A Review of Generating Vascular Network

For generating vascular networks by means of computer simulation, the establishment of appropriate model is the first step. Given the main structure of vascular network, several approaches have been made to give the details or substructures of the vascular system. The models in this review are based on hypothetical (mathematical) theory, not image-based.

1. Constructive Optimization Models

1.1 Constrained Constructive Optimization Models (CCO)

The basic principle of CCO is to construct a strictly binary tree by adding one leaf node at a time to an initial tree, each time introducing an optimal bifurcation. Thus, CCO can be seen to be driven by the assumption of equal in- or outflow at all leaf nodes representing constant supply/drainage for each lobulus. Moreover, at bifurcations the radii are balanced such that the flow resistance according to the Hagen-Poiseuille law is equal for both subtrees. This finally results in equal exit pressures at the leaf nodes.

1.1.1 Different PRNSs (Rudolf Karch, Friederike Neumann, Martin Neumann, 1999)

Use different pseudo random number sequence to generate locations of the terminal segments, each PRNS induces a different topological structure but optimized according to the same target function and meet the same boundary conditions (pressures, flows, bifurcation laws). This generates the different in topological (connective) structures. It can be correlated to variability of vascular trees in different individuals of same species, which does not induce drastic difference in properties intimately related to blood flow.

1.1.2 The shape of perfusion volume (W Schreiner, R Karch, F Neumann)

Different shapes of perfusion areas induce different branching patterns, location of feeding vessels also influence the degree of optimality (convex or nonconvex). The measure of bifurcation symmetry depends largely on the shapes.

1.1.3 Different target function

Minimize intravascular volume, since blood is costly substance to maintain; minimize total intravascular surface, total vascular length, total vascular hypervolume by inserting different bifurcation exponent. The larger the exponent of the radius the more optimization weight is put on large caliber vessels which as a result, become more streamlined. In physiological terms, this effect the distance of access for biochemical substance to be carried by blood into the tissue.

There are also some implementations to CCO models. (Wolfgang Schreiner, Rudolf Karch, Martin Neumann, 2005)

The first aspect is the blood flow.

In conventional CCO model, vascular trees are generated with equal terminal flows, whereas in implemented one, they are obtained by choosing a uniform probability distribution for the terminal flows with a set mean value. This model allows adaptation of arterial diameters to changes in blood flow rate, formation of different patterns of angiogenesis induced by changing needs of blood supply and vascular reactions due to therapeutic interventions such as shunting and revascularization.

The second aspect is the improved condition for non-convex case.

CCO is generalized to cope also with non-convex organ shapes, possibly featuring external as well as internal concavities. The key issue is the mathematical formulation and the algorithmic implementation of appropriate constraints for geometric optimization: arterial segments must stay within surrounding tissue and not cross any concavity (i.e. they must not leave and reenter the tissue) during optimization. While this behavior is intrinsically guaranteed in convex domains it has to be enforced within non-convex domains.

Approaches: analytical representation (shapes may be represented by analytical mathematical functions); finite elements triangulation (supported by numerous software packages, allowing for instant processing of segmented CT-images and interactive model manipulation via a GUI); potential surfaces (generated as finite sums over single-point potentials)

1.2 Global Constructive Optimization (Manfred Georg, Tobias Preusser, Horst K. Hahn, 2010)

Given the position and flow distribution of end points of a vascular system, we construct the topology and positions of internal nodes to complete the vascular system in a realistic manner. Optimization is driven by intravascular volume minimization with constraints derived from physiological principles. Direct optimization of a vascular system, including topological changes, is used instead of simulating vessel growth. The main consideration in the optimization of a vascular system is the energy required in creating and maintaining the system. Specifically, the minimization of the intravascular volume has been identified as a driving design principle. Rather than growing these trees through the simulation of angiogenesis, we make the assumption that vascular systems, regardless of how they are formed, tend toward a state of optimality.

Assumption: First, assume a laminar, perfect fluid of constant viscosity flows through the vessel system. Additionally, disregard the effects of angle of bifurcation and vessel curvature on pressure drops in the vascular system. Finally, assume that the flow is steady, not pulsatile. There is literature suggesting that geometrically optimizing a vascular system is similar regardless of steady or pulsatile flow.

The difference between CCO and GCO is that Constrained Constructive Optimization finds an optimal tree by adding one branch one at a time whereas Global Constructive Optimization performs a multiscale optimization finding an optimal tree for all leaf nodes at the same time.

Optimization models are widely used in generating vascular systems, which works well comparing to the actual model. Although for individuals, the vascular network of organs could differ, the same species often have similar structure. The main issue here is how to design the objective function, since the factors to be considered would affect the result and computational costs.

2. Compartment Model (M. Karlsson, P. Bruinsma, T. Arts, J. Dankelman, and J. A. E. Spaan)

   Large group of vessels are lumped together into compartments, each of which is then characterized globally (e.g., by a global resistance and pressure-volume relation). In this method, models of the arterial tree with distributed parameters has been developed in order to quantify hydro-mechanical effects in the arterial system. Such model must resolve the characteristic features of the arterial tree such as distributed resistance and the ability to incorporate local variations in segmental compliance. The topology of the arterial tree must also be included in order to resolve wave reflections adequately. Furthermore, an effective and robust numerical method is necessary in order to keep the computational time as low as possible.

   This kind of models avoid the problem when consider the non-convex case (for example, heart), but factors taken into consideration are limited, mainly in the field of bio-fluid dynamics.

3. Fractural Model (H. K. Hahn, C. J. G. Evertsz, J. H. D. Fasel, and H.-O. Peitgen)

This kind of models are self-similar models based on some generating rules, constructed over successive orders of bifurcations, the generating rules either be constant or stochastic in the nature. Generators can be chosen either based on statistical evaluation of experimental data, or can be derived from optimization principle. Since many biological structures possess structural similarities independent of scale and thus may be considered fractal (lungs), this feature helps to introduce an algorithm that constructs organs' airway structures and to use computer simulations of growth based on fractal concepts. Under some conditions, limits imposed by simple boundary constraints generate structures that are in good agreement with actual morphometric data.[i]

Such approaches benefit (in terms of algorithmic complexity) from avoiding optimization problems. The resulting vascular networks, however, suffer from an artificial overall structure, which is even visually perceptible, so that they are not appropriate for general models.

4. Angiogenesis-Based Construction (M. Kretowski, Y. Rolland, J. Bézy-Wendling, Lars Ole Schwen1 and Tobias Preusser)

   Another approach is modeling angiogenesis, the actual process by which vascular structures grow. This type of approach requires more involved models and algorithms than basic optimality conditions.

   Earlier results in this area exhibit a visually artificial structure or "somewhat stylized appearance". More recent work includes, the latter combining introducing new vascular edges due to angiogenesis with subsequent geometric optimization of the vascular tree. Grid-based methods typically produce visual geometric artifacts reflecting the grid used. To us there seems no easy way of introducing parameters in these algorithms for being able to calibrate them to better match geometric features measured in the vasculature of human livers. One could combine angiogenesis models with geometric parameters by changing the way how new vascular edges form. Instead of only considering gradients of angiogenetic factors generated by ischemic cells, also properties of the existing vascular edge for which a new bifurcation is to be introduced, on the branching angles of that bifurcation or on other properties of the existing vascular structure, could be taken into account.

   This would, however, involve additional assumptions for the model and parameters in the algorithmic implementation which are not easily observed experimentally.[ii]

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[i] Nelson T R , Manchester D K . Modeling of lung morphogenesis using fractal geometries.[J]. IEEE Transactions on Medical Imaging, 1988, 7(4):321-7.

[ii] Schwen L O , Preusser T . Analysis and Algorithmic Generation of Hepatic Vascular Systems[J]. International Journal of Hepatology, 2012, 2012:1-17.