Spatial Dependence

SERGIO REY

GPH 483/598
Geographic Information Analysis
School of Geographical Sciences and Urban Planning
Arizona State University
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Outline

Spatial Dependence

There is no question with respect to emergent geospatial science. The important harbingers were Geary's article on spatial autocorrelation, Dacey's paper about two- and K-color maps, and that of Bachi on geographic series.
– Berry, Griifth, Tiefelsdorf (2008)

Spatial Dependence

Working Concept

- what happens at one place depends on events in nearby places
- all things are related but nearby things are more related than distant things (Tobler)

Goodchild 1991

- a world without positive spatial dependence would be an impossible world
- impossible to describe
- impossible to live in
- hell is a place with no spatial dependence

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Spatial Dependence

Categorizing

- Type: Substantive versus nuisance
- Direction: Positive versus negative

Issues

- Time versus space
- Inference

Substantive Spatial Dependence

Process Based

- Part of the process under study
- Leaving it out
  - Incomplete understanding
  - Biased inferences

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Nuisance Spatial Dependence

Not Process Based

- Artifact of data collection
- Process boundaries not matching data boundaries
- Scattering across pixels
- GIS induced

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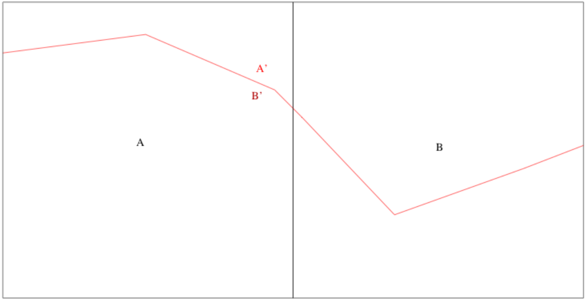
Boundary

Boundary Mismatch



- Even if A and B are independent
- A' and B' will be dependent

Nusiance vs. Substantive Dependence

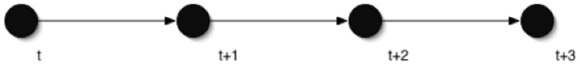
Issues

- Not always easy to differentiate from substantive
- Different implications for each type
- Specification strategies (Econometrics)
- Both can be operating jointly

Space versus Time

Temporal Dependence

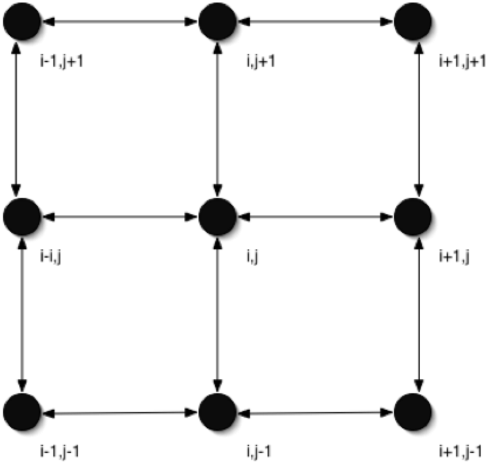
- Past influences the future
- Recursive
- One dimension



Space versus Time

Spatial Dependence

- Multi-directional
- Simultaneous



Terminology

Related Concepts

- Spatial Dependence
- Spatial Autocorrelation
- Spatial Association

Spatial Dependence

Distributional Characteristic

- Multivariate density function
- difficult/impossible to verify empirically

Dependent Distribution

- does not factor in marginal densities

Spatial Autocorrelation

- Auto = same variable
- Correlation = scaled covariance
- Spatial - geographic pattern to the correlation

Spatial Autocorrelation

Measurement of Moment of Distribution

- off-diagonal elements of variance-covariance matrix
- autocovariance
- $C[y_i, y_j] \neq 0 \ \forall i \neq j$
- can be estimated

Spatial Autocorrelation Coefficient

- significance test on coefficient = 0

Spatial Autocorrelation

Joint multivariate distribution function

$$f(y) = \frac{\exp\left[-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)\right]}{\sqrt{(2\pi)^n|\Sigma|}} \tag{1}$$

Variance-Covariance Matrix

$$\Sigma = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_{n,n} \end{bmatrix} \tag{2}$$

- covariance: $\sigma_{i,j} = E[(y_i - \mu_i)(y_j - \mu_j)]$
- symmetry: $\sigma_{i,j} = \sigma_{i,j}$
- variance: $\sigma_{i,i} = E[(y_i - \mu_i)(y_i - \mu_i)]$

Correlation

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_i^2}\sqrt{\sigma_j^2}} \tag{3}$$

$$-1.0 \leq \rho_{ij} \leq 1.0 \tag{4}$$

Data Types and Autocorrelation

Point Data

- focus on geometric pattern
- random vs. nonrandom
- clustered vs. uniform

Geostatistics

- 2-D modeling of spatial covariance (pairs of observations in function of distance)
- kriging, spatial prediction

Data Types and Autocorrelation

Lattice Data

- areal units: states, counties, census tracts, watersheds
- points: centroids of areal units
- focus on the spatial nonrandomness of attribute values

Spatial Association

Not a Rigorously Defined Term

- Usually the same as spatial autocorrelation
- often used in non-technical discussion
- avoid unless meaning is clear

Spatial Dependence

Good News (for geographers)

- Space matters
- Suggestive of underlying process

Bad news

- invalidates random sampling assumption
- necessitates new methods = spatial statistics and spatial econometrics

Spatial Dependence: Implications

The specific process we are simulating is as follows:

$$\begin{aligned} y &= X\beta + \epsilon \\ \epsilon &= \lambda W\epsilon + \nu \end{aligned} \tag{5}$$

where $\nu \sim N(0, \sigma^2 I)$, λ is a spatial autocorrelation parameter (scalar) and W is a spatial weights matrix. We will shortly explain these new entities, but for now we simply note that they allow us to simulate a process whereby the ϵ's, and therefore the y's are spatially autocorrelated. If $\lambda = 0$ then the *i.i.d.* assumption holds, otherwise there is spatial dependence.

$\beta = 40,\ \sigma^2 = 16,\ x = [1, 1, \ldots]$
$\lambda = [0.0, 0.25, 0.50, 0.75],\ n = 25$

Spatial Dependence: Implications

For each D.G.P. we are going to generate 500 samples of size $n = 25$ for our map. You can think of this as generating 500 maps using the same D.G.P.. For each sample we will then do the following:

1. Estimate μ with $\bar{y}$
2. Test the hypothesis that $\mu = 40$

Implications

Table: Monte Carlo Results

λ	0.00	0.25	0.50	0.75
$\hat{\mu}$	39.947	39.931	39.901	39.814
$\sigma_{\bar{x}}$	0.816	1.090	1.641	3.304
p	0.056	0.148	0.278	0.492

Spatial Randomness

Null Hypothesis

- observed spatial pattern of values is equally likely as any other spatial pattern
- values at one location do no depend on values at other (neighboring) locations
- under spatial randomness, the location of values may be altered without affecting the information content of the data

Spatial Autocorrelation on a Grid

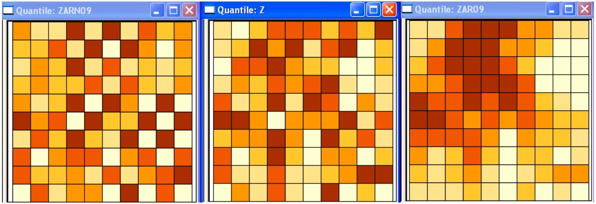


Figure: Negative, Random, Positive

Positive Spatial Autocorrelation

Clustering

- like values tend to be in similar locations

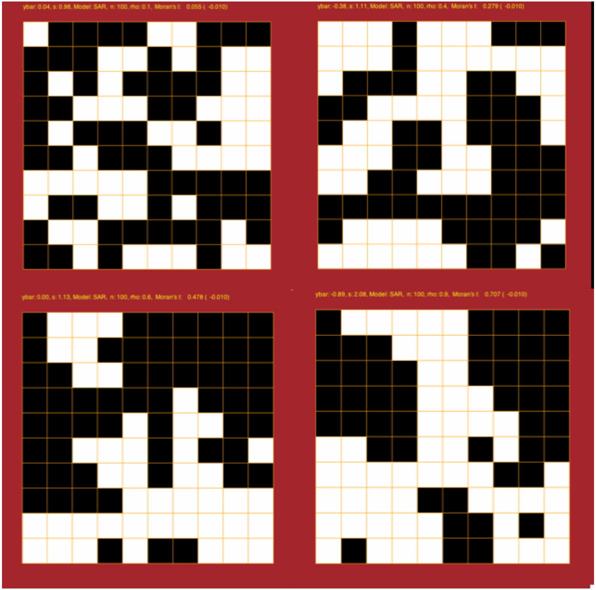
Neighbor similarity

- more alike than they would be under spatial randomness

Compatible with Diffusion

- but not necessarily caused by diffusion

Positive Spatial Autocorrelation



Negative Spatial Autocorrelation

Checkerboard pattern

- anti-clustering

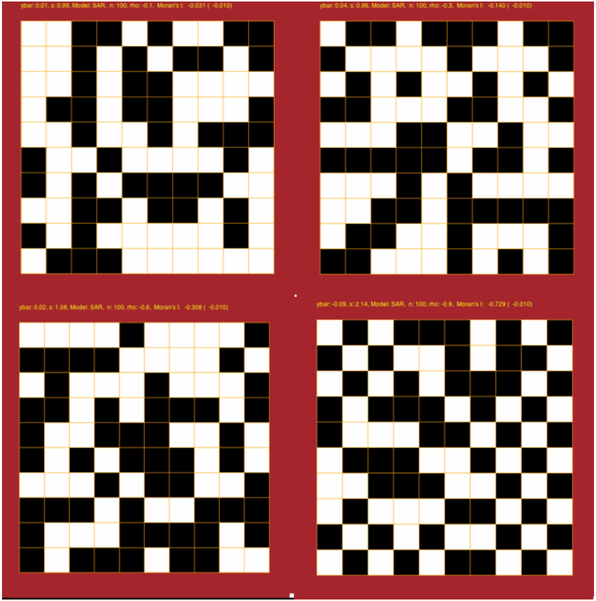
Neighbor dissimilarity

- more dissimilar than they would be under spatial randomness

Compatible with Competition

- but not necessarily caused by competition

Negative Spatial Autocorrelation



Autocorrelation and Diffusion

One does not necessarily imply the other

- diffusion tends to yield positive spatial autocorrelation but the reverse is not necessary
- positive spatial correlation may be due to structural factors, without contagion or diffusion

True vs. Apparent Contagion

What is the Cause behind the clustering?

- True contagion
  - result of a contagious process, social interaction, dynamic process
- Apparent contagion
  - spatial heterogeneity
  - stratification
- Cannot be distinguished in a pure cross section
- Equifinality or Identification Problem

Clustering

Global characteristic

- property of overall pattern = all observations
- are like values more grouped in space than random
- test by means of a global spatial autocorrelation statistic
- no location of the clusters determined

Clusters

Local characeristic

- where are the like values more grouped in space than random?
- property of local pattern = location-specific
- test by means of a local spatial autocorrelation statistic
- local clusters may be compatible with global spatial randomness

Spatial Autocorrelation Statistic

Structure

- Formal Test of Match between Value Similarity and Locational Similarity
- Statistic Summarizes Both Aspects
- Significance
  - how likely is it (p-value) that the computed statistic would take this (extreme) value in a spatially random pattern

Attribute Similarity

- Summary of the similarity or dissimilarity of a variable at different locations
  - variable y at locations i, j with $i \neq j$
- Measures of similarity
  - cross product: $y_i y_j$
- Measures of dissimilarity
  - squared differences: $(y_i - y_j)^2$
  - absolute differences: $|y_i - y_j|$

Locational Similarity

- Formalizing the notion of Neighbor
  - when two spatial units a-priori are likely to interact
- Spatial Weights
  - not necessarility geographical
  - many approaches

Summary

Spatial Dependence

- Core of Lattice Analysis
- Spatial Autocorrelation More Complex than Temporal Autocorrelation
- Combine Value and Locational Similarities