1 Introduction

Let E be a finite set, a *chain* on E is a mapping f of E into $\mathbb{F}_2$

If $x \in E$ then fx is the *coefficient* of x in f

Chains can be added to give new chains by adding coefficients for each $x \in E$

Thus chains form an *additive group*

Any subgroup of this group is called a *chain-group* on E

The zero element in a chain group is the chain in which each coefficient is zero, this is the *zero chain*

Let N be a chain-group on E, the elements of E are *cells* of N

A cell x is *filled* if $fx = 1$ for some chain f of N and *empty* otherwise

N is a *full chain group* if it has no empty cells

A colouring of N is a pair $\{f, g\}$ of chains of N such that for each $x \in E$ either $fx = 1$ or $gx = 1$

N is *chromatic* if it has a colouring, and *achromatic* otherwise

A chromatic chain-group is necessarily full

2 The Cycles and Coboundaries of a Graph

An edge is a *loop* if the ends are the same, a *link* if the ends are different

Graphs are assumed to be *finite* - both $V(G)$ and $E(G)$ are finite sets

A *path* is a sequence $(a_0, A_1, a_1, A_2, a_2, \ldots, A_n, a_n)$ of vertices a_i and edges A_j such that:

1. If $1 \leq i \leq n$ the ends of A_i are a_{i-1} and a_i

2. If $1 \leq i \leq n$ then $a_{i-1} = a_i$ iff A_i is a loop

If all terms in path are distinct, then the path is *simple*

If all terms are distinct except that $a_0 = a_n$, then the path is *circular*