Complex Data - lab3

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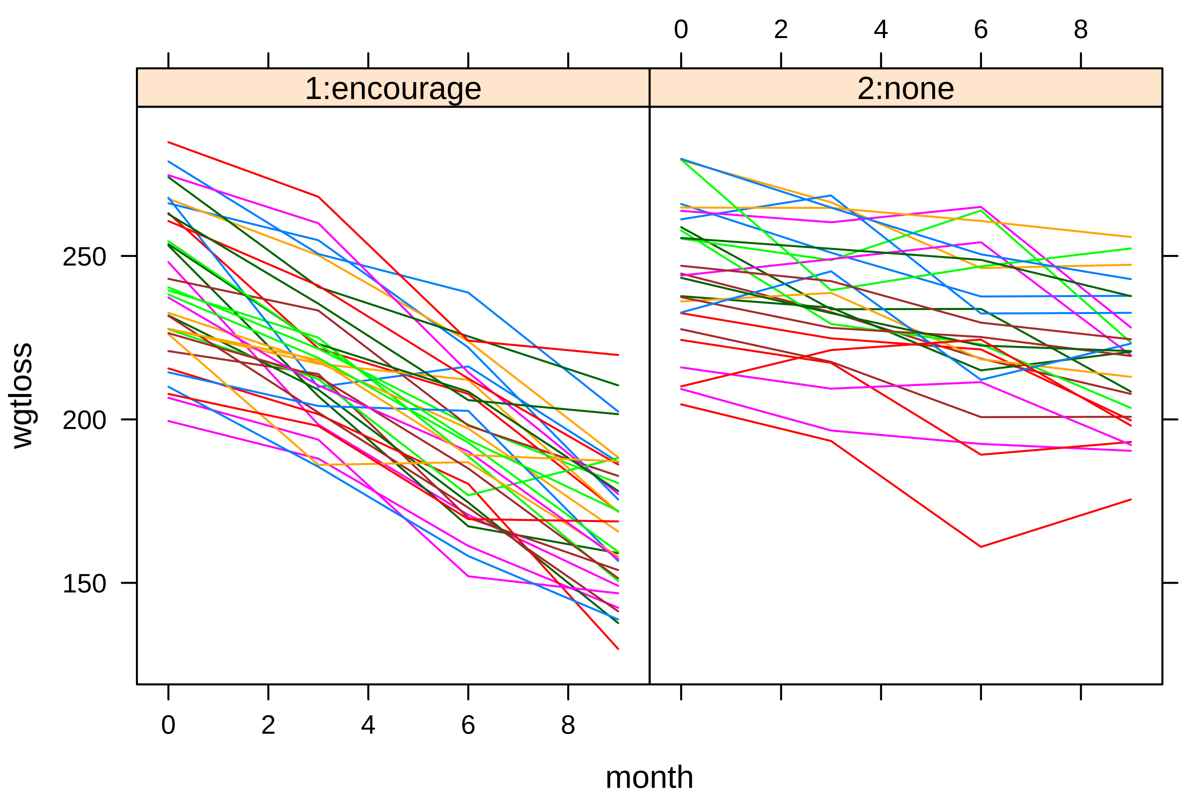
Data input and Calculating Means

```
wtloss <- read.table("../data/weightloss.dat",
header=F)
## Give names to variables
names(wtloss) <- c("id", paste("y", 1:4, sep=""), "program")
## Univariate format
wtloss.uni <- data.frame(id=rep(wtloss$id, each=4),
wgtloss=as.numeric(t(as.matrix(wtloss[,2:5]))),
program=rep(wtloss$program, each=4),
month=seq(0,9,3),
time.cat=rep(1:4))
wtloss.uni$prog.fac <- factor(wtloss.uni$program, labels=c("1:encourage", "2:none"))
attach(wtloss.uni)
wgt.mean <- tapply(wgtloss, list(month, program), mean)
wgt.sd <- tapply(wgtloss, list(month, program), sd)
detach(wtloss.uni)
```

Plotting Response Profiles

```
xyplot(wgtloss~month|prog.fac, type='l',groups=id,data=wtloss.uni)
```



Choice of covariance structure

```
## CATEGORICAL time, UN, REML
wtloss.un.cat <- gls(wgtloss~factor(month)*prog.fac,
correlation=corSymm(form= ~1 | id),
weights=varIdent(form= ~1 | month),
data=wtloss.uni)
#summary(wtloss.un.cat)

## CATEGORICAL time, CS, REML
wtloss.cs.cat <- gls(wgtloss~factor(month)*prog.fac,
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni)
#summary(wtloss.cs.cat)

## CATEGORICAL time, AR1, REML
wtloss.ar1.cat <- gls(wgtloss~factor(month)*prog.fac,
correlation=corAR1(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni)
#summary(wtloss.ar1.cat)
```

Unstructured vs compound symmetry

H_0 : compound symmetry is adequate for the data
H_1 : unstructured is required

```
un_vs_cs <- anova(wtloss.un.cat, wtloss.cs.cat)
```

CS vs. UN

Structure	-2 REML Log-Likelihood	Number of Cov. Parameters
Compound Symmetry	1905.8232573	10
Unstructured	1900.8766893	18
Difference	4.9465681	8

LRT yields $G^2 = 4.9465681$ with 8 df (p-value $= 0.7632715$) so we do not reject the null hypothesis at $\alpha = 0.05$ and conclude that the assumption of compound symmetry covariance structure is adequate for the data.

Unstructured vs autoregressive

H_0 : autoregressive is adequate for the data H_1: unstructured is required

```
un_vs_ar <- anova(wtloss.un.cat, wtloss.ar1.cat)
```

AR-1 vs. UN

Structure	-2 REML Log-Likelihood	Number of Cov. Parameters
Autoregressive	1931.246837	10
Unstructured	1900.8766893	18
Difference	30.3701477	8

LRT yields $G^2 = 30.3701477$ with 8 df (p-value = 1.8178034×10^{-4}), so we reject the null hypothesis at $\alpha = 0.05$ and conclude that the assumption of autoregressive covariance structure is not adequate when compared to unstructured.

Autoregressive vs compound symmetry

```
cs_vs_ar1 <- anova(wtloss.cs.cat, wtloss.ar1.cat)
```

Since CS and AR-1 have the same number of parameters = 10 , no LRT is necessary. We can directly compare their likelihoods, or -2*log(likelihood):

- $-2 * log(likelihood)$ for CS = 1905.8232573
- $-2 * log(likelihood)$ for AR-1 = 1931.246837

Since $-2 * log(likelihood)$ for CS is smaller than for AR-1, CS has a higher likelihood and we conclude that CS is an adequate model for the covariance structure when compared to AR-1.

The most adequate covariance structure is compound symmetry model.

Are all these tests correct?

No the last test is theoretically not correct, because for non nested models we can't just compare log-likelihoods or perform LRT. We have to comapre AIC or BIC instead. However, in this case the number of parameters is the same, so comparing likelihoods is equivalent to comparing AICs.

Structure	-2 REML Log-Likelihood	Number of Parameters	AIC
Compound Symmetry	1905.8232573	10	1925.8232573
Autoregressive	1931.246837	10	1951.246837
Unstructured	1900.8766893	18	1936.8766893

Thus, we will use a compound symmetry covariance structure for the remainder of the lab.

Single Degree of Freedom Contrasts

AUC - test for equality of the area under the curve in two groups.

```
t <- wtloss.uni$month
L2 <- 0.5*c(t[1] + t[2] - 2*t[4], t[3] - t[1], t[4]-t[2], t[4]-t[3])
coefs <- summary(wtloss.cs.cat)$coefficients
AUC_enc <- sum(L2 * wgt.mean[,1])
AUC_none <- sum(L2 * wgt.mean[,2])
y_mean_enc <- mean(wtloss.uni$wgtloss[which(wtloss.uni$program==1)])
```

The obtained values of AUCs are:

Estimated mean AUC in encouragement program is -319.5705882.
Estimated mean AUC in no encouragement program is -117.7038462.

As written in the textbook to test for the equality of AUCs, we test for the contrast $(-L_2, L_2)$.

```
L <- c(-L2, L2)
anova(wtloss.cs.cat, L=L)
```

```
## Denom. DF: 232
##  F-test for linear combination(s)
##                   (Intercept)                factor(month)3
##                           7.5                          -3.0
##                factor(month)6                factor(month)9
##                          -3.0                          -1.5
##                prog.fac2:none factor(month)3:prog.fac2:none
##                          -7.5                           3.0
## factor(month)6:prog.fac2:none factor(month)9:prog.fac2:none
##                           3.0                           1.5
##   numDF  F-value p-value
## 1     1 759.3707  <.0001
```

The p-value is almost 0, so we reject the null hypothesis that the response profile is the same for two treatments.

Parametric curves

Quadratic time trend

```
## QUADRATIC time, CS, ML,
wtloss.cs.quad <- gls(wgtloss~month*prog.fac + I(month^2)*prog.fac,
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni,
method="ML")
#summary(wtloss.cs.quad)

## CATEGORICAL time, CS, ML
wtloss.cs.cat.ml <- gls(wgtloss~factor(month)*prog.fac,
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni,
method="ML")
#summary(wtloss.cs.cat.ml)
```

H_0 : quadratic model
H_1 : saturated mode

```
qm_vs_sm <- anova(wtloss.cs.cat.ml, wtloss.cs.quad)
```

Testing the Quadratic trend

Structure	-2 Log Likelihood	Number of Parameters
Quadratic model	1935.7000029	8
Saturated model	1935.3313128	10
Difference	0.3686901	2

LRT yields $G^2 = 0.3686901$ with 2 df (p-value $= 0.8316488$), so we fail to reject the null hypothesis at $\alpha = 0.05$ and conclude that the model with Month as a quadratic effect seems to fit the data adequately. (Note: $\chi^2_{2,0.95} = 5.99$)

Linear time trend

```
wtloss.cs.lin <- gls(wgtloss~month*prog.fac,
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni,
method="ML")
#summary(wtloss.cs.lin)
```

H_0 : linear model
H_1 : quadratic model

```
qm_vs_lm <- anova(wtloss.cs.quad, wtloss.cs.lin)
```

Linear vs. Quadratic

Structure	-2 Log Likelihood	Number of Parameters
Linear model	1936.295409	6
Quadratic model	1935.7000029	8
Difference	0.5954061	2

LRT yields $G^2 = 0.5954061$ with 2 df (p-value = 0.7425218), so we fail to reject the null hypothesis at $\alpha = 0.05$ and conclude that the model with Month as a linear effect fits the data better than the model with Month as a quadratic effect. (Note: $chi^2_{2,0.95} = 5.99$)

Testing for intersections

```
## Linear model, NO interacations
wtloss.cs.lin.noint <- gls(wgtloss~month + prog.fac,
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni,
method="ML")
#summary(wtloss.cs.lin.noint)
```

$H_0 : \beta_4 = 0$
$H_1 : \beta_4 \neq 0$

```
anova(wtloss.cs.lin)
```

```
## Denom. DF: 236
##                 numDF  F-value p-value
## (Intercept)         1 6970.808  <.0001
## month               1 1011.995  <.0001
## prog.fac            1   24.301  <.0001
## month:prog.fac      1  184.368  <.0001
```

Since the p-value for month*program is < 0.0001, we reject the null hypothesis and conclude that there is a significant interaction between month and program.

Thus, our final model is the linear model with interaction.

```
final.wtloss.cs.lin <- gls(wgtloss~month*prog.fac,
correlation=corCompSymm(form= ~1 | id),
```

```
weights=varIdent(form= ~1),
data=wtloss.uni)
summary(final.wtloss.cs.lin)
```

```
## Generalized least squares fit by REML
##   Model: wgtloss ~ month * prog.fac
##   Data: wtloss.uni
##        AIC      BIC    logLik
##   1941.074 1961.857 -964.5368
##
## Correlation Structure: Compound symmetry
##  Formula: ~1 | id
##  Parameter estimate(s):
##       Rho
## 0.8083168
##
## Coefficients:
##                         Value Std.Error   t-value p-value
## (Intercept)         242.37676  3.655475  66.30513  0.0000
## month                -8.10431  0.245748 -32.97814  0.0000
## prog.fac2:none        2.99477  5.553065   0.53930  0.5902
## month:prog.fac2:none  5.08329  0.373318  13.61650  0.0000
##
##  Correlation:
##                      (Intr) month  prg.2:
## month                -0.303
## prog.fac2:none       -0.658  0.199
## month:prog.fac2:none  0.199 -0.658 -0.303
##
## Standardized residuals:
##        Min         Q1        Med          Q3         Max
## -3.0172539 -0.6942911 -0.0369479  0.7699925  2.2892673
##
## Residual standard error: 21.95552
## Degrees of freedom: 240 total; 236 residual
```

What can you conclude about the two weight programs in terms of their effectiveness?

Interpreting quadratic trends:

```
wtloss.cs.quad.noint <- gls(wgtloss~month + prog.fac + I(month^2),
correlation=corCompSymm(form= ~1 | id),
weights=varIdent(form= ~1),
data=wtloss.uni)
ncoefs <- wtloss.cs.quad.noint$coefficients
coefs <- c(ncoefs[1], ncoefs[2], ncoefs[4], ncoefs[3])
change_rates <- coefs[2] + 2*coefs[3]*t[1:4]
expected1 <- coefs[1] + coefs[2]*t[1:4] + coefs[3]*t[1:4]*t[1:4]
```

$\hat{\beta_1} = 232.0118529$ expected mean response at baseline of subjects in program 1.
$\hat{\beta_4} = 25.8695701$ change in expected response at baseline of subjects in program 2 vs. program 1.
Rate of change in program 1 is $\beta_2 + 2\beta_3 time_{ij}$.
Plugging in the above estimates,

Program 1

time	Rate of Change	Expected Response
0	-5.4490556	232.0118529
1	-5.7507222	215.2121863
2	-6.0523889	197.5075196
3	-6.3540556	178.8978529

Thus, the mean response for Program 1 decreases over time. For the second program it is exactly the same - in this case for every time point 25.8695701 is added to the expected respons, because in our model we don't include any interactions and the group effect is incorporated just by adding a constant.