Local Spatial Autocorrelation

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Outline

1. Principles
   - Global v. Local Analysis
   - Local Indicator of Spatial Association (LISA)
2. Local Moran
   - Inference
   - Visualization
3. Issues
   - Multiple Comparisons
   - Sampling Distribution

Outline

Global vs. Local Analysis

Global Analysis

- one statistic to summarize pattern
- clustering
- homogeneity

Local Analysis

- location-specific statistics
- clusters
- heterogeneity

Outline

Local Indicators of Spatial Association: LISA

Anselin (1995)

- Local Spatial Statistic
- indicate significant spatial autocorrelation for each location

Local-Global Relation

- sum of LISA proportional to a corresponding global indicator of spatial autocorrelation

LISA Forms of Global Statistics

Decomposition

- if $global = a.\sum_i component_i$
- then $local = component_i$

General Case: Local Gamma

- $\Gamma = \sum_i \left(\sum_j w_{i,j} a_{i,j}\right)$
- Local $\Gamma_i = \sum_j w_{i,j} a_{i,j}$
- $\Gamma = \sum_i \Gamma_i$

Examples

- Local Moran: $I_i = m \sum_j w_{i,j} y_j$
- Local Geary: $c_i = m \sum_j w_{i,j} (y_i - y_j)^2$
- Local Sokal: $s_i = m \sum_j w_{i,j} |y_i - y_j|$

Outline

Local Moran

Statistic

- $I_i = (z_i/m_2) \sum_j w_{i,j} z_j$
- $m_2 = \sum_i z_i^2$ does not vary with i
- $\sum_i I_i = nI$ local-global link
- $I = \sum_i I_i / n$ global is mean of locals

Inference

Analytical

- equal probability assumption
- normal approximation to statistic
- poor in small samples: do not use

Computational

- conditional permutation
- hold value at *i* fixed, permute others

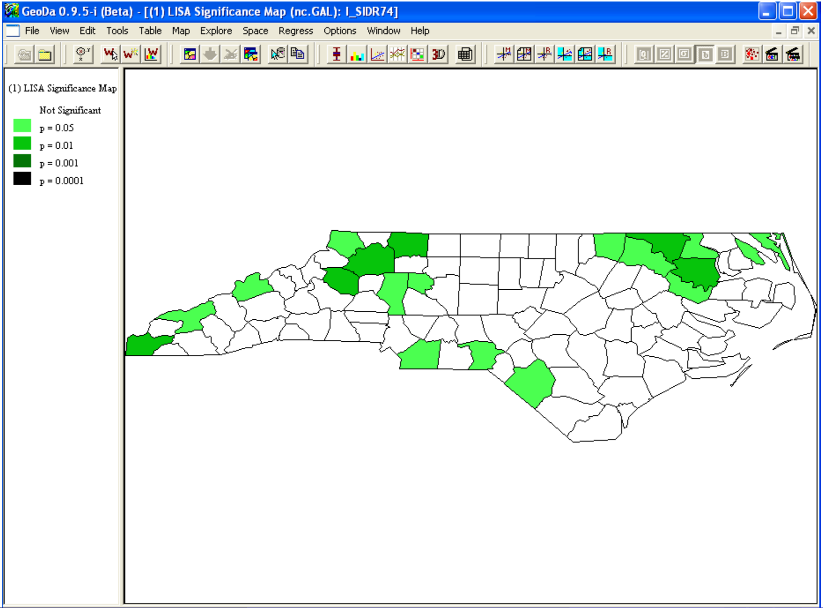
Outline

LISA Significance Map

Locations with Significant Local Statistics

- Choropleth Map
- shading by significance
- non-significant locations

LISA Significance Map



LISA Cluster Map

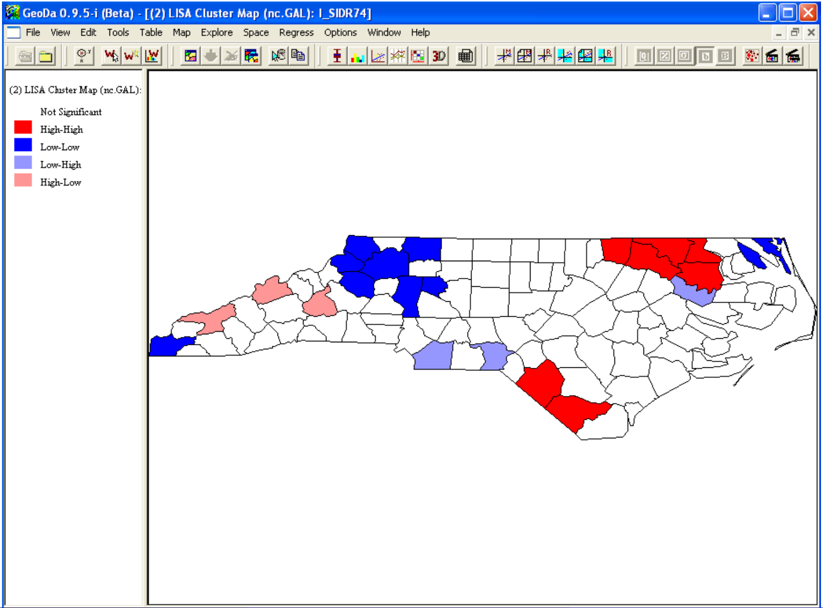
Only the Significance Locations

- Matches LISA Significance Map

Types of Spatial Autocorrelation

- spatial clusters
  - high-high (red)
  - low-low (blue)
- spatial outliers
  - high-low (light red)
  - low-high (light blue)

LISA Cluster Map



LISA Box Plot

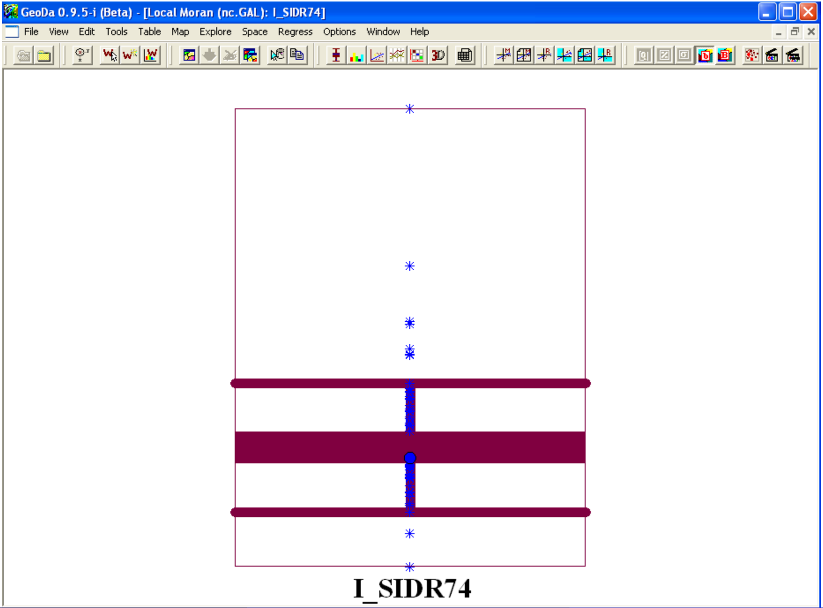
Distribution of Local Statistics

- global is related to average of the locals
- check for asymmetry

Outliers

- individual locations that may unduly influence the global statistic

LISA Box Plot with Outliers



Spatial Clusters and Spatial Outliers

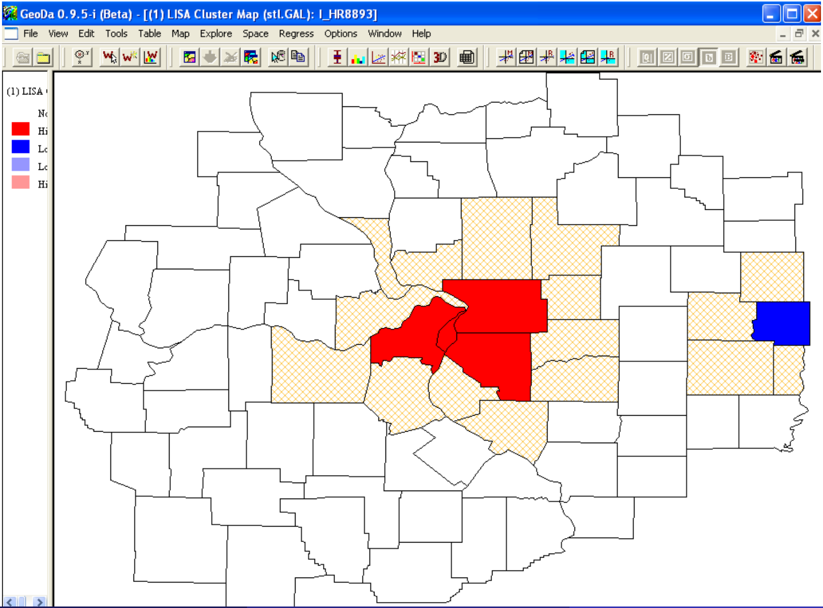
Spatial Clusters

- core of the cluster in LISA map
- cluster itself also includes neighbors
- us $p < 0.001$ to identify meaningful cluster cores and neighbors

Spatial Outliers

- individual locations
- no significant neighbors

Spatial Cluster Cores and Neighbors



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Multiple Comparisons

n local tests are being carried out

- Need to maintain overall significance level
- How to adjust local significance levels
- Overall versus marginal

Conservative approach

$$\alpha = 1 - (1 - \alpha_0)^{1/n} \tag{1}$$

where α_0 is the overall (joint) significance level
α is the marginal significance level
n is the number of comparisons

Multiple Comparisons

Conservative approach

$$\alpha = 1 - (1 - \alpha_0)^{1/n} \tag{2}$$

where α_0 is the overall (joint) significance level
α is the marginal significance level
n is the number of comparisons

Example: $n = 48, \alpha_0 = 0.05$

$$\alpha = 1 - (1 - 0.05)^{1/48} = 0.00107 \tag{3}$$

Outline

Sampling Distribution

Under the global null

- No global spatial autocorrelation
- Analytical distribution
- Local tests will not be independent

Under the global alternative

- Global spatial autocorrelation is present
- sampling distributions for local statistics unknown

Caveats

LISA Clusters and Hot Spots

- suggest interesting locations
- suggest significant spatial structure
- do no explain

Multivariate Relations

- univariate spatial autocorrelation may be due to other covariates
- scale mismatch