Kepler Problem Error Convergence, 99.0% Confidence Weak/Hamiltonian Strong/Mean Square  $10^{-2}$  $10^{-1}$ 10<sup>-3</sup>  $10^{-4}$  $E[H(x_{exact})]$  $10^{-6}$  $10^{-5}$ 10-8  $10^{-7}$  $^{\prime}E[||x_{exact}|]$  $E[H(x_{approx})]$  $10^{-10}$  $10^{-9}$  $10^{-12}$  $10^{-11}$  $10^{-13}$  $10^{-14}$  $10^{-16}$  $10^{-15}$ 10<sup>-3</sup> 10<sup>-3</sup> Δt  $\Delta t$ Slope order 1 Gauss-2 or HBVM(1,1) Gauss-4 or HBVM(2,2)  $\longrightarrow$  Gauss-6 or HBVM(3,3) Slope order 2 Gauss-HBVM(2,1) Gauss-HBVM(3,2)  $\longrightarrow$  Gauss-HBVM(4,3) Slope order 3 Gauss-HBVM(3,1) Gauss-HBVM(4,2)  $\longrightarrow$  Gauss-HBVM(5,3) Slope order 4 Gauss-HBVM(4,1)  $\leftarrow$  Gauss-HBVM(5,2)  $\longrightarrow$  Gauss-HBVM(6,3) Slope order 5  $\longrightarrow$  Gauss-HBVM(5,1)  $\leftarrow$  Gauss-HBVM(6,2)  $\longrightarrow$  Gauss-HBVM(7,3) Slope order 6