MACM 316 Computing Assignment 1 301313489 Jeongwoon Suh

(a) To solve a general quadratic equation, I computed 3 parameters, a, b, and c in the function. To get the most accurate approximation formula from Equations (1.1), (1.2), and (1.3) with four-digit rounding arithmetic, I have computed the solution by separating several conditions. Assuming that b^2 - 4ac >= 0, the goal was to get as smallest relative error as possible (so as absolute error). If the value of the b is negative, I have used Equation (1.3) for x2, and (1.2) for x1. If b is 0 or positive value, I used Equation (1.2) for x1, and Equation (1.1) for x2 to minimize the error and to get the solutions.

(b) Using the standard quadratic formula by Equation (1.1) to solve the quadratics:

$x^2 - \sqrt{7}x + \sqrt{2} = 0$: $x_1 = \frac{\sqrt{7} + \sqrt{(\sqrt{7})^2 - 4\cdot 1\cdot \sqrt{2}}}{2\cdot 1} = 1.902$, $x_2 = \frac{\sqrt{7} - \sqrt{(\sqrt{7})^2 - 4\cdot 1\cdot \sqrt{2}}}{2\cdot 1} = 0.7434$

$fl(x_1) = \frac{2.646 + 1.160}{2.000} = \frac{3.806}{2.000} = 1.903$, Relative error: $\frac{|1.902 - 1.903|}{|1.902|} = \frac{0.001}{1.902} = 0.0005258$, Abs. error: 0.001000

$fl(x_2) = \frac{2.646 - 1.160}{2.000} = \frac{1.486}{2.000} = 0.7430$, Relative error: $\frac{|0.7434 - 0.7430|}{|0.7434|} = \frac{0.0004000}{0.7434} = 0.0005381$,
Abs. error = 0.0004000

$\pi x^2 + 13x + 1 = 0$: $x_1 = \frac{-13 + \sqrt{13^2 - 4\cdot\pi\cdot 1}}{2\cdot\pi} = -0.07841$, $x_2 = \frac{-13 - \sqrt{13^2 - 4\cdot\pi\cdot 1}}{2\cdot\pi} = -4.060$

$fl(x_1) = \frac{-13.00 + 12.51}{2.000\cdot 3.142} = \frac{-0.4900}{6.284} = -0.07798$, Relative error: $\frac{|-0.07841 + 0.07798|}{|-0.07841|} = \frac{0.0004300}{0.07841} = 0.005484$
Abs. error = 0.0004300

$fl(x_2) = \frac{-13.00 - 12.51}{2.000\cdot 3.142} = \frac{-25.51}{6.284} = -4.060$, Relative error: $\frac{|-4.060 + 4.060|}{|-4.060|} = \frac{0}{4.060} = 0$, Abs. error = 0

$x^2 + x - e = 0$: $x_1 = \frac{-1 + \sqrt{1^2 - 4\cdot 1\cdot(-e)}}{2\cdot 1} = 1.223$, $x_2 = \frac{-1 - \sqrt{1^2 - 4\cdot 1\cdot(-e)}}{2\cdot 1} = -2.223$

$fl(x_1) = \frac{-1.000 + 3.445}{2.000} = 1.223$, Relative error: $\frac{|1.223 - 1.223|}{|1.223|} = \frac{0}{1.223} = 0$, Abs. error = 0

$fl(x_2) = \frac{-1.000 - 3.445}{2.000} = -2.223$, Relative error: $\frac{|-2.223 + 2.223|}{|-2.223|} = \frac{0}{2.223} = 0$, Abs. error = 0

$x^2 - \sqrt{35}x - 2 = 0$: $x_1 = \frac{\sqrt{35} + \sqrt{(\sqrt{35})^2 - 4\cdot 1\cdot(-2)}}{2\cdot 1} = 6.237$, $x_2 = \frac{\sqrt{35} - \sqrt{(\sqrt{35})^2 - 4\cdot 1\cdot(-2)}}{2\cdot 1} = -0.3207$

$fl(x_1) = \frac{5.916 + 6.557}{2.000} = \frac{12.47}{2.000} = 6.235$, Relative error: $\frac{|6.237 - 6.235|}{|6.237|} = 0.0003207$, Abs. error = 0.002000

$fl(x_2) = \frac{5.916 - 6.557}{2.000} = \frac{-0.641}{2.000} = -0.3205$

Relative error: $\frac{|-0.3207 + 0.3205|}{|-0.3207|} = 0.0006236$

Abs. error = 0.0002000

Quadratic equations	My code (MatLab)	Standard quadratic formula
$x^2 - \sqrt{7}x + \sqrt{2} = 0$	x1 = 1.9020 x2 = 0.7434	x1 = 1.902 x2 = 0.7434
$\pi x^2 + 13x + 1 = 0$	x1 = - 0.0784 x2 = - 4.0600	x1 = - 0.07840 x2 = - 4.060
$x^2 + x - e = 0$	x1 = 1.2230 x2 = - 2.2230	x1 = 1.223 x2 = - 2.223
$x^2 - \sqrt{35}x - 2 = 0$	x1 = 6.2370 x2 = - 0.3207	x1 = 6.237 x2 = - 0.3207

Quadratic equations	My code (MatLab) Absolute Error / Relative Error	Standard quadratic formula Absolute Error / Relative Error
$x^2 - \sqrt{7}x + \sqrt{2} = 0$	For x1 : 0.0005258 / 0.001000 For x2 : 0.0005381 / 0.0004000	For x1 : 0.0005258 / 0.001000 For x2 : 0.0005381 / 0.0004000
$\pi x^2 + 13x + 1 = 0$	For x1 : 0.00001000 / 0.0001275 For x2 : 0 / 0	For x1 : 0.0004300 / 0.005484 For x2 : 0 / 0
$x^2 + x - e = 0$	For x1 : 0 / 0 For x2 : 0 / 0	For x1 : 0 / 0 For x2 : 0 / 0
$x^2 - \sqrt{35}x - 2 = 0$	For x1 : 0.003000 / 0.0004810 For x2 : 0.0001000 / 0.0003118	For x1 : 0.002000 / 0.0003207 For x2 : 0.0002000 / 0.0006236

(c) From the chart, for the second quadratic equation, the absolute and relative errors for x1 are both roughly 40 times smaller from my algorithm compared to the absolute and relative errors for x1 from standard quadratic formula.
Also, for the last quadratic equation, the absolute and relative errors for x2 are both roughly halved from my algorithm compared to the absolute and relative errors for x2 from standard quadratic formula.