Ayudantía 9 MAT033

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Pregunta 1

Sea X una variable aleatoria, con función generadora de momentos:

$$\varphi_x(t) = \frac{1}{2}e^{-t} + \frac{1}{2}$$

Sean $X_1, X_2, \ldots, X_n$ variables aleatorias independientes distribuidas de igual forma que X, encuentre $E(\bar{X})$ y $V(\bar{X})$.

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$$\frac{n^2 - n + 2n}{4n^2} = \frac{n+1}{4n}$$

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$$\frac{n^2 - n + 2n}{4n^2} = \frac{n+1}{4n}$$

$$V(\bar{X}) = E(\bar{X}^2) - E(\bar{X})^2 = \frac{n+1}{4n} - \frac{1}{4} =$$

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$$V(\bar{X}) = E(\bar{X}^2) - E(\bar{X})^2 = \frac{n+1}{4n} - \frac{1}{4} = \frac{1}{4n}$$

Pregunta 2

Sean $X_1, X_2, \ldots, X_n$ una m.a. de tamaño n, de una población con la siguiente función de densidad:

$$f_{X_i}(x_i) = \begin{cases} \frac{1}{x_i \sigma \sqrt{2\pi}} e^{\frac{-1(\ln(x_i)-\mu)^2}{2\sigma^2}} & x_i > 0; i = 1, \ldots, n \\ 0 & e.o.c. \end{cases}$$

a. Encuentre el EMV de μ, con σ^2 conocido.
b. Si $n = 3$ y $X_1 = e$, $X_2 = e^2$ y $X_3 = e^3$. Evalué el μ_{EMV} encontrado en a.

Solución:

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a. Tenemos que:

$$f_{X_i}(x_i) = \frac{1}{x_i\sigma\sqrt{2\pi}} e^{\frac{-1(\ln(x_i)-\mu)^2}{2\sigma^2}}$$

Solución:

a. Tenemos que:

$$
\begin{aligned}
f_{X_i}(x_i) &= \frac{1}{x_i\sigma\sqrt{2\pi}} e^{\frac{-1(\ln(x_i)-\mu)^2}{2\sigma^2}} \\
L &= \prod_{i=1}^{n} \frac{1}{x_i\sigma\sqrt{2\pi}} EXP\left\{ -\frac{1}{2}\left(\frac{\ln(x_i)-\mu}{\sigma}\right)^2 \right\}
\end{aligned}
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L &= \prod_{i=1}^{n}\frac{1}{x_i\sigma\sqrt{2\pi}}EXP\left\{-\frac{1}{2}\left(\frac{\ln(x_i)-\mu}{\sigma}\right)^2\right\} \\
&= (\frac{1}{\sigma\sqrt{2\pi}})^n\frac{1}{\prod_{i=1}^{n}x_i}EXP\left\{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\ln(x_i)-\mu}{\sigma}\right)^2\right\}
\end{aligned}
$$

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$$\ln(L) \quad = \quad -n\ln(\sigma\sqrt{2\pi}) - \ln(\prod_{i=1}^{n} x_i) - \Big(-\frac{1}{2}\sum_{i=1}^{n}(\frac{\ln(x_i)-\mu}{\sigma})^2\Big)$$

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\ln(L) &= -n\ln(\sigma\sqrt{2\pi}) - \ln(\prod_{i=1}^{n} x_i) - \Big(-\frac{1}{2}\sum_{i=1}^{n}(\frac{\ln(x_i)-\mu}{\sigma})^2\Big) \\
\Rightarrow \frac{\partial \ln L}{\partial \mu} &= \sum_{i=1}^{n}\Big(\frac{\ln(x_i)-\mu}{\sigma}\Big)
\end{aligned}
$$

Como buscamos la EMV se debe cumplir;

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\sum_{i=1}^{n} \left(\ln(x_i) - \mu\right) &= 0 \\
\Rightarrow \hat{\mu} &= \frac{\displaystyle\sum_{i=1}^{n} \ln x_i}{n}
\end{aligned}
$$

- b Sabemos que $X_1 = e$, $X_2 = e^2$ y $X_3 = e^3$.
  Por lo que se tendrá que $\ln X_1 = 1$, $\ln X_2 = 2$ y $\ln X_3 = 3$.

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\hat{\mu} &= \sum_{i=1}^{3} \frac{lnx_i}{3} \\
&= \frac{1+2+3}{3} \\
&= 2
\end{aligned}$$

Pregunta 3

Sea X_1, X_2 una muestra aleatoria de tamaño 2 de X con media μ y varianza σ^2. Si disponemos de dos estimadores para $\hat{\mu}_1 = \dfrac{X_1 + X_2}{2}$ y $\hat{\mu}_2 = \dfrac{X_1 + 2X_2}{3}$. Cual de los dos estimadores es el mejor?.

Solución:

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&= \frac{1}{3}(\mu + 2\mu) = \mu
\end{aligned}$$

Solución: Para definir cual estimador es mejor, debemos partir calculando el sesgo de los 2 estimadores.

$$\begin{aligned} E[\hat{\mu}_1] &= E\left[\frac{X_1 + X_2}{2}\right] \\ &= \frac{1}{2}(\mu + \mu) = \mu \\ E[\hat{\mu}_2] &= E\left[\frac{X_1 + 2X_2}{2}\right] \\ &= \frac{1}{3}(\mu + 2\mu) = \mu \end{aligned}$$

Obteniendo así que ambos estimadores son insesgados, por lo cual resta estudiar la variavilidad de estos.

Ahora calculando la varianza de los estimadores obtenemos.

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Debido a que $\hat{\mu}_1$ posee una menor varianza y de acuerdo al criterio de eficiencia, $\hat{\mu}_1$ es preferido a $\hat{\mu}_2$.