U Wroclaw, Fall 2015
Applied Stats
DISCUSSION/LAB 12: Analysis of variance ANOVA
Two –Way ANOVA with Multiple Comparisons

We will use data set: yield for different catalysts and reagents catreagdata.*.
Yield data for different levels of catalyst and reagent. We already know that the additive model holds. We can now compare the means for different levels of catalyst and different levels of reagent.

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Analysis of Variance for Yield

Source     DF        SS       MS      F       P
Catalyst   3    877.56   292.52   9.58   0.000
Reagent    2    327.14   163.57   5.36   0.008
Error     42   1282.31    30.53
Total     47   2487.02


S = 5.52551   R-Sq = 48.44%   R-Sq(adj) = 42.30%
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We computed row effects and column effects earlier:

Row/catalyst effects: $\widehat{\alpha_1} = 6.81, \widehat{\alpha_2} = 0.19, \widehat{\alpha_3} = -4.56, \widehat{\alpha_4} = -2.44$.

Column/reagent effects: $\widehat{\beta_1} = -3.69, \widehat{\beta_2} = 1.96, \widehat{\beta_3} = 1.73$.

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Row differences in means: Need q4, 36, 0.5 – not in the table, so use what is close in the table: q4, 30, 0.05 = 3.85.

Then, compute q4,30,0.05$\sqrt{MSE/JK}$= 3.85$\sqrt{31.259/12}$= 6.21

Check which pairs of rows have differences of row effects larger than 6.21: $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$, $\widehat{\alpha_1}$ and $\widehat{\alpha_3}$, and $\widehat{\alpha_1}$ and $\widehat{\alpha_4}$.
Thus, the mean yield of catalyst A differs from the mean yields of catalysts B, C and D. No other differences.

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Similar work for column/reagent effects:

Need q3,36,0.05 – not in the table, so use q3,30,0.05=3.49.
Compute q3,30,0.05$\sqrt{MSE/IK}$= 3.49$\sqrt{31.259/16}$= 4.88.

Check which pairs of columns have differences of column effects larger than 4.88: $\widehat{\beta_1}$ and $\widehat{\beta_2}$, $\widehat{\beta_1}$ and $\widehat{\beta_3}$. Thus, the mean yield of reagent 1 differs from the mean yields of reagents 2 and 3. No other differences.

Exercise 2. The effect of curing pressure on bond strength (in MPa) was tested for two different adhesives. There were three levels of curing pressure. Three replications were performed for each combination of curing pressure and adhesive. The resuls are in the data set: bonddata.*.

    a) Construct an ANOVA table.

    b) Is the additive model plausible? Provide hypotheses you test, value of the test statistic and p-value.

    c) Can the effect of adhesive on the bond strength be described by interpreting the main effects of adhesive? If so, interpret the main effects. If not, explain why not.

    d) Can the effect of curing pressure on the bond strength be described by interpreting the main effects of curing pressure? If so, interpret the main effects. If not, explain why not.

ANOVA: Bond Strength versus Adhesive, Pressure

1) Analysis of Variance for Bond Strength

Source	DF	SS	MS	F	P
Adhesive	1	17.014	17.014	10.12	0.008
Pressure	2	35.663	17.832	10.61	0.002
Adhesive*Pressure	2	4.354	2.177	1.30	0.310
Error	12	20.173	1.681		
Total	17	77.205			

S = 1.29658   R-Sq = 73.87%   R-Sq(adj) = 62.98%

Means

Adhesive	N	Bond Strength
A	9	6.0556
B	9	4.1111

Pressure	N	Bond Strength
High	6	3.1167
Low	6	6.3333
Medium	6	5.8000

Adhesive	Pressure	N	Bond Strength
A	High	3	3.4000
A	Low	3	7.7333
A	Medium	3	7.0333
B	High	3	2.8333
B	Low	3	4.9333
B	Medium	3	4.5667

b) Ho: no interaction effects vs. Ha: some interaction effects
F=1.3, p-value=0.31 – do not reject Ho, conclusion: no interaction effects. We can use additive model.

c) The effect of adhesive can be interpreted using the main effects of adhesive, because there is no interaction.

Ho: no adhesive effect ($\alpha_1 = \alpha_2 = 0$) vs Ha: some adhesive effect (at least one $\alpha_i \neq 0$)
F-stat= 10.12, p-value=0.008 < 0.05 = significance level. Reject Ho, there is a significant main effect of adhesive. Adhesive A produces larger bond strength.

d) The effect of curing can be interpreted using the main effects of curing pressure, because there is no interaction.

Ho: no curing pressure effect (β_1(low)=β_2 (medium)= β_2 (high)= 0) vs Ha: some curing pressure effect (at least one $\beta_i \neq 0$)

F-stat= 10.61, p-value=0.002 < 0.05 = significance level. Reject Ho, there is a significant main effect of curing pressure. The strength of bond seems to increase with lowering curing pressure.