H∞ control of Markov jump LPV systems for a four degree-of-freedom active magnetic bearing system

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ABSTRACT

This paper addresses the H_∞ state feedback control of continuous Markov jump LPV systems. A separating technique is employed to tackle the coupling among Lyapunov variable, system matrix, and controller parameter. Worst-case analysis is used for LPV systems to design gain-scheduled controllers. Based on these strategies, new sufficient conditions for the closed-loop system to be stochastically stable are formulated in the framework of linear matrix inequalities. Finally, the derived stabilization condition is applied to the control of a four-freedom active magnetic bearing.

CCS Concepts

• **Information systems→Data warehouses**

Keywords

Markov jump systems (MJSs); linear parameter-varying (LPV); separating technique; active magnetic bearing （AMBs）

1 INTRODUCTION

Compared with conventional mechanical bearings, magnetic bearings (MBs) possess several remarkable advantages, such as no friction and wear, no need of lubrication, long life span, high potential of high control precision, as well as the ability of long-term high speed running [1]. Therefore, MBs have been attracting considerable interests in various high-performance applications including flywheel energy and storage devices, bearingless motors, artificial heart pumps, vacuum pumps, numerical control machines, especially in special high-purity applications. As a traditional control scheme, the proportional plus integral plus differential (PID) control scheme has been already applied in the MB system in a general way owing to its simple realization [2]. However, the control performance of the MB system by using the conventional PID control scheme could not be very satisfied owing to the unmeasured parameters variations and unavoidable external disturbances. Therefore, with the development of modern control theories, all kinds of advanced control methods have been recently proposed for MB systems in [3-6] and the references therein, such as fuzzy control, intelligent control, sliding mode control, predictive control, robust control, adaptive control, fractional order control, back-stepping control, optimum control. These advanced control methods not only enrich the control theory of MB systems but also improve their performance in different aspects.

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Besides aforementioned nonlinear control methods, the linearization and decoupling control schemes have been widely employed for MB systems in recent years [7]. However, all these control methods were carried out at a specific speed of MBs. There is no research focused on the stability control of MBs under full speed range.

Markov jump systems (MJSs) belong to the category of stochastic hybrid systems with state and jump mode modeled by differential equations. A great deal of effort has been made to research the analysis and synthesis of MJSs [8–9], and the involved results have been successfully applied to a variety of practical applications, such as fault-tolerant systems, biology systems, distributed network systems, robotic manipulator systems and wireless communication systems . As an application field, under the full speed range, the MBs model can be established as a Markov jump system. Since different speed causes different unbalanced vibration and gyro effect, thus, considering the MB speed is essentially needed. Accordingly, this paper deals with the MB stability problem by modeling it with some measurable time-varying parameters coupled with multiple switchable modes. Subsequently, by establishing the time-varying parameters for each mode, the real- time information on the speed is exactly applied to the control of Markov jump linear parameter-varying (MJLPV) systems.

In this paper, we further consider the $H\infty$ state feedback control of MJLPV systems for the active magnetic bearing system. Considering the unbalanced vibration and gyro effect, the MB model was described as an MJS mode. Then the coupling among controller gains and system matrices is removed by a constructive method. To make the closed-loop system be stochastically stable with a prescribed $H\infty$ performance index, sufficient conditions are established by means of linear matrix inequalities. Finally, a numerical MB example is provided to demonstrate the validity of the established results.

Notation : The notation R > 0 stands for R is symmetric and positive (negative) definite. (•) T indicates the transpose of a vector or matrix (•). ∗ represents the symmetry. He (M) = M ^{T} + M.

2 SYSTEM DESCRIPTION AND PRELIMINARIES

Let us consider the following MJLPV system:

$$\dot{x}(t) = A(r_t, \theta(t))x(t) + B(r_t)u(t) + F(r_t)\omega(t)$$
$$z(t) = C(r_t)x(t) \quad (1)$$

Where $x(t) \in R^n$ is the state vector of the system; $u(t) \in R^m$ is control input; $z(t) \in R^p$ is controlled output; $\omega(t) \in R^q$ is the noise signal which is assumed to be an arbitrary signal; $A(r_t,\theta(t))$, $B(r_t)$, $C(r_t)$ and $F(r_t)$ are system matrices; $\theta(t)$ denotes the time varying parameter satisfying that

$$\theta(t) \in Co(\Theta), \Theta = \{\theta \in R^k : \theta_j \in [\underline{\theta}_j, \overline{\theta}_j], j = 1, \ldots, k\}.$$

r_t is a continuous Markov process and takes values in $I = \{1, 2, ..., s\}$ and satisfies

$$\Pr\{r_{t+h} = j \mid r_t = i\} = \begin{cases} \pi_{ij}h + o(h), i \neq j \\ 1 + \pi_{ii}h + o(h), i = j \end{cases}$$

Where h>0, $\pi_{ij} \geq 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{i=1, j\neq i}^{S} \pi_{ij}$ for each mode i, $\lim_{h\to 0} o(h)/h = 0$. As a sequence, the corresponding transition probability matrix is

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \cdots & & & \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix}$$

For notation simplicity, when $r_t = i$, the system matrices of the ith mode can be simplified as Ai, Bi, Ci, and Fi. This paper aims at designing a state-feedback controller

$$u(t) = K_i x(t) \quad (2)$$

such that the following closed-loop system

$$\dot{x}(t) = (A_i(\theta(t)) + B_i K_i)x(t) + F_i\omega(t)$$
$$z(t) = C_i x(t) \quad (3)$$

to be stochastically stable and meets the required H_∞ performance level γ.

Before ending this section, the definition of stochastic stability and technical lemmas are given as follows:

Definition 1 [10]. *System (1) is said to be stochastically stable if, when u (t) = 0 , for any finite* $\psi(t) \in R^n$ *defined on* $[-\tau, 0]$, *and* $r_0 \in S$ *the following condition is satisfied*

$$\lim_{t\to\infty} E\{\int_0^t x^T(t)x(t)dt \mid \psi, r_0\} <$$

Lemma 1 [10]. *For given matrices H , F (t) and E of appropriate dimensions with F (t) satisfying* $F(t)^T F(t) \leq I$, *for any* λ> 0 *the following inequality holds*

$$HF(t)E + E^T F(t) H^T \leq \lambda H H^T + \lambda^{-1} E^T E$$

Lemma 2 [10]. *The following two inequalities are equivalent:*

(a) *there exists a symmetric and positive-definite matrix P satisfying*

$$\begin{bmatrix} -P & A^T \\ A & -P^{-1} \end{bmatrix} < 0$$

(b) *there exists a symmetric and positive-definite matrix P and matrix X satisfying*

$$\begin{bmatrix} -P & (XA)^T \\ XA & He(-X) + P \end{bmatrix} < 0$$

3 MAIN RESULTS

In this section, with the assumption of known TPs, sufficient conditions for the closed-loop system (3) to be stochastically stable are presented in Theorem 1 .

Theorem 1. The closed-loop system (3) with known transition probabilities is stochastically stable and meets the required H ∞ performance index γ, if there exist symmetric and positive definite matrices Pi and matrices J, Zi , Ni satisfying the following LMIs:

$$\begin{bmatrix} \Delta_{11} & P_iF_i & C_i^T & N_i^T & 0 \\ * & -\gamma^2 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & He(Z_i) & (P_iB_i - B_iZ_i)^T \\ * & * & * & * & -J \end{bmatrix} < 0 \quad (4)$$

and

$$\begin{bmatrix} \Delta_{11}' & P_iF_i & C_i^T & N_i^T & 0 \\ * & -\gamma^2 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & He(Z_i) & (P_iB_i - B_iZ_i)^T \\ * & * & * & * & -J \end{bmatrix} < 0 \quad (5)$$

Where

$$\Delta_{11} = 2P_iA_i(\underline{\theta}) + \Sigma_{j=1}^S \pi_{ij}P_j + 2(B_iN_i)^T + J$$
$$\Delta_{11}' = 2P_iA_i(\overline{\theta}) + \Sigma_{j=1}^S \pi_{ij}P_j + 2(B_iN_i)^T + J$$

Moreover, the controller gain matrix Ki can be calculated as $K_i = Z_i^{-1}N_i$.

Proof: Consider the following Lyapunov–Krasovskii functional candidate:

$$V(t,i) = x^T(t)P_ix(t)$$

Then the derivative of $V(t,i)$ is

$$\dot{V} = 2x^T(t)P_iA_i(\theta)x(t) + 2x^T(t)P_iB_iK_ix(t) \quad +2x^T(t)P_iE_i\omega(t) + x^T(t)\Sigma_{j=1}^S\pi_{ij}P_jx(t)$$

Considering the $H\infty$ performance γ under zero initial conditions for any nonzero $\omega(t)$, the system (3) is stochastically stable if the following equalities hold

$$\begin{bmatrix} 2P_iA_i(\theta) + 2P_iA_iK_i + \sum \pi P_i & P_iE_i & C_i^T \\ * & -r^2I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (6)$$

Rewritten the left of (6) as

$$\begin{bmatrix} 2P_iA_i(\theta) + 2P_iA_iK_i + \sum \pi P_i & P_iE_i & C_i^T \\ * & -r^2I & 0 \\ * & * & -I \end{bmatrix}$$
$$= \Delta + \begin{bmatrix} [2(P_iB_i - B_iZ_i)Z_i^{-1}N_i]^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \Delta + \begin{bmatrix} (Z_i^{-1}N_i)^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (P_iB_i - B_iZ_i) & 0 & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} P_iB_i - B_iZ_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} Z_i^{-1}N_i & 0 & 0 \end{bmatrix}$$
$$= \Delta + \begin{bmatrix} (Z_i^{-1}N_i)^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (P_iB_i - B_iZ_i)^T & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P_iB_i - B_iZ_i \end{bmatrix} \begin{bmatrix} Z_i^{-1}N_i & 0 & 0 \end{bmatrix}$$
$$= \Phi$$

Wh ere

$$\Delta = \begin{bmatrix} 2P_iA_i(\theta) + 2(B_iN_i)^T & P_iE_i & C_i^T \\ * & r^2 & 0 \\ * & * & -I \end{bmatrix}$$

According to Lemma 1 , the following inequality holds

$$\Phi \le \Delta + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} J \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}^T$$

$$+ \begin{bmatrix} (Z_i^{-1}N_i)^T \\ 0 \\ 0 \end{bmatrix} \left[(P_iB_i - B_iZ_i)^T \right] J^{-1} \left[P_iB_i - B_iZ_i \right] \left[Z_i^{-1}N_i \quad 0 \quad 0 \right]$$

$$= \begin{bmatrix} 2P_iA_i(\theta) + \sum \pi P_i + 2(B_iN_i)^T & P_iE_i & C_i^T \\ * & -r^2 & 0 \\ * & * & -I \end{bmatrix}$$

$$+ \begin{bmatrix} (Z_i^{-1}N_i)^T \\ 0 \\ 0 \end{bmatrix} \left[(P_iB_i - B_iZ_i)^T \right] J^{-1} \left[P_iB_i - B_iZ_i \right] \left[Z_i^{-1}N_i \quad 0 \quad 0 \right]$$

Adopting the Schur complement, one has

$$\begin{bmatrix} \varphi_{11} & P_iE_i & C_i^T & (Z_i^{-1}N_i)^T \\ * & -r^2 & 0 & 0 \\ * & * & -I & 0 \\ * & 0 & 0 & \varphi_{44} \end{bmatrix} < 0 \tag{7}$$

Where

$\varphi_{11} = 2P_iA_i(\theta) + \sum \pi P_i + 2(B_iN_i)^T$, $\varphi_{44} = -\left[(P_iB_i - B_iZ_i)^T J^{-1} (P_iB_i - B_iZ_i) \right]^{-1}$

Adopting Lemma 2 to (7), one has

$$\begin{bmatrix} \lambda_{11} & P_iE_i & C_i^T & N_i^T \\ * & -r^2 & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & \lambda_{44} \end{bmatrix} < 0 \tag{8}$$

where

$\lambda_{11} = 2P_iA_i(\theta) + \sum \pi P_i + 2(B_iN_i)^T + J$

$\lambda_{44} = He(Z_i) + (P_iB_i - B_iZ_i)^T J^{-1} (P_iB_i - B_iZ_i)$

Adopting the Schur complement to (8)

$$\begin{bmatrix} \lambda_{11} & P_iE_i & C_i^T & N_i^T & 0 \\ * & -r^2 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & He(Z_i) & (P_iB_i - B_iZ_i)^T \\ * & * & * & * & -J \end{bmatrix} < 0 \tag{9}$$

If the $A_i(\theta)$ in the LPV systems linear affine depends on parameter θ, (9) can be equivalent to (4) and (5) . Consequently, the system (3) is stochastically stable with the required $H\infty$ performance once (4) and (5) hold.

4. APPLICATION TO ACTIVE MAGNETIC BEARING

In this paper, the rotor of the MB is assumed to be a rigid and symmetric body. Fig.1 shows the relationship of the coordinate system and the rotor forces of the 4-degree of freedom (DOF) AMBs. In Fig.1, O is the geometrical center of the rotor; Ol and Or denote the two centers of the two 2-DOF radial AMBs, respectively; x_a, x_b and y_a, y_b are the radial displacements in X- and Y-axes, respectively; θ_x and θ_y are the rotor angular displacements about the X- and Y-axes, respectively; F_{xa}, F_{xb}, F_{ya}, and F_{yb} are the radial AMB forces along the aX–, bX–, aY–, and bY– axes, respectively; ω is the rotational mechanical speed of the rotor. F_{ex} (F_{ey}) and $M_{\varepsilon x}$ ($M_{\varepsilon y}$) are the unbalanced forces and unbalanced moments of the rotor in the x (y) directions.

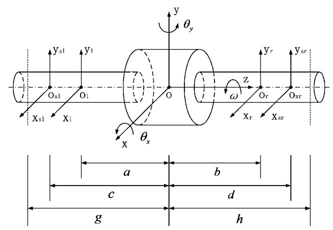

Figure 1. Sketch map of rotor motion

By Newton's second law, the dynamic model of 4-DOF AMB system can be obtained according to its structure as follows:

$$m\ddot{x} = F_{xa} + F_{xb} + F_{ex} \qquad m\ddot{y} = F_{ya} + F_{yb} + F_{ey}$$

$$J_x\ddot{\theta}_x + J_z\omega\dot{\theta}_y = l_aF_{ya} - l_bF_{yb} + M_{\varepsilon x}$$

$$J_y\ddot{\theta}_y - J_z\omega\dot{\theta}_x = l_bF_{xb} - l_aF_{xa} + M_{\varepsilon y}$$

where m is the mass of the rotor; g is the gravity constant; la and lb are the distance from O to the centers of the two 2-DOF radial AMB, respectively; Jx, Jy, and Jz are the moments of inertia of the rotor in the X–, Y–, and Z–axes, respectively, and Jx=Jy; $F_{xa} = k_ii_{xa} - k_sx_a$; $F_{xb} = k_ii_{xb} - k_sx_b$; $F_{ya} = k_ii_{ya} - k_sy_a$; $F_{yb} = k_ii_{yb} - k_sy_b$; k_i and k_s are current stiffness coefficient and displacement stiffness coefficient.

According to the system matrix differential equation described in equation above, we can get the state space description of the magnetic bearing rotor with respect to the state variable X , Input variable u and disturbance variable v . The state space of the system is described as

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{Bu} + \boldsymbol{Ev}$$
$$\boldsymbol{Y} = \boldsymbol{CX} \tag{10}$$

Where $\boldsymbol{X} = [\boldsymbol{q}_L \quad \dot{\boldsymbol{q}}_L]^T$, $\boldsymbol{v} = \left[\boldsymbol{F}_{ex}^T \quad \boldsymbol{F}_{ey}^T \quad \boldsymbol{M}_{\varepsilon x}^T \quad \boldsymbol{M}_{\varepsilon y}^T \right]^T$

$\boldsymbol{q}_c = [x\ y\ \theta_x\ \theta_y]^T$, $\boldsymbol{C} = \left[\boldsymbol{T}_L^{-T} \quad \boldsymbol{0}_{4\times4} \right]$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0}_{4\times4} & \boldsymbol{I}_{4\times4} \\ -\boldsymbol{T}_L^T\boldsymbol{M}^{-1}\boldsymbol{T}_L\boldsymbol{K}_s & -\boldsymbol{T}_L^T\boldsymbol{M}^{-1}\boldsymbol{G}\boldsymbol{T}_L^{-T} \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0}_{4\times4} \\ \boldsymbol{T}_L^T\boldsymbol{M}^{-1}\boldsymbol{T}_L\boldsymbol{K}_i \end{bmatrix}$$

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{0}_{4\times4} & \boldsymbol{0}_{4\times4} \\ \boldsymbol{T}_L^T\boldsymbol{M}^{-1}\boldsymbol{T}_d & \boldsymbol{T}_L^T\boldsymbol{M}^{-1} \end{bmatrix}, \qquad \boldsymbol{M} = \begin{bmatrix} m & 0 & 0 & \\ 0 & m & 0 & \\ 0 & 0 & J_x & \\ 0 & 0 & 0 & J_y \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_z\omega \\ 0 & 0 & -J_z\omega & 0 \end{bmatrix}, \qquad \boldsymbol{T}_L = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & l_a & -l_b \\ -l_a & l_b & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{q}_L = \boldsymbol{T}_L{}^T\boldsymbol{q}_c = \begin{bmatrix} 1 & 0 & 0 & -l_a \\ 1 & 0 & 0 & l_b \\ 0 & 1 & l_a & 0 \\ 0 & 1 & -l_b & 0 \end{bmatrix} \boldsymbol{q}_c .$$

From formula (10) we can see that the unbalanced vibration and gyro effect are functions of the speed. So corresponding to the three modes, the speed can be divided into low speed, medium speed and high speed.

To verify the proposed control scheme for the 4-DOF AMB system, some simulation studies are carried out under the speed of 0~40000r/min. With different speed range, the A and E in equation (10) are varied using the worst-case analysis of LPV.

Table 1 illustrates the system parameters, and Fig. 2-3 show the photograph of the simulation results as well as the parameters of the control schemes.

Table 1. Parameter of symmetrical system

Parameter	Value	Parameter	Value
m	100 kg	a	0.4 m
J_x	8.3333 kg m^2	b	0.4 m
J_y	8.3333 kg m^2	c	0.45 m
J_z	0.75 kg m^2	d	0.45 m
k_s	-1×10^7 N/m	g	0.604 m
k_i	250 N/A	h	0.604 m

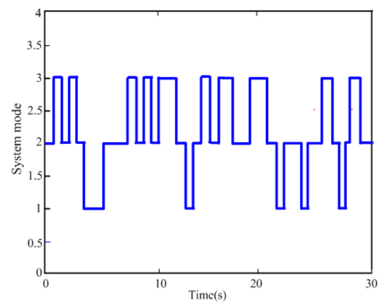

Figure2. A possible modes evolution

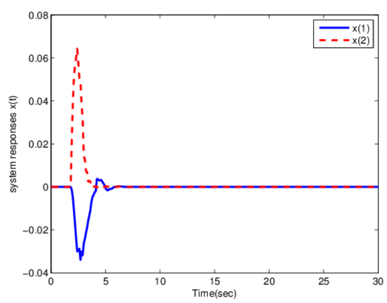

Figure 3. State response curves

From Fig. 3, it is seen that the MB system (10) is stochastically stable and meets the required $H\infty$ performance with γ=1.4235, v(t) = 0.2sin (t) , K1、K2 and K3.

$$K1=\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & -43.0166 & 15.0684 & 0.0620 & -0.0725 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 7.8444 & -40.3407 & 0.0307 & -0.0385 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0620 & 0.0725 & -43.0166 & 15.0684 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0307 & 0.0385 & 7.8444 & -40.3407 \end{bmatrix}$$

$$K2=\begin{bmatrix} -0.0021 & 0.0024 & 0.0001 & -0.0001 & -42.8819 & 14.5148 & -0.2243 & 0.2554 \\ 0.0033 & -0.0036 & -0.0001 & 0.0002 & 26.5088 & -62.0858 & 0.4788 & -0.5488 \\ -0.0001 & 0.0001 & -0.0021 & 0.0023 & 0.2243 & -0.2554 & -42.8820 & 14.5148 \\ 0.0001 & -0.0002 & 0.0033 & -0.0036 & -0.4788 & 0.5488 & 26.5089 & -62.0858 \end{bmatrix}$$

$$K3=\begin{bmatrix} -0.0004 & 0.0005 & -0.0001 & 0.0002 & -41.5347 & 13.0846 & -0.0457 & 0.0451 \\ 0.0005 & -0.0006 & 0.0002 & -0.0002 & 17.8021 & -52.0520 & 0.2266 & -0.2263 \\ 0.0001 & -0.0002 & -0.0004 & 0.0005 & 0.0457 & -0.0451 & -41.5347 & 13.0846 \\ -0.0002 & 0.0002 & 0.0005 & -0.0006 & -0.2266 & 0.2263 & 17.8021 & -52.0520 \end{bmatrix}$$

5. CONCLUSION

$H\infty$ control of MJLPV is discussed in this paper. Since a scaling method is developed to deal with controller design, how to reduce the induced scaling conservativeness needs further research.

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