Part A: "Exercise Set 2.2 using Matlab"

iteration \ p-value	(a)	(b)	(c)	(d)
1	343	6	2.2	1.5
2	$-2.25393386 \cdot 10^{25}$	-344	1.819764	1.1450521
3	$-3.33838 \cdot 10^{253}$	$4.023390 \cdot 10^{7}$	1.583475	1.498750
4	NaN	$-6.512933 \cdot 10^{22}$	1.489461	1.451904
5	NaN	$2.762675 \cdot 10^{68}$	1.476022	1.497577
6	NaN	$-2.108578 \cdot 10^{205}$	1.475773	1.453192
. .	. .	. .	. .	. .
$\lvert p_6 - 7^{1/5} \rvert$	$>10^{253}$	$>10^{205}$	$3.16159 \cdot 10^{-6}$	0.0225809

Hence, the algorithm does not converges for the formulas (a) and (b) and the algorithm converges for the formulas (c) with the solution 1.475773 and (d) with the solution 1.480571. From the initial condition $p_0 = 1$, we could possibly discover the algorithm converges much faster for (c) compared with (d) since there are much more iterations needed to converge for part (d).

Part B: Table of the absolute error $\lvert p_n - p \rvert$ against n

iteration \ abs. error	(a)	(b)	(c)	(d)
1	342	5	1.2	0.5
2	$2.2539 \cdot 10^{25}$	350	0.380236	0.049479
3	$3.33838 \cdot 10^{253}$	$4.02342 \cdot 10^{7}$	0.236289	0.048229
4	NaN	$6.5129333 \cdot 10^{22}$	0.094014	0.046846
5	NaN	$2.7626756 \cdot 10^{68}$	0.013439	0.045674
6	NaN	$2.108578 \cdot 10^{205}$	0.000249	0.044385
. .	. .	. .	. .	. .
66	NaN	NaN	NaN	0.009779

Part C: We know that (a) and (b) are divergent from **Part A**. Then, to estimate the α and λ for (c) and (d), we need $\lim_{n\to\infty} \frac{\lvert p_{n+1} - p \rvert}{\lvert p_n - p \rvert^{\alpha}} = \lambda$, where $\{p_n\}_{n=0}^{\infty}$ converges to p of order α with λ. Also, $\alpha = \frac{\log \lvert p_{n+1} - p \rvert - \log \lambda}{\log \lvert p_n - p \rvert}$ by logging on both side of the equation.

Then, using the above equations we obtain α = 1.9242 and λ = 0.82.

Also, $g(x) = x - \frac{x^2 - 7}{5x^4} \Rightarrow g'(p) = 1 - \frac{p^5 + 28}{5p^5} = -4.37883 \cdot 10^{-7} \Rightarrow g''(p) = \frac{28}{p^6} = 2.71044$ for (c), meaning the iteration will have a quadratic rate of convergence.

For (d), α = 0.999 and λ = 0.97 and $g(x) = x - \frac{x^5 - 7}{12} \Rightarrow g'(p) = 1 - \frac{5p^4}{12} = -2.00219 \Rightarrow g''(p) = -\frac{5p^3}{3}$ $= -5.40924$, meaning the iteration will have a linear rate of convergence.