Matrix Solver

```
with(LinearAlgebra) :
MatrixSolve := proc(A, b)
  local i, j, n, L, U, v, const, det;
    n := RowDimension(A);
    L := Matrix(n);
    U := A;
    v := b;
    det := 1;
    print(A, b);
  for i from 1 to n − 1 do
    for j from i + 1 to n do

      if (U[i, i] = 0) then
        error "zero along main diagonal";
      end if;
      const := U[j, i] / U[i, i];
      if (const ≠ 0) then
        L[j, i] := const;
        U := RowOperation(U, [j, i], −const);
        v[j] := v[j] − const·v[i];
        print(R(j) − const·R(i), U, v);
      end if;

    end do;
  end do;

  for i from 1 to n do
    det := det·U[i, i];
    L[i, i] := 1;
  end do;

  print(L, U, v, det);

end proc:
```

Problem #3

$A := Matrix([[6,-2,2,4],[12,-8,6,10],[3,-13,9,3],[-6,4,1,-18]]);$

$b := Vector([16,26,-19,-34]);$

$$A := \begin{bmatrix} 6 & -2 & 2 & 4 \\ 12 & -8 & 6 & 10 \\ 3 & -13 & 9 & 3 \\ -6 & 4 & 1 & -18 \end{bmatrix}$$

$$b := \begin{bmatrix} 16 \\ 26 \\ -19 \\ -34 \end{bmatrix} \quad (6)$$

$MatrixSolve(A, b);$

$$\begin{bmatrix} 6 & -2 & 2 & 4 \\ 12 & -8 & 6 & 10 \\ 3 & -13 & 9 & 3 \\ -6 & 4 & 1 & -18 \end{bmatrix}, \begin{bmatrix} 16 \\ 26 \\ -19 \\ -34 \end{bmatrix}$$

$$R(2) - 2\,R(1), \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 3 & -13 & 9 & 3 \\ -6 & 4 & 1 & -18 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -19 \\ -34 \end{bmatrix}$$

$$R(3) - \frac{R(1)}{2}, \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & -12 & 8 & 1 \\ -6 & 4 & 1 & -18 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -27 \\ -34 \end{bmatrix}$$

$$R(4) + R(1), \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & -12 & 8 & 1 \\ 0 & 2 & 3 & -14 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -27 \\ -18 \end{bmatrix}$$

$$R(3) - 3\,R(2), \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & 0 & 2 & -5 \\ 0 & 2 & 3 & -14 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -9 \\ -18 \end{bmatrix}$$

$$R(4) + \frac{R(2)}{2}, \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & 4 & -13 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -9 \\ -21 \end{bmatrix}$$

$$R(4) - 2\,R(3), \begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & 0 & -3 \end{bmatrix}, \begin{bmatrix} 16 \\ -6 \\ -9 \\ -3 \end{bmatrix}$$

$$\boxed{\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ \frac{1}{2} & 3 & 1 & 0 \\ -1 & -\frac{1}{2} & 2 & 1 \end{bmatrix}}_{L}, \underbrace{\begin{bmatrix} 6 & -2 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & 0 & -3 \end{bmatrix}}_{U}}, \begin{bmatrix} 16 \\ -6 \\ -9 \\ -3 \end{bmatrix}, 144 \tag{7}$$

Ly=b

$y1 := 16 :$

$y2 := -2 \cdot y1 + 26 :$

$y3 := -\frac{1}{2} \cdot y1 - 3 \cdot y2 - 19 :$

$y4 := y1 + \frac{1}{2} \cdot y2 - 2 \cdot y3 - 34 :$

$y := Vector([y1, y2, y3, y4]);$

$$y := \begin{bmatrix} 16 \\ -6 \\ -9 \\ -3 \end{bmatrix} \tag{8}$$

Ux=y

$x4 := -\frac{1}{3}(-3) :$

$x3 := \frac{1}{2}(5 \cdot x4 - 9) :$

$x2 := -\frac{1}{4}(-2 \cdot x3 - 2 \cdot x4 - 6) :$

$x1 := \frac{1}{6}(2 \cdot x2 - 2 \cdot x3 - 4 \cdot x4 + 16) :$

$x := Vector([x1, x2, x3, x4]);$

$$\boxed{x := \begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}} \tag{9}$$

Jacobi

```
with(Student[LinearAlgebra]) :
Jacobi := proc(T, c, α, ε, N)
    local i, x, err, temp;
        x := α;

    for i from 1 to N do
        temp := T · x + c;
        err := Norm(temp − x, infinity);
        x := temp;
        if (err < ε) then
            break;
        end if;
    end do;

 print(evalf(x), i);
 end proc:
```

$T := Matrix\left(\left[\left[0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right], \left[-\frac{1}{8}, 0, -\frac{1}{4}, -\frac{3}{8}\right], \left[\frac{1}{5}, \frac{2}{5}, 0, 0\right], \left[\frac{1}{4}, 0, -\frac{1}{2}, 0\right]\right]\right);$

$$T := \begin{bmatrix} 0 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{8} & 0 & -\frac{1}{4} & -\frac{3}{8} \\ \frac{1}{5} & \frac{2}{5} & 0 & 0 \\ \frac{1}{4} & 0 & -\frac{1}{2} & 0 \end{bmatrix} \tag{1}$$

$c := Vector\left(\left[-\frac{5}{4}, \frac{23}{8}, -\frac{9}{5}, 1\right]\right);$

$$c := \begin{bmatrix} -\frac{5}{4} \\ \frac{23}{8} \\ -\frac{9}{5} \\ 1 \end{bmatrix} \tag{2}$$

$\alpha := Vector([0, 0, 0, 0]);$

$$\alpha := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$Jacobi(T, c, \alpha, 10^{-4}, 100);$

$$\boxed{\begin{bmatrix} -1.999988563 \\ 3.000005627 \\ -1.000036062 \\ 0.9999687134 \end{bmatrix}}, 20 \tag{4}$$

Gauss-Seidel

```
with(LinearAlgebra) :
GaussSeidel := proc(A, b, α, ε, N)
   local i, j, n, L, D, U, T, c, x, err, temp;
      n := RowDimension(A);
      L := Matrix(n);
      D := Matrix(n);
      U := Matrix(n);
      x := α;

   # Create L
   for j from 1 to n − 1 do
      for i from j + 1 to n do
         L[i, j] := A[i, j];
      end do;
   end do;

   # Create D
   for i from 1 to n do
      D[i, i] := A[i, i];
   end do;

   # Create U
   for i from 1 to n − 1 do
      for j from i + 1 to n do
         U[i, j] := A[i, j];
      end do;
   end do;

   T := − (L + D)^(−1) · U;
   c := (L + D)^(−1) · b;

   for i from 1 to N do
      temp := T · x + c;
      err := Norm(temp − x, infinity);
      x := temp;
      if (err < ε) then
         break;
      end if;
   end do;

 print(evalf(x), i);
 return T;
 end proc:
```

$A := Matrix([[4, 1, 1, 1], [1, 8, 2, 3], [1, 2, -5, 0], [-1, 0, 2, 4]]);$

$$A := \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 8 & 2 & 3 \\ 1 & 2 & -5 & 0 \\ -1 & 0 & 2 & 4 \end{bmatrix} \tag{5}$$

$b := Vector([-5, 23, 9, 4]);$

$$b := \begin{bmatrix} -5 \\ 23 \\ 9 \\ 4 \end{bmatrix} \tag{6}$$

$T2 := GaussSeidel(A, b, \alpha, 10^{-4}, 100);$

$$T2 := \boxed{\begin{bmatrix} -2.000010143 \\ 2.999995446 \\ -1.000003850 \\ 0.9999993894 \end{bmatrix}}, 8, \begin{bmatrix} 0 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{32} & -\frac{7}{32} & -\frac{11}{32} \\ 0 & -\frac{3}{80} & -\frac{11}{80} & -\frac{3}{16} \\ 0 & -\frac{7}{160} & \frac{1}{160} & \frac{1}{32} \end{bmatrix} \tag{7}$$

Jacobi

$evalf(Eigenvalues(T));$

$$\begin{bmatrix} 0.3637795831 \\ 0.0960484587 \\ -0.2299140209 + 0.5521692700\,\mathrm{I} \\ -0.2299140209 - 0.5521692700\,\mathrm{I} \end{bmatrix} \tag{8}$$

$eval3 := \mathrm{sqrt}(0.2299140209^2 + 0.552169270^2);$

$eval4 := \mathrm{sqrt}(0.2299140209^2 + 0.552169270^2);$

$$eval3 := 0.5981231978$$

$$eval4 := 0.5981231978 \tag{9}$$

ρ(T) =0.5981231978 < 1, therefore the sequence will converge.

Gauss-Seidel

$evalf(Eigenvalues(T2));$

$$\begin{bmatrix} 0. \\ 0. \\ 0.1388341998 \\ -0.2138341998 \end{bmatrix} \tag{10}$$

$$eval3 := 0.2053959591$$

$$eval4 := 0.2053959591 \tag{11}$$

ρ(T) =0.2138341998 < 1, therefore the sequence will converge.

Actual

$with(Student[NumericalAnalysis])$:

$evalf(IterativeApproximate(A, b, method = jacobi, initialapprox = \alpha, tolerance = 10^{-4}, maxiterations = 100));$

$evalf(IterativeApproximate(A, b, method = gaussseidel, initialapprox = \alpha, tolerance = 10^{-4}, maxiterations = 100));$

$$\begin{bmatrix} -1.999988563 \\ 3.000005627 \\ -1.000036062 \\ 0.9999687134 \end{bmatrix}$$

$$\begin{bmatrix} -2.000010143 \\ 2.999995446 \\ -1.000003850 \\ 0.9999993894 \end{bmatrix} \quad \textbf{(12)}$$