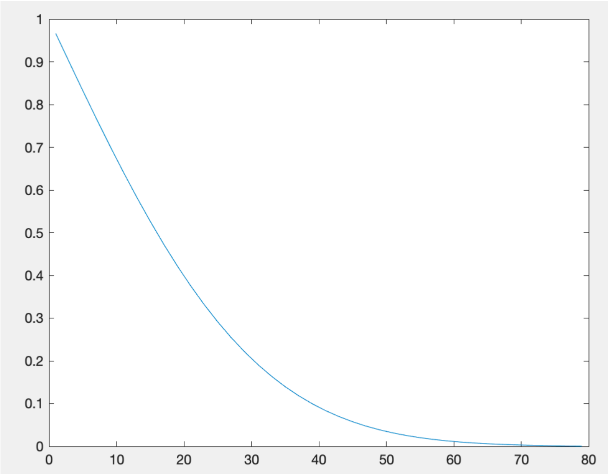
(a) “Jacobi code”



(a) approximation for n = 80

n	Number of iteration
20	227
40	457
80	502

part (a) table

(b) “Backslash operator”

To find a mathematical relationship between n and the number of iterations, we first have to consider that the number of iteration is the k^{th} iterate of a numerical approximation for $p^{(\text{k})}$. We have used Matlab’s “backslash operator” for n = 20, 40, 80 and iterate until the relative error is less than TOL = 10^{-3}. From the stopping criterion by part (a), we know that $\frac{||p^{(\text{k})}|| - ||p^{(\text{k-1})}||}{||p^{(\text{k})}||} < 10^{-3} = ||T||^{(\text{k})}$.

By corollary, there exists an error bound $||p|| - ||p^{(\text{k})}|| \le ||T||^{(\text{k})}||x^0 - x||$ if $||T|| < 1$ for all natural norm and ‘a’ as a given vector. Thus, we get $\frac{||p|| - ||p^{(\text{k})}||}{||p||} \le 10^{-3} = ||T||^{(\text{k})}$.

From the fact that, for all natural norm, $\rho(\text{T}) \le ||T||,\ \rho(T)^k \le 10^{-3} = ||T||^{(\text{k})}$.

In addition, we can obtain$(1 - \frac{1}{2}(\frac{\pi}{n})^2)^k \le 10^{-3}$ from the spectral radius of matrix by :

$\Rightarrow a - 2\sqrt{bc}\cos(\frac{k\pi}{n+1}) - \cos(\frac{k\pi}{n+1})$

$\Rightarrow | -\cos\frac{k\pi}{n+1}|$

$\Rightarrow |\cos\frac{k\pi}{n+1}|$

$\Rightarrow (1 - \frac{1}{2}(\frac{k\pi}{n+1})^2)$

n	Number of iteration
20	526
40	2100
80	8388

part (b) table

Thus, $\text{k} \le \log_{(1-\frac{1}{2}(\frac{\pi}{n}))^2} 10^{-3}$.

$\Rightarrow$For n = 20, number of iteration = 556

$\Rightarrow$For n = 40, number of iteration = 2236

$\Rightarrow$For n = 80, number of iteration = 8955, which are slightly bigger than the actual k from using the ‘backslash operator’.

Hence, we have to obtain $-\frac{1}{2}(\frac{\pi}{n})^2 k \le \ln 10^{-3}$ by approximating $(1 - \frac{1}{2}(\frac{\pi}{n}))^2 \Rightarrow e^{-\frac{1}{2}(\frac{\pi}{n})^2}$

Therefore, we can finally obtain $k \le \frac{-2\ln 10^{-3}}{\pi}n^2$, and results 526, 2100, and 8388.

(c) The number of iterations in part (a) and (b) does not agree, and as the number of n gets doubled, they don’t agree more and more. As a comparison, the number of iteration approximately got doubled for n = 20, got quadruple for n = 40, and got 15 times larger for n = 80.

We shall assume that the backslash operator gives the exact solutions, which is part (b), and also iterated until the relative error is less than TOL = 10^{-3} whereas used pure Jacobi code with I_∞ < TOL = 10^{-3} in part (a).

Meaning that in part (b), it calculates the relative error against the exaction solution but in part (a), it calculates the difference between the iterations by isolating k.