Fibrations of toposes

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The notions of (op)fibration of toposes have close connections to topological properties. For instance, every local homeomorphism is an opfibration. This connection is in line with the conception of toposes as generalized spaces.

To study fibrations of toposes, Johnstone defined fibrations internal to 2-categories ([2]). If toposes are taken to be bounded over some fixed base $\mathscr{S}$, the analysis of fibrations and opfibrations in the 2-category $\mathscr{B}\mathfrak{Top}/\mathscr{S}$ of bounded toposes over base $\mathscr{S}$ is much easier than the general case where there is no canonical choice of base topos and one has to work in the 2-category $\mathscr{B}\mathfrak{Top}$. Indeed, Johnstone proved several important (op)fibrational results in $\mathscr{B}\mathfrak{Top}$.

I will introduce the 2-category $\mathfrak{Con}$ of contexts developed by Vickers ([3],[4]). It provides a language to reason about geometric construction within the predicative fragment of internal language of toposes, that is within the language of Arithmetic Universes.

Borrowing from work of Street ([1]), we introduce a syntactic notion of (op)fibration in $\mathfrak{Con}$ which is based on Chevalley's internal characterization of fibrations obtained as a theorem in there. Note that Johnstone's definition of internal (op)fibrations is more genral than Chevalley's definition: neither strictness of 2-categories nor the existence of the structure of strict pullbacks and comma objects are assumed.

I shall explain our result that gives a recepie for obtaining (op)fibrations of toposes from the finitary syntactic (op)fibrations of contexts ([5]). The scaffolding of the proof of this result is based on a certain comprehension bicategory involving fibred bicategory of generalized spaces over elementary toposes.

I will discuss the application of this result to construction of colimits of topos and bag toposes from generalized point-free point of view.

References

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