Signed Area of a Polygon

$$A = \frac{1}{2} \sum_{i=1}^{n+1} x_i y_{i+1} - x_{i+1} y_i$$

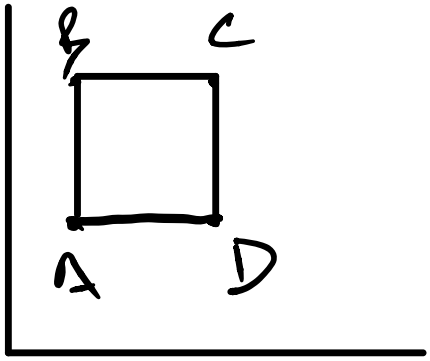
w/ $(x_1, y_1) = (x_{n+1}, y_{n+1})$

closed form

$(x_i, y_i) \rightarrow$ Vertex i

Polygon:

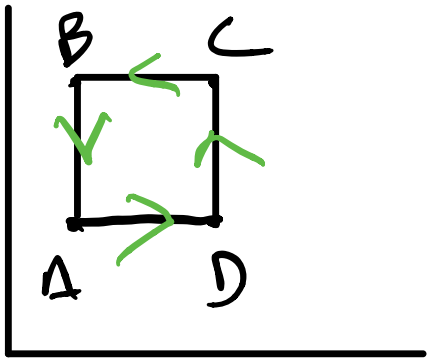
$$\left((x_1, y_1), (x_2, y_2), \cdots, (x_{n+1}, y_{n+1}) \right)$$



A 2,2
B 2,4
C 4,4
D 4,2
A 2,2

$2.4 - 2.2$ $\quad 4$
$2.4 - 4.4$ $\quad -8$
$4.2 - 4.4$ $\quad -8$
$4.2 - 2.2$ $\quad 4$
$\quad -8$

$\frac{1}{2}(-8) = -4$ $\qquad |1-4| = 4$

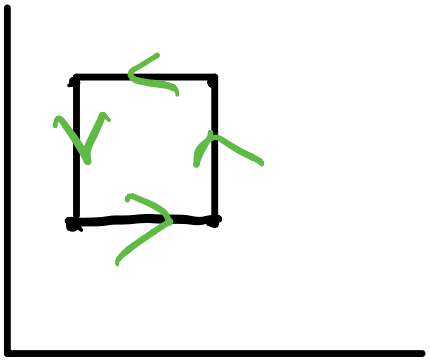


A	2	2
D	4	2
C	4	4
B	2	4
A	2	2

$$2 \cdot 2 - 2 \cdot 4 = -4$$
$$4 \cdot 4 - 2 \cdot 4 = 8$$
$$4 \cdot 4 - 4 \cdot 2 = 8$$
$$2 \cdot 2 - 4 \cdot 2 = -4$$
$$8 \rightarrow \frac{1}{2}(8) = 4$$

Sign is positive

+ Area → counterclockwise
− Area → clockwise



2	2
4	2
4	4
2	4
2	2

$$2 \cdot 2 - 2 \cdot 4 = -4$$
$$4 \cdot 4 - 2 \cdot 4 = 8$$
$$4 \cdot 4 - 4 \cdot 2 = 8$$
$$2 \cdot 2 - 4 \cdot 2 = -4$$
$$8 \rightarrow \frac{1}{2}(8) = 4$$

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Ring Ordering

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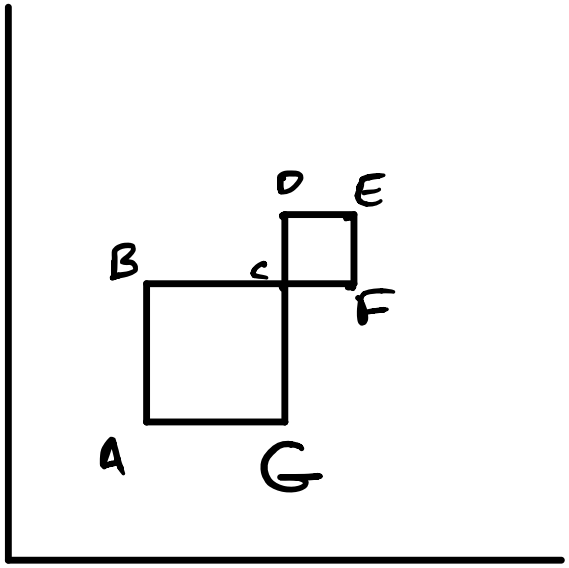
((2,2), (4,2), (4,4), (2,4), (2,2))

((2,2), (2,4), (4,4), (4,2), (2,2))



	x	y
A	2	2
B	2	4
C	4	4
D	4	5
E	5	5
F	5	4
C	4	4
G	4	2
A	2	2

Area should be $5 = (4+1)$

2.4	−	2.2	4
2.4	−	4.4	−8
4.5	−	4.4	4
4.5	−	5.5	−5
5.4	−	5.5	−5
5.4	−	4.4	4
4.2	−	4.4	−8
4.2	−	2.4	0

$|-14|/2 = 7$

Self-intersecting ring

→ Bad

$7 \neq 5$