$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\Big[\underbrace{p + \left(\frac{\partial u}{\partial V}\right)_T}_{=?}\Big]$$

<u>State functions</u>

$$\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V = \left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)_V\right)_T$$

$$\cancel{\frac{\partial S}{\partial T} = \frac{C_V}{\partial T} \to \frac{\partial}{\partial V}\left(\frac{C_V}{T}\right)_T = }$$

$$\frac{\partial S}{\partial T} = \frac{C_V}{T} = \frac{1}{T}\left(\frac{\partial u}{\partial T}\right)_V$$

$$\left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)\right) = \frac{1}{T}\frac{\partial^2 u}{\partial V \partial T}$$

$$\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V = \frac{1}{T}\left[\left(\frac{\partial p}{\partial T}\right)_V + \frac{\partial^2 u}{\partial T \partial V}\right]$$ product rule

$$- \frac{1}{T^2}\left[p + \left(\frac{\partial u}{\partial V}\right)_T\right]$$

$$dS = \frac{1}{T}\, dU + \frac{P}{T}\, dV$$

also, since state fncts

Maybe do this first

$$dS = \underbrace{\left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV}$$

T dependence is what we want to calculate

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$= C_v\, dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$dS = \frac{1}{T}\left[C_v\, dT + \left(\frac{\partial U}{\partial V}\right)_T dV\right] + \frac{P}{T}\, dV$$

$$= \frac{C_v\, dT}{T} + \frac{1}{T}\left(P + \left(\frac{\partial U}{\partial V}\right)_T\right) dV$$