Spatial Effects in the Analysis of Regional Income Convergence and Inequality

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October 12, 2001

Organization

1. Regional Convergence Studies

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2. Spatially Explicit Approaches

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2. Spatially Explicit Approaches

   - Spatial Markov Matrix

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   - Spatial Dependence in Distributional Mixing

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3. Demo: Space Time Analysis of Regional Systems (STARS)

Background

empiric (ĕm-pîr' ĭk) n. 1. That depending upon the observation of phenomena. 2. An unqualified or dishonest practitioner; a charlatan.

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Theoretical Basis

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Theoretical Basis

The rich are not like us. *F. Scott Fitzgerald*

Yes, they have more money. *Ernest Hemingway*

Regional Convergence Studies

- σ Convergence (**?**)
  - measures dispersion $=$ income gap
  - 2nd moment of distribution
  - ignores mixing
  - ignores geographical distribution

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- β Convergence (**?**)

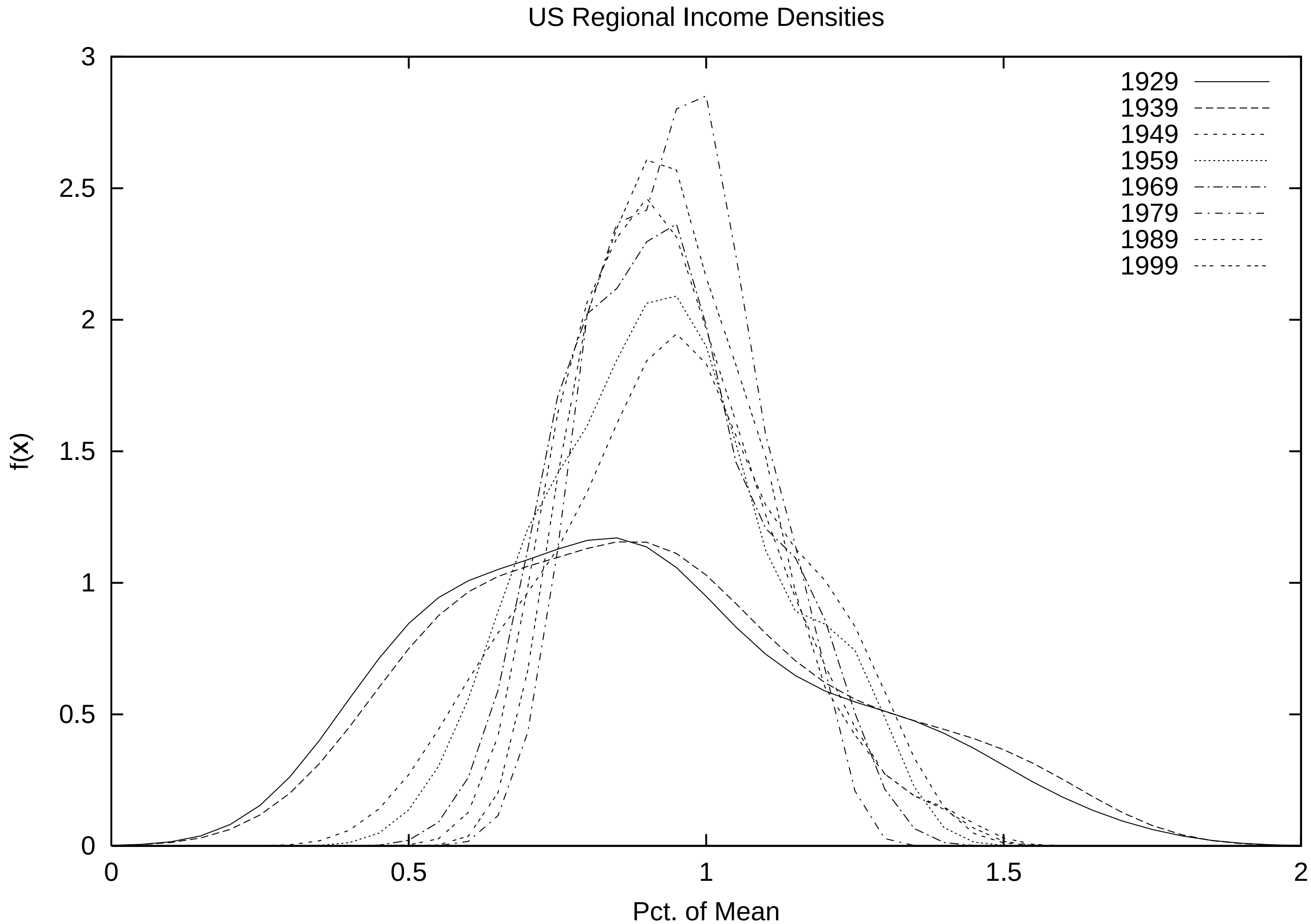
$$\frac{1}{k} \cdot \ln \left(\frac{y_{i,t+k}}{y_{i,t}} \right) = \alpha - \beta \cdot \ln \left(y_{i,t} \right) + \psi X_{i,t} + \epsilon_{i,t} \tag{1}$$

  - conditional vs. unconditional
  - neoclassical basis

- “Newer Approaches”
  - Stochastic convergence (**?**)
  - Markov Chains (**???**)
  - Mixture Models (**?**)

- “Newer Approaches”
  - Stochastic convergence (**?**)
  - Markov Chains (**???**)
  - Mixture Models (**?**)

- Treatment of Space?
  - Spatial dependence
    β: (**??**)
  - Spatial heterogeneity
    β: (**??**)
  - Spatial scale



Space Time Information Systems, 2002

Markov Chains

- focus on transitional dynamics
- higher order moments of distribution
- accommodates shocks, discontinuities and turbulence

$$P_{t+b} = P_t M^b \tag{2}$$

$$P_t = \left[p_{1,t}, p_{2,t} \ldots p_{k,t}\right], \sum_{i=1}^{k} p_{i,t} = 1 \tag{3}$$

$$M = \begin{bmatrix} m_{1,1} & m_{1,2} & \ldots & m_{1,k} \\ m_{2,1} & m_{2,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ m_{k,1} & m_{k,2} & \ldots & m_{k,k} \end{bmatrix} \tag{4}$$

$$\sum_{j=1}^{k} m_{i,j} = 1 \ \forall i$$

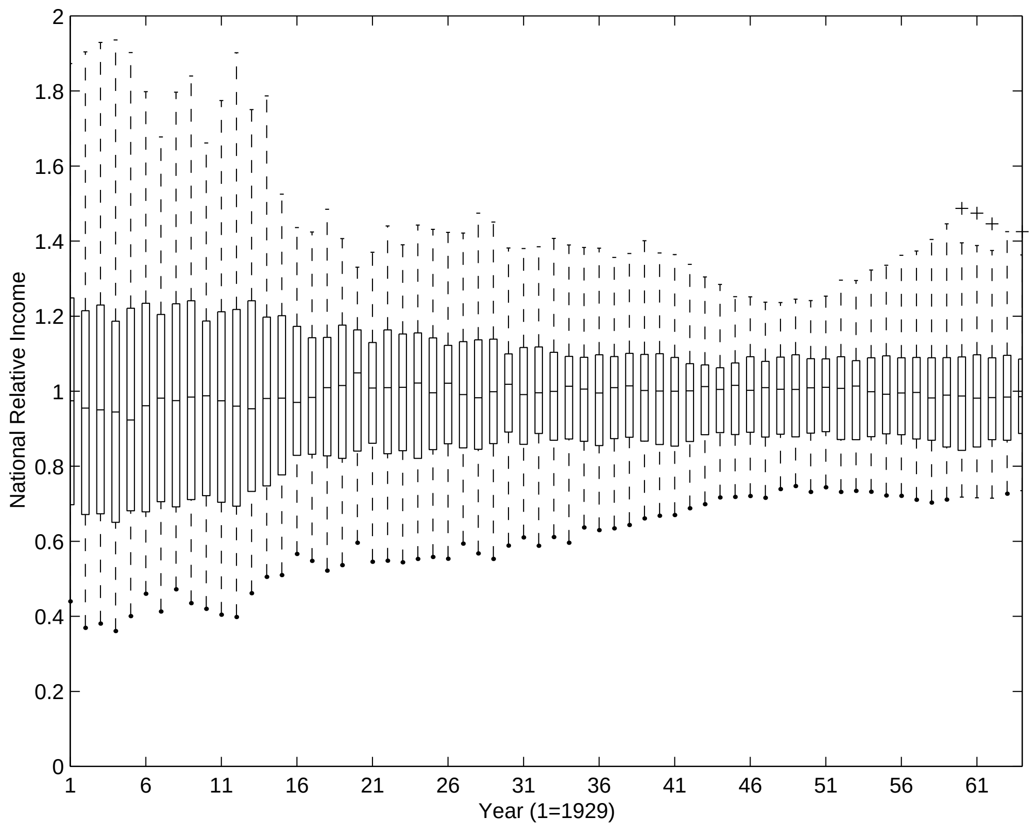
Space and Markov Chains

- spatial effects
  - dependence
    - chains based on independence assumption
    - $+$ dependence $=$ inflated deviance
  - heterogeneity?

- Quah's regional conditioning
  - national-relative distribution
  - regional-relative distribution
  - results
    - regional conditioning $=$ less dispersion
    - accounts for variation in (national)distribution

– national-regional relative
– the two should be identical under spatial independence assumption

Nationally Conditioned Distributions



Regionally Conditioned Distributions

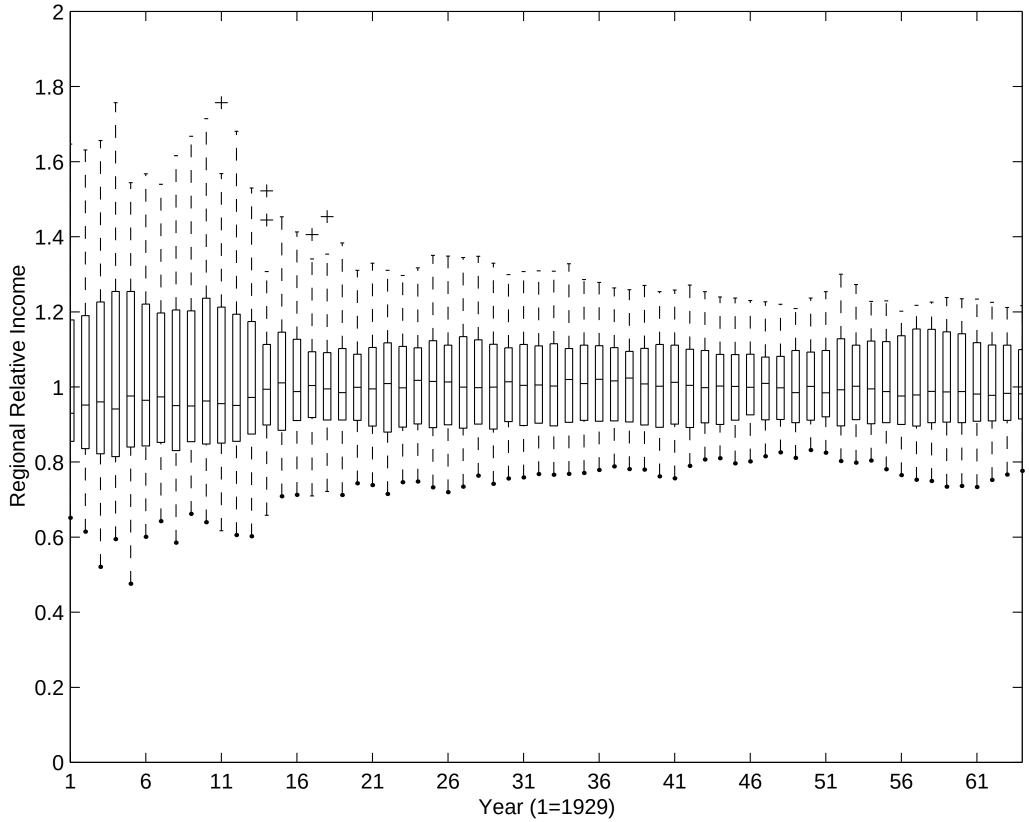


Table 1: US Markov Matrices (Annual) 1929-94

		t_1		US-Relative		
t_0	N	P	L	M	U	R
P	335	0.919	0.072	0.006	0.003	0.000
L	597	0.022	0.889	0.085	0.003	0.000
M	667	0.000	0.070	0.813	0.112	0.004
U	881	0.000	0.001	0.091	0.873	0.035
R	640	0.000	0.000	0.000	0.067	0.933
		t_1		Neighbor-Relative		
t_0	N	P	L	M	U	R
P	103	0.786	0.194	0.019	0.000	0.000
L	600	0.028	0.870	0.098	0.003	0.000
M	908	0.000	0.061	0.872	0.066	0.001
U	947	0.000	0.000	0.062	0.886	0.052
R	562	0.000	0.000	0.000	0.103	0.897
US		t_0		US to Neighbor-Relative		
t_0	N	P	L	M	U	R
P	335	0.221	0.439	0.269	0.039	0.033
L	597	0.028	0.387	0.422	0.124	0.039
M	667	0.009	0.234	0.390	0.256	0.111
U	881	0.007	0.072	0.285	0.460	0.177
P	640	0.000	0.005	0.086	0.444	0.466

Issues

- Problem
  - Quah's national-regional transition is static but spatial
  - dynamics?
- Solution
  - Local Indicators of Spatial Association (LISAs) (**?**)
  - Integrate into Markov Chain framework

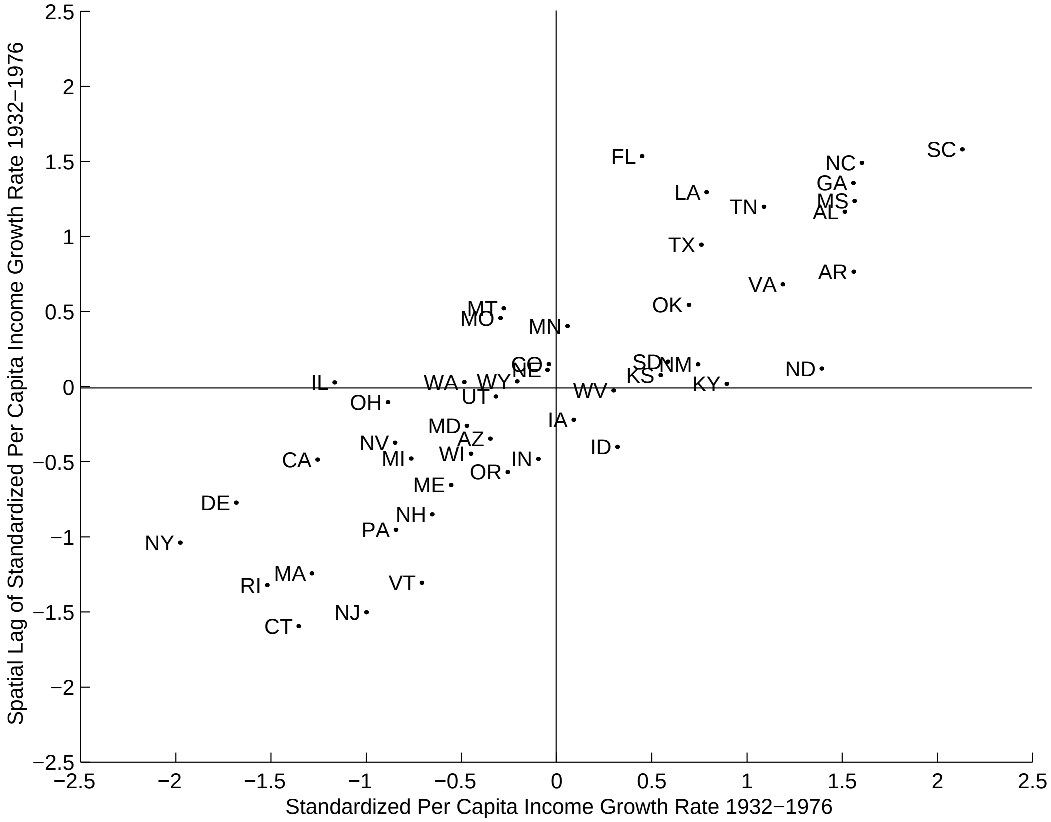
LISAs

$$I_i = z_i \sum_{i=1}^{n} w_{i,j} z_j \tag{5}$$

Table 2: LISA Classifications

Class	Own Value	Neighbors' Value
HH	Above Average	Above Average
HL	Above Average	Below Average
LH	Below Average	Above Average
LL	Below Average	Below Average

Moran Scatterplot



Space Time Information Systems, 2002

Table 3: Spatial Transitions

Transition	State t_0	State t_1	Mover	Type
1	HH	HL	Neighbors	II
2	HH	LH	State	I
3	HH	LL	Both	III
4	HL	HH	Neighbors	II
5	HL	LH	Both	III
6	HL	LL	State	I
7	LH	HH	State	I
8	LH	HL	Both	III
9	LH	LL	Neighbors	II
10	LL	LH	Neighbors	II
11	LL	HL	State	I
12	LL	HH	Both	III

Spatial Flux:

$$n = F_{0,t} + F_{I,t} + F_{II,t} + F_{III,t}. \tag{6}$$

$$RF_0 < RF_I < RF_{II} < RF_{III} \tag{7}$$

Spatial Stability:

$$S_t = \frac{F_{0,t}}{n}. \tag{8}$$

Spatial Cohesion

$$C_t = \frac{F_{IIIA,t}}{n} \tag{9}$$

$$C_t^* = \frac{F_{IIIA,t} + F_{0,t}}{n} \tag{10}$$

Table 4: A Spatial Markov Matrix

	State	t_1		
Lag	t_0	Low	Med.	High
Low	Low	$m_{LL\|L}$	$m_{LM\|L}$	$m_{LH\|L}$
	Med.	$m_{ML\|L}$	$m_{MM\|L}$	$m_{MH\|L}$
	High	$m_{HL\|L}$	$m_{HM\|L}$	$m_{HH\|L}$
Med.	Low	$m_{LL\|M}$	$m_{LM\|M}$	$m_{LH\|M}$
	Med.	$m_{ML\|M}$	$m_{MM\|M}$	$m_{MH\|M}$
	High	$m_{HL\|M}$	$m_{HM\|M}$	$m_{HH\|M}$
High	Low	$m_{LL\|H}$	$m_{LM\|H}$	$m_{LH\|H}$
	Med.	$m_{ML\|H}$	$m_{MM\|H}$	$m_{MH\|H}$
	High	$m_{HL\|H}$	$m_{HM\|H}$	$m_{HH\|H}$
	Low	m_{LL}	m_{LM}	m_{LH}
	Med.	m_{ML}	m_{MM}	m_{MH}
	High	m_{HL}	m_{HM}	m_{HH}

Under the Null:

$$m_{ij|1} = m_{ij|2} = \ldots = m_{ij|k} = m_{ij} \;\forall\; i, j. \tag{11}$$

Table 5: Classification of US Spatial Transitions 1929-94

Interval	TYPE 0	I	II	IIIA	IIIB	Cohesion	Flux
1-year	0.822	0.115	0.058	0.004	0.000	0.826	0.178
	UP	0.058	0.030	0.002	0.000		
	DOWN	0.057	0.028	0.002	0.000		
5-year	0.643	0.231	0.107	0.017	0.002	0.660	0.357
	UP	0.123	0.053	0.009	0.000		
	DOWN	0.108	0.054	0.009	0.002		
10-year	0.547	0.297	0.114	0.041	0.003	0.587	0.453
	UP	0.166	0.060	0.017	0.000		
	DOWN	0.131	0.054	0.023	0.003		
15-year	0.475	0.351	0.117	0.049	0.007	0.524	0.525
	UP	0.210	0.057	0.020	0.000		
	DOWN	0.142	0.060	0.029	0.007		
25-year	0.384	0.458	0.115	0.031	0.012	0.415	0.616
	UP	0.288	0.057	0.007	0.000		
	DOWN	0.170	0.058	0.024	0.012		

Table 6: US Spatial Markov Matrix

LAG	State t_o	N	t_1 P	L	M	U	R
P	P	150	0.960	0.040	0.000	0.000	0.000
	L	40	0.050	0.775	0.175	0.000	0.000
	M	27	0.000	0.148	0.741	0.111	0.000
	U	3	0.000	0.000	0.667	0.333	0.000
	R	0	0.000	0.000	0.000	0.000	0.000
L	P	136	0.912	0.081	0.007	0.000	0.000
	L	171	0.035	0.936	0.029	0.000	0.000
	M	108	0.000	0.028	0.907	0.065	0.000
	U	48	0.000	0.000	0.146	0.854	0.000
	R	3	0.000	0.000	0.000	0.000	1.000
M	P	45	0.867	0.111	0.022	0.000	0.000
	L	266	0.004	0.898	0.090	0.008	0.000
	M	220	0.000	0.086	0.755	0.155	0.005
	U	364	0.000	0.000	0.085	0.882	0.033
	R	85	0.000	0.000	0.000	0.165	0.835
U	P	4	0.250	0.500	0.000	0.250	0.000
	L	116	0.034	0.862	0.103	0.000	0.000
	M	253	0.000	0.067	0.818	0.107	0.008
	U	346	0.000	0.003	0.092	0.861	0.043
	R	269	0.000	0.000	0.000	0.082	0.918
R	P	0	0.000	0.000	0.000	0.000	0.000
	L	4	0.000	0.250	0.750	0.000	0.000
	M	59	0.000	0.068	0.864	0.068	0.000
	U	120	0.000	0.000	0.067	0.900	0.033
	R	283	0.000	0.000	0.000	0.025	0.975

Table 7: Transition Probabilities Conditioned on Neighbors Income

		MOVE		
LAG	N	Down	None	Up
Poorer	1249	0.0697	0.9047	0.0256
Same	723	0.0858	0.8465	0.0678
Richer	1148	0.0348	0.8676	0.0976

Table 8: Ergodic Income Distributions

Lag	P	L	M	U	R
P	0.345	0.276	0.326	0.054	0.000
L	0.130	0.326	0.377	0.168	0.000
M	0.006	0.206	0.234	0.456	0.098
U	0.008	0.167	0.271	0.345	0.209
R	0.000	0.026	0.288	0.292	0.394
Below Avg.	0.187	0.311	0.354	0.148	0.000
Above Avg.	0.006	0.132	0.256	0.324	0.283

Mobility

1. Regional Income Mobility
   - Class Mobility
   - Mixing (Rank Mobility)

2. Spatial Issues

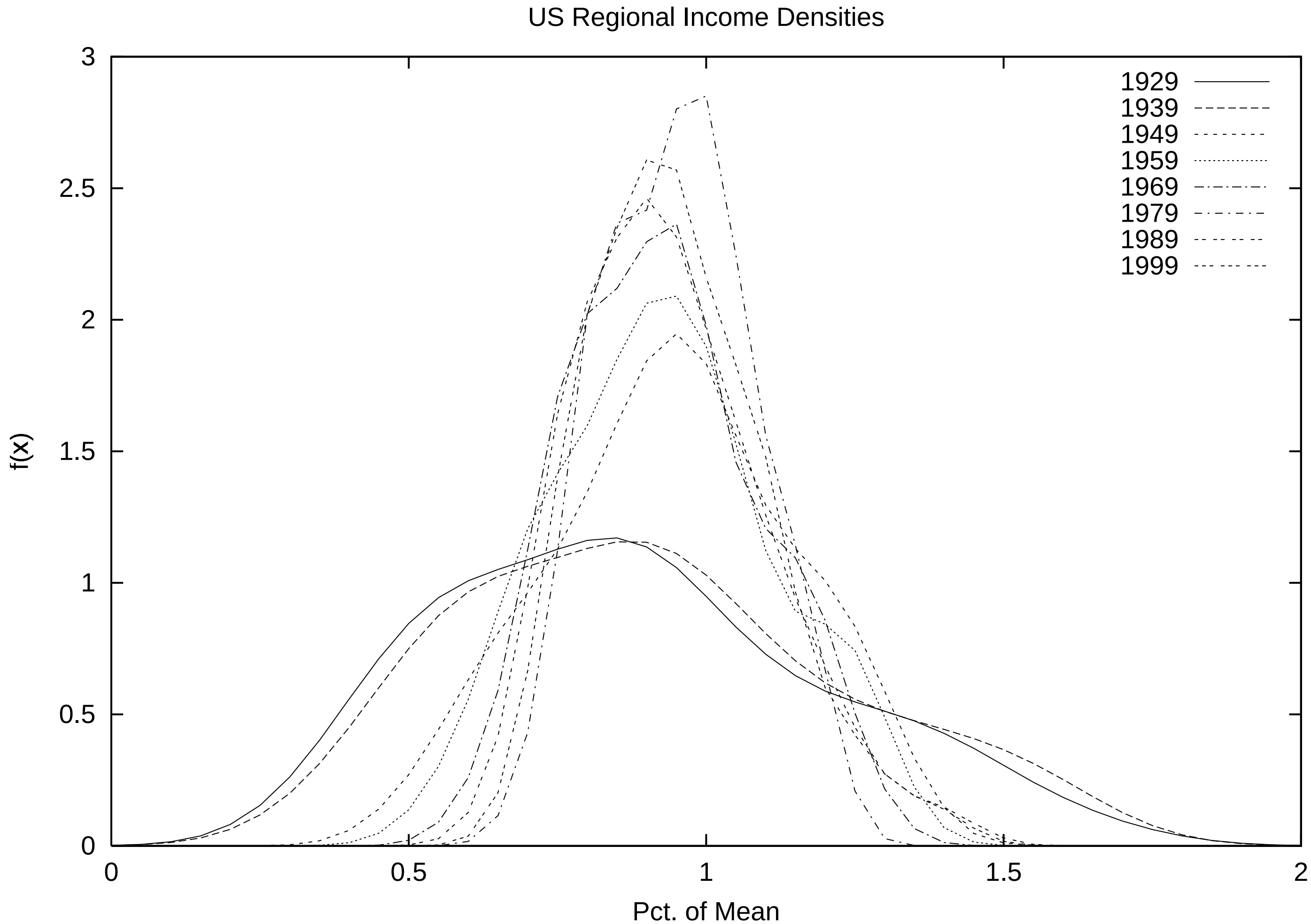
3. New Measures
   (a) Trace = Spatial Dependence (Static)
   (b) External Regional Cohesion = Dependence (Dynamic)
   (c) Internal Regional Cohesion = Dependence (Dynamic)

4. Illustration: Lower 48 States 1929-99

Why Mobility Matters

- Class Mobility
  - σ convergence
  - twin peaks (**?**)
  - mixtures?

- Rank Mobility
  - Leap-frogging
  - Cohesion versus flux


US Regional Income Densities
3
2.5
2
1.5
1
0.5
0
f(x)
0
0.5
1
1.5
2
Pct. of Mean
1929
1939
1949
1959
1969
1979
1989
1999

Spatial Issues

- Income distributions vs. Spatial distributions
- Moments - robust spatial pattern?
- Modality - robust to spatial pattern?
  - at one point in time
  - changes in moments vs. changes in pattern?
  - $\rightarrow$ dynamics

- Inference and spatial dependence
  - stochastic kernels
  - mixture modeling
  - $i.i.d$ versus space?

Class Mobility

$$M_{t,t+s} = \begin{pmatrix} m_{11} & \dots & m_{1k} \\ m_{21} & \dots & m_{2k} \\ \vdots & \vdots & \vdots \\ m_{k1} & \dots & m_{kk} \end{pmatrix} \tag{12}$$

where m_{ij} are the probabilities of a region making the transition from income class i to class j over the period t to $t+s$.

Table 9: Estimated Annual Transition Matrix, US 1929-99

	Class				
n	0.59	0.79	0.89	1.10	∞
158	0.87	0.13	0.00	0.00	0.00
565	0.02	0.88	0.09	0.01	0.00
635	0.00	0.07	0.82	0.11	0.00
1409	0.00	0.00	0.05	0.93	0.02
593	0.00	0.00	0.00	0.06	0.94

Shorrock (1978) mobility index:

$$SI = \frac{k - Tr\left(M_{t,t+s}\right)}{k-1} \tag{13}$$

where Tr indicates the trace operator.

- $0 <= SI <= 1.25$
- US: 0.141
- 1929 quintiles mapped to 1999:
  0, 5, 11, 25, 6
- upward convergence

Role of Space

More attention needed on two fronts.

- Substantive
  - Do growth spill-overs exist?
  - Is growth regionally competitive or regionally cooperative? Or, both?
  - Interregionally competitive vs. intraregionally cooperative?
  - Poverty Traps, Convergence Clubs, Regional Cohesion
- Measures
  - spatial dependence in regional income distributions.
  - the role of regional context in income mobility

Regional Conditioning

- **?**
- one point in time
- relative incomes
  - nationally conditioned income

$$y_{r,t}^{N} = \frac{y_{r,t}}{\sum_s y_{s,t}/n} \tag{14}$$

– regionally conditioned income

$$y^R_{r,t} = \frac{y_{r,t}}{\sum_s w_{r,s} y_{s,t}} \quad (15)$$

$$M_{R,N} = \begin{pmatrix} m_{R1,N1} & \dots & m_{R1,Nk} \\ m_{R2,N1} & \dots & m_{R2,Nk} \\ \vdots & \vdots & \vdots \\ m_{Rk,Nk} & \dots & m_{Rk,Nk} \end{pmatrix} \tag{16}$$

- diagonality $=$ spatial randomness

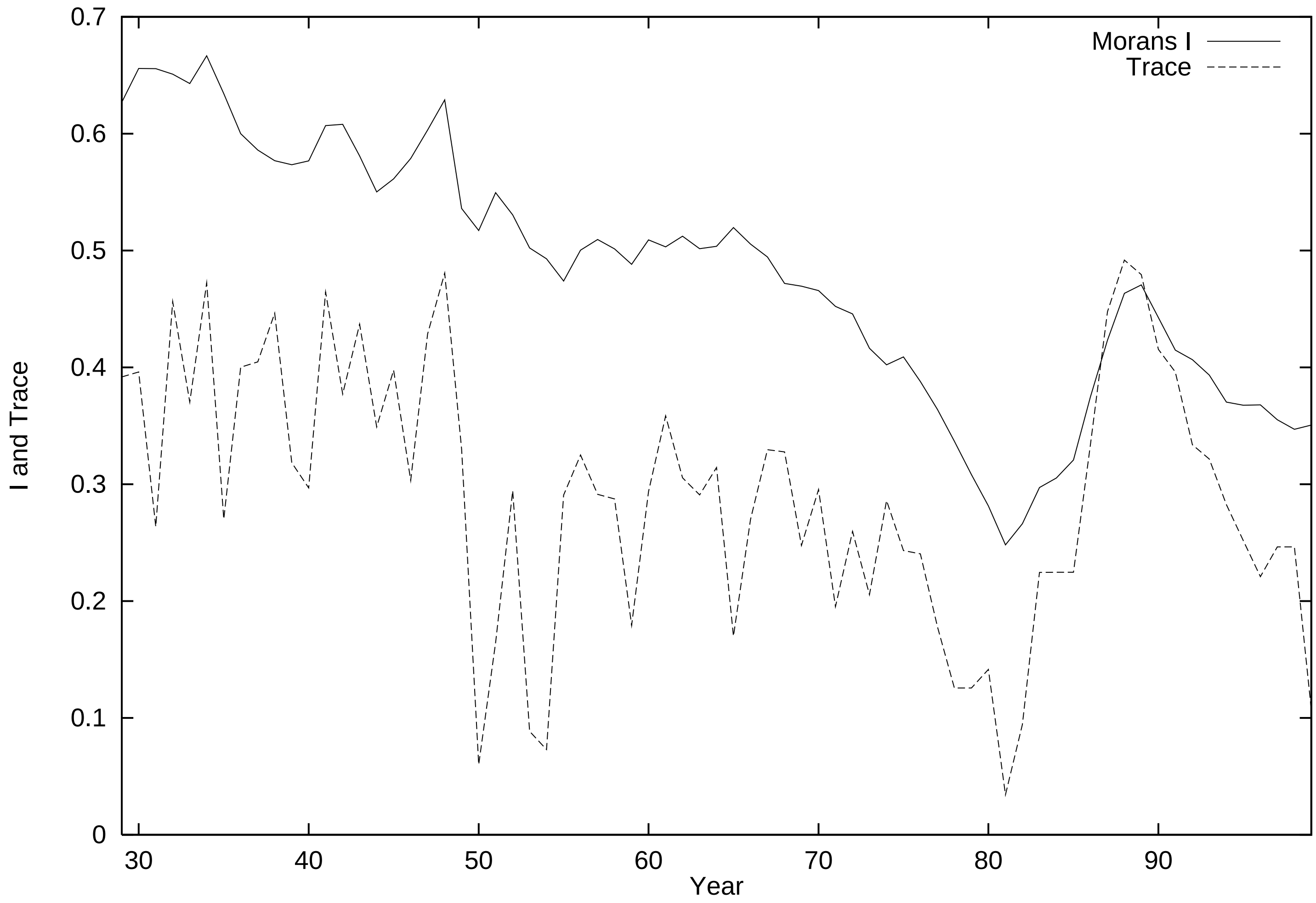
- intuitive, no formal statistic suggested

$$\zeta = 1 - \sum_{l}^{k} M_{l,l} \tag{17}$$

$$0 \leq \zeta \leq 1$$

- inference based on random spatial permutations.

Trace Test for Spatial Autocorrelation



Space Time Information Systems, 2002

Results: ζ

- Both tests display general downward trend
- Both tests pick up during 1980s
- Moran's I significant **each** year
- ζ significant 68 (0.1), 62 (0.05), 53 (0.01)

Spatial Rank Mobility

Let $\theta_{i,t}$ represent the rank of state i's per-capita income in year t. Then between any two periods, a scalar measure of spatial clustering of distributional transitions is:

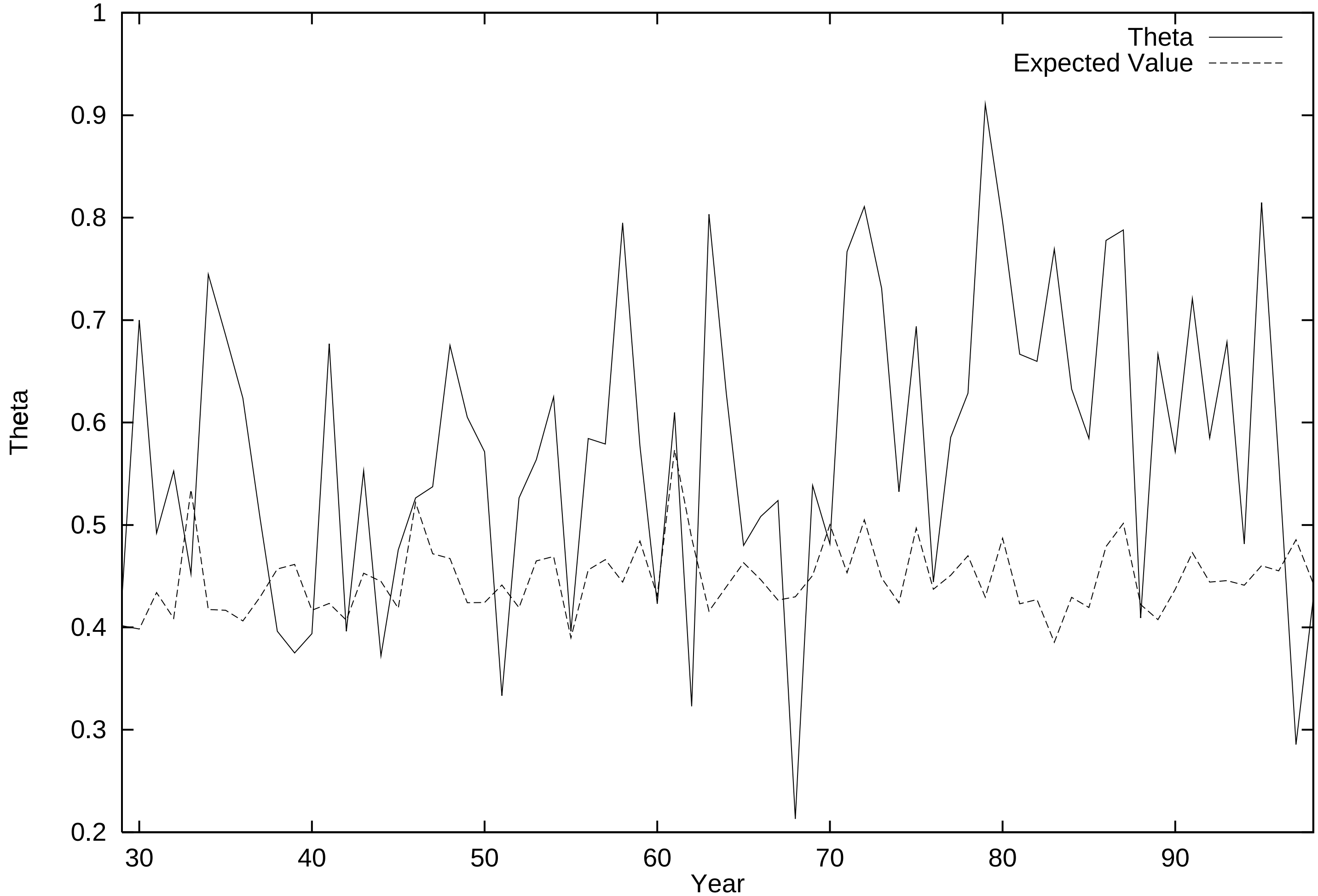
$$\Theta_{t1-t0} = \frac{\sum_R \left|\sum_{i \in R} \theta_{i,t1} - \theta_{i,t0}\right|}{\sum_i |\theta_{i,t1} - \theta_{i,t0}|} \tag{18}$$

where R is one of a set of exhaustive and mutually exclusive groups of states.

$$0 \leq \Theta_{t1-t0} \leq 1$$

$$Random \Longleftrightarrow Cohesion$$

Spatial Cohesion Index: 1-year Interval



Space Time Information Systems, 2002

Results: Θ

- $\Theta > E[\Theta]$ (56 of 70)
- Significant differences in 27, 23, 12 years
- Before 1968 Θ averages 0.54, after 0.61
- Before 1968, $E[\Theta]$ averages 0.445, after 0.451

Decomposition of Spatial Rank Mobility

Kendall's τ is given as:

$$\tau = \frac{N_c - N_d}{(n^2 - n)/2} \tag{19}$$

where N_c is the number of concordant pairs, and N_d the number of discordant pairs. If all pairs are concordant, then $N_c = (n^2 - n)/2$, $N_d = 0$ and $\tau = 1$, while if all pairs are discordant, then $\tau = -1$.

$$N_c = N_{c,r} + N_{c,o} \tag{20}$$

where $N_{c,r}$ is the number of concordant pairs involving locations belonging to the same region, and $N_{c,o}$ are the number of concordant pairs for observations belonging to different regions. A similar decomposition holds for the discordant pairs N_d. To develop a spatial version of the τ statistic from equation (**??**), define:

$$\omega = (n^2 - n)/2 \tag{21}$$

which is the denominator in the original form of the statistic. This is also equal to the number of elements above (or below) the main diagonal in the spatial weights matrix. Each element in that matrix specifies whether observations i and j are considered members of the same region (or more generally neighbors). The sum of these elements can also be decomposed:

$$\omega = \omega_r - \omega_o \tag{22}$$

where ω_r is the number of pairs of observations that are members of the same region. From this a spatial version of the rank mobility measure is:

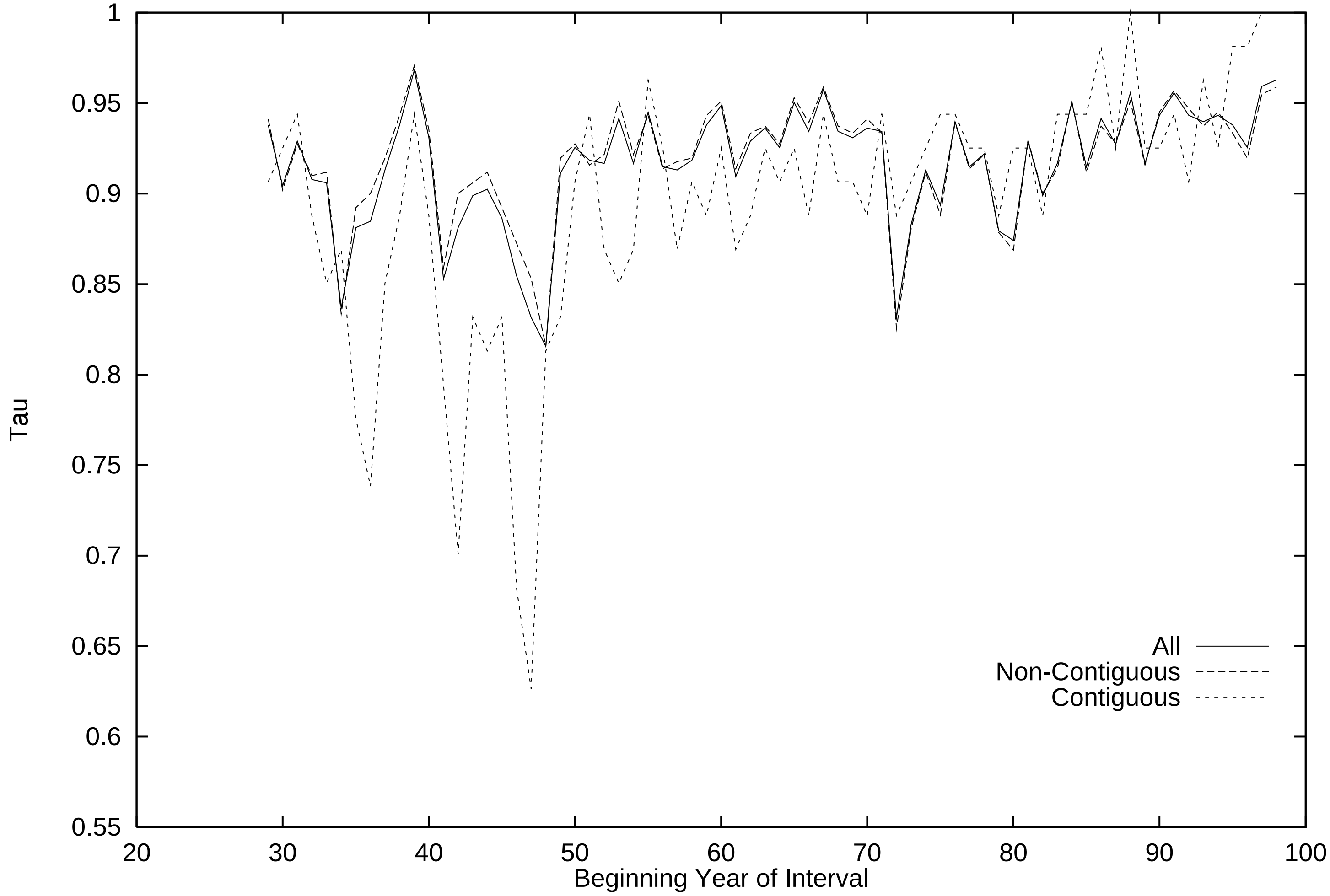
$$\tau_r = \frac{N_{c,r} - N_{d,r}}{\omega_r} \tag{23}$$

A similar measure can be derived for the non-neighbor pairs of observations τ_o. The original measure of rank mobility can then be decomposed as follows:

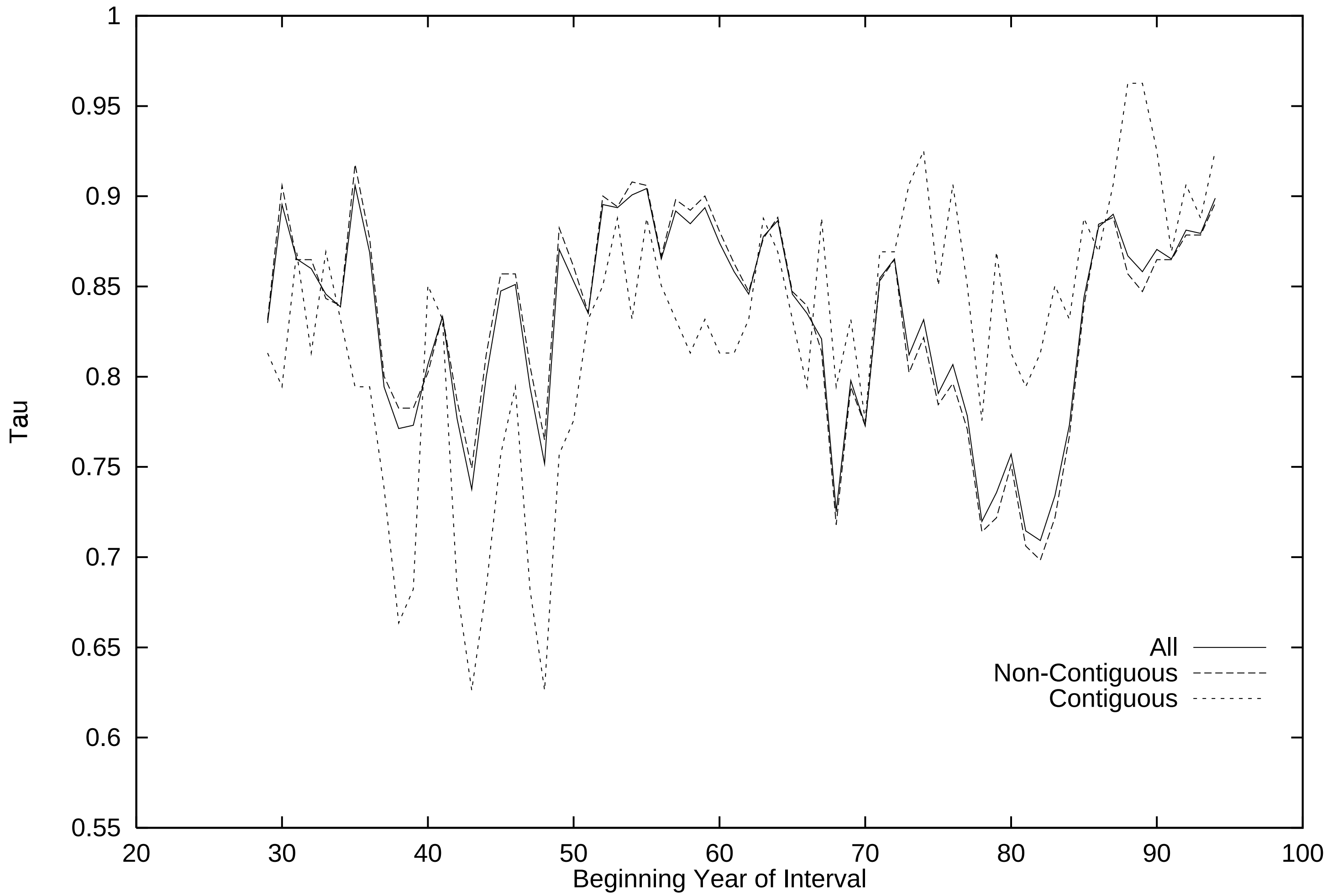
$$\tau = \psi\tau_r + (1 - \psi)\tau_o \tag{24}$$

with $\psi = \omega_r/\omega$.

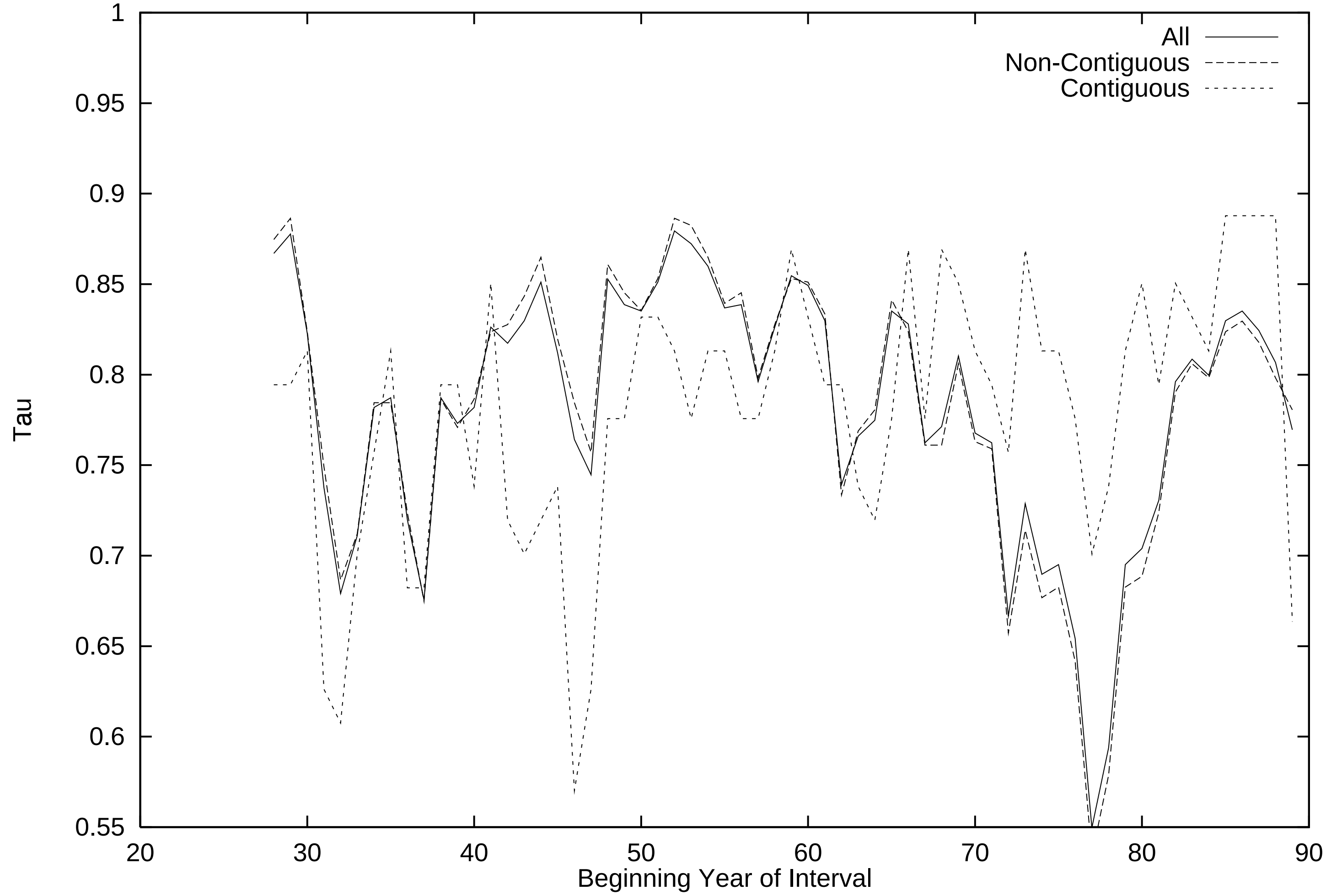
Spatial τ : 1-year interval



Spatial τ : 5-year interval



Spatial τ : 10-year interval



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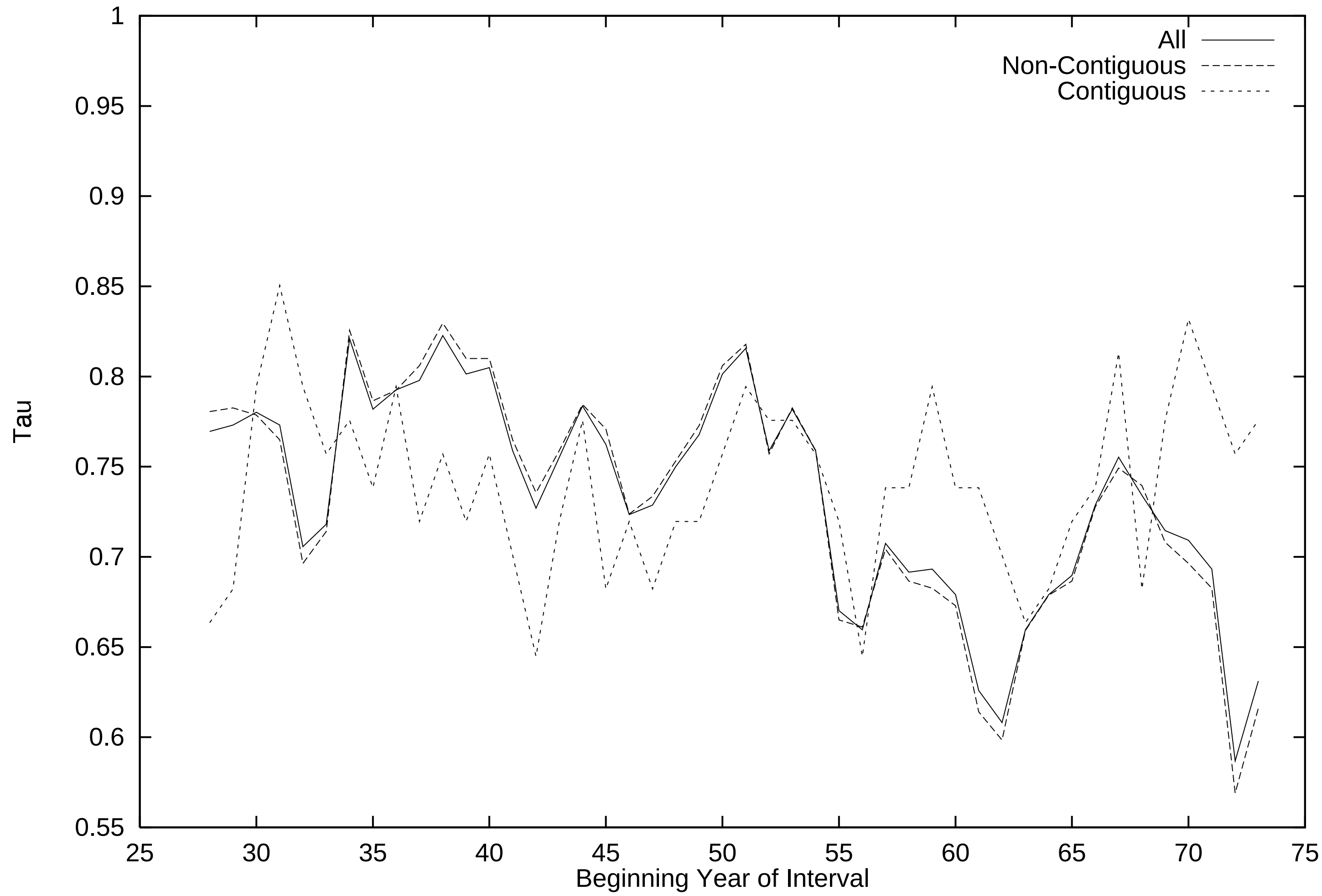
Spatial τ : 25-year interval



Results: Spatial τ

- Mobility increases with interval length
- Early, $\tau_r < \tau_o$
- Late, $\tau_r > \tau_o$
- Spatial dependence and dispersion greater in beginning of period.

Future Directions

- Distributional Properties of Measures

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- Software: STARS