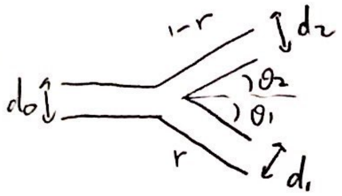
1st principle: the amount of fluid delivery through a branch $\propto$ the volumn of region it supplies

2nd principle: the terminal branches of the tree are homogeneously arranged within the organ



$d_0^n = d_1^n + d_2^n$

$d_1 = d_0 r^{1/n}$, r = flow-dividing ratio *

$d_2 = d_0(1-r)^{1/n}$, $0 < r \leq 0.5$, $d_1 \leq d_2$

Aside:

$Q = Cd^n$

$Q = (Cd')^n$

$r = \frac{Q_{daughter}}{Q_{parent}} = \frac{d_1^n}{d_0^n}$

Assumption: when the total volume of three branches in a bifurcation is minimized. (Kamiya et al.)

$$\frac{d_0^2}{\sin(\theta_1+\theta_2)} = \frac{d_1^2}{\sin\theta_1} = \frac{d_2^2}{\sin\theta_2}$$

$$\therefore \cos\theta_1 = \frac{1 + r^{4/n} - (1-r)^{4/n}}{2r^{2/n}}$$ eq. (6)

$$\cos\theta_2 = \frac{1 + (1-r)^{4/n} - r^{4/n}}{2(1-r)^{2/n}}$$

* $n = 3$ by Kamiya et al.

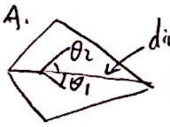
* $n = 3$ for laminar flow, $n = 2.333$ for turbulent flow, by Uylings

length & diameter ratios — Normal-like distribution with $\bar{x} = 2.8$ & std $= 1.0$

9 rules:
1. Branching is dichotomous
2. parents and two daughter branches in the same plane
3. $\dot{V}$ conserved. $\dot{V}_0 = \dot{V}_1 + \dot{V}_2$
4. "space-diving" plane $\perp$ branching plane, seperate parent region & daughter region
5. ☆ Flow dividing ratio = volume-dividing ratio
6. Diameter & angle determined by substituting r
7. The length of each daughter is assigned value that is three times its diameter
8. Two successive branching planes are $\perp$ to each other
9. Branching stops whenever flow rate becomes less than a specified threshold or the branch extends beyond its own region

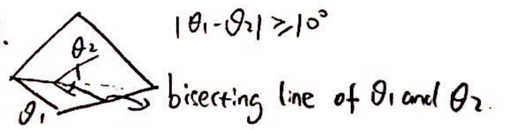
Supplementary Rules:

4a. A.



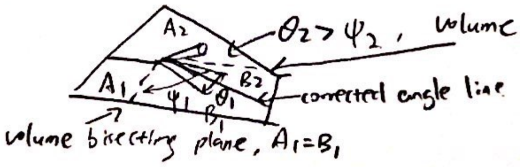
$|\theta_1 - \theta_2| < 10°$

B. $|\theta_1 - \theta_2| \geq 10°$



6a. ① Define a 'volume-bisecting' plane including the branching point
② Define ' ' ', angle, ψ, between the parent branch and volume-bisecting plane
③ If θ_1 or $\theta_2 < \psi$

If $\psi_1 > \theta_1$, $\theta_1 = \frac{(\psi_1 + \theta_1)}{2}$, if $\theta_1 > 90°$, $\theta_1 = 90°$



Rule 7a. $\frac{\text{length}}{\text{Diameter}} = 3.0$ (originally)

① Define distance-to-length ratio ranging from 3.0 ~ 6.0, with a step of 0.25 increase / decrease should not smaller than 1.0

$\frac{P_0P_2}{P_0P_1}$

where P_0: starting point of the branch

P_1 : end point of branch

P_2 : boundary point.

Rule 8a. ① Set a thresh-hold value for volume-dividing ratio, r

② If r < threshold value, the rotation angle of branching plane is increased or decreased in steps of 9° until its r > threshold

r = 0.05

③ Additionaly, $\dot{Q}_{daught} < 1.5\ \dot{Q}_{threshold}$, r = 0.35

For each branch,

① 3D position

② Unit vector (to detemine direction)

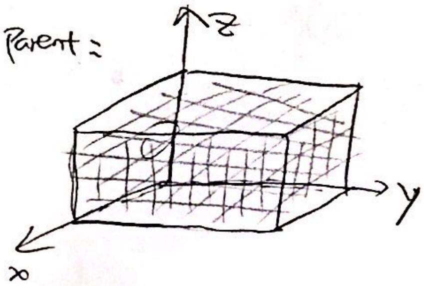
③ Flow rate

④ Normal vector

⑤ sets of planes that define the boundaries

- Store the ancestor region, exclude this region for other new branches

Parent :



volume dividing ratio = $\frac{\text{\# rectangles in daughter}}{\text{\# rectangles in parents}}$