Ayudantía 3 MAT033

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Pregunta 1

Hay estudios que relacionan la falta de silicona disuelta en agua de mar con una productividad decreciente de productos primarios. Las dos variables estudiadas son la distancia de la costa y la concentración de silicona. Los resultados de 62 mediciones fueron los siguientes:

	Concentración de silicona			
Distancia	[3.5-4.5)	[4.5-5.5)	[5.5-6.5)	[6.5-7.5]
[2.5 - 7.5)			3	4
[7.5 - 13.5)			7	1
[13.5 - 18.5)	2	7	1	
[18.5 - 28.5)	8	13		
[28.5 - 38.5]	13	3		

Pregunta 1

a. Calcule las distribuciones marginales y determine cuál de ellas es más homogénea.
b. Determine la distribución condicional y la moda para la concentración de silicona cuando se encuentran a 16 kilómetros de la costa.
c. ¿Es posible establecer que la distancia de la costa y la concentración de silicona son variables correlacionadas linealmente?
de la costa?

Solución:

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a. $\overline{X} = 21.1048 \; S_X = 9.5425$

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$CV_Y = \dfrac{S_Y}{\overline{X}} = 0.1893$

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$CV_X = \frac{S_X}{\overline{X}} = 0.4521$
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Por lo que la distribución de silicona es mas homogénea.

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$Cor(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} =$

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$Cor(X,Y) = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y} = -0.8048$

Por lo que se puede decir que tienen una alta correlación negativa.

Pregunta 2

A continuación se presenta una tabla con dos variables, Número de días en los que llovío en el año (X) y la variable del Número de miles de visitas anuales al zoológico (Y).

Año	2008	2009	2010	2011	2012
X	18	26	30	33	38
Y	107	105.5	105	104.4	104.3

Año	2013	2014	2015	2016	2017
X	39	42	44	46	49
Y	104	103.7	103,4	103.1	103

Pregunta 2

a. Verifique si hay alguna relación lineal entre estos datos con algún coeficiente conocido en esta área y comente la relacion que tengan.
b. Calcule la correlación y defínala según algún criterio visto en clases.

Solución:

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a. $\overline{X} = 36.5$

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Solución:

a) $\overline{X} = 36.5$
$\overline{Y} = 104.3$
$$Cov(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) =$$

Solución:

a) $\overline{X} = 36.5$
$\overline{Y} = 104.3$
$$Cov(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) = -10.6$$

Solución:

a. $\overline{X} = 36.5$
   $\overline{Y} = 104.3$
   $$Cov(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) = -10.6$$

b. $$\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 = 84.85$$

Solución:

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$$Cov(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) = -10.6$$

b. $$\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 = 84.85$$

$$\sigma_Y^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 1.36$$

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$$\sigma_Y^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 1.36$$
$\sigma_X = 9.21$

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$$Cor(X,Y) = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} =$$

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$\sigma_X = 9.21$
$\sigma_Y = 1.16$
$$Cor(X,Y) = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = -0.98$$
Por lo que podemos decir que se tiene una alta correlación negativa.

Pregunta 3

En el prestigioso hospital de Talca, a 50 pacientes se les administra una sustancia que se identifica con la letra C en miligramos, considerando como segunda variable la edad E medida en años, tal como se muestra en la siguiente tabla:

MC_E/MC_C	15	20	25	30	35
20	4	2	2		
30	2	6	3	1	
40		2	5	4	3
50		2	3	6	
60			2	2	1

Pregunta 3

a. Calcule el promedio de las variables y explique cuanto varían estas y que tanto esta relacionada la cantidad de sustancia C con la edad de cada paciente .
b. Calcule las medias y varianzas de la sustancia C con respecto a la edad E y de igual manera para la edad con respecto a C.

Solución:

Solución:

MC$_E/MC_C$	15	20	25	30	35	Total
20	4	2	2			8
30	2	6	3	1		12
40		2	5	4	3	14
50		2	3	6		11
60			2	2	1	5
Total	6	12	15	13	4	50

MC_E	n_i
20	8
30	12
40	14
50	11
60	5

MC_E	n_i
20	8
30	12
40	14
50	11
60	5

$\overline{E} = 38.6$

MC$_E$	n_i
20	8
30	12
40	14
50	11
60	5

$\overline{E} = 38.6$
$S_E = 12.29$

MC_E	n_i
20	8
30	12
40	14
50	11
60	5

$\overline{E} = 38.6$
$S_E = 12.29$
$S_E^2 = 146$

MC_E	n_i
20	8
30	12
40	14
50	11
60	5

$\overline{E} = 38.6$
$S_E = 12.29$
$S_E^2 = 146$

MC_C	n_j
15	6
20	12
25	15
30	13
35	4

$\overline{C} = 24.7$

MC_C	n_j
15	6
20	12
25	15
30	13
35	4

$\overline{C} = 24.7$
$S_C = 5.75$

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15	6
20	12
25	15
30	13
35	4

$\overline{C} = 24.7$
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$S_C^2 = 33.06$

MC_C	n_j
15	6
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$\overline{C} = 24.7$
$S_C = 5.75$
$S_C^2 = 33.06$

$Cov(E, C) =$

MC_C	n_j
15	6
20	12
25	15
30	13
35	4

$\overline{C} = 24.7$
$S_C = 5.75$
$S_C^2 = 33.06$

$$Cov(E,C) = \sum_{i=1}^{r}\sum_{j=1}^{s} f_{ij}(m_{E_i} - \overline{E})(m_{C_j} - \overline{C}) =$$

MC_C	n_j
15	6
20	12
25	15
30	13
35	4

$\overline{C} = 24.7$
$S_C = 5.75$
$S_C^2 = 33.06$

$$Cov(E,C) = \sum_{i=1}^{r}\sum_{j=1}^{s} f_{ij}(m_{E_i} - \overline{E})(m_{C_j} - \overline{C}) = 41.41$$

$\overline{C}_1 =$

$$\overline{C}_1 = \overline{C}_{C/E=20} =$$

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$$V_1(C) =$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 =$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 17.19$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 17.19$$

$$\overline{C}_2 = \overline{C}_{C/E=30} = \sum_{j=1}^{s} f_{j/i} mc_j = 21.25$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 17.19$$

$$\overline{C}_2 = \overline{C}_{C/E=30} = \sum_{j=1}^{s} f_{j/i} mc_j = 21.25$$

$$V_2(C) = V_{C/E=30}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 17.19$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 17.19$$

$$\overline{C}_2 = \overline{C}_{C/E=30} = \sum_{j=1}^{s} f_{j/i} mc_j = 21.25$$

$$V_2(C) = V_{C/E=30}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 17.19$$

$$\overline{C}_3 = \overline{C}_{C/E=40} = \sum_{j=1}^{s} f_{j/i} mc_j = 27.85$$

$$\overline{C}_1 = \overline{C}_{C/E=20} = \sum_{j=1}^{s} f_{j/i} mc_j = 18.75$$

$$V_1(C) = V_{C/E=20}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 17.19$$

$$\overline{C}_2 = \overline{C}_{C/E=30} = \sum_{j=1}^{s} f_{j/i} mc_j = 21.25$$

$$V_2(C) = V_{C/E=30}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 17.19$$

$$\overline{C}_3 = \overline{C}_{C/E=40} = \sum_{j=1}^{s} f_{j/i} mc_j = 27.85$$

$$V_3(C) = V_{C/E=40}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 23.98$$

$$\overline{C}_4 = \overline{C}_{C/E=50} = \sum_{j=1}^{s} f_{j/i} mc_j = 26.82$$

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$$V_4(C) = V_{C/E=50}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 14.88$$

$$\overline{C}_4 = \overline{C}_{C/E=50} = \sum_{j=1}^{s} f_{j/i} mc_j = 26.82$$

$$V_4(C) = V_{C/E=50}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 14.88$$

$$\overline{C}_5 = \overline{C}_{C/E=60} = \sum_{j=1}^{s} f_{j/i} mc_j = 29$$

$$\overline{C}_4 = \overline{C}_{C/E=50} = \sum_{j=1}^{s} f_{j/i} mc_j = 26.82$$

$$V_4(C) = V_{C/E=50}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=30} - mc_j)^2 = 14.88$$

$$\overline{C}_5 = \overline{C}_{C/E=60} = \sum_{j=1}^{s} f_{j/i} mc_j = 29$$

$$V_5(C) = V_{C/E=60}(C) = \sum_{j=1}^{s} f_{j/i} (\overline{C}_{C/E=20} - mc_j)^2 = 14$$

$$\overline{E}_1 =$$

$$\overline{E}_1 = \overline{E}_{E/C=15} =$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i =$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

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$$V_1(E) =$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 =$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 = 22.22$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 = 22.22$$

$$\overline{E}_2 = \overline{E}_{E/C=20} = \sum_{i=1}^{r} f_{i/j} me_i = 33.33$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 = 22.22$$

$$\overline{E}_2 = \overline{E}_{E/C=20} = \sum_{i=1}^{r} f_{i/j} me_i = 33.33$$

$$V_2(E) = V_{E/C=20}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=20} - me_i)^2 = 88.89$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 = 22.22$$

$$\overline{E}_2 = \overline{E}_{E/C=20} = \sum_{i=1}^{r} f_{i/j} me_i = 33.33$$

$$V_2(E) = V_{E/C=20}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=20} - me_i)^2 = 88.89$$

$$\overline{E}_3 = \overline{E}_{E/C=25} = \sum_{i=1}^{r} f_{i/j} me_i = 40$$

$$\overline{E}_1 = \overline{E}_{E/C=15} = \sum_{i=1}^{r} f_{i/j} me_i = 23.33$$

$$V_1(E) = V_{E/C=15}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=15} - me_i)^2 = 22.22$$

$$\overline{E}_2 = \overline{E}_{E/C=20} = \sum_{i=1}^{r} f_{i/j} me_i = 33.33$$

$$V_2(E) = V_{E/C=20}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=20} - me_i)^2 = 88.89$$

$$\overline{E}_3 = \overline{E}_{E/C=25} = \sum_{i=1}^{r} f_{i/j} me_i = 40$$

$$V_3(E) = V_{E/C=25}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=25} - me_i)^2 = 146.67$$

$$\overline{E}_4 = \overline{E}_{E/C=30} = \sum_{i=1}^{r} f_{i/j} me_i = 46.92$$

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$$V_4(E) = V_{E/C=30}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=30} - me_i)^2 = 67.46$$

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$$\overline{E}_5 = \overline{E}_{E/C=35} = \sum_{i=1}^{r} f_{i/j} me_i = 45$$

$$\overline{E}_4 = \overline{E}_{E/C=30} = \sum_{i=1}^{r} f_{i/j} me_i = 46.92$$

$$V_4(E) = V_{E/C=30}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=30} - me_i)^2 = 67.46$$

$$\overline{E}_5 = \overline{E}_{E/C=35} = \sum_{i=1}^{r} f_{i/j} me_i = 45$$

$$V_5(E) = V_{E/C=35}(E) = \sum_{i=1}^{r} f_{i/j} (\overline{E}_{E/C=35} - me_i)^2 = 75$$