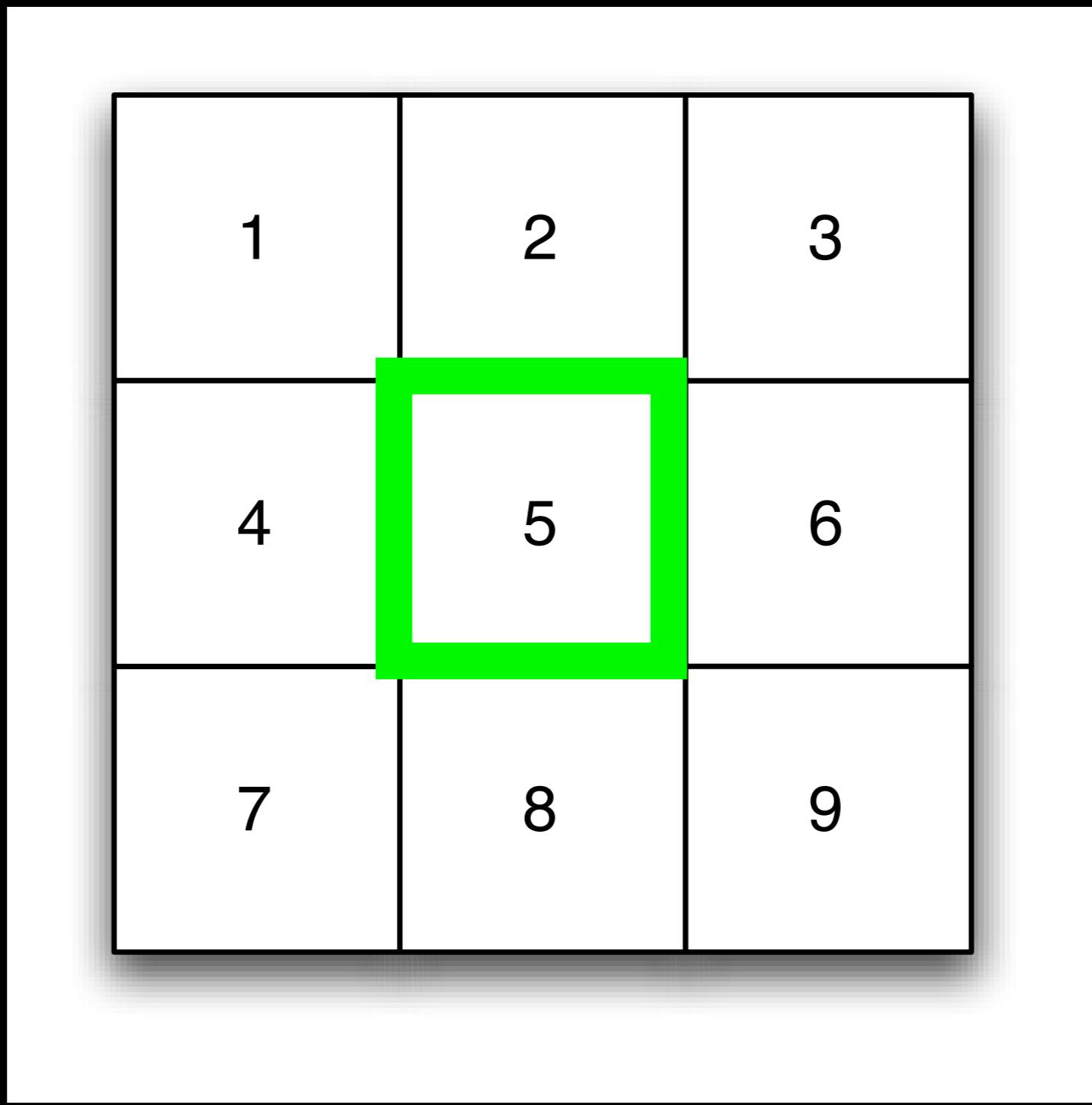


Rook Weights

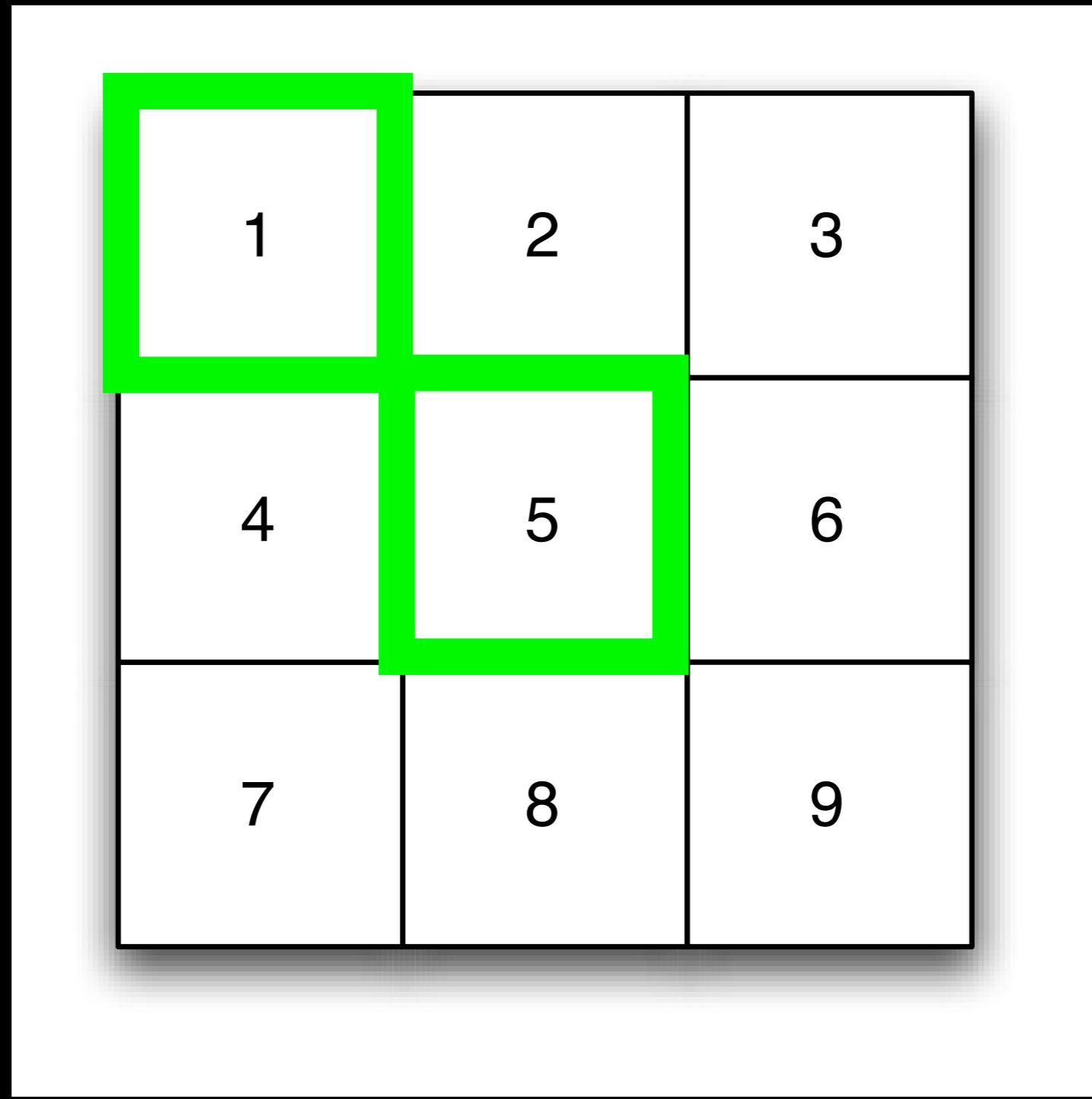
$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Neighbors for 5: 2, 4, 6, 8

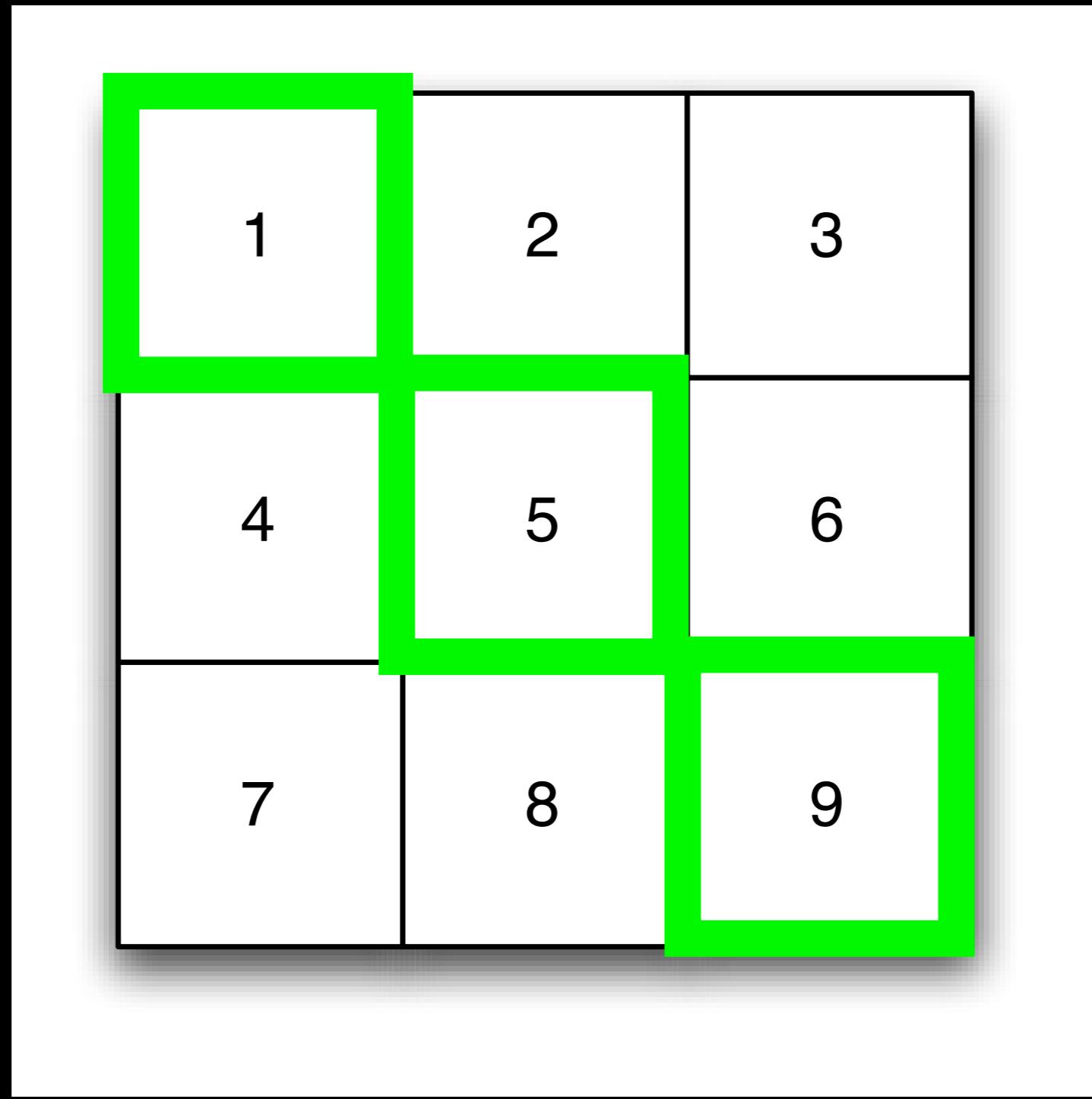
Bishop Contiguity



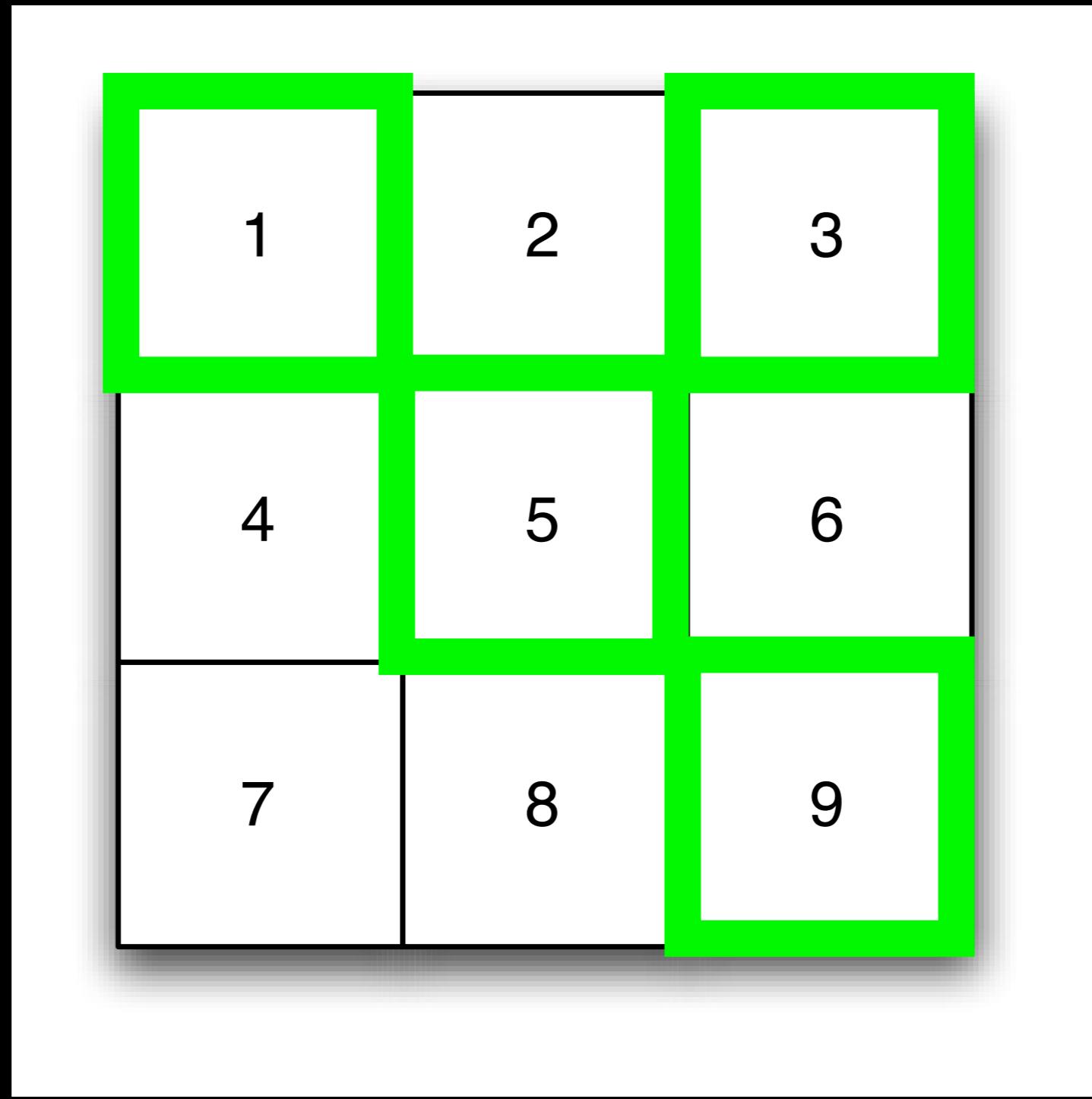
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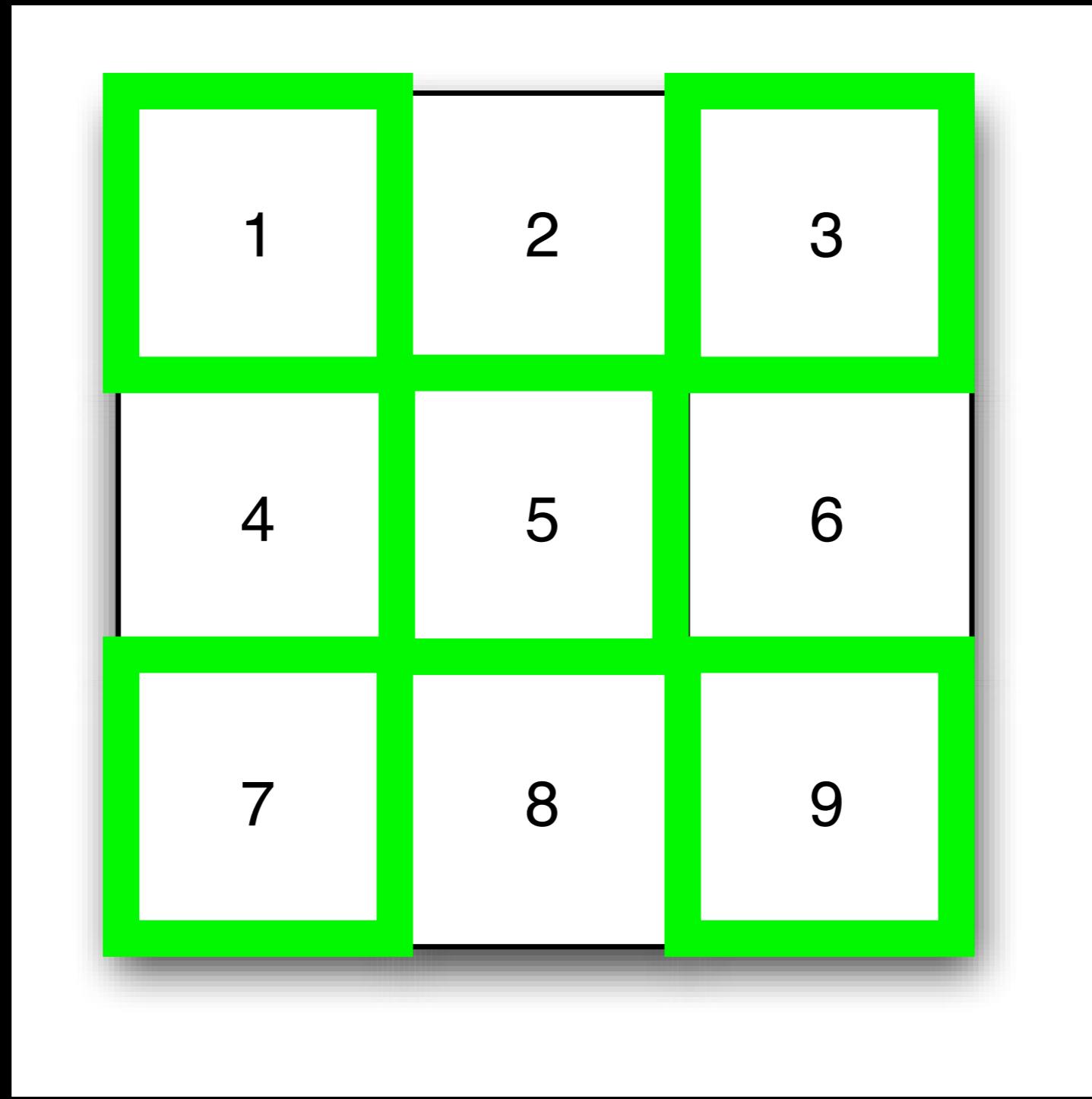
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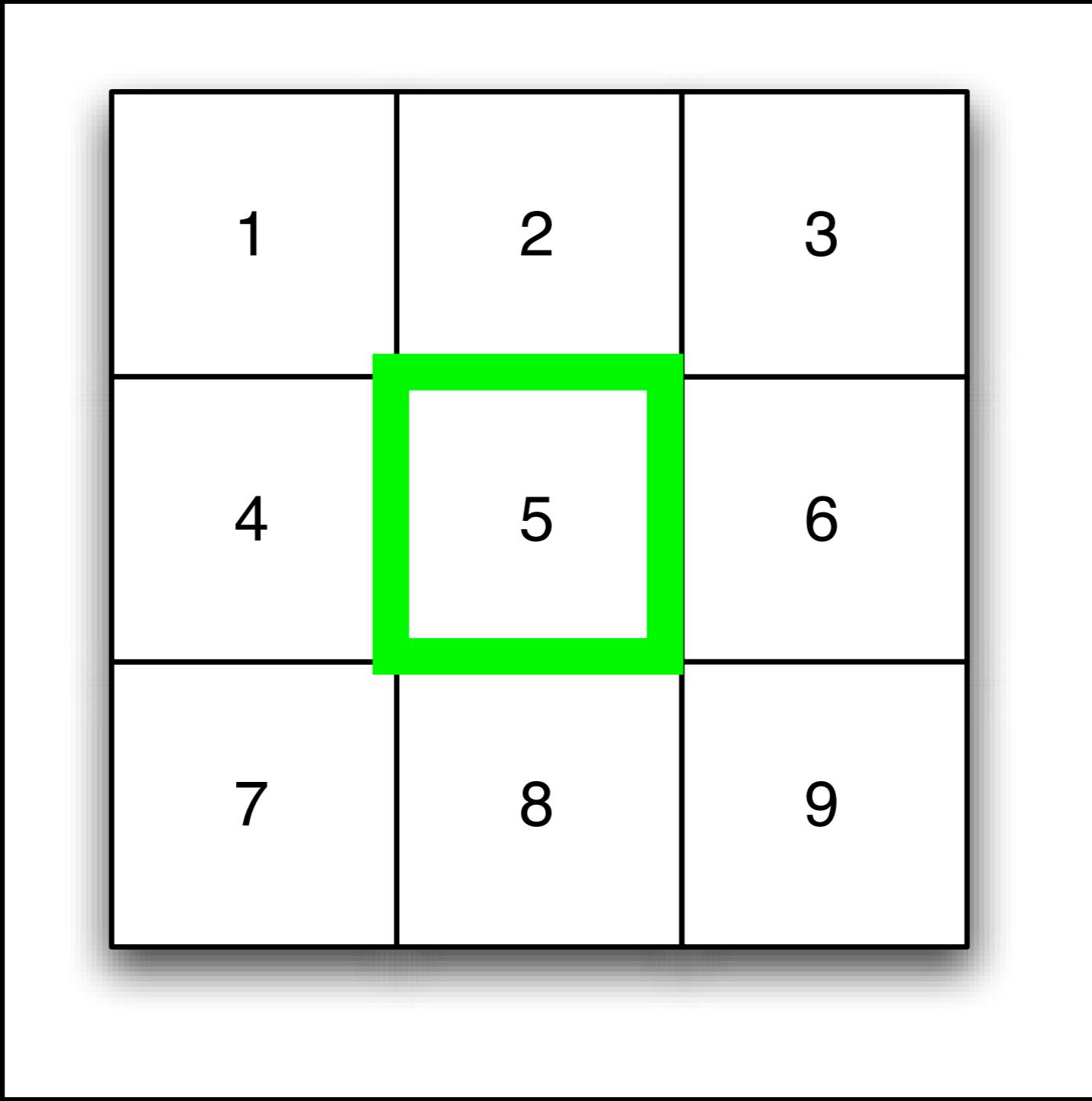


Bishop Weights

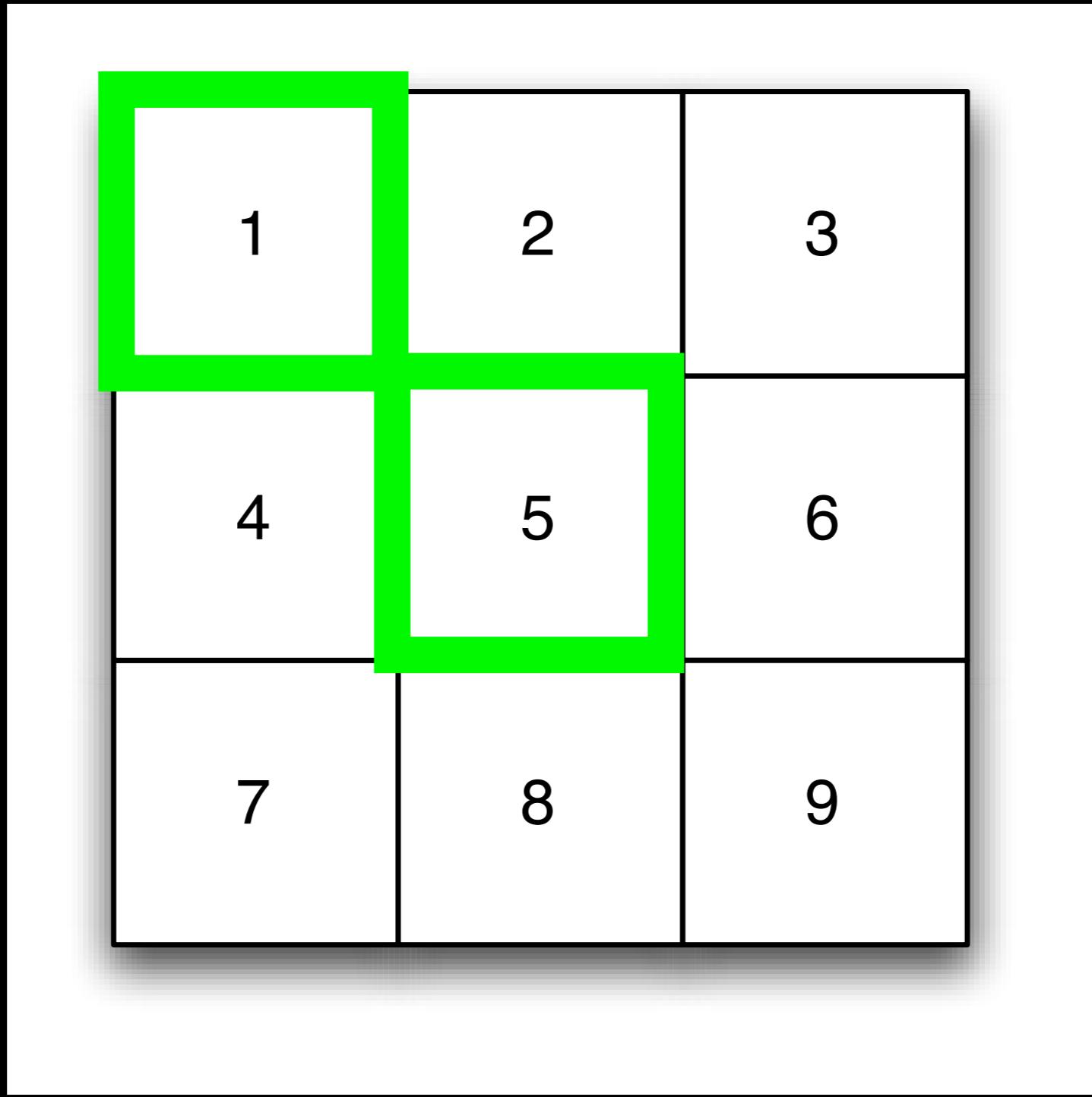
$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Neighbors for 5: 1, 3, 7, 9

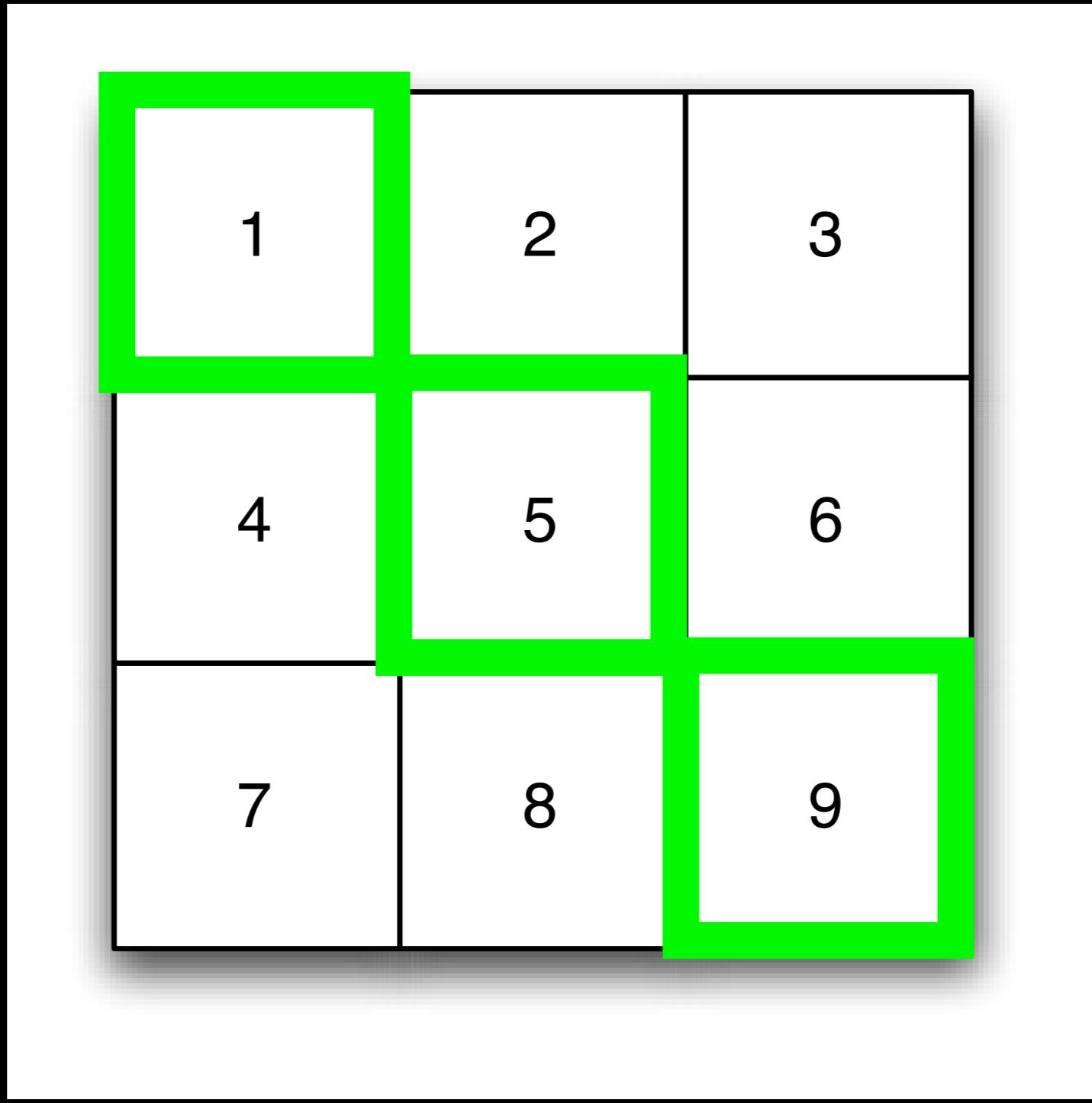
Queen Contiguity



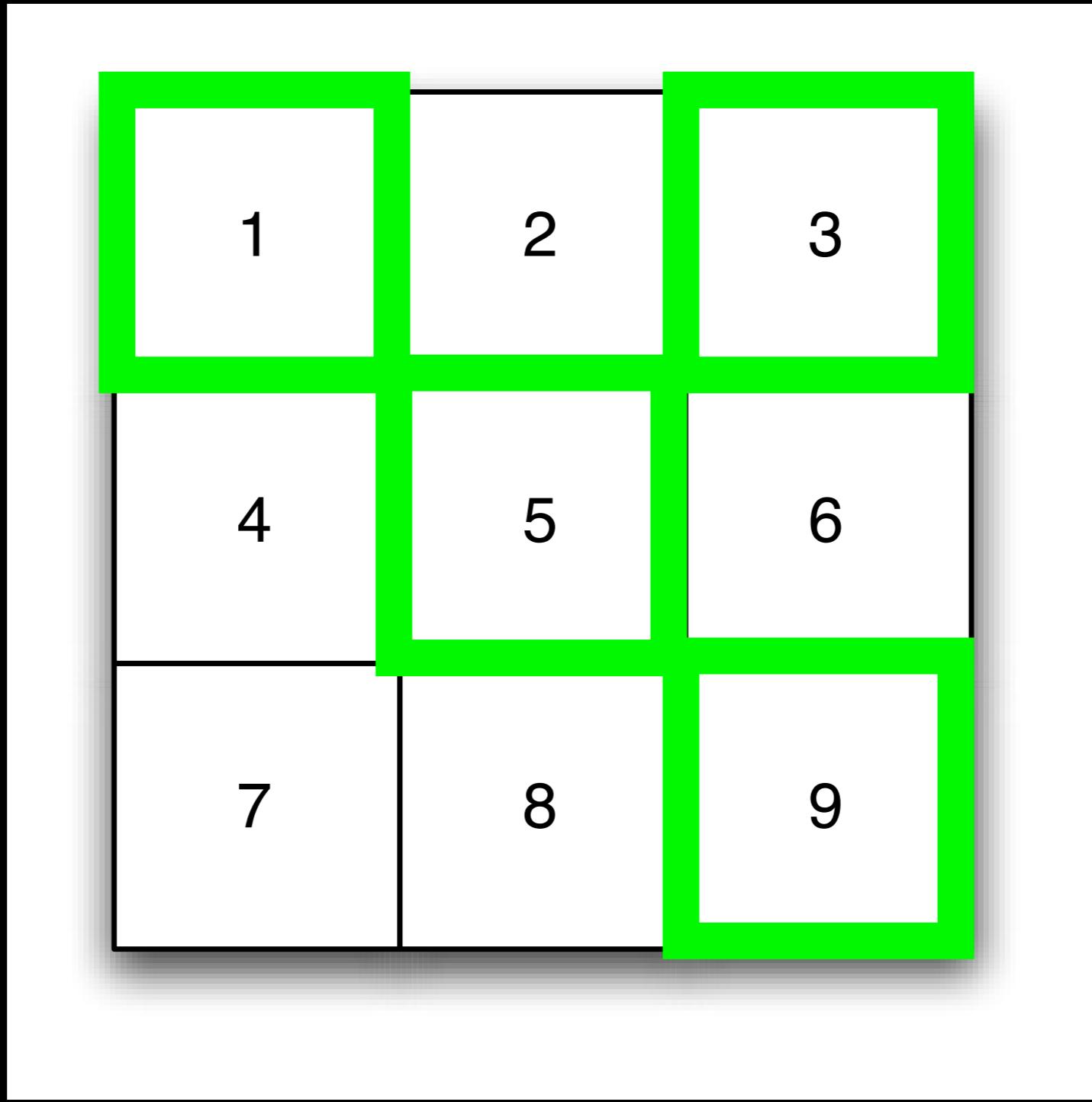
Queen Contiguity



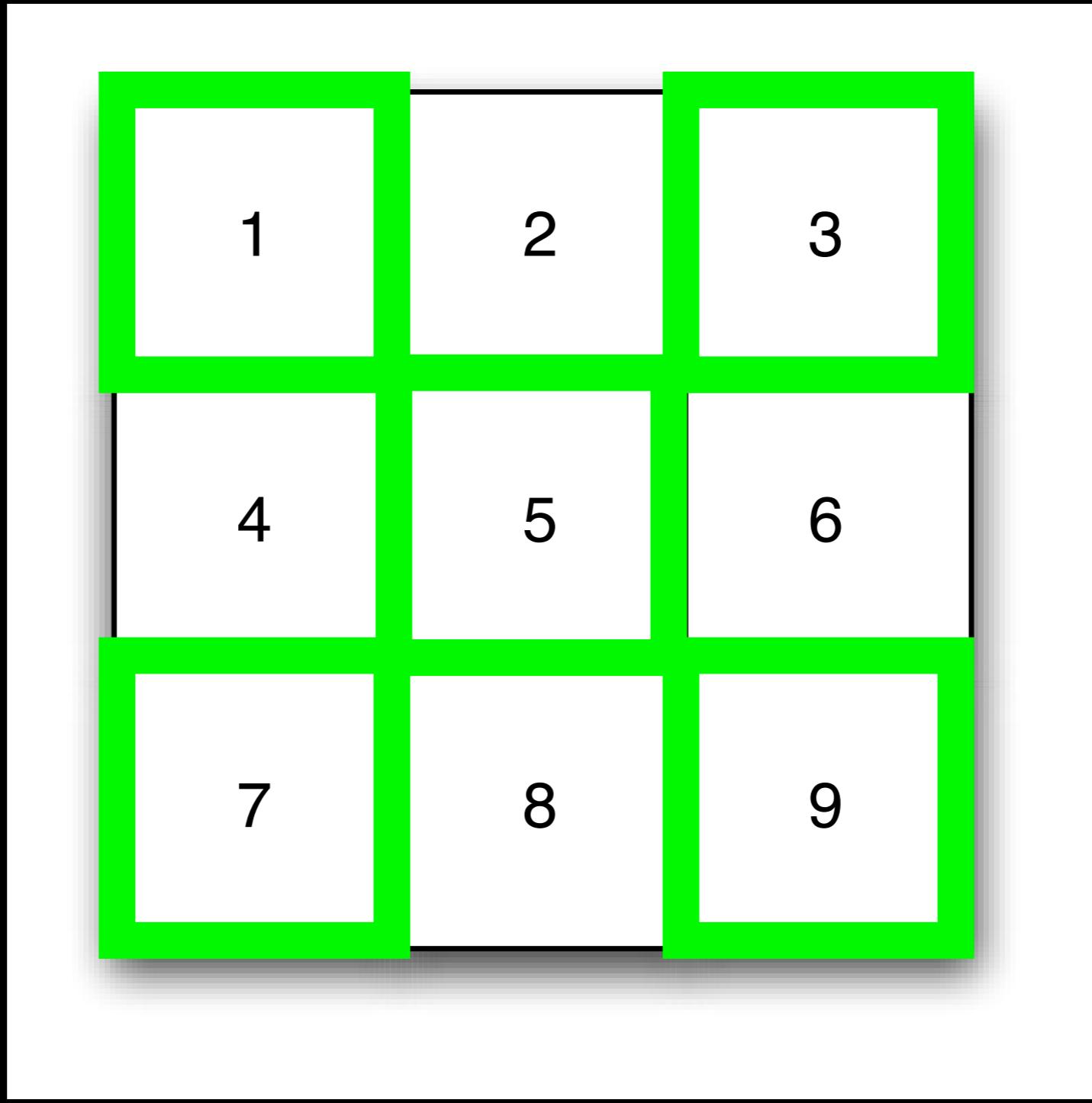
Queen Contiguity



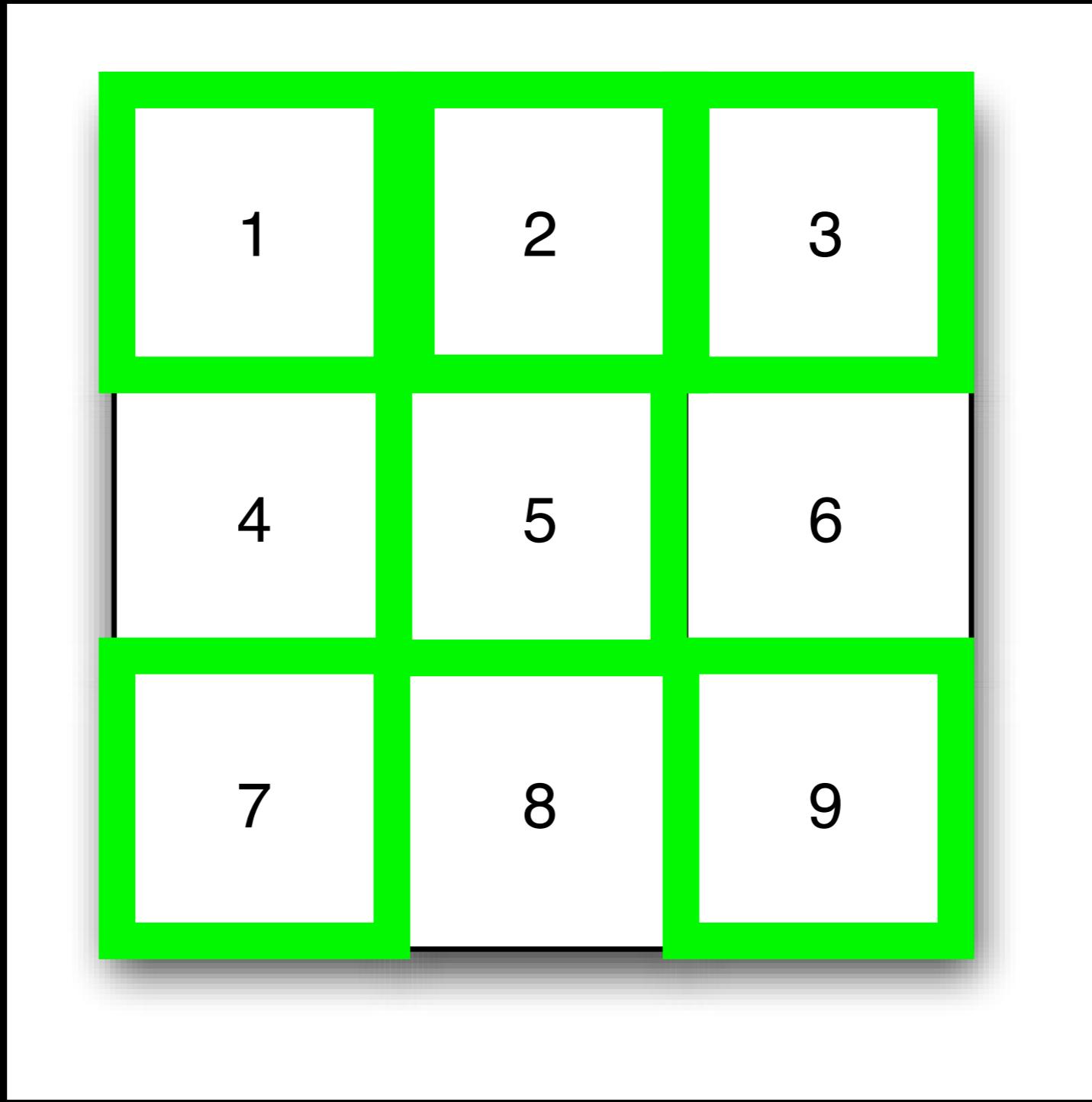
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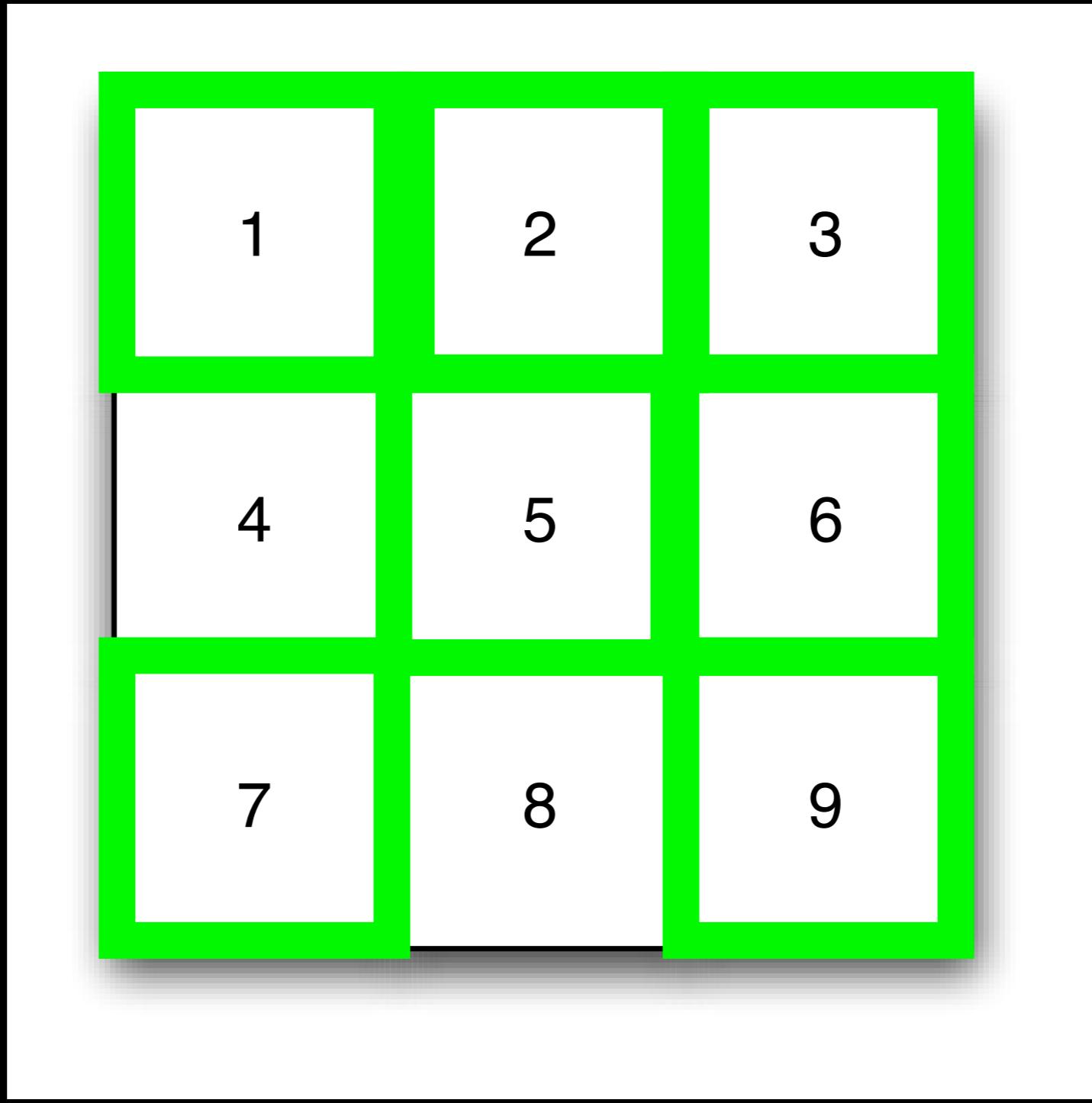
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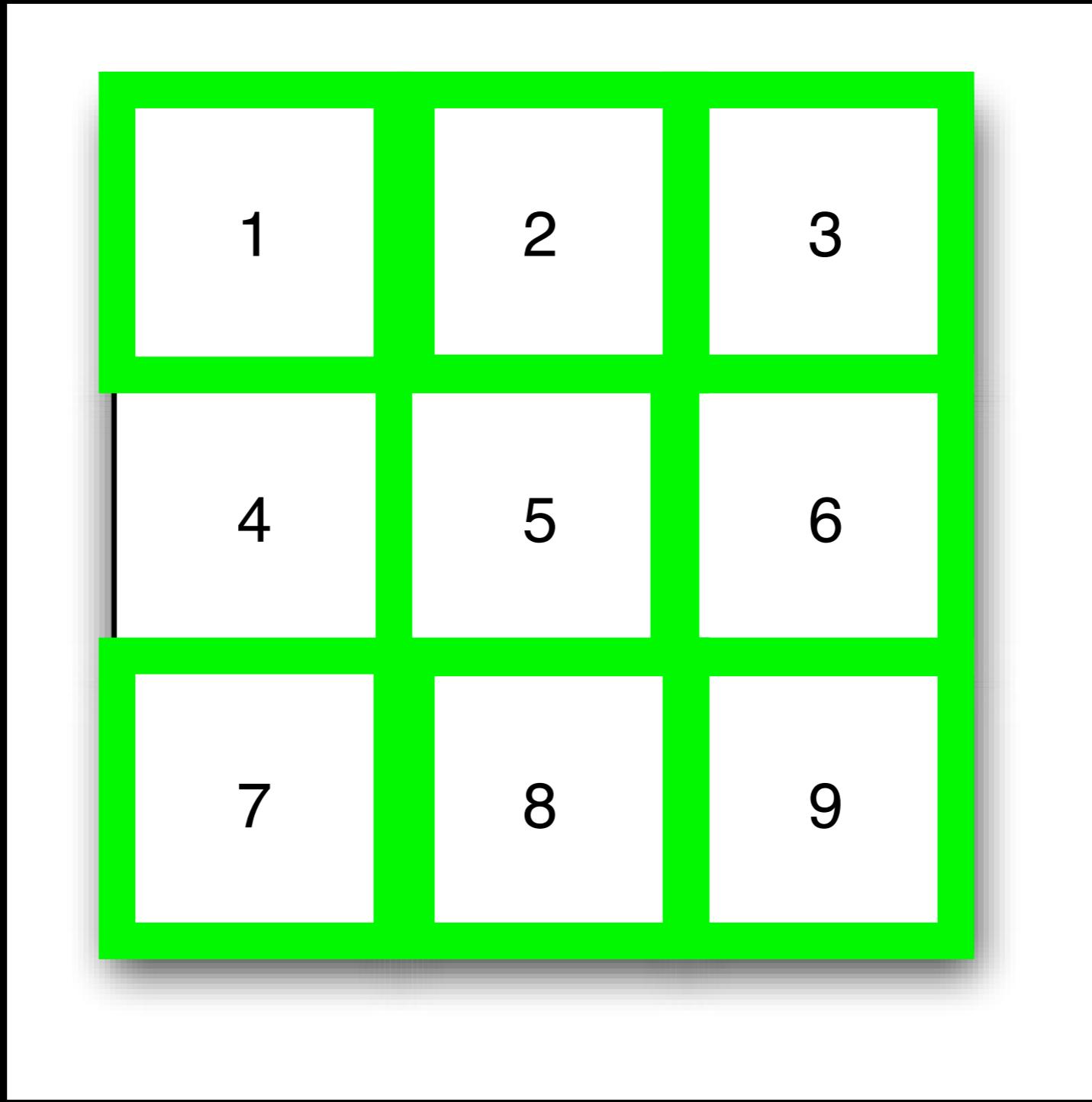
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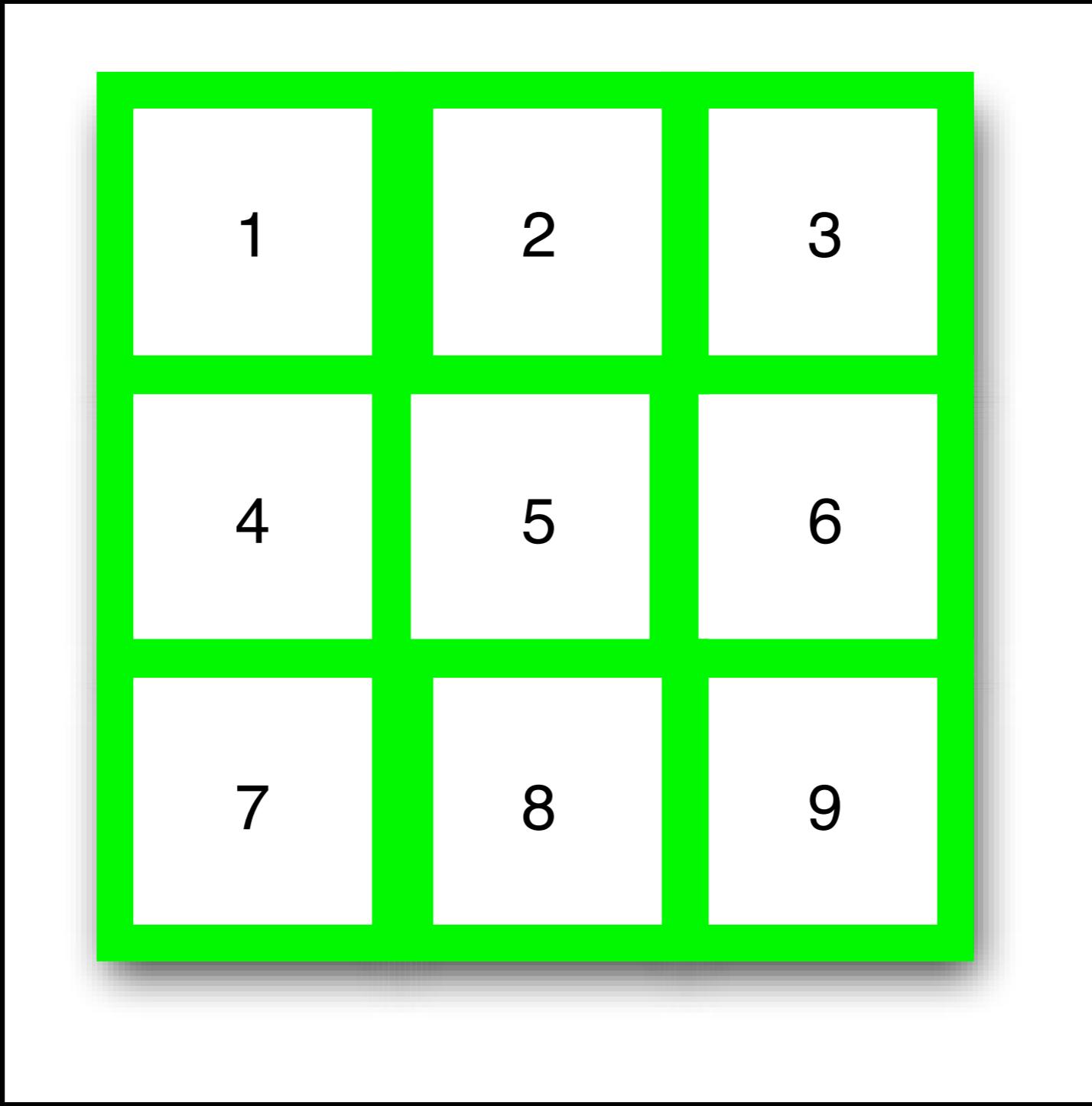
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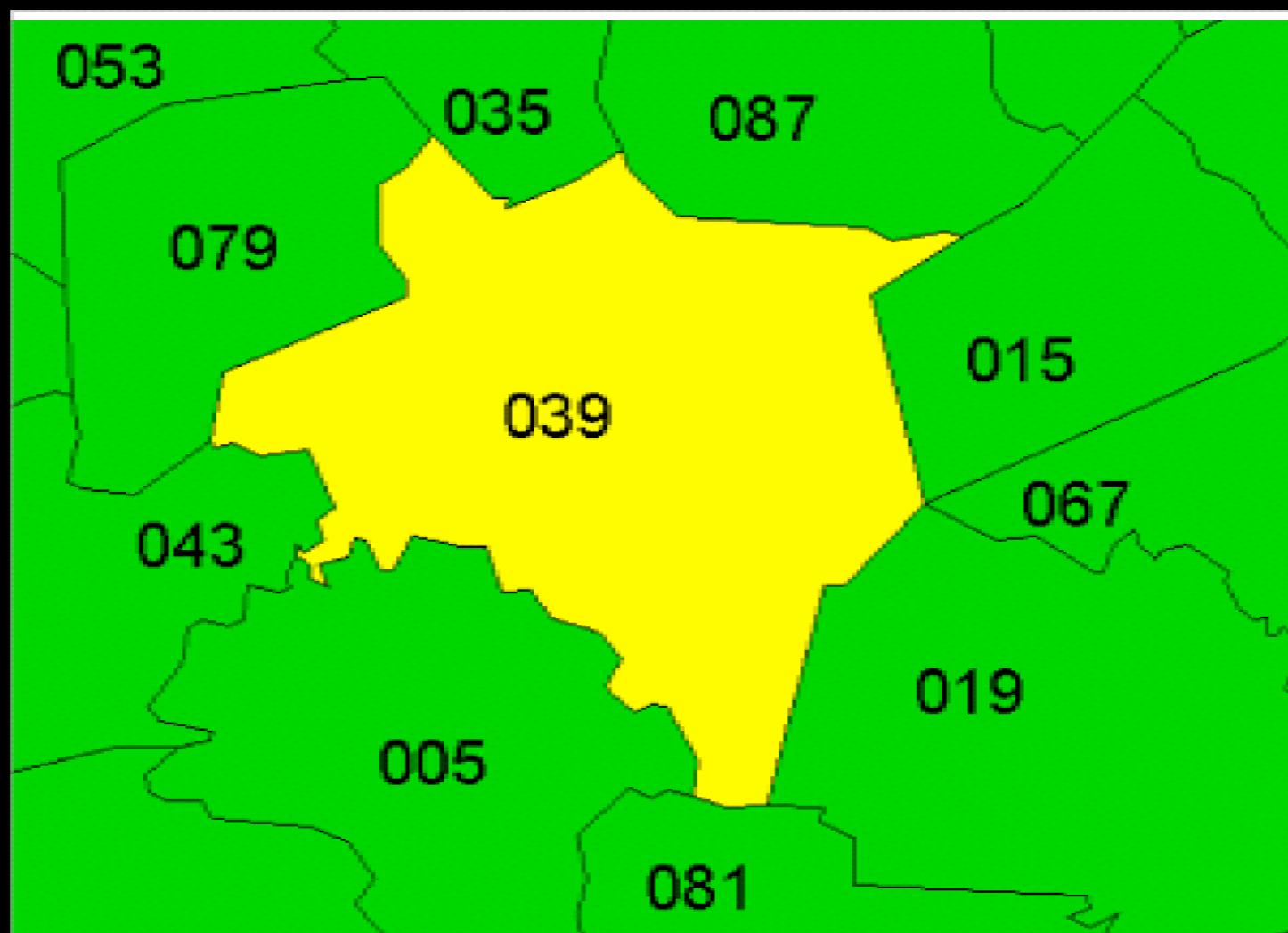


Queen Weights

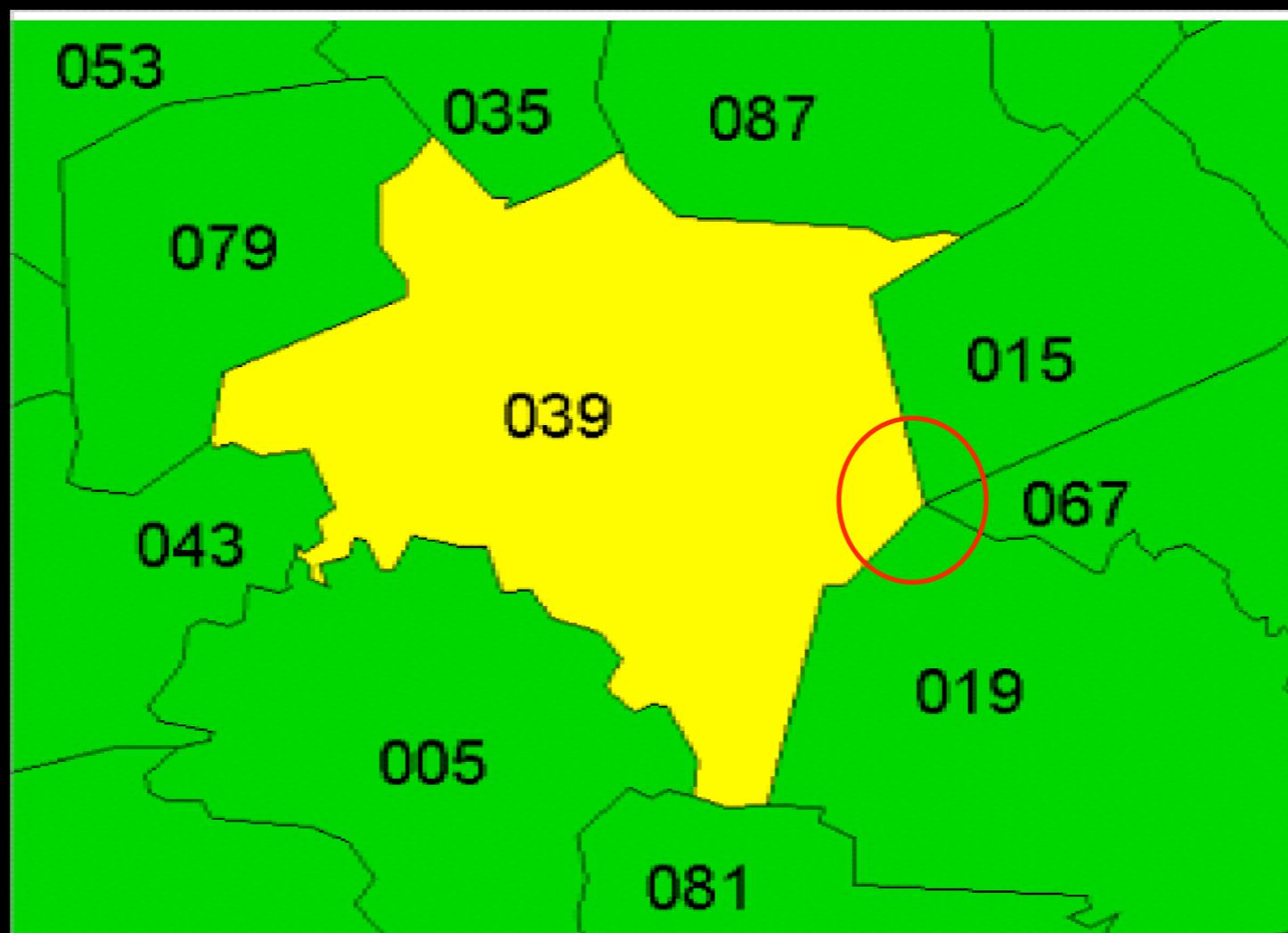
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8 Neighbors for 5
Both Border and Verex

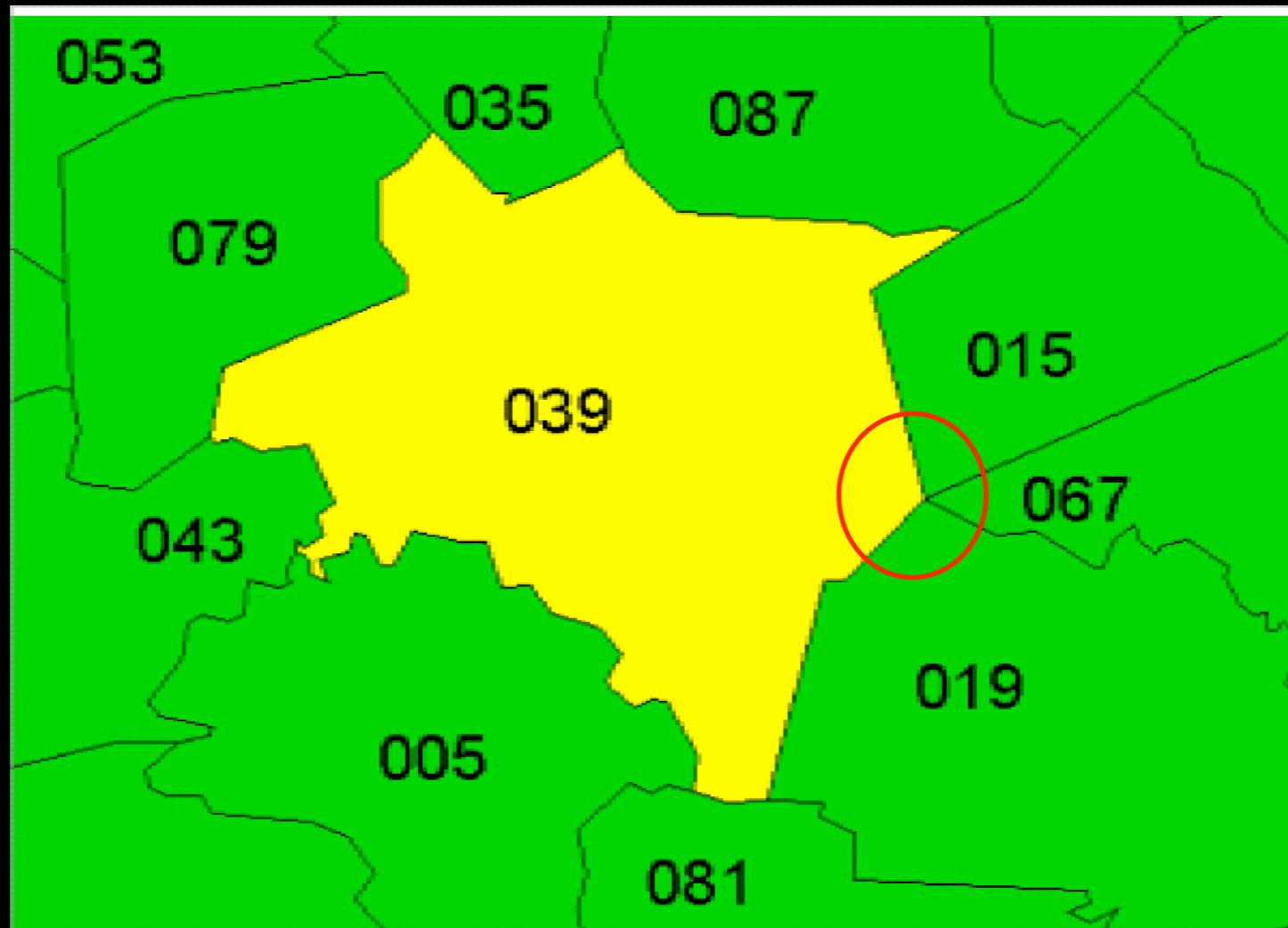
Irregular Lattice Contiguity



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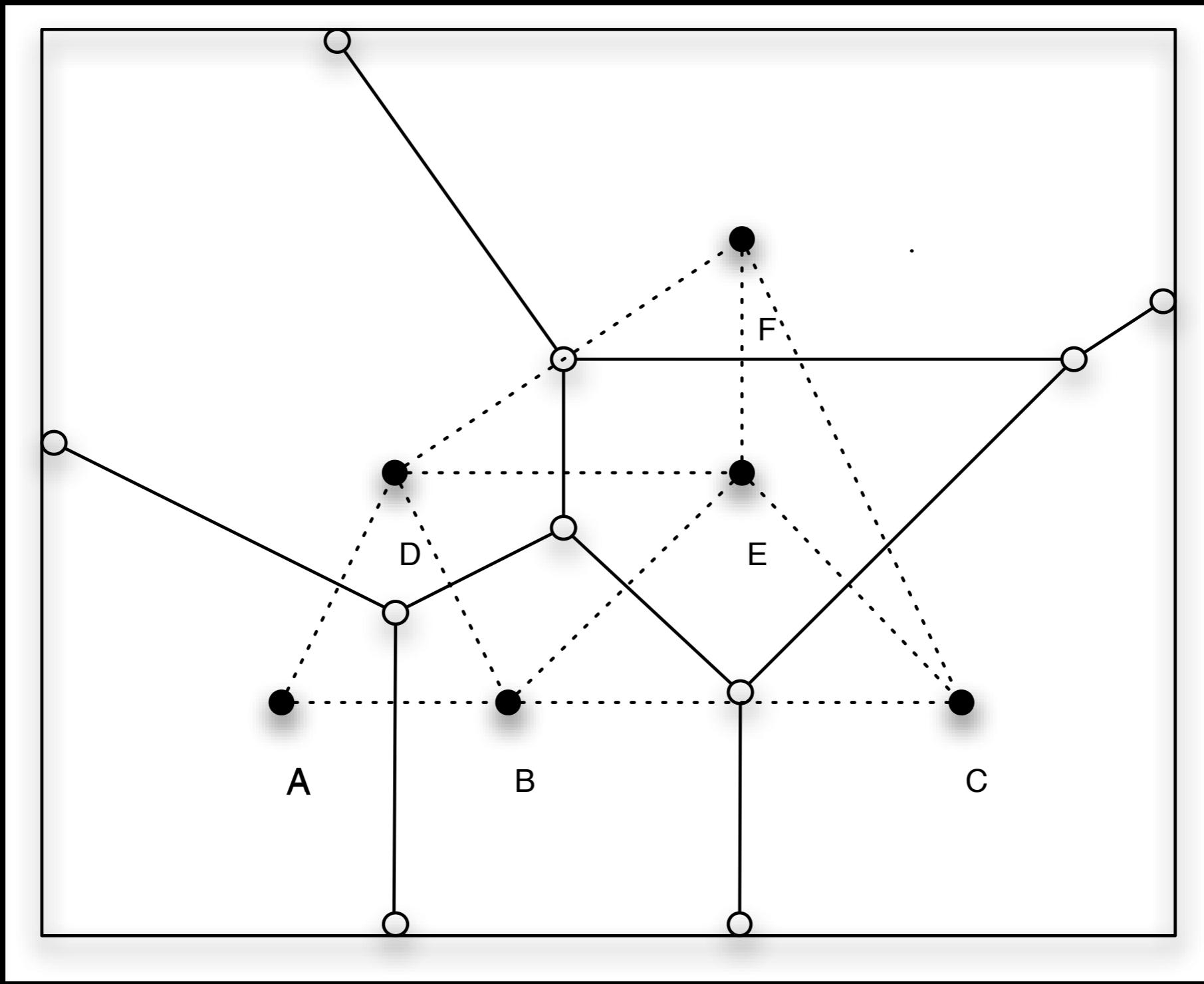
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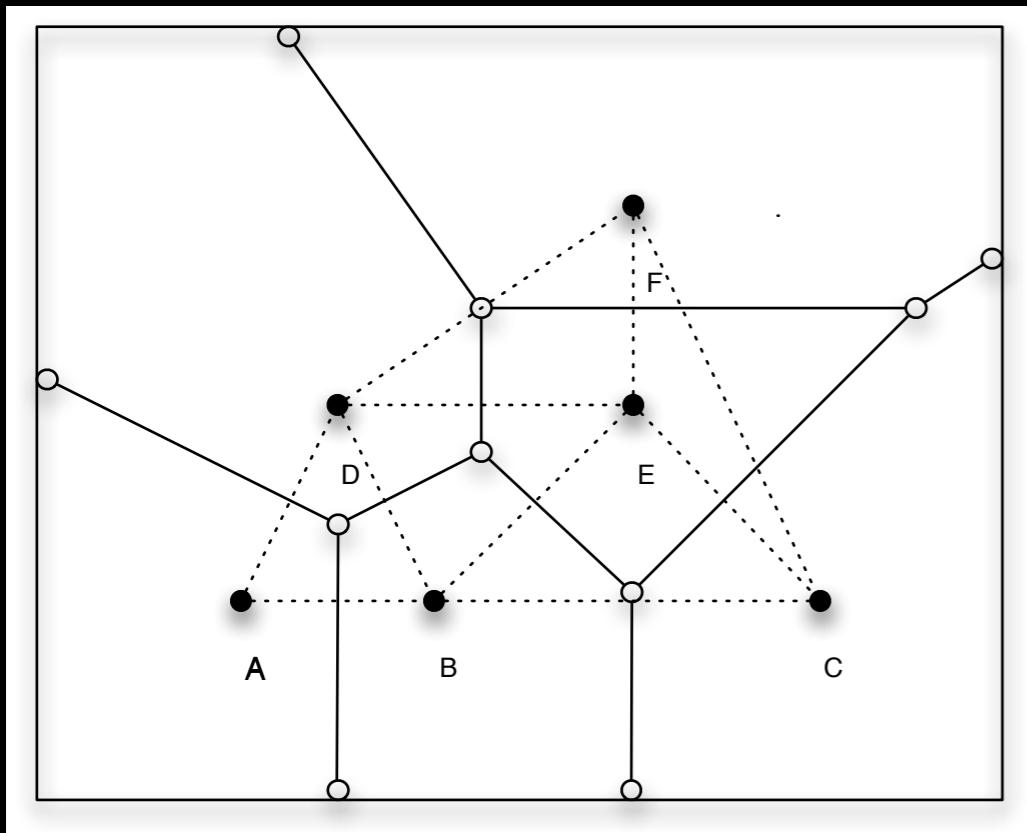
Rook Contiguity: common boarder only

Queen Contiguity: border and vertices - 039 and 067

Point Contiguity

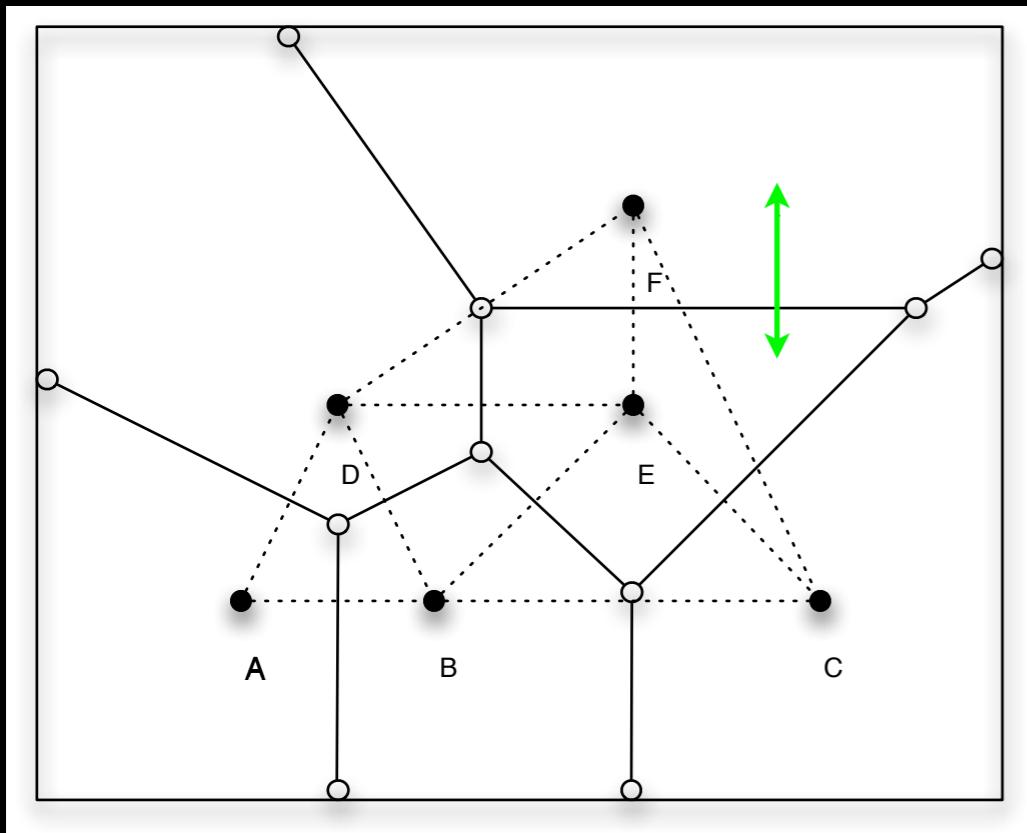


Point Contiguity



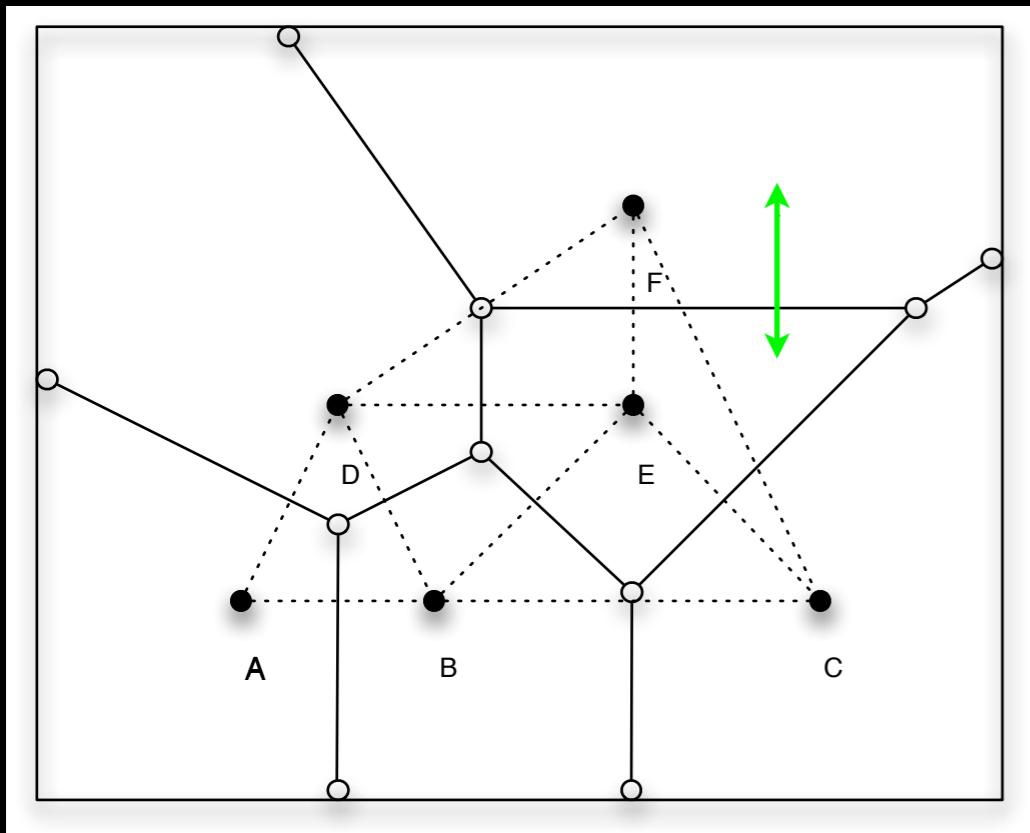
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Point Contiguity



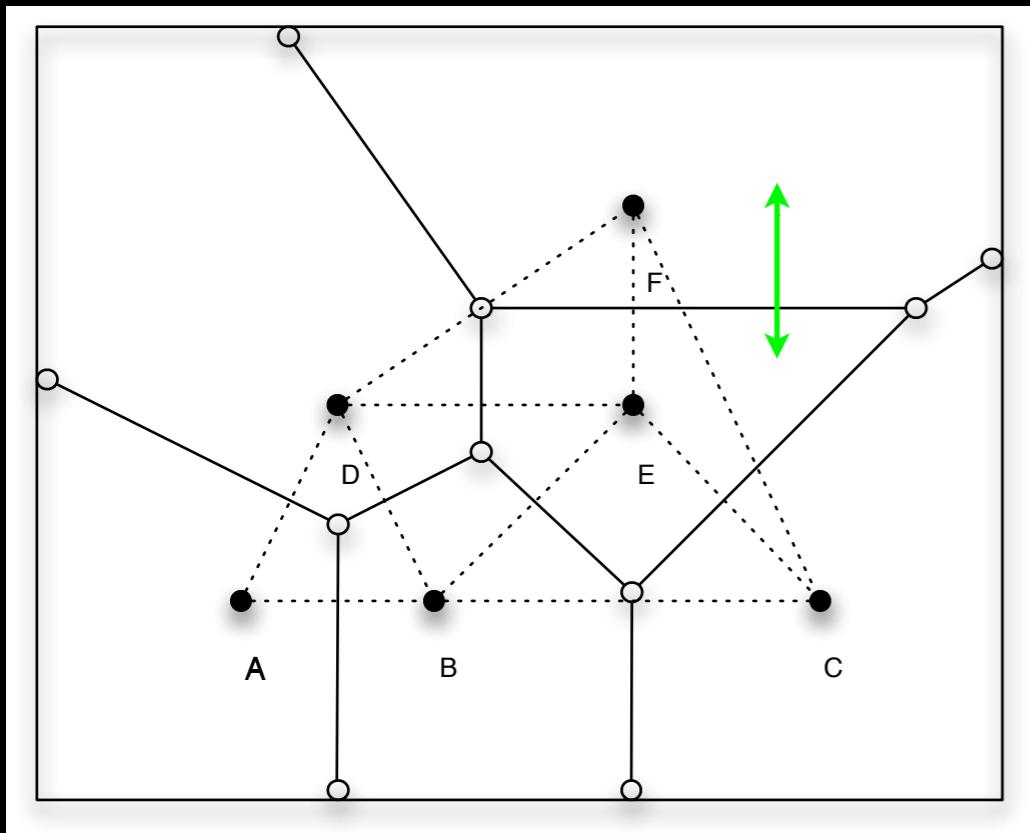
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Distance-Based Weights

Distance Measures

- Point (x_i, y_i)
- Interpoint Distances
- Metrics
 - Euclidean $d_{ij}^e = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
 - Manhattan $d_{ij}^m = |x_i - x_j| + |y_i - y_j|$
 - Minkowski $d_{ij}^p = (|x_i - x_j|^p + |y_i - y_j|^p)^{(1/p)}$

Distance Measures

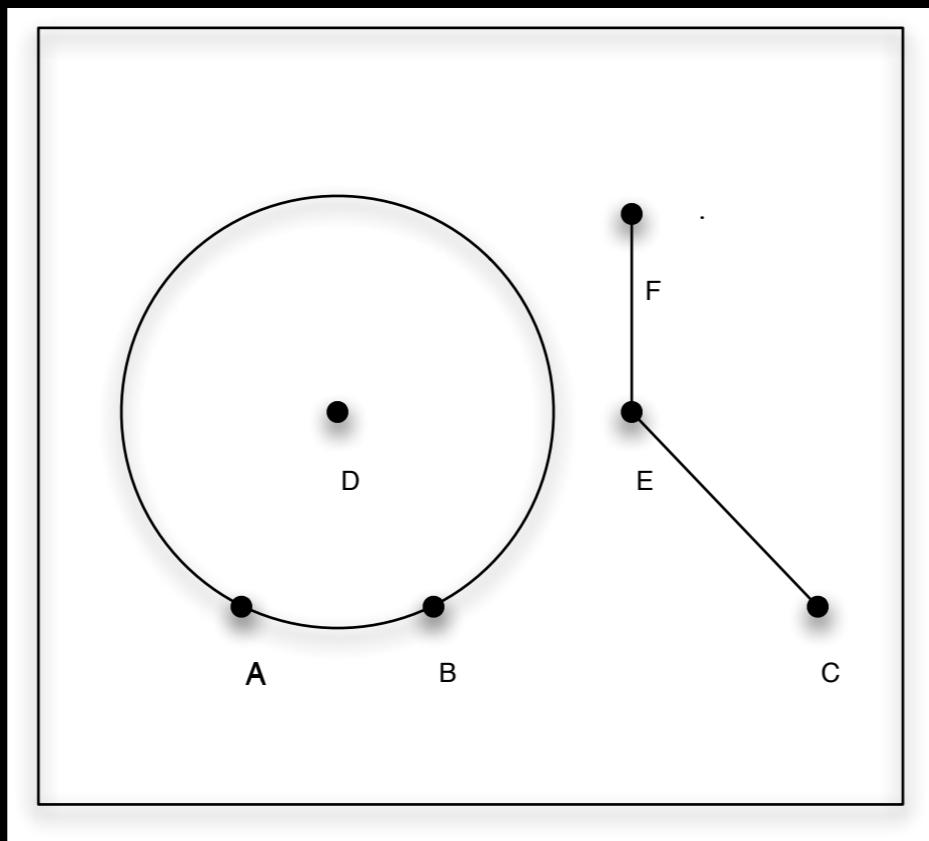
- Other
 - Road network
 - Actual travel
- Unprojected coordinates
 - Straight line distance measure inappropriate
 - Use great circle distance

Distance Measures

- Unprojected coordinates
- Straight line distance measure inappropriate
- Use great circle distance

$$d_{ij}^c = R \cdot \arccos[\sin(\text{lat}_i) \cdot \sin(\text{lat}_j) + \cos(\text{lon}_i) \cdot \cos(\text{lon}_j) \cdot \cos(\text{lon}_i - \text{lon}_j)]$$

Interpoint Distance



$A(10, 10), B(20, 10), C(40, 10), D(15, 20), E(30, 20), F(30, 30)$

Interpoint Euclidean Distance

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Distance Bands

$w_{ij} = 1$ when $d_{ij} \leq \delta$, and $w_{ij} = 0$ otherwise

δ is a preset critical distance cutoff

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C is an island

Nearest Neighbor Distances

|0 (A-B)
|0 (B-A)
|4.1 (C-E)
|11.2 (D-A and D-B),
|0 (E-F)
|0 (F-E)

Take **Maximum** nn-distance

$$\delta = 14.1$$

Max nn-distance Band

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C is connected

Max nn-distance Band

- Symmetric Matrix
- Avoids islands
- Too many neighbors for clustered locations
- Driven by maximum nn-distance

K Nearest Neighbors

- Avoids islands
- select k nearest neighbors for each location
- e.g., k=3

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Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
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| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Not symmetric

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Not symmetric

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Not symmetric

B is a 3nn to F

F is not a 3nn to B

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Not symmetric

B is a 3nn to F

F is not a 3nn to B

Ties

$$d_{FB} = d_{FC} = 22.4$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

| | B | C | D | E | F |
|---|------|------|------|------|------|
| A | 10.0 | 30.0 | 11.2 | 22.4 | 28.3 |
| B | | 20.0 | 11.2 | 14.1 | 22.4 |
| C | | | 26.9 | 14.1 | 22.4 |
| D | | | | 15.0 | 18.0 |
| E | | | | | 10.0 |

Not symmetric

B is a 3nn to F

F is not a 3nn to B

Representative Points

- Represent a polygon by a point
- Measure distance between representative points
- Construct distance based W

Representative Points

- Capital, County Seat
- Central Point
- Polygon Centroid
- Minimum Bounding Rectangle Center

Centroid

- **Polygon** $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$

$$(x_n, y_n) = (x_1, y_1)$$

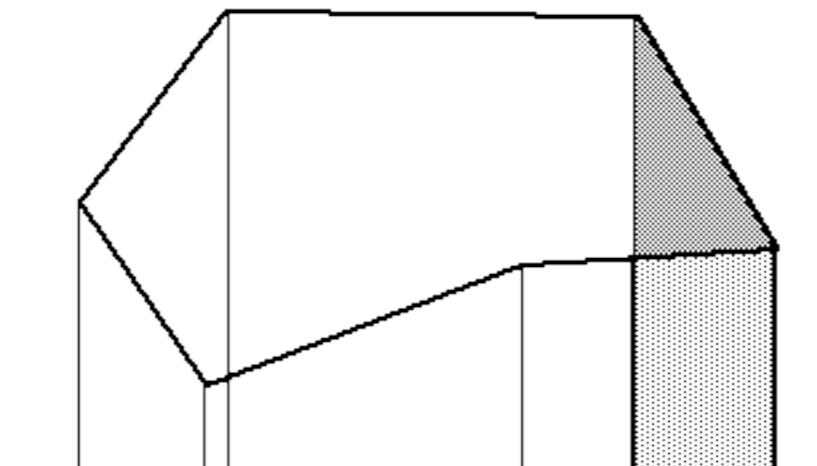
$$x_c = (1/6A) \sum_{i=1}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$y_c = (1/6A) \sum_{i=1}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

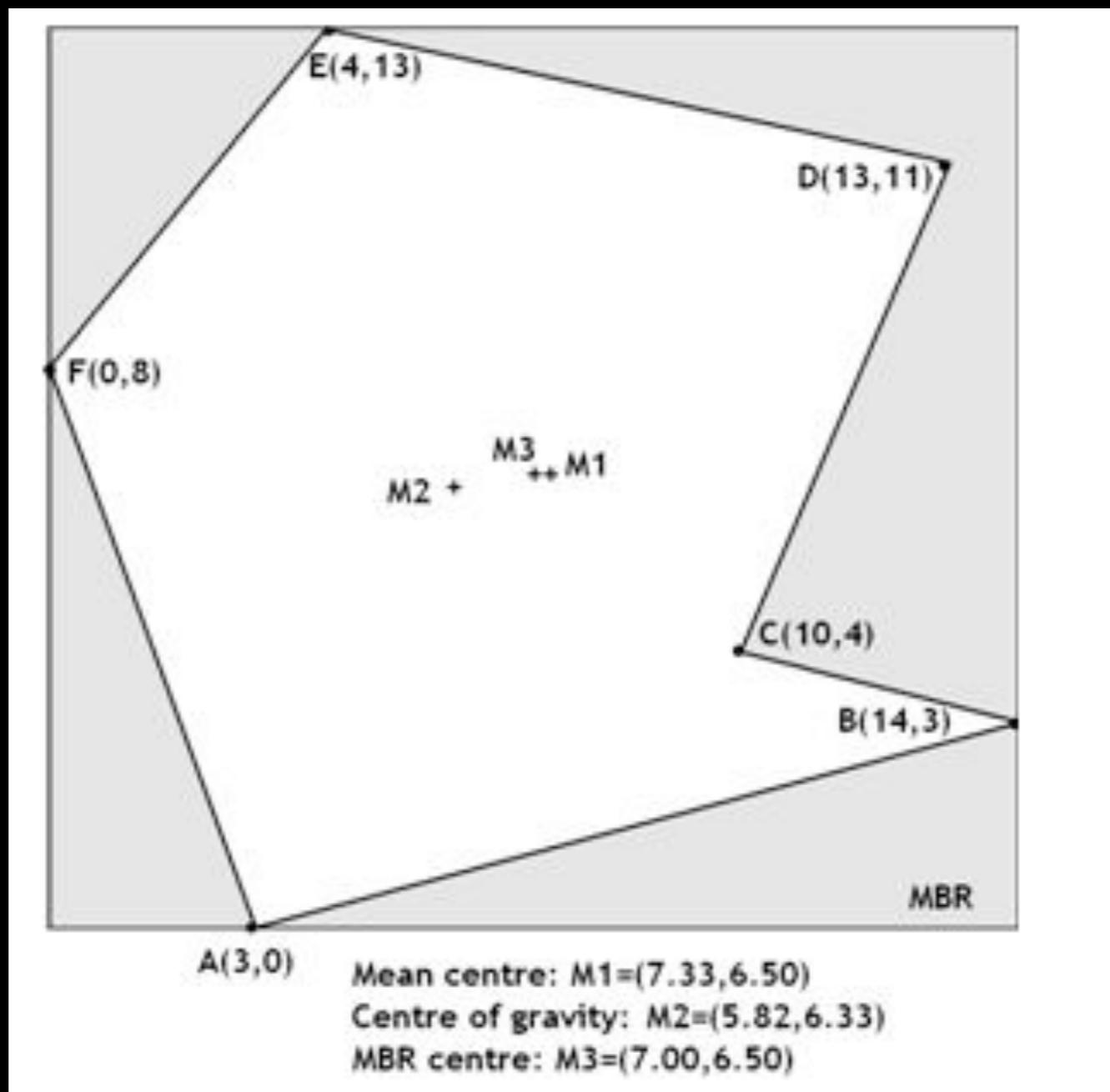
$$A = (1/2) \sum_{i=1}^{n-1} x_i y_{i+1} - x_{i+1} y_i$$

Polygon Area

- Negative if digitized ccw
- Trapezoids

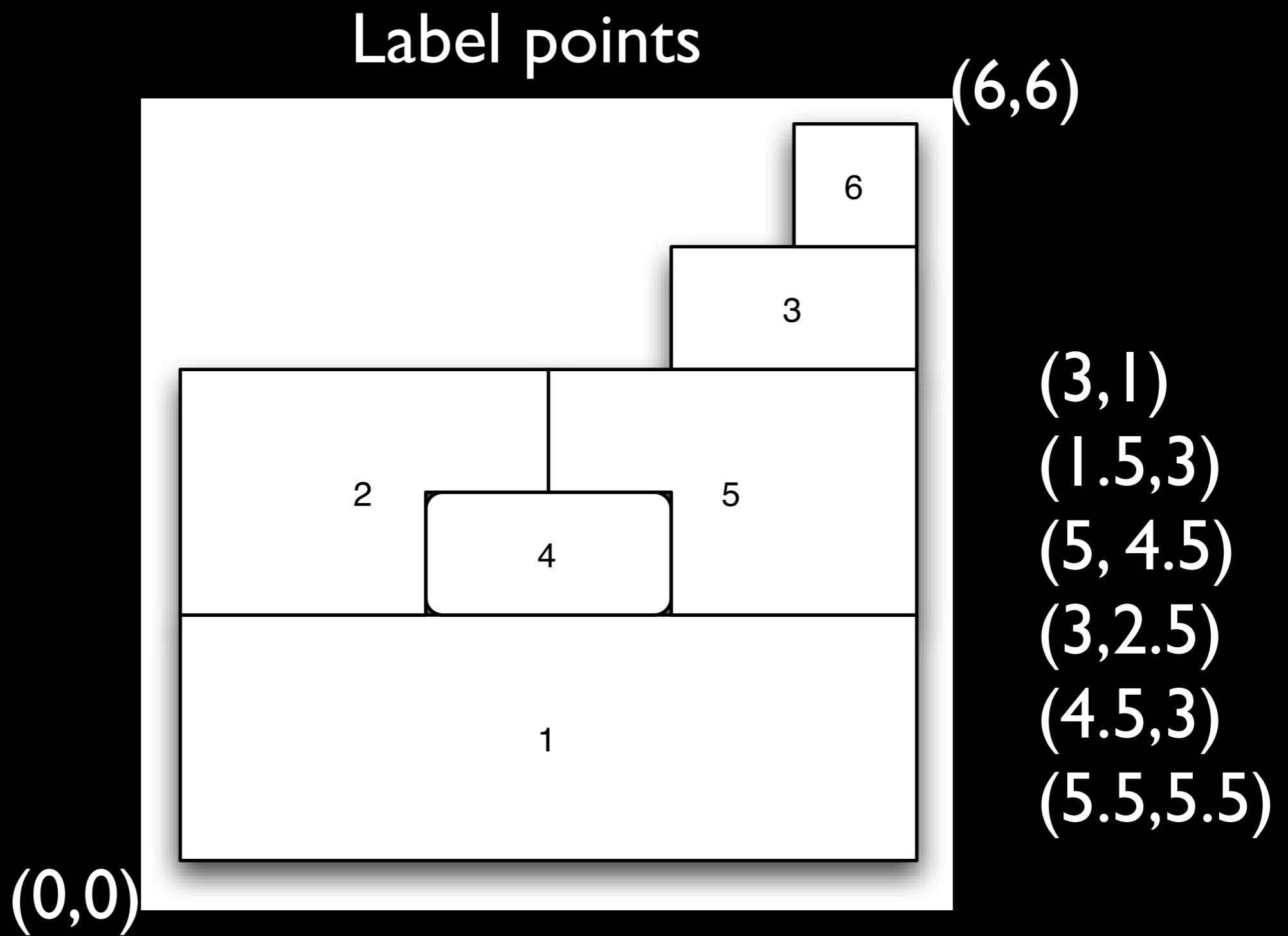


Representative Points



Source: <http://www.spatialanalysisonline.com>

Representative Points



Distance Band

| | 2 | 3 | 4 | 5 | 6 |
|---|-----|-----|-----|-----|-----|
| 1 | 2.5 | 4.0 | 1.5 | 2.5 | 5.1 |
| 2 | | 3.8 | 1.6 | 3.0 | 4.7 |
| 3 | | | 2.8 | 1.6 | 1.1 |
| 4 | | | | 1.6 | 3.9 |
| 5 | | | | | 2.7 |

$$\delta = 2.5$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Distance Band

| | 2 | 3 | 4 | 5 | 6 |
|---|-----|-----|-----|-----|-----|
| 1 | 2.5 | 4.0 | 1.5 | 2.5 | 5.1 |
| 2 | | 3.8 | 1.6 | 3.0 | 4.7 |
| 3 | | | 2.8 | 1.6 | 1.1 |
| 4 | | | | 1.6 | 3.9 |
| 5 | | | | | 2.7 |

$$\delta = 2.5$$

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Distance Band

| | 2 | 3 | 4 | 5 | 6 |
|---|-----|-----|-----|-----|-----|
| 1 | 2.5 | 4.0 | 1.5 | 2.5 | 5.1 |
| 2 | | 3.8 | 1.6 | 3.0 | 4.7 |
| 3 | | | 2.8 | 1.6 | 1.1 |
| 4 | | | | 1.6 | 3.9 |
| 5 | | | | | 2.7 |

$$\delta = 2.5$$

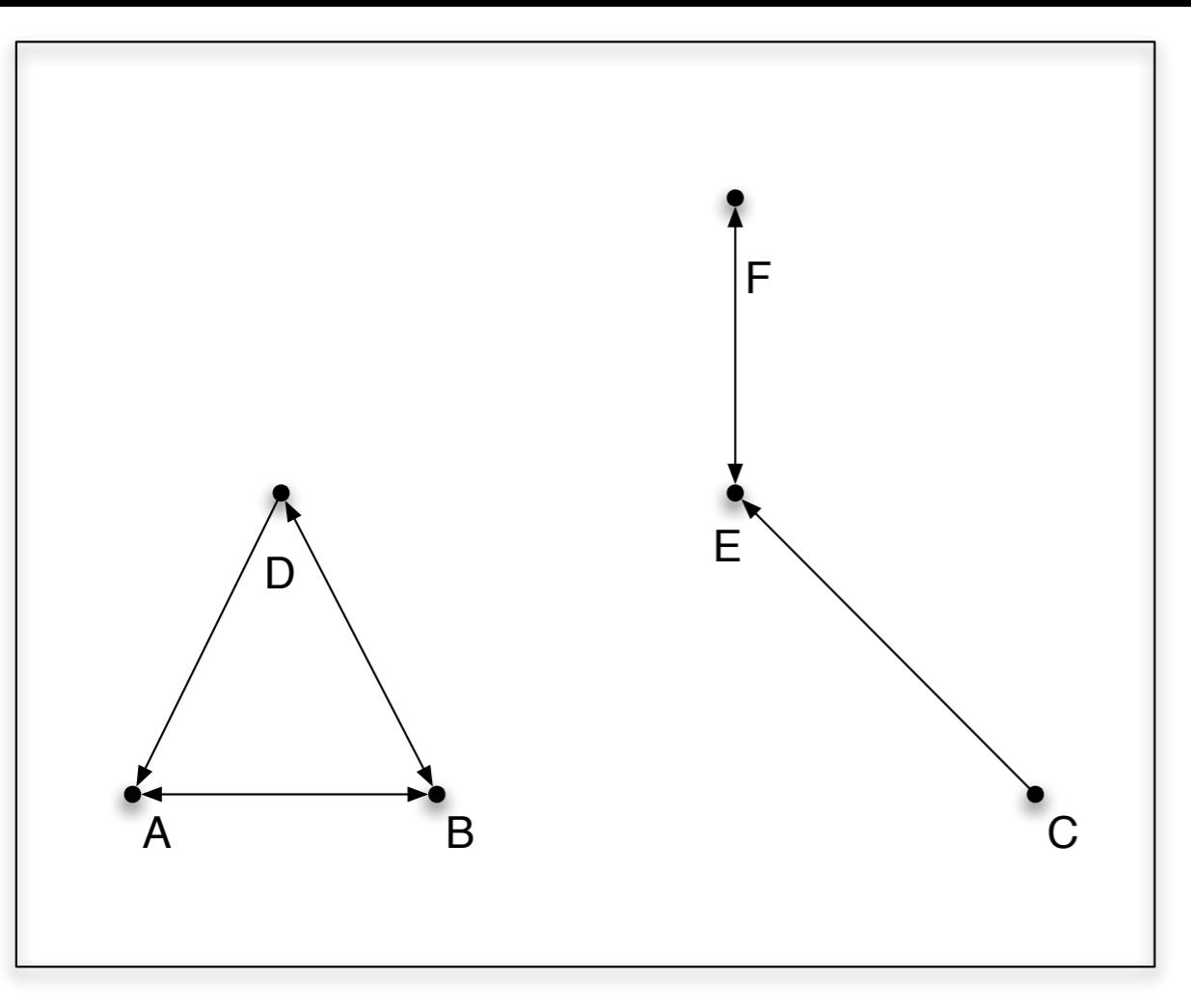
Variance in area leads
to variance in
connectivity

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Graph-Based Weights

- Points = Vertices
- Edges = Contiguity
- Definition of Edges based on Distance Criterion

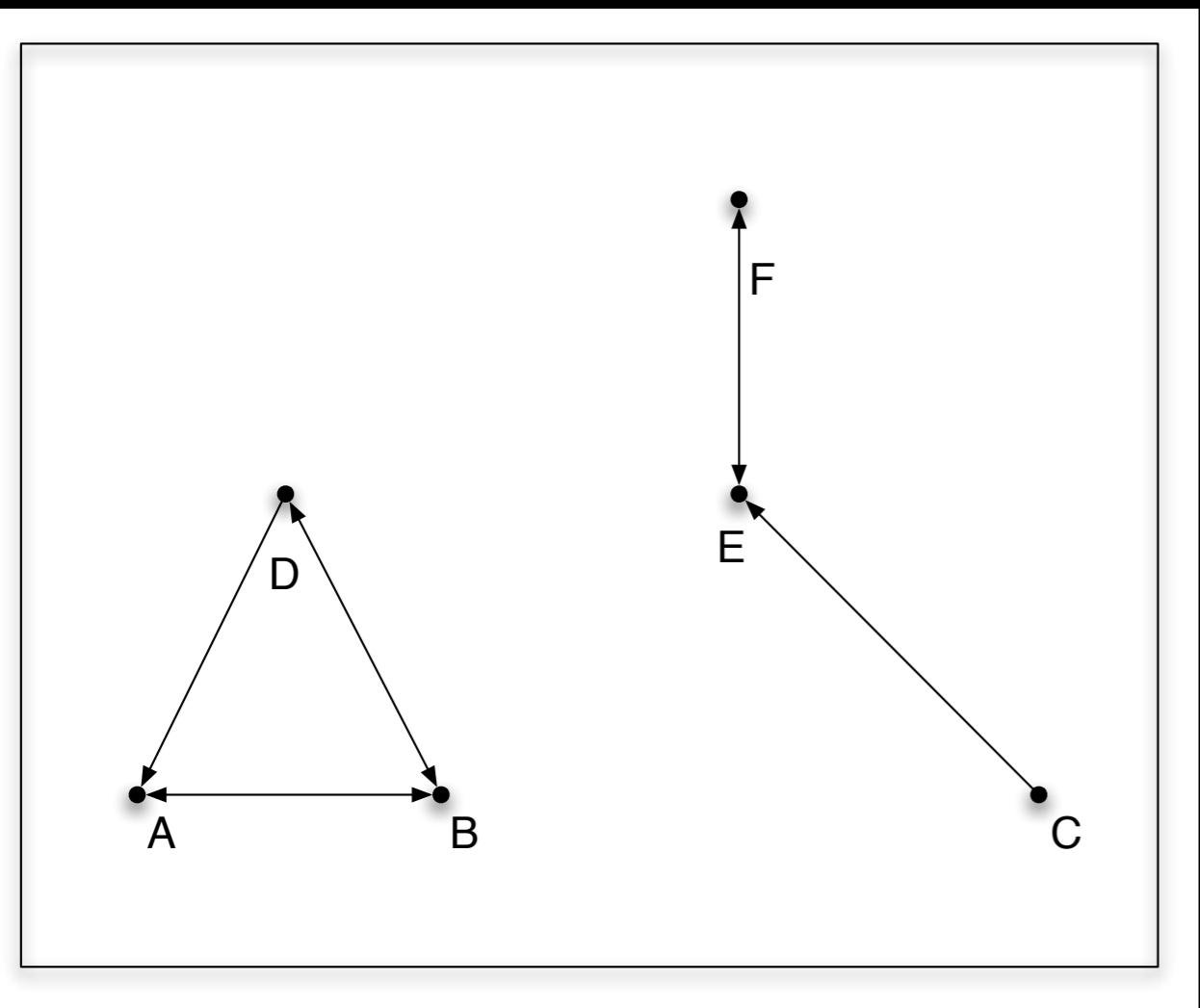
Nearest Neighbor Graph



$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Directed Graph

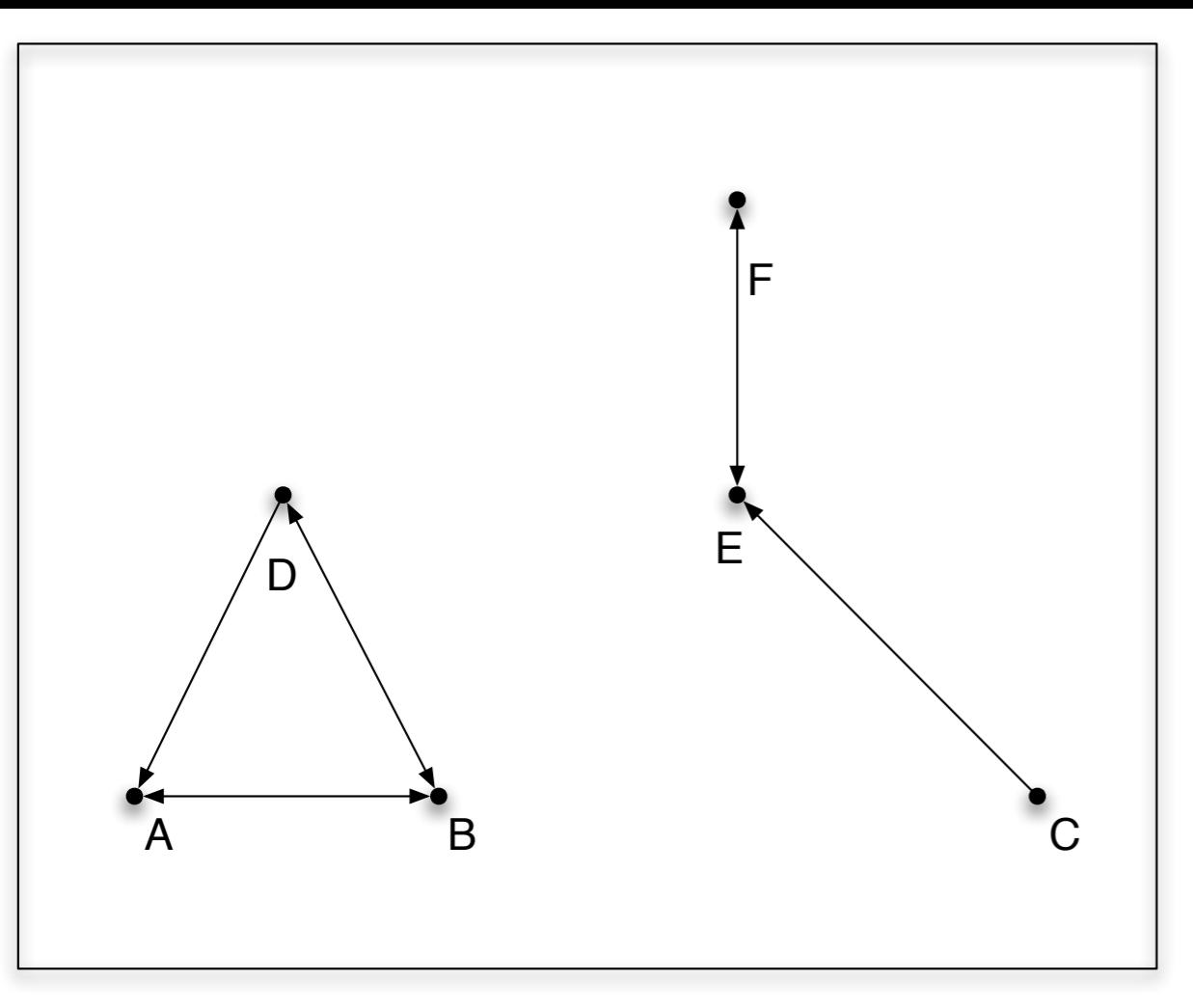
Nearest Neighbor Graph



$$W = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Directed Graph

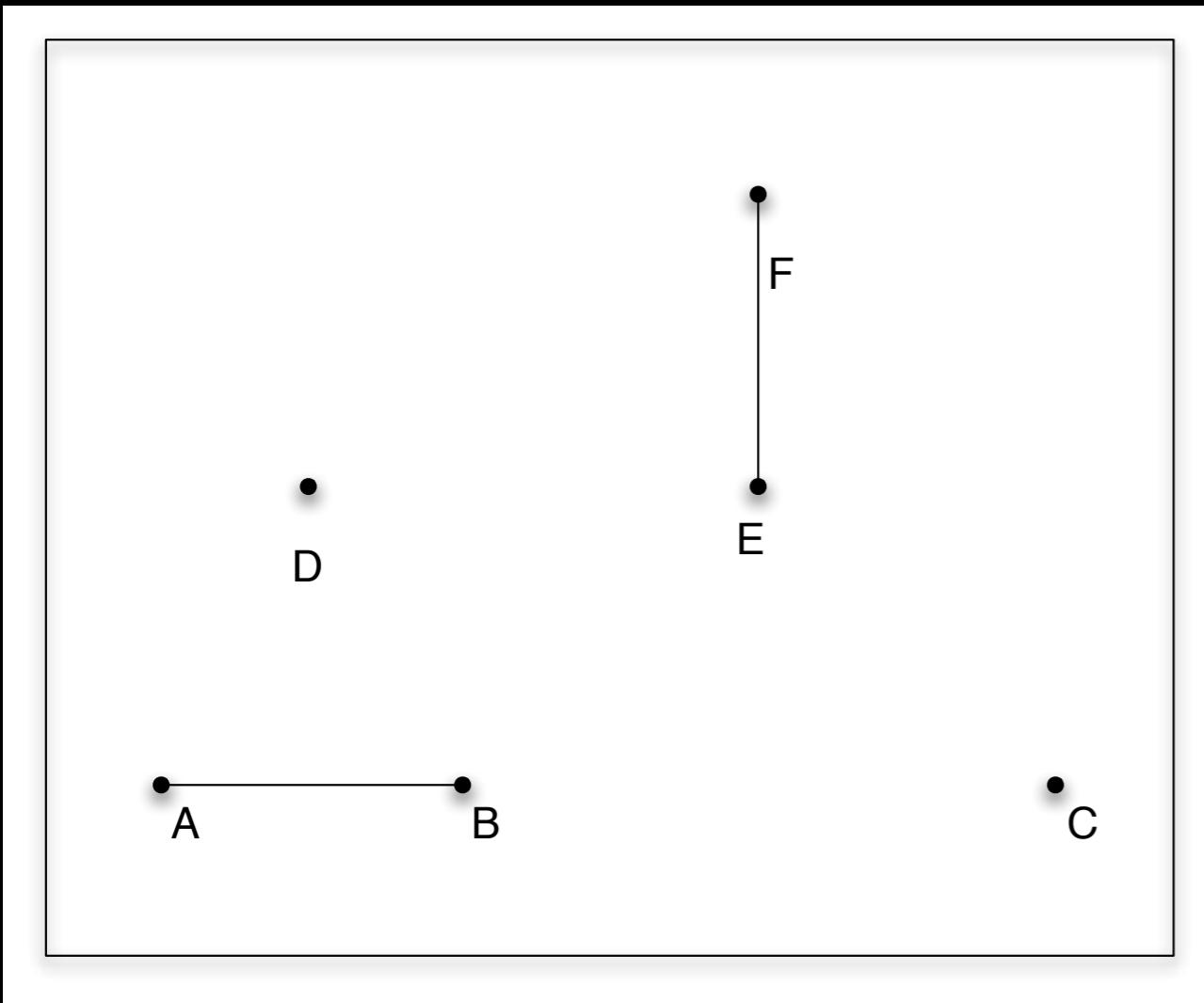
Nearest Neighbor Graph



Directed Graph

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Mutual Nearest Neighbors

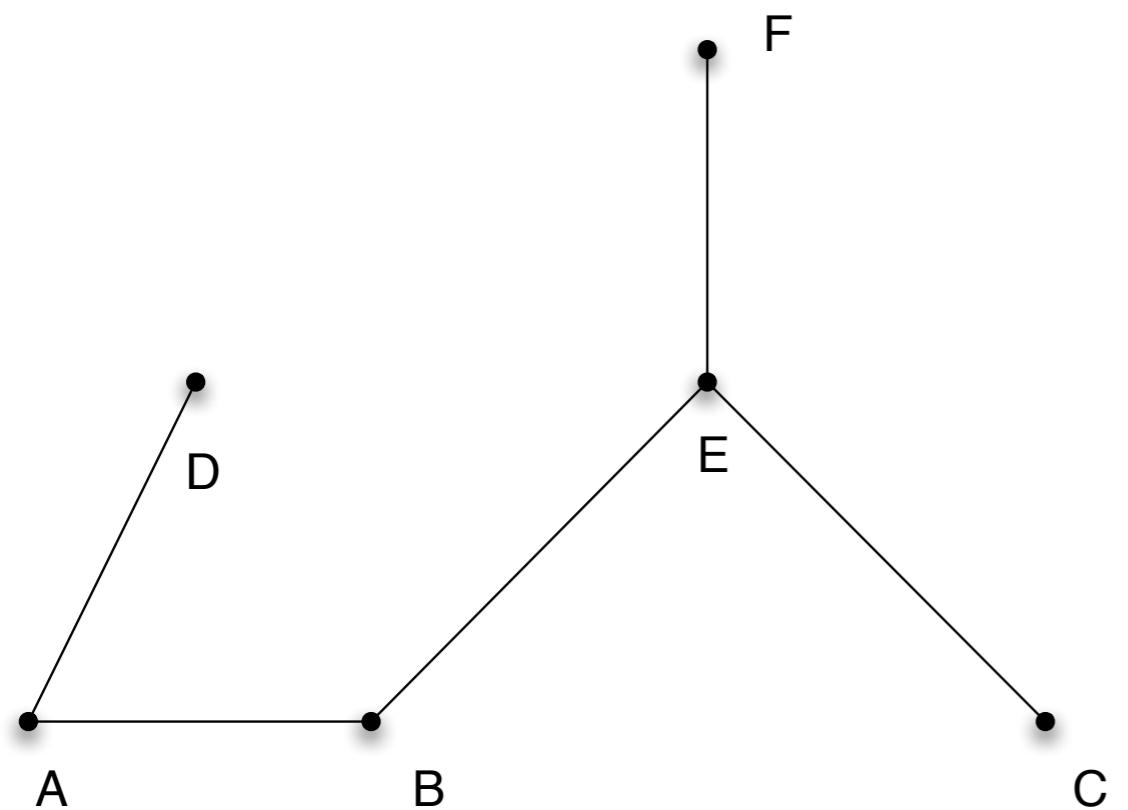


$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Sparse

Non-directed Graph

Minimum Spanning Tree

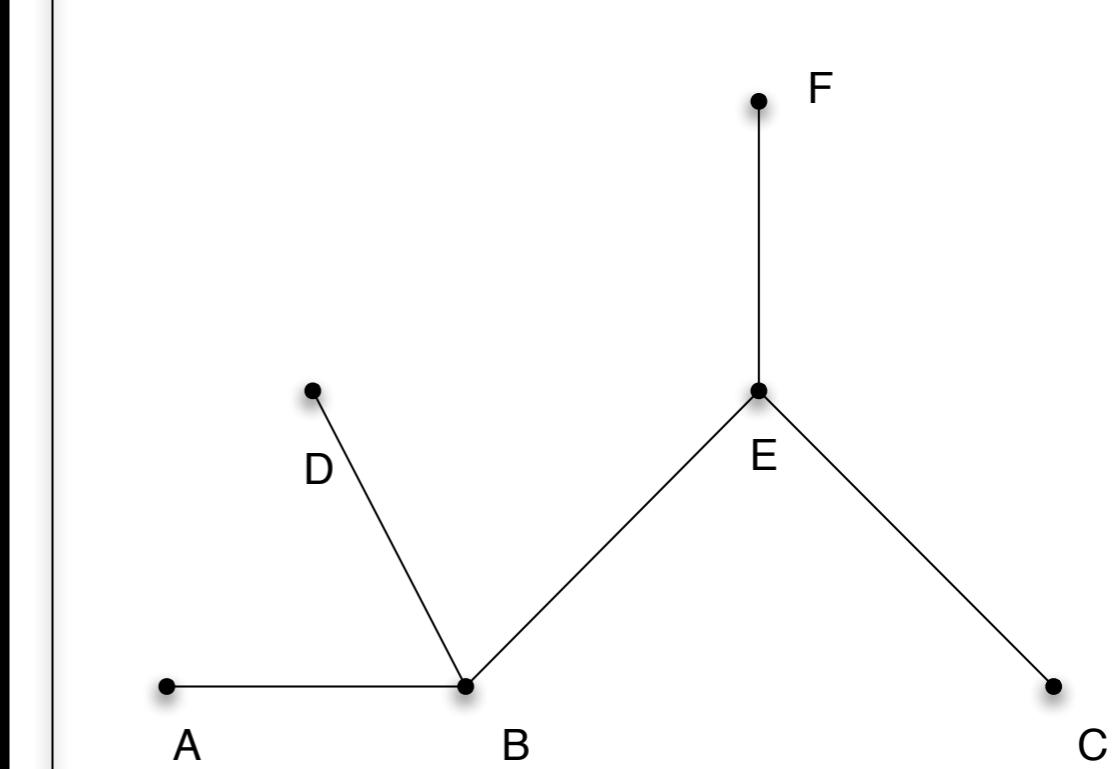
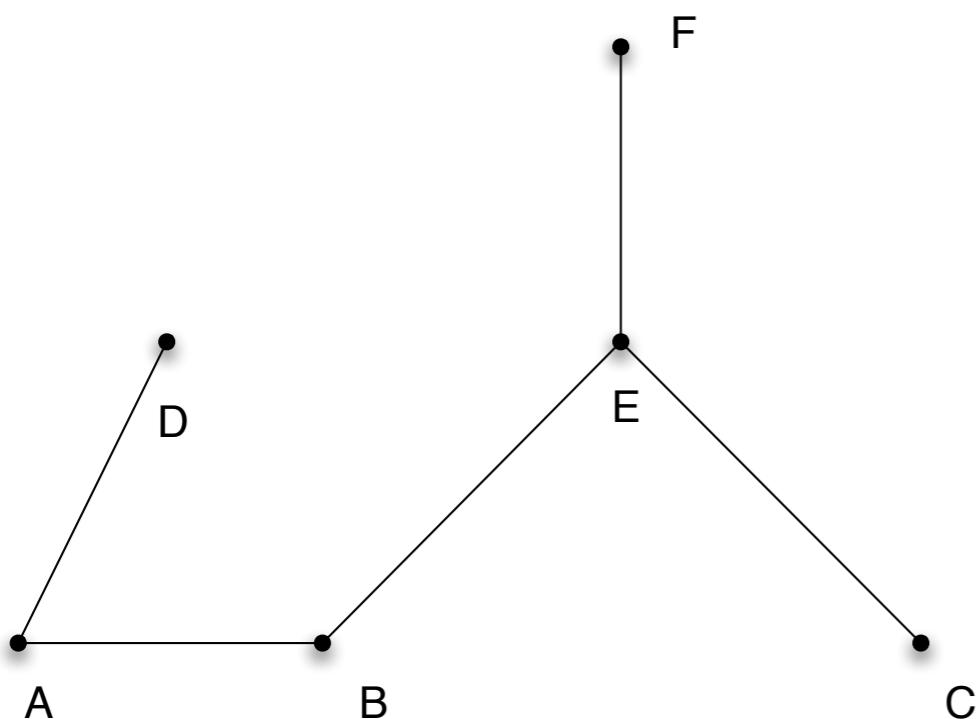


$$W = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Non-directed Graph

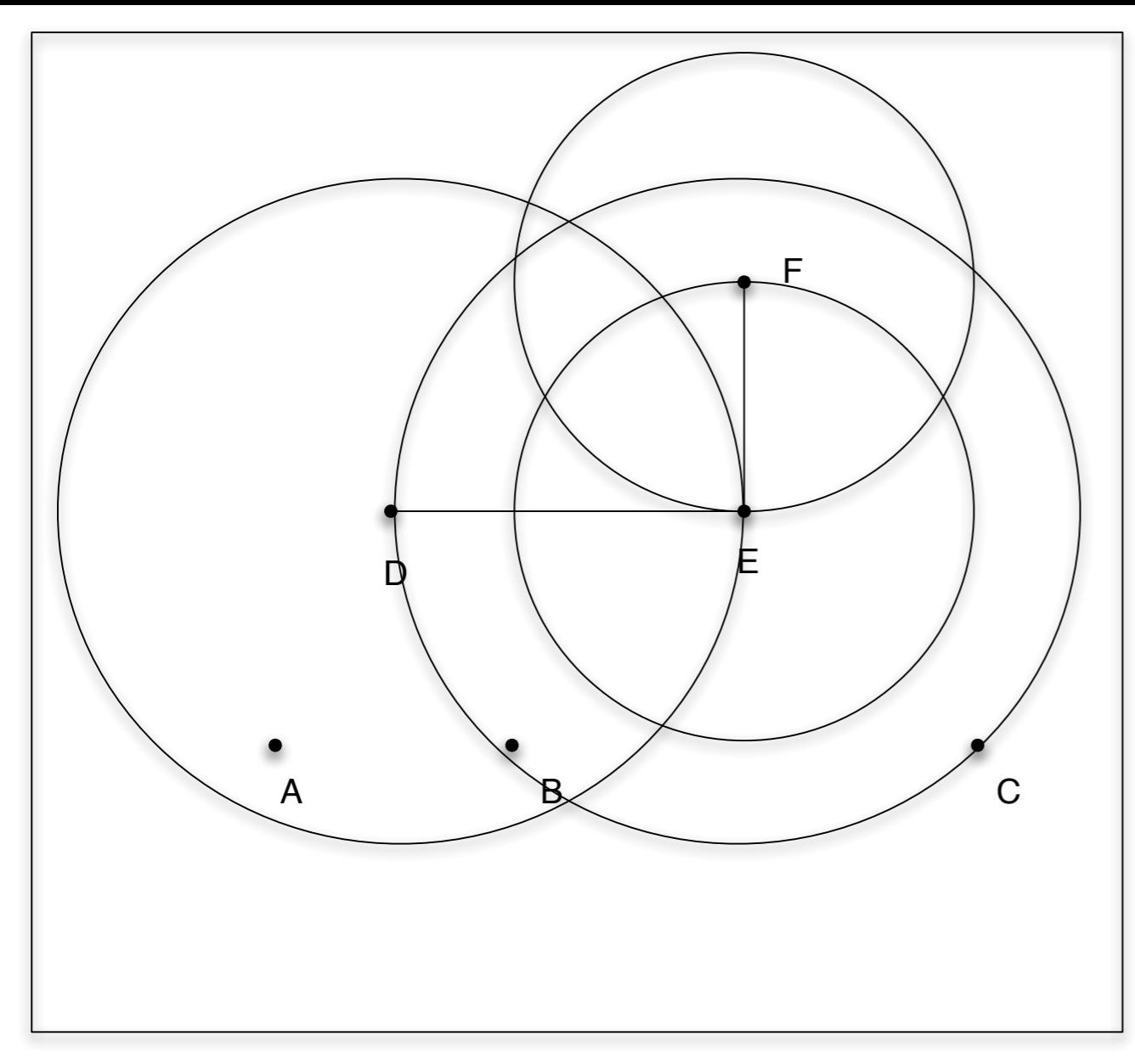
Minimum Spanning Tree

- No Islands
- May not be unique



Relative Neighbors

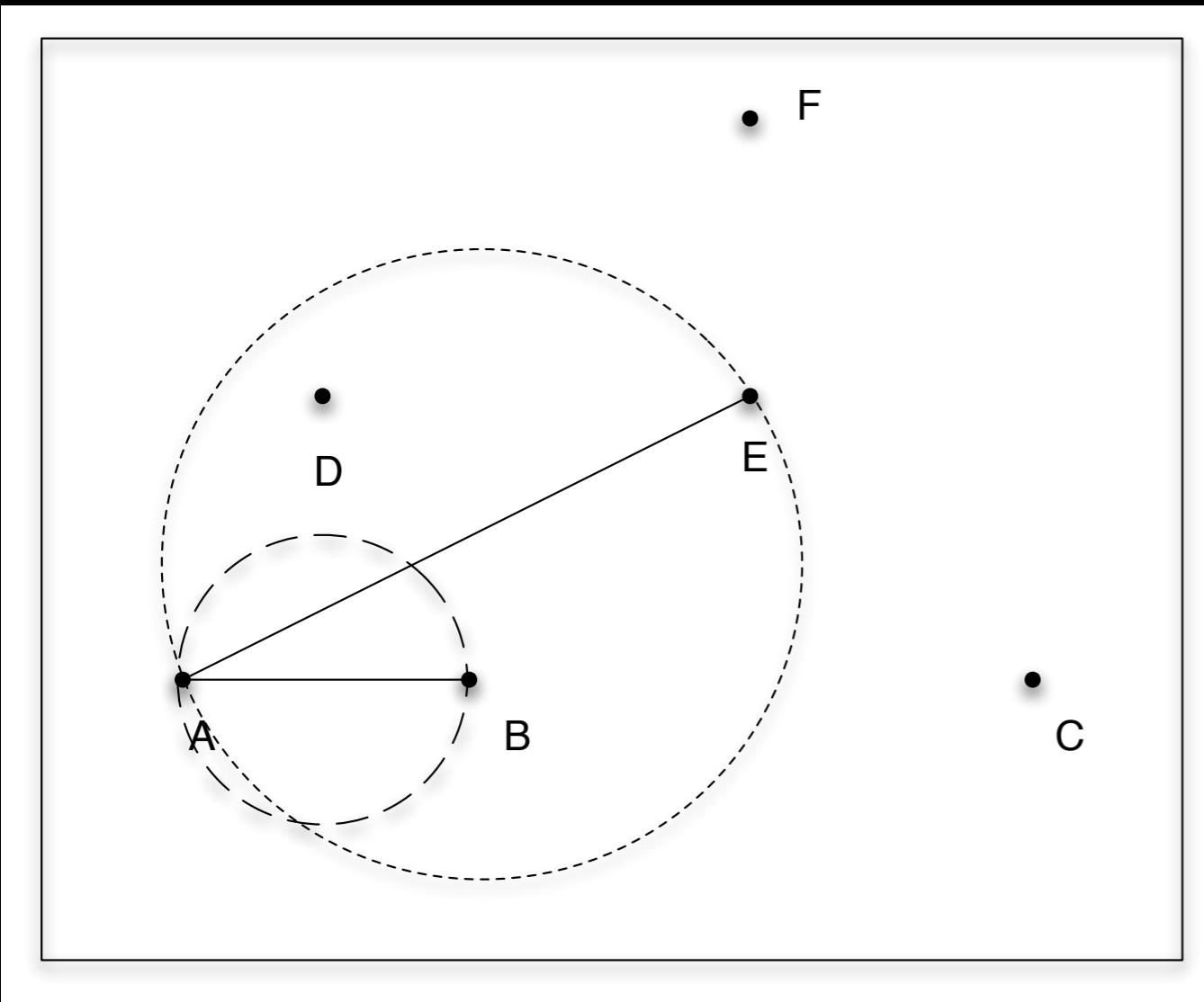
$$d_{i,j} \leq \min(\max(d_{i,k}, d_{j,k})) \quad \forall k \neq i, j.$$



$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

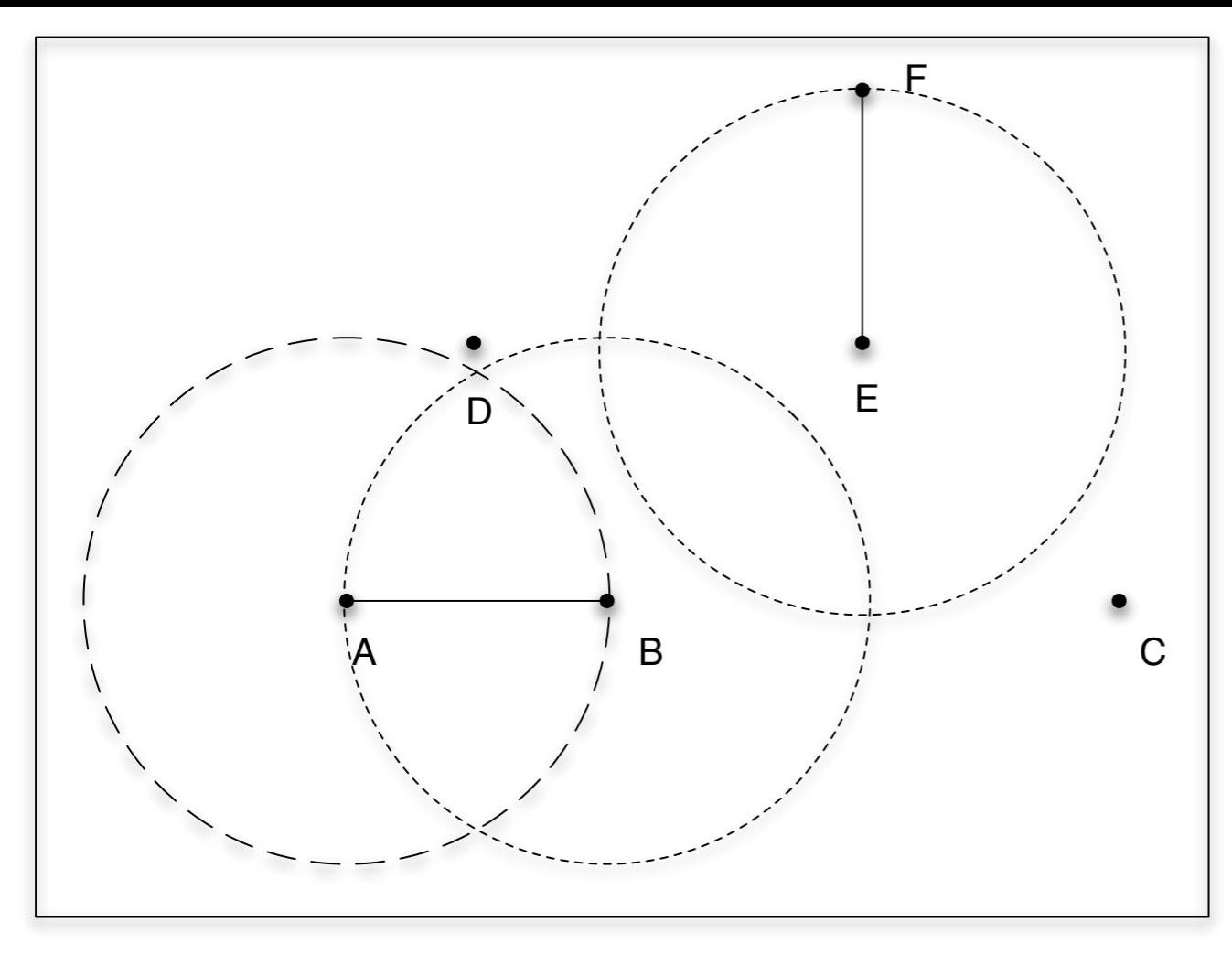
Gabriel Neighbors

$$d_{i,j} \leq \min \left(\sqrt{d_{i,k}^2 + d_{j,k}^2} \right) \quad \forall k \neq i, j.$$



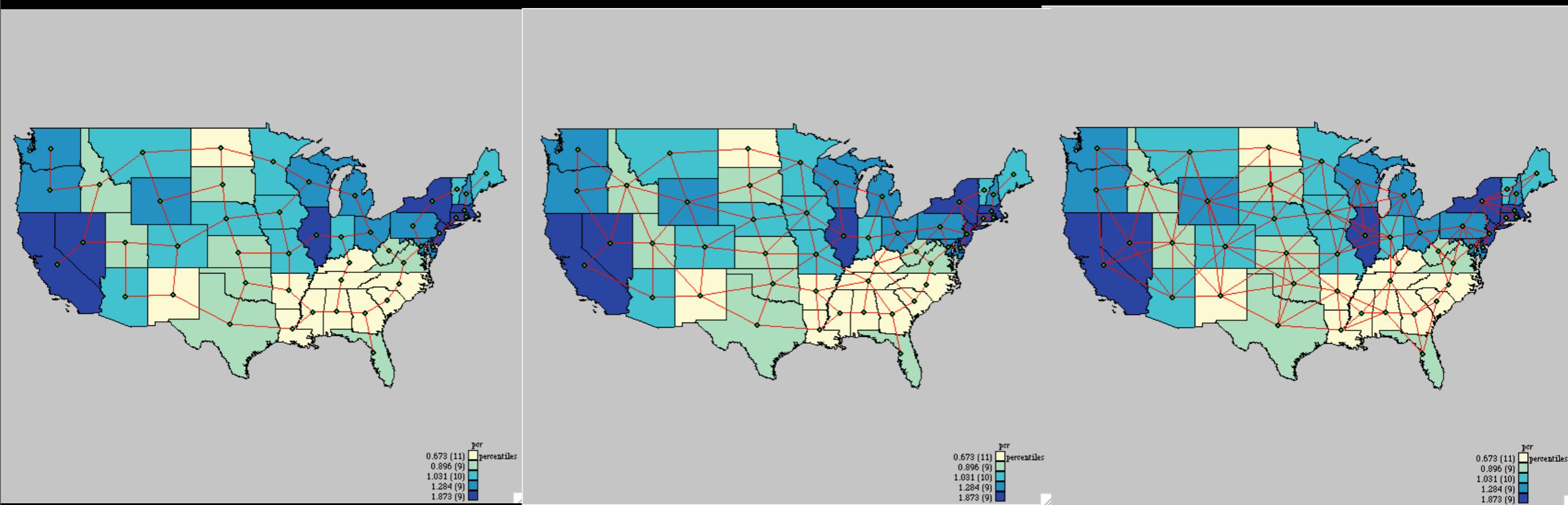
$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Sphere of Influence Neighbors



$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

States

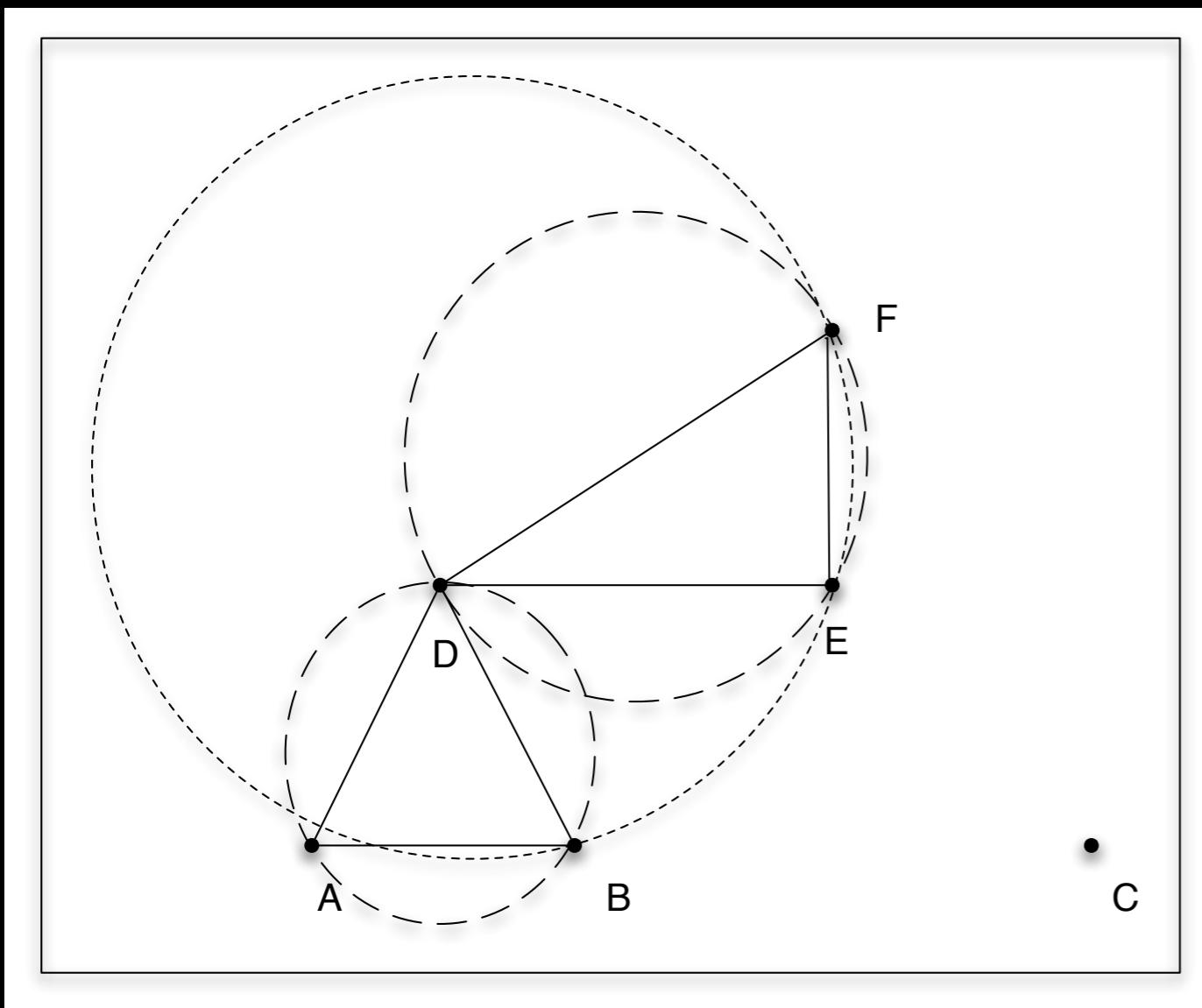


Relative

Gabriel

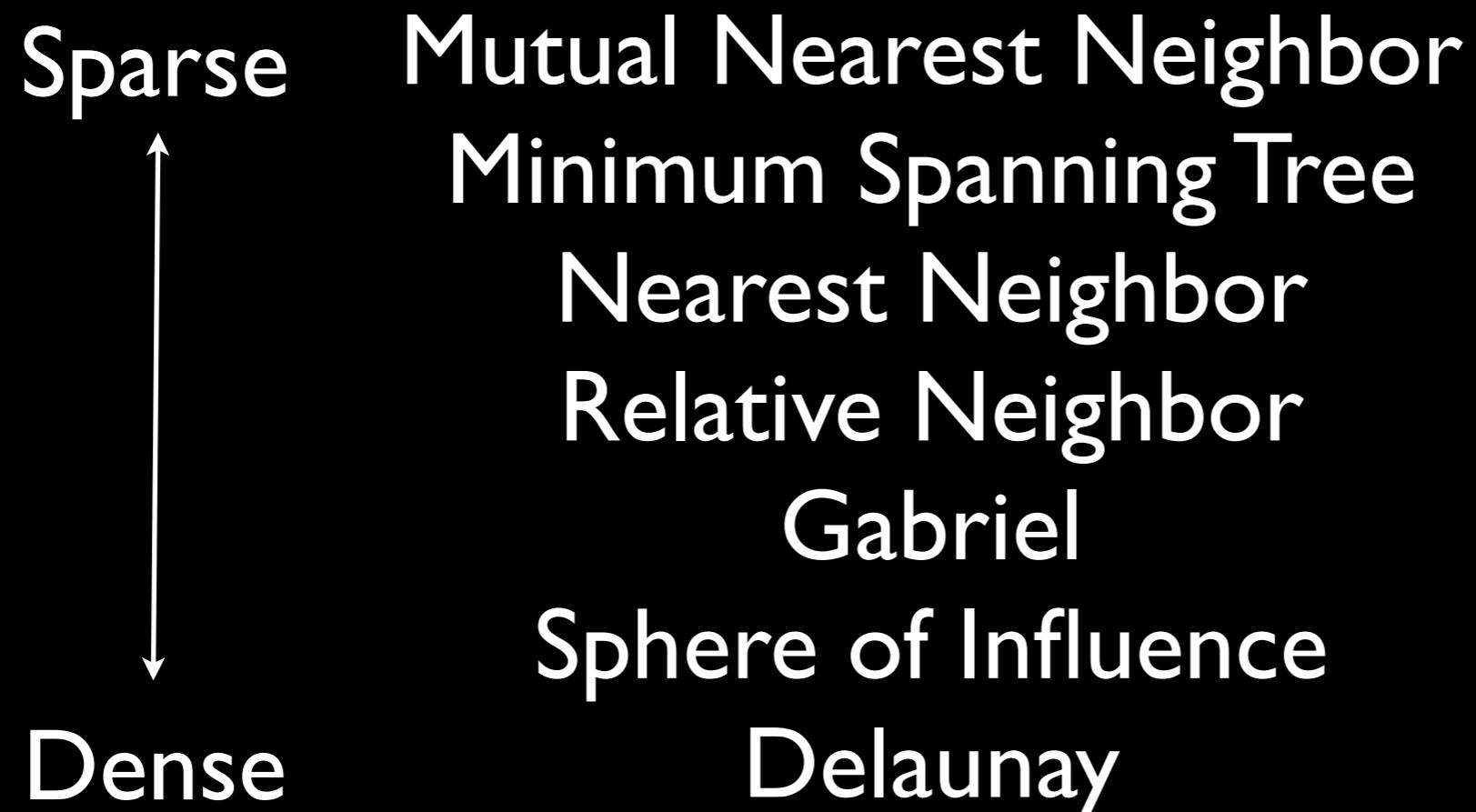
SOI

Delaunay Neighbors



$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Connectivity

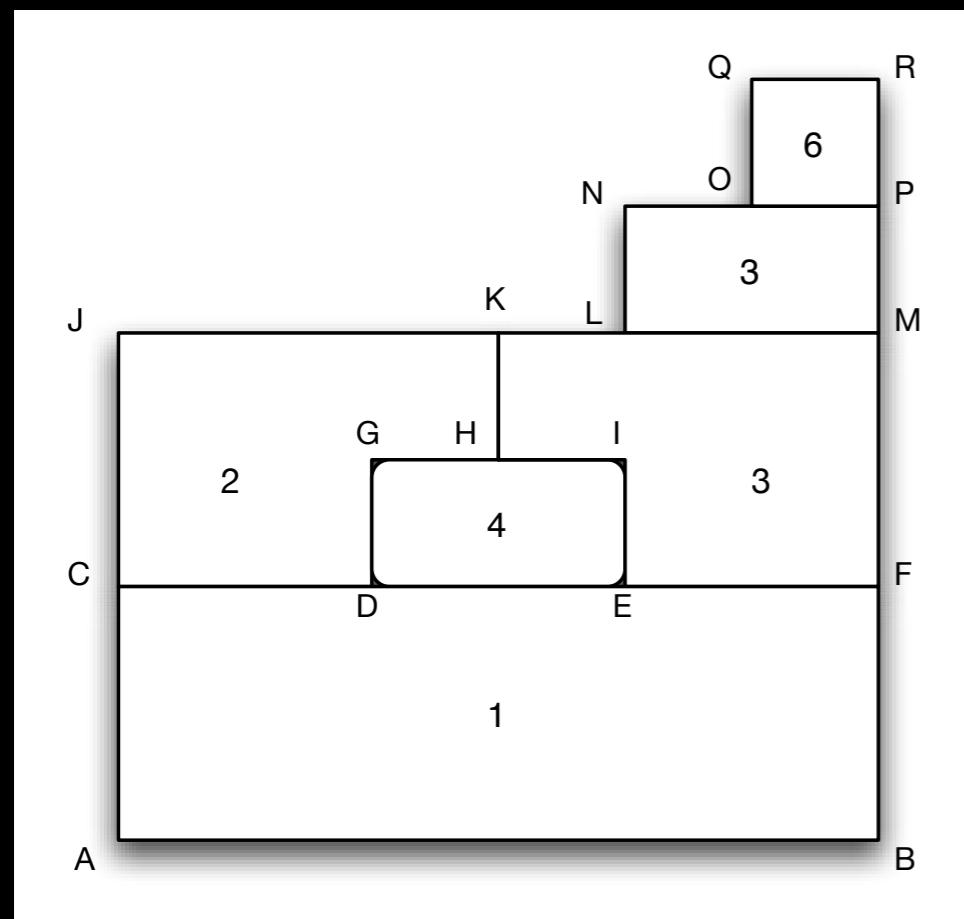


Practical Considerations

- Contiguity not determined via visual inspection
 - Highly error prone
- Specialized software
 - Small problems ($n < 1000$) brute force ok
 - Large problems different story

Using GIS to Derive Contiguity

- Topological database
- ArcInfo Arc Attribute Table

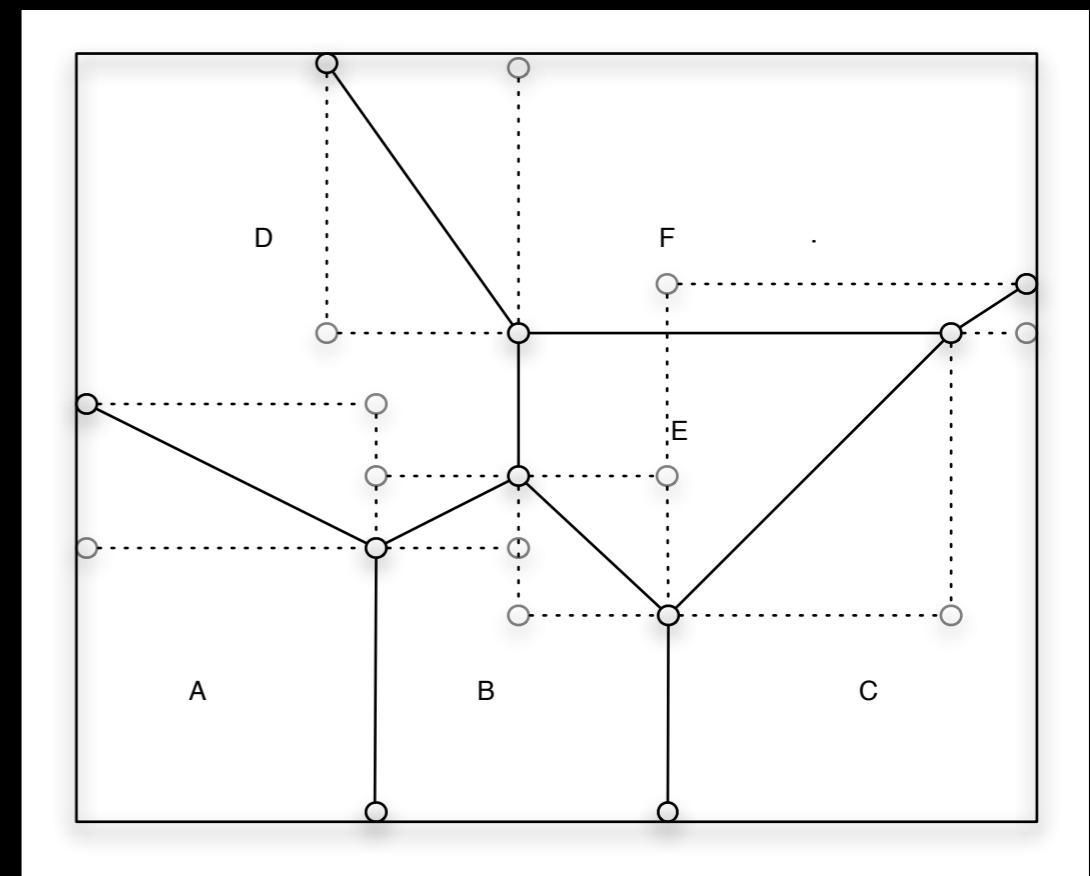
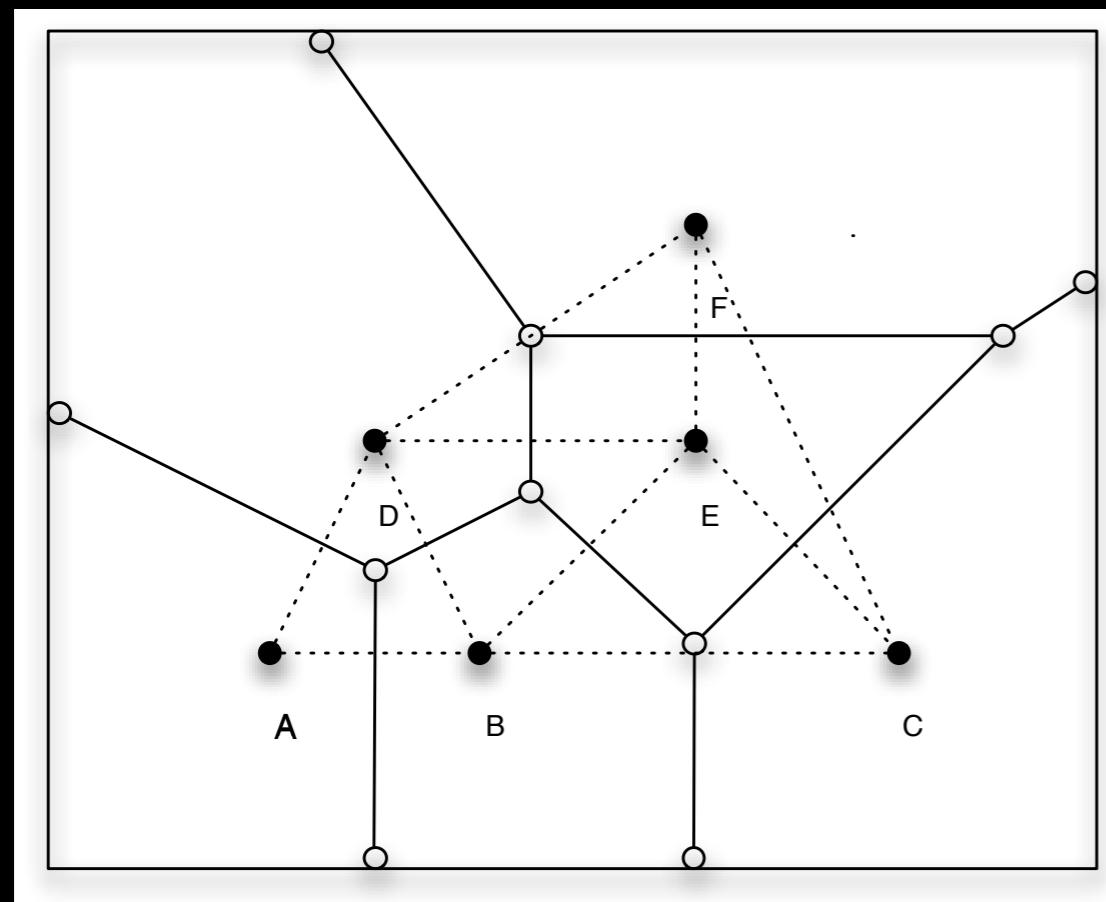


| ArcID | FromNode | ToNode | RightPoly | LeftPoly | Length |
|-------|----------|--------|-----------|----------|--------|
| AB | A | B | 0 | 1 | 6 |
| BF | B | F | 0 | 1 | 3 |
| FE | F | E | 3 | 1 | 2 |
| ED | E | D | 4 | 1 | 2 |
| DC | D | C | 2 | 1 | 2 |
| CA | C | A | 0 | 1 | 3 |
| ... | ... | ... | ... | ... | ... |

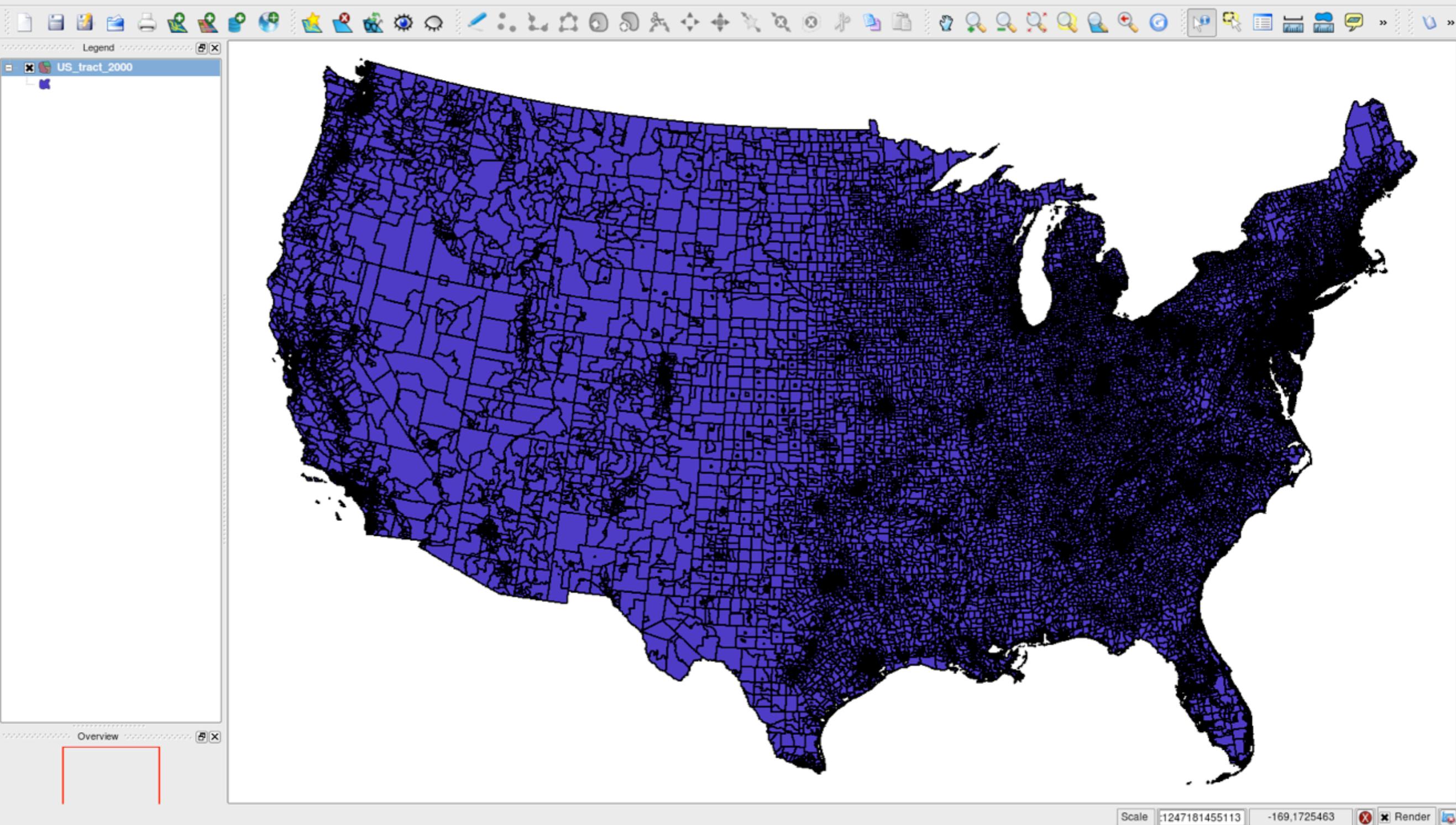
Using GIS to Derive Contiguity

- Nontopological database
 - Shapefile = spaghetti
 - Polygon as a list of vertices
- PySAL
 - Filter-refine + plane sweep algorithm

Minimum Bounding Box to Derive Contiguity



File View Layer Settings Plugins Help



US Tracts

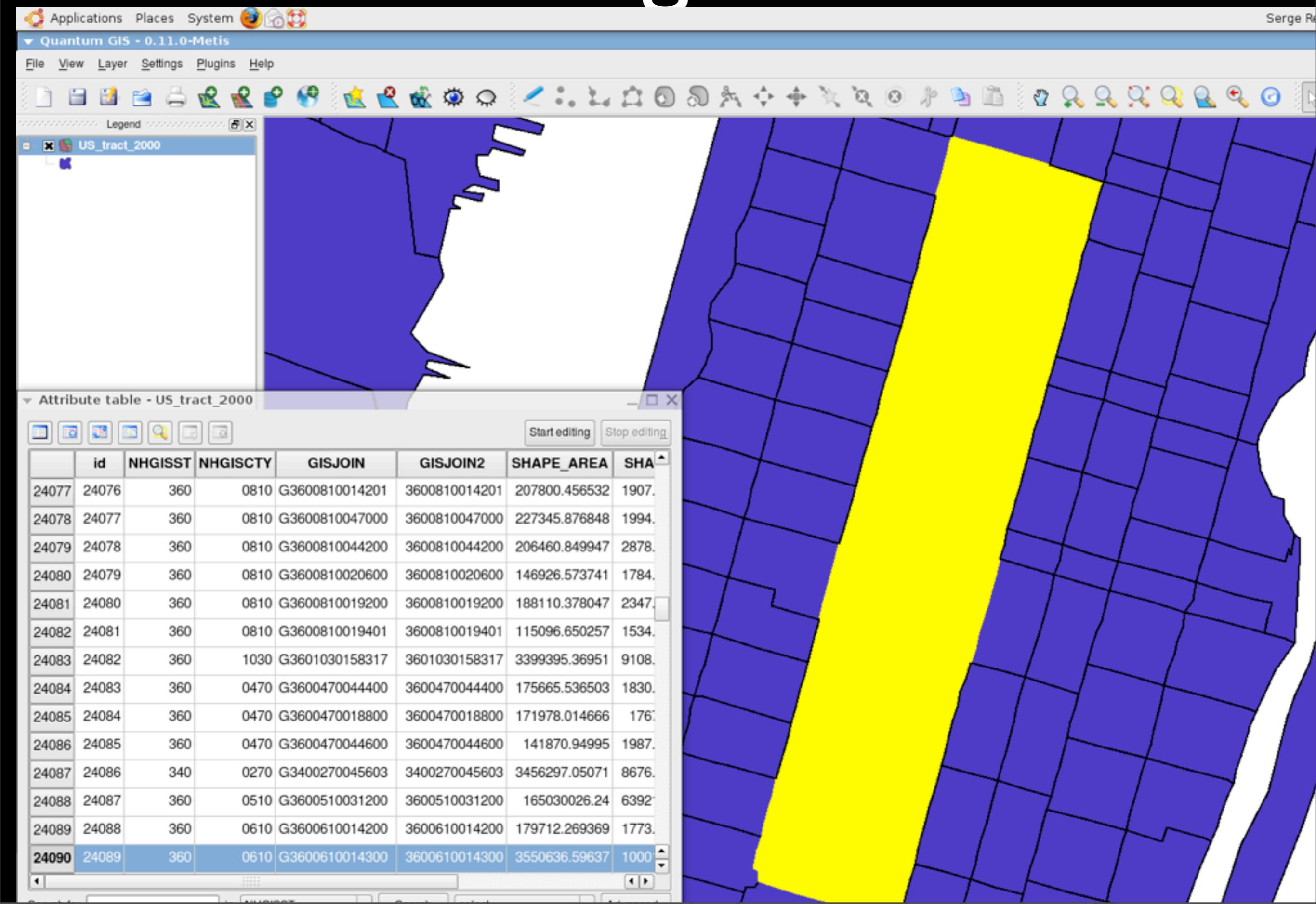
ContiguityWeights

```
961 fname="/Users/serge/Desktop/nhgis/us_tract_2000/us_tract_2000.shp"
962 shpFile=shpIO.shpFile(fname)
963
964 def run():
965     c=ContiguityWeights(shpFile, None, None)
966
967 import cProfile
968 t0=time.time()
969 cProfile.run('run()')
970 t1=time.time()
971 print t1-t0
972
973 c=ContiguityWeights(shpFile, None, None)
974 c.writeGAL("tracts2000.gal")
975 g=GAL(c.w)
```

Cardinalities

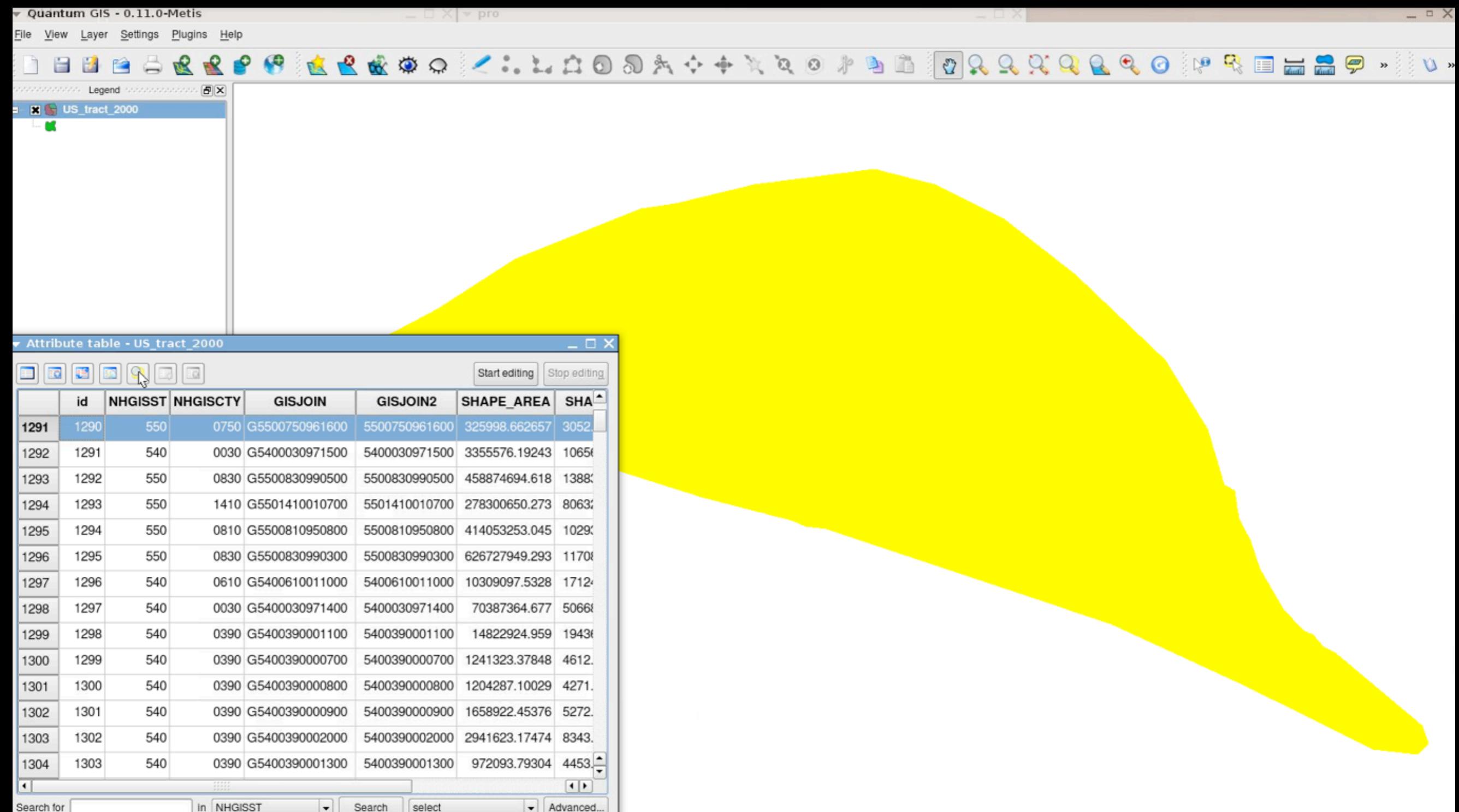
```
>>> cards=[ len(nn) for i,nn in c.w.items()]
>>> max(cards)
30
>>> cards.index(30)
24089
>>>
```

30 neighbors



Islands

```
>>> len(g.islands)
46
>>> g.islands[0:10]
[1290, 2859, 2906, 2916, 2996, 3070, 3125, 10554, 10622, 11142]
>>>
```



Garbage In

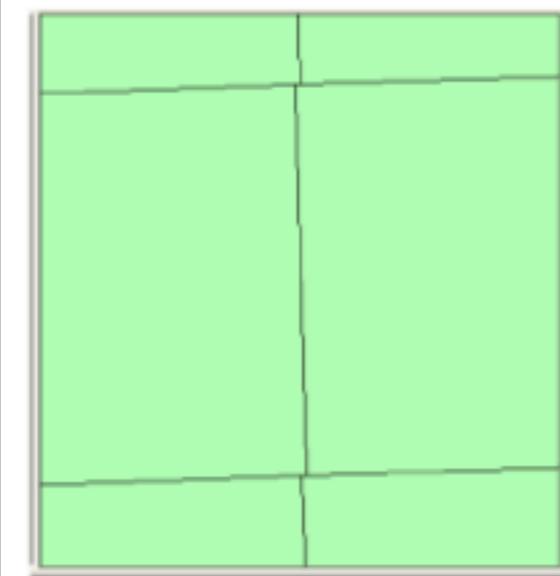


Figure 3.6: Misaligned grid cells

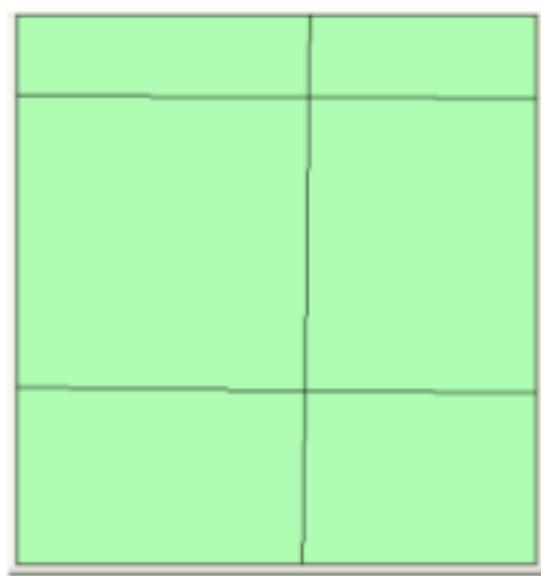


Figure 3.7: Aligned grid cells

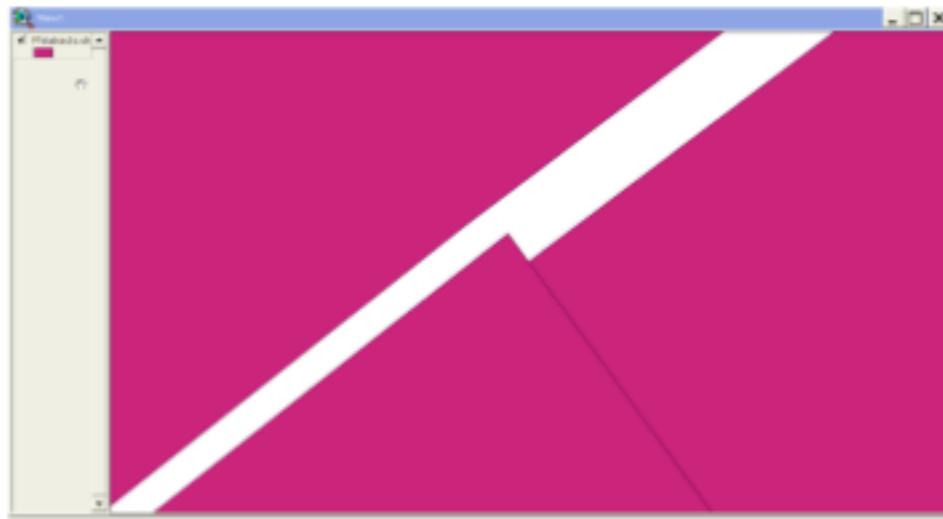


Figure 3.8: Non-touching census tracts

General Weights

Distance Function

- $w_{ij} = f(d_{ij}, \Theta)$
- $\partial w_{ij}/\partial d_{ij} < 0$
- Tobler's first law
- distance **decay** effect

Distance Functions

- $w_{ij} = f(d_{ij}, \Theta)$
 - $w_{ij} = 1 / d_{ij}^\alpha$
 - $w_{ij} = 0 \quad \forall d_{ij} > \delta$
 - α
 - $\alpha = -1$ for inverse distance weights
 - $\alpha = -2$ for gravity weights

Inverse Distance

| | | | | | |
|-------|-------|-------|-------|-------|-------|
| 0 | 0.100 | 0.033 | 0.089 | 0.045 | 0.035 |
| 0.100 | 0 | 0.050 | 0.089 | 0.071 | 0.045 |
| 0.033 | 0.050 | 0 | 0.037 | 0.071 | 0.045 |
| 0.089 | 0.089 | 0.037 | 0 | 0.067 | 0.056 |
| 0.045 | 0.071 | 0.071 | 0.067 | 0 | 0.100 |
| 0.035 | 0.045 | 0.045 | 0.056 | 0.100 | 0 |

Gravity Weights

$$\mathbf{W} = \begin{vmatrix} 0 & 0.010 & 0.001 & 0.008 & 0.002 & 0.001 \\ 0.010 & 0 & 0.003 & 0.008 & 0.005 & 0.002 \\ 0.001 & 0.003 & 0 & 0.001 & 0.005 & 0.002 \\ 0.008 & 0.008 & 0.001 & 0 & 0.004 & 0.003 \\ 0.002 & 0.005 & 0.005 & 0.004 & 0 & 0.010 \\ 0.001 & 0.002 & 0.002 & 0.003 & 0.010 & 0 \end{vmatrix}$$

Distance-Perimeter Weights

- $w_{ij} = b_{ij}^\beta / d_{ij}^\alpha$
- b_{ij} is the share of the common border between i and j

Distance-Perimeter

| Shared Perimeter | | | | | | | Perimeters |
|---------------------|---|---|---|---|---|--|------------|
| | 2 | 3 | 4 | 5 | 6 | | |
| 1 | 2 | 0 | 2 | 2 | 0 | | 16 |
| 2 | | 0 | 2 | 1 | 0 | | 10 |
| 3 | | | 0 | 2 | 1 | | 6 |
| 4 | | | | 2 | 0 | | 6 |
| 5 | | | | | 0 | | 10 |
| | | | | | | | 4 |

$b = \begin{pmatrix} 0 & 0.125 & 0 & 0.125 & 0.125 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.333 & 0.167 \\ 0.333 & 0.333 & 0 & 0 & 0.333 & 0 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \end{pmatrix}$

Distance-Perimeter

$$\mathbf{b} = \begin{vmatrix} 0 & 0.125 & 0 & 0.125 & 0.125 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.333 & 0.167 \\ 0.333 & 0.333 & 0 & 0 & 0.333 & 0 \\ 0.2 & 0.1 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \end{vmatrix}$$

$$\mathbf{w} = \begin{vmatrix} 0 & 0.05 & 0 & 0.08 & 0.05 & 0 \\ 0.08 & 0 & 0 & 0.13 & 0.07 & 0 \\ 0 & 0 & 0 & 0 & 0.21 & 0.15 \\ 0.22 & 0.21 & 0 & 0 & 0.21 & 0 \\ 0.08 & 0.03 & 0.13 & 0.13 & 0 & 0 \\ 0 & 0 & 0.23 & 0 & 0 & 0 \end{vmatrix}$$

Kernel Weights

- $w_{ij} = K(u/h)$
 - K : kernel, h : bandwidth
 - $k(u/h) = 0$ for $u > h$
 - $\int K(u) du = 1$

Kernel Functions

| Functions | Expression |
|-----------------------------|---|
| Truncated, Uniform | $K(z) = 1 \times \mathbf{1}(z \leq 1)$ |
| Triangular, Bartlett | $K(z) = (1 - z) \times \mathbf{1}(z \leq 1)$ |
| Epanechnikov, Quadratic | $K(z) = (3/4)(1 - z^2) \times \mathbf{1}(z \leq 1)$ |
| Quartic, Biweight, Bisquare | $K(z) = (15/16)(1 - z^2)^2 \times \mathbf{1}(z \leq 1)$ |
| Parzen | $K(z) = (1 - 6z^2 + 6 z ^3) \times \mathbf{1}(0 \leq z \leq 1/2)$ $K(z) = 2(1 - z)^3) \times \mathbf{1}(1/2 < z \leq 1)$ |
| Gaussian, Normal | $K(z) = (2\pi)^{-1/2} \exp(-z^2/2)$ |

Kernel $\delta = 15.0$

| | B | C | D | E | F |
|---|-------|---|-------|-------|-------|
| A | 0.667 | — | 0.747 | — | — |
| B | — | — | 0.747 | 0.940 | — |
| C | | | — | 0.940 | — |
| D | | | | 1.000 | — |
| E | | | | | 0.667 |

| | | | | | | |
|------------------|-------|-------|-------|-------|-------|-------|
| Arguments | 1.000 | 0.333 | 0 | 0.253 | 0 | 0 |
| | 0.333 | 1.000 | 0 | 0.253 | 0.060 | 0 |
| | 0 | 0 | 1.000 | 0 | 0.060 | 0 |
| | 0.253 | 0.253 | 0 | 1.000 | 0.000 | 0 |
| | 0 | 0.060 | 0.060 | 0.000 | 1.000 | 0.333 |
| | 0 | 0 | 0 | 0 | 0.333 | 1.000 |

$$\mathbf{W} = \begin{pmatrix} 1.000 & 0.333 & 0 & 0.253 & 0 & 0 \\ 0.333 & 1.000 & 0 & 0.253 & 0.060 & 0 \\ 0 & 0 & 1.000 & 0 & 0.060 & 0 \\ 0.253 & 0.253 & 0 & 1.000 & 0.000 & 0 \\ 0 & 0.060 & 0.060 & 0.000 & 1.000 & 0.333 \\ 0 & 0 & 0 & 0 & 0.333 & 1.000 \end{pmatrix}$$

Statistical Weights

- AMOEBA (Aldstadt and Getis 2006)

$$w_{ij} = \frac{P[G_i^*(k_{max})] - P[G_i^*(k_j)]}{P[G_i^*(k_{max})] - P[G_i^*(0)]}$$

where $P[]$ is the cumulative probability of a standard normal variate.

Socioeconomic Weights

- Many Different Types
 - Socioeconomic Similarity
 - Input-Output Tables
 - Social Networks
- Issues
 - W now endogenous
 - Regularity conditions may be violated

Row Standardization

$$w_{ij}^* = \frac{w_{ij}}{\sum_j w_{ij}}$$

Number of Neighbors $K = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Row Standardization

$$W^* = K^{-1}W$$

$$\mathbf{W}^* = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{W}^* = \begin{bmatrix} 0 & 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Guidance

How to Select the Right W?

- Theoretical priors
 - exceptionally rare
- Typically no “best” solution
- Relation to spatial interaction theory

Rules of Thumb (Griffith 1996)

- “It is better to posit some reasonable geographic weights matrix specification than to assume all entries are zero.
- “..use a surface partitioning that falls somewhere between a regular square and a regular hexagonal tessellation”
- “ ..better to employ a somewhat under-specified than a somewhat over-specified geographic weights matrix.”

Choice of W

- For **testing** for autocorrelation
 - power matters
 - contiguity may be sufficient
 - power robust to misspecification
- For **modeling** and parameter **estimation**
 - more complex and fragile