

group 1 20 acres 1.1 tons per acre
group 2 30 acres 0.9 tons per acre
group 3 20 acres 0.8 tons per acre
group 4 10 acres 0.65 tons per acre

Grain can be bought for £90 per ton and sold for £75 per ton. Sugar beet can be bought for £70 per ton and sold for £58 per ton.

The labour requirements are:

each heifer	10 hours per year
each milk-producing cow	42 hours per year
each acre put to grain	4 hours per year
each acre put to sugar beet	14 hours per year

Other costs are:

each heifer	£50 per year
each milk-producing cow	£100 per year
each acre put to grain	£15 per year
each acre put to sugar beet	£10 per year

Labour costs for the farm are at present £4000 per year and provide 5500 hours labour. Any labour needed above this will cost £1.20 per hour.

How should the farmer operate over the next five years to maximize profit? Any capital expenditure would be financed by a 10 year loan at 15% annual interest. The interest and capital repayment would be paid in 10 equally sized yearly instalments. In no year can the cash flow be negative. Lastly, the farmer would not wish to reduce the total number of dairy cows at the end of the five year period by more than 50% nor increase the number by more than 75%.

12.9 Economic Planning

An economy consists of three industries: coal, steel and transport. Each unit produced by one of the industries (a unit will be taken as £1's worth of value of production) requires inputs from possibly its own industry as well as other industries. The required inputs as well as the manpower requirements (also measured in £) are given in Table 12.1. There is a time lag in the economy so that output in year $t + 1$ requires an input in year t .

Output from an industry may also be used to build productive capacity for itself or other industries in future years. The inputs required to give unit increases (capacity for £1's worth of extra production) in productive capacity are given in Table 12.2. Input from an industry in year t results in a (permanent) increase in productive capacity in year $t + 2$.

Table 12.1
Outputs (year $t + 1$), production

Inputs (year t)	Coal	Steel	Transport
Coal	0.1	0.5	0.4
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.6	0.3	0.2

Table 12.2
Outputs (year $t + 2$), productive capacity

Inputs (year t)	Coal	Steel	Transport
Coal	0.0	0.7	0.9
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.4	0.2	0.1

Table 12.3
Year 0

	Stocks	Productive capacity
Coal	150	300
Steel	80	350
Transport	100	280

Stocks of goods may be held from year to year. At present (year 0) the stocks and productive capacities (per year) are given in Table 12.3 (in £m). There is a limited yearly manpower capacity of £470m.

It is wished to investigate different possible growth patterns for the economy over the next five years. In particular it is desirable to know the growth patterns which would result from pursuing the following objectives:

- (i) Maximizing total productive capacity at the end of the five years while meeting an exogenous consumption requirement of £60m of coal, £60m of steel, and £30m of transport in every year (apart from year 0).
- (ii) Maximizing total production (rather than productive capacity) in the fourth and fifth years, but ignoring exogenous demand in each year.
- (iii) Maximizing the total manpower requirement (ignoring the manpower capacity limitation) over the period while meeting the yearly exogenous demands of (i).