Localic completions in constructive analysis

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Abstract. Formal topology may be regarded as locale theory developed within a neutral constructivist framework, such as that of Bishop's constructive mathematics [1], avoiding impredicativity and the principle of excluded middle. A systematic investigation was started by Martin-Löf and Sambin [5] using type theory as foundation. A link between formal topology and Bishop's theory of compact metric spaces was provided by Vickers' seminal notion of *localic completion* [3, 4]. With Vickers' method it can been shown [2] that the category of open subspaces of locally compact metric spaces embed fully faithfully into the category of locales (or formal topologies). This gives the possibility to apply methods of locale theory to constructive analysis, and conversely prove certain results about locales using point-based methods.

References

[1] E. Bishop and D.S. Bridges, *Constructive Analysis.* Springer 1985.

[2] E. Palmgren. Open sublocales of localic completions. *Journal of Logic and Analysis.* 2:1(2010), 1 – 22.

[3] S. Vickers. Localic completion of generalized metric spaces I, *Theory and Applications of Categories* 14(2005), 328 – 356.

[4] S. Vickers. Localic completion of generalized metric spaces II, *Journal of Logic and Analysis* 1:11(2009), 1 – 49

[5] G. Sambin. Intuitionistic formal spaces — a first communication, in: D. Skordev (ed.), *Mathematical logic and its Applications, Plenum Press* 1987, pp. 187 – 204.