MACM 316 - Computing Assignment 4 Jeongwoon Suh (301313489)

Part A: "Approximate the population at January 1, 2015 using n data points where $2 \le n \le 9$"

Following table shows that $P_{1,2,3,4,5,6,7,8,9,10} = 4747931.4217$ represents the approximated population at January 1, 2015 using the year of 2014, and 2016 of July 1, since the average of July 1, 2014 and July 1, 2016 could possibly approximate the population of January 1, 2015. Thus, $n = 4.5$ gives the best estimate of the population at January 1, 2015.

2011	$P_1 = 4{,}502{,}104.0000$								
2012	$P_2 = 4{,}566{,}769.0000$	$P_{1,2} = 4{,}728{,}431.5000$							
2013	$P_3 = 4{,}630{,}077.0000$	$P_{2,3} = 4{,}725{,}039.0000$	4,722,494.6250						
2014	$P_4 = 4{,}707{,}103.0000$	$P_{3,4} = 4{,}745{,}616.0000$	4,750,760.2500	4,755,471.1875					
2016	$P_6 = 4{,}859{,}250.0000$	$P_{4,6} = 4{,}745{,}139.7500$	4,745,377.8750	4,747,396.2656	4,749,818.7422				
2017	$P_7 = 4{,}929{,}384.0000$	$P_{6,7} = 4{,}754{,}049.0000$	4,746,624.6250	4,745,845.4062	4,746,620.8359	4,747,953.2969			
2018	$P_8 = 5{,}010{,}476.0000$	$P_{7,8} = 4{,}726{,}654.0000$	4,774,595.2500	4,750,120.9531	4,747,128.0703	4,746,832.1836	4,747,392.7402		
2019	$P_9 = 5{,}090{,}955.0000$	$P_{8,9} = 4{,}728{,}799.5000$	4,723,972.1250	4,799,906.8125	4,755,099.5391	4,749,120.9375	4,747,649.5957	4,747,505.1145	
2020	$P_{10} = 5{,}147{,}712.0000$	$P_{9,10} = 4{,}835{,}548.5000$	4,541,988.7500	4,875,624.9375	4,771,512.5156	4,756,467.2871	4,750,695.1553	4,748,601.3331	4747931.4217

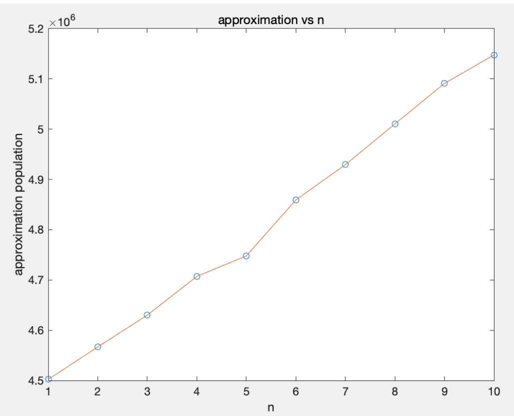

Figure 1. Plot approximation of the population at January 1, 2015 using n = 4.5 (approximation vs. n)

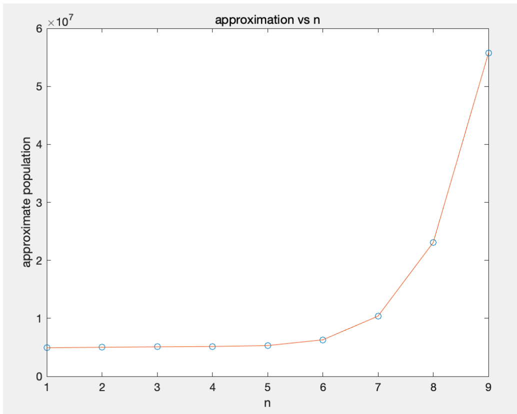

Figure 2. Plot approximation of the population at July 1, 2025 using n = 9 (approximation vs. n)

Part B: "Approximate the population at July 1, 2025 using n data points where $2 \le n \le 9$"

From an implemented Neville's method code in Matlab, following table shows that $P_{7,8,9,10,11,12,13,14} = 55{,}715{,}538.2386$ represents the approximated population at July 1, 2025 using the year of 2024, and 2025 of July 1. Thus, $n = 9$ gives the best estimate of the population at July 1, 2025.

$x_7 = 2017$	$P_7 = 4{,}929{,}384.0000$							
$x_8 = 2018$	$P_8 = 5{,}010{,}476.0000$	$P_{7,8} = 5{,}578{,}120.0000$						
$x_9 = 2019$	$P_9 = 5{,}090{,}955.0000$	$P_{8,9} = 5{,}573{,}829.0000$	5,560,956.0000					
$x_{10} = 2020$	$P_{10} = 5{,}147{,}712.0000$	$P_{9,10} = 5{,}431{,}497.0000$	5,075,667.0000	4,266,852.0000				
$x_{11} = 2021$	$P_{11} = 5{,}305{,}782.6667$	$P_{10,11} = 5{,}938{,}065.3335$	6,951,202.0005	9,451,915.3345	14,636,978.669			
$x_{12} = 2022$	$P_{12} = 6{,}293{,}674.0003$	$P_{11,12} = 9{,}257{,}348.0011$	14,236,272.0025	21,521,342.0045	30,573,412.007	40,135,272.0098		
$x_{13} = 2023$	$P_{13} = 10{,}387{,}320.0015$	$P_{12,13} = 18{,}574{,}612.0039$	27,891,876.0067	36,995,612.0095	44,732,747.012	50,396,481.014	53,816,884.0154	
$x_{14} = 2024$	$P_{14} = 23{,}079{,}998.1166$	$P_{13,14} = 35{,}772{,}676.2317$	44,371,708.3456	49,864,985.7919	53,082,329.2375	54,752,245.6826	55,478,206.4607	55,715,538.2386

Part C: "Accuracy of the approximation in Part A and Part B"

From using the spline interpolation, it approximates as 4747111, which is also reasonable as Part A.

Table 1.

n	2	3	4	5	6	7	8	9
Rel. err.	3.6000×10^{-4}	3.0000×10^{-4}	2.1000×10^{-4}	4.2000×10^{-5}	8.4000×10^{-7}	1.2000×10^{-4}	1.4000×10^{-4}	2.3000×10^{-4}

Table 2.

n	2	3	4	5	6	7	8	9
Rel. err.	0.02000	0.08000	0.2300	0.3800	0.2500	0.8100	3.8900	8.9800

Table 1. represents the relative errors for Part A of the approximation versus the actual value which we obtained from the spline interpolation since its robustness to oscillation (assuming its the actual value). Except for n = 6 where the error is relatively larger compared to the other n values, our approximation is pretty accurate.

Table 2. represents the relative errors for Part B, and n = 2 is the only accurate approximation compare to the other n values.