Geostatistics Basics

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GEOG 384
Spatial Data Analysis
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Outline

Spatial Random Field

Continuous Spatial Process

$$\{Z(s) : s \in D\} \tag{1}$$

- s is spatial index, continuous in R^2 (R^3)
- Sample of spatial locations
  - $\{s_1, s_2, \ldots, s_n\}$: sample locations
  - $\{Z(s_1), Z(s_2), \ldots, Z(s_n)\}$: random variable at sample locations

Geostatistical Data

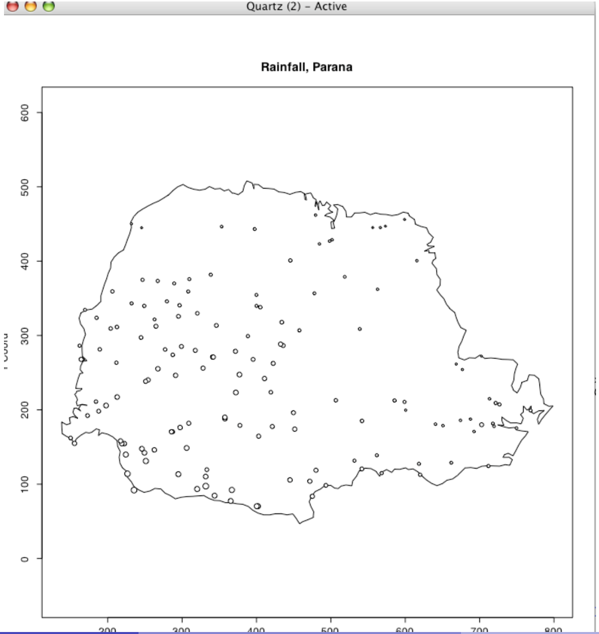
Spatial Domain: D

- A continuous and fixed set.
- Meaning $Z(s)$ can be observed everywhere within D.
- Between any two sample locations s_i and s_j you can theoretically place an infinite number of other samples.
- By fixed: the points in D are non-stochastic

Continuous Variation

- Because of the continuity of D
- Geostatistical data is referred to as "spatial data with continuous variation."
- Continuity is associated with D.
- Attribute Z may, or may not, be continuous.

Geostatistical Data: Rainfall in Parana State Brazil



Geostatistical Data

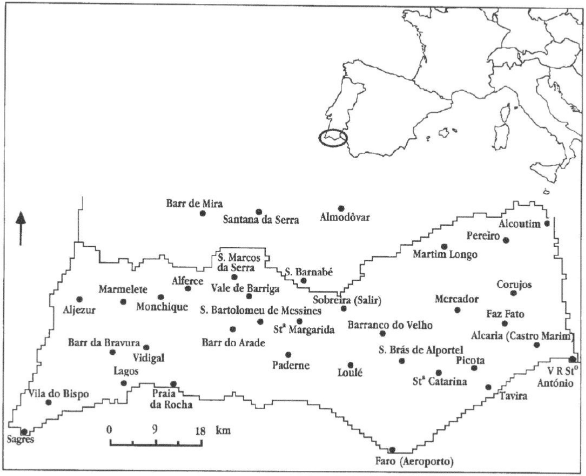
Continuous variation

- Potentially measurable anywhere in D
- Impossible to sample D exhaustively

Reconstruction of the surface from observed sites

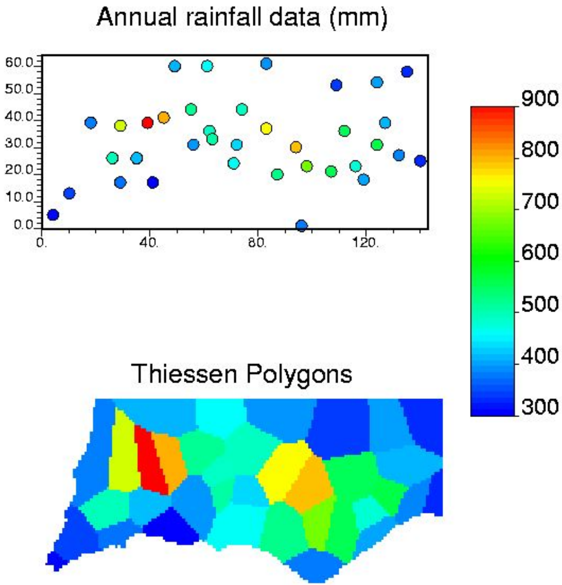
- Tessellation based methods
- Interpolation
- Kriging

Surface Reconstruction: Example[1]

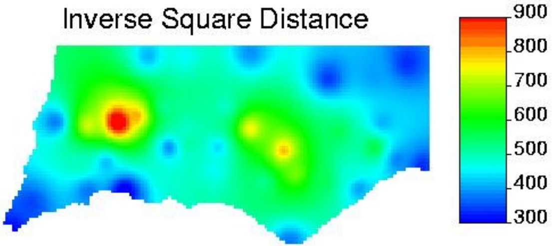


[1] From Goovaerts, P. (1999)"Performance comparison of geostatistical algorithms for incorporating elevation into the mapping of precipitation". *Geocomputation '99*.

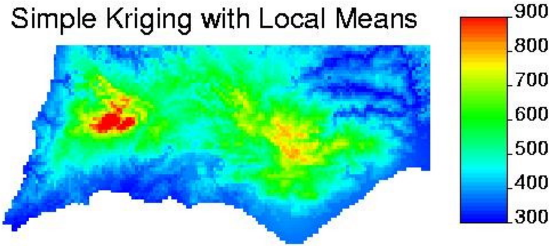
Surface Reconstruction: Tessellation Based Method



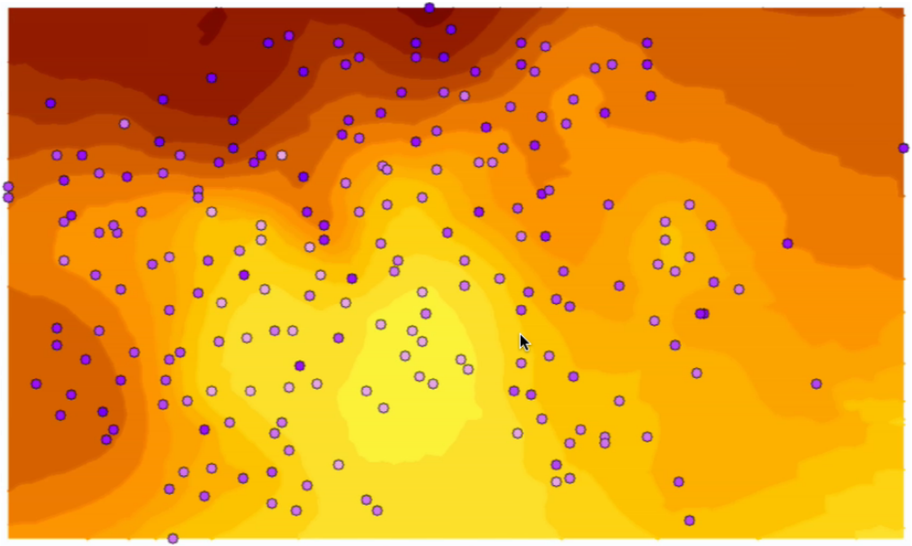
Surface Reconstruction: Spatial Interpolation



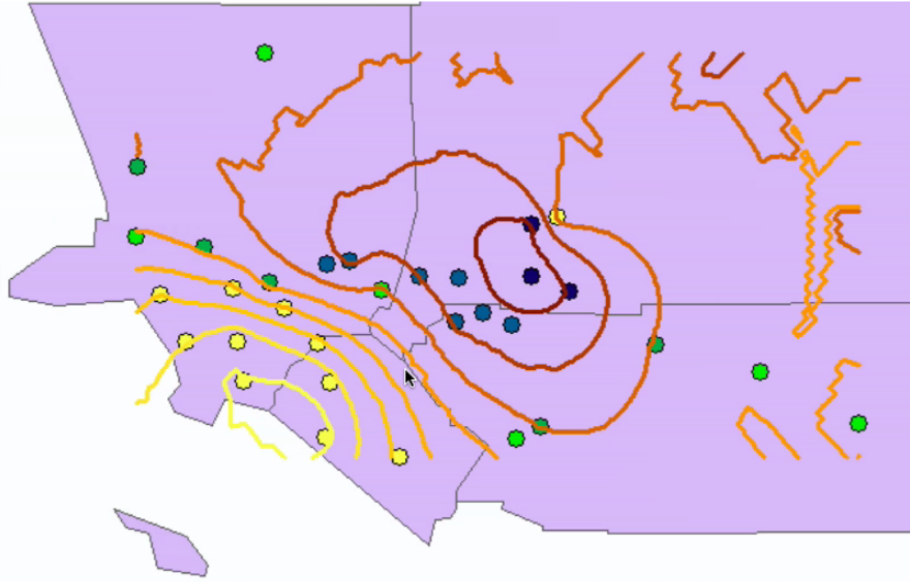
Surface Reconstruction: Kriging



Baltimore Sales Price, Baltimore MD



LA Basin Ozone Levels

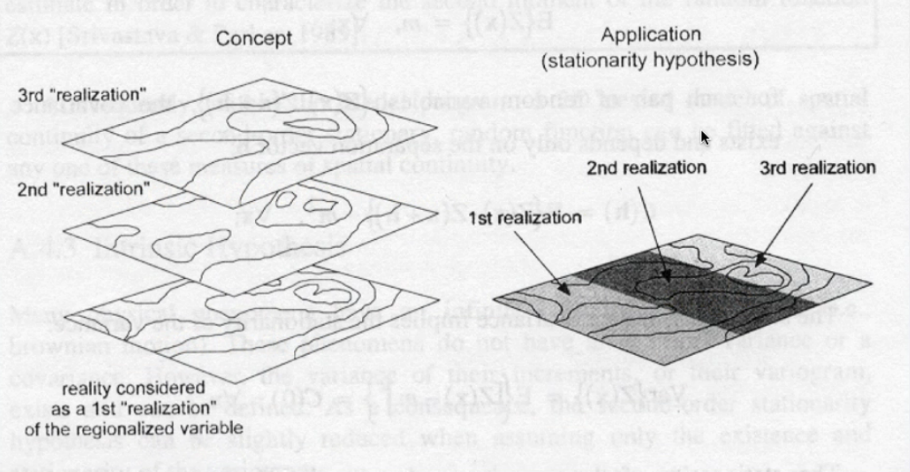


Conceptual Framework

Equilibrium

- Equilibrium = Stationarity
- Stochastic Process
  - not multiple realizations, but a single realization
  - the map is a single data point
- Notion of Stability
  - go from a single data point to acting as if there are multiple observations

Spatial Stationarity



Moment Conditions

- Constrain Variability
- Moments Must Exist
  - no infinite variance
- Moments Must Be Regular Over Space
  - restrictions on heterogeneity
  - restrictions on range of dependence

Ergodicity

- Sample Realization is Representative
  - average obtained using a single realization is same as over all possible realizations
  - whether you observer one or many maps, the information should be the same

Strict Stationarity

- Pertains to the Complete Distribution
- Invariance Under Spatial Shift
  - joint density for two different spatial subsets is same
  - $\{z(s_l), \ldots, z(s_k)\}$ *and* $\{z(s_{l+h}), \ldots, z(s_{k+h})\}$
  - information about process is the same no matter where it is obtained
- Very Strict Requirement

Moment Stationarity

- Moments Invariant Under Spatial Shift
- Mean
  - no spatial trend
- Variance
  - no heteroskedasticity (no spatial regimes)
- Covariance
  - not a function of location
  - only spatial separation, angle

Intrinsic Hypothesis

No Spatial Trend

- if there is a trend, take it out
- residuals have no trend (mean = 0

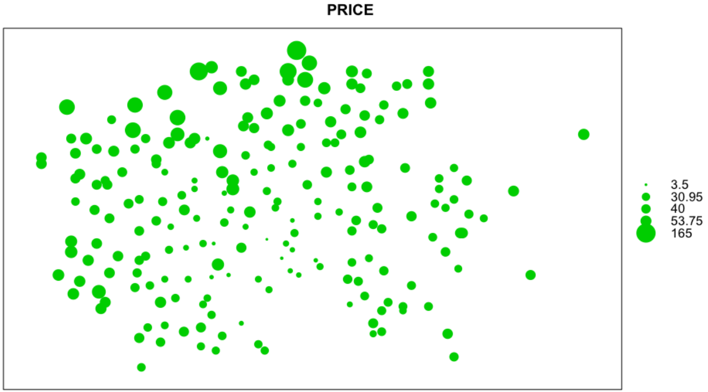
Constant Variance

- Variance of first difference only a function of displacment

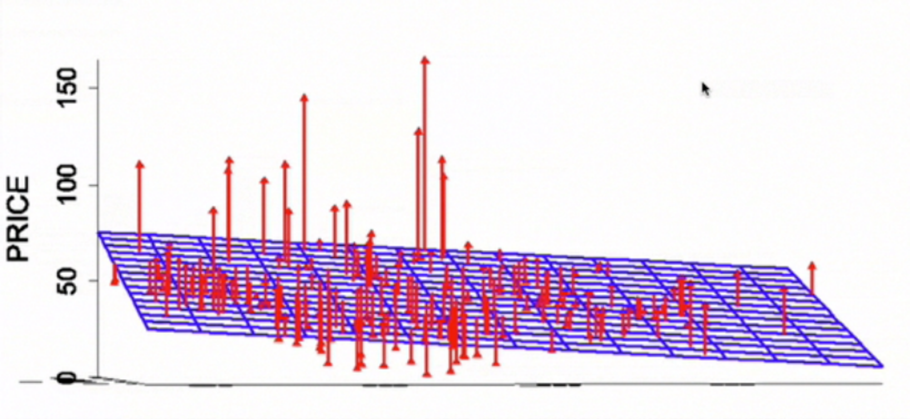
$$V\{Z_{s+h} - Z_s\} \tag{2}$$

a variability of the difference

Bubble Plot Baltimore House Sales Prices



Detrending Baltimore House Sales Prices



$$P = -166.02 - 0.148x + 0.634y$$

Semi-Variogram

Variogram Function

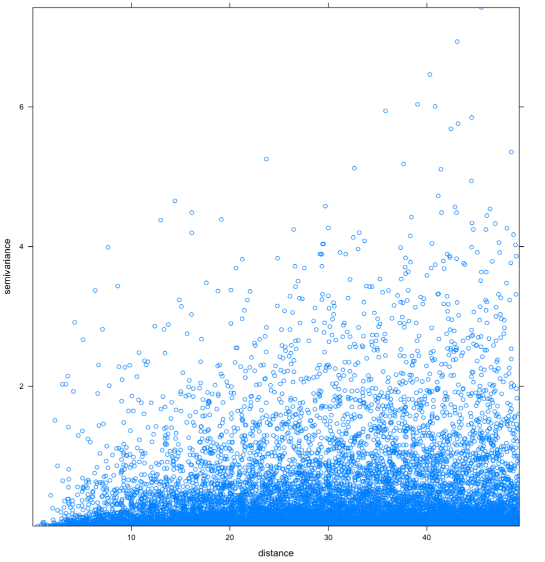
$$2\gamma(h) = V[Z_{s+h} - Z_s] \tag{3}$$

factor 2, so $\gamma(h)$ is half of the variogram, or semi-variogram

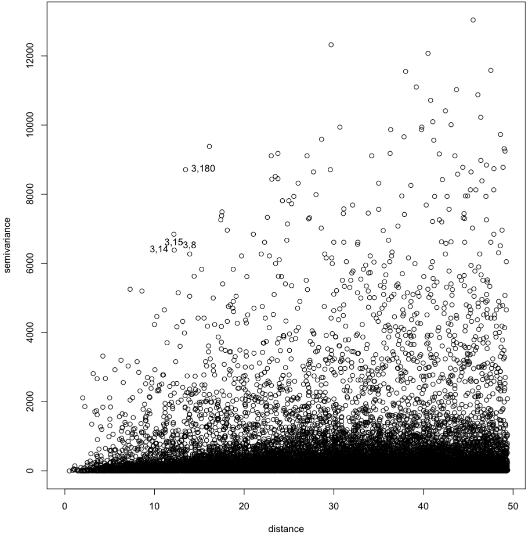
Constant Mean Assumption

- $E[Z_{s+h} - Z_s] = E[Z_{s+h}] - E[Z_s] = 0$
- $V[Z_{s+h} - Z_s] = E[Z_{s+h} - E[Z_s]]^2 - 0$
- $\gamma(h) = (1/2)E[Z_{s+h} - E[Z_s]]^2$
- average of squared differences

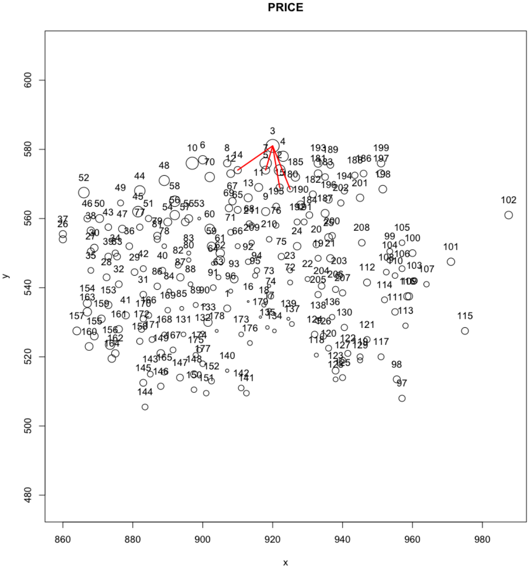
Variogram Cloud



Outliers in Variogram Cloud



Outliers in Bubble Plot



Estimating a Variogram

Methods of Moments

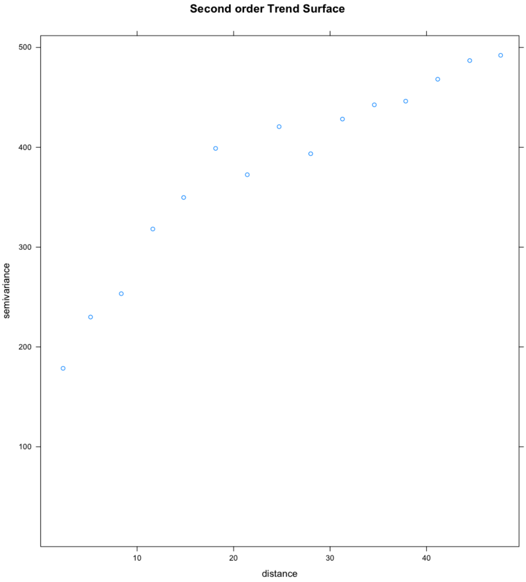
$$2\gamma(h) = (1/|N(h)|) \sum_{h} [Z_{s+h} - Z_s]^2 \quad (4)$$

- average of squared differences by distance bin h
- $N(h)$ number of pairs in distance bin h

Rules of Thumb

- at least 30 pairs in each bin
- max $h < D/2$ (D is maximum distance)
- distance of reliability

Variogram on 2nd Order Trend Surface



Covariogram

Second Order Stationarity

- covariance regular over space
- stronger assumptions: regularity second order moments, cross-product

Covariance at h

$$C(h) = Cov[Z_{s+h}, Z_s] \tag{5}$$

Process Variance

- covariance at distance zero
- $C(0) = V[Z_s]$

Correlogram

Autocorrelation Function

$$\rho(h) = C(h)/C(0) \tag{6}$$

covariance standardized by proces variance

Distance Decay

- Correlogram decreases with distance
- Tobler's law

Semi-Variogram and Covariogram

Two approaches to same concept

$$
\begin{aligned}
2\gamma(h) &= E[Z_{s+h} - Z_s]^2 && (7)\\
&= 2E[Z_s^2] - 2E[Z_{s+h}Z_s] && (8)
\end{aligned}
$$

Semi-variance

- $\gamma(h) = C(0) - C(h)$
- semi-variance is variance of process less covariance at distance h