misalignment

March 6, 2025

1 List of Experiments

This set of experiment simulate the cases where the human and algorithm are misaligned:

- **Experiment 1**: Complementarity of human and algorithm with **Equal Accuracy** and **Single best**
- **Experiment 2**: Aligned human and misaligned human with **Equal Accuracy** and **Single best**
- **Experiment 3**: Comparison between type-i and type-(i+1)
- **Experiment 4**: Complementarity of human and algorithm with **Different Accuracy** and **Single best**
- **Experiment 5**: Extension of our framework

Denote the ground truth ranking of the algorithm and the human by π_a^* and π_h^*.

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.lines as mlines
from matplotlib.pyplot import text
from model.mallows import Mallows
from model.rum import SuperStar, gRUMBel
from human_ai import HumanAI
```

2 Experiment 1 (Complementarity under Misaligned and Equal-Acc Setting)

There exist cases where it is possible to achieve complementarity with **misaligned** and **equally accurate** algorithm (potential example?)

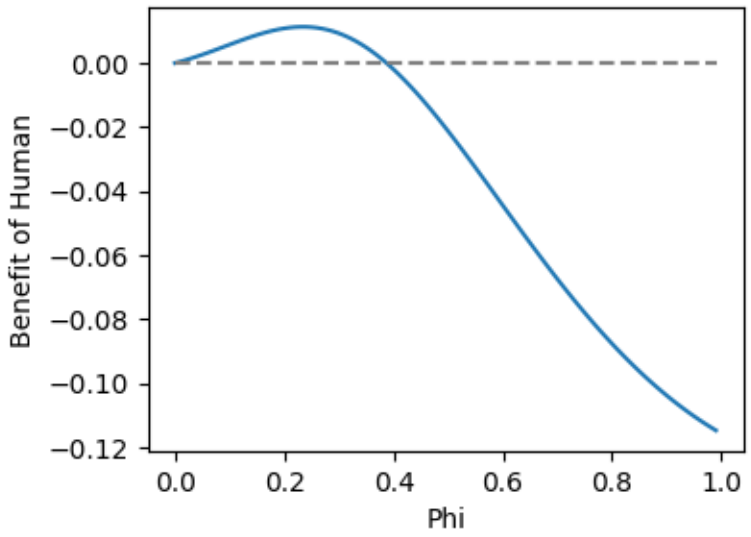
2.1 Mallows Model

Answer: Yes, the following is an example, where

- $\pi_a^* = [x_1, x_2, x_3, \dots, x_{10}]$,
- $\pi_h^* = [x_2, x_1, x_{10}, \dots, x_3]$
- $0 < \phi_a = \phi_h \leq 0.3$

[18]:
```
m = 10
k = 2
```

```
slight_misaligned_pi_h_star = [2, 1] + [i for i in range(m, 2, -1)]

## Accuracy from 0 to 1
phis, benefits = [], []
for phi in np.arange(0, 1, 0.01):
    D_a = Mallows(m, phi, list(range(1, m+1)))
    D_h = Mallows(m, phi, slight_misaligned_pi_h_star)
    joint_system = HumanAI(m, D_a, D_h)

    benefit = joint_system.benefit_of_human_single_best(k)
    phis.append(phi)
    benefits.append(benefit)

plt.figure(figsize=(4, 3))
plt.plot(phis, benefits)
plt.plot(phis, [0 for _ in range(len(phis))], linestyle='--', color='gray')
plt.xlabel('Phi')
plt.ylabel('Benefit of Human')
plt.show()
```



Another Example:

- $m = 5$
- $\pi_h^* = (x_2, x_5, x_4, x_3, x_1)$
- $\phi_a = \phi_h = 1$

[22]:
```
m = 5
k = 2
```

```
slight_misaligned_pi_h_star = [2, 5, 4, 3, 1]

phi = 1
D_a = Mallows(m, phi, list(range(1, m+1)))
D_h = Mallows(m, phi, slight_misaligned_pi_h_star)
joint_system = HumanAI(m, D_a, D_h)

benefit = joint_system.benefit_of_human_beyond_single_best(m, k, [1,0, 0, 0,␣
↪0], verbose=True)
print(benefit)
```

```
Utility of joint system :  0.6383893495452974
Utility of human        :  0.6364086465588308
Joint - Human           :  0.001980702986466576


0.001980702986466576
```

2.2 gRUMbel Model (Superstar Setting)

However, it may **not** hold in the gRUMbel model.

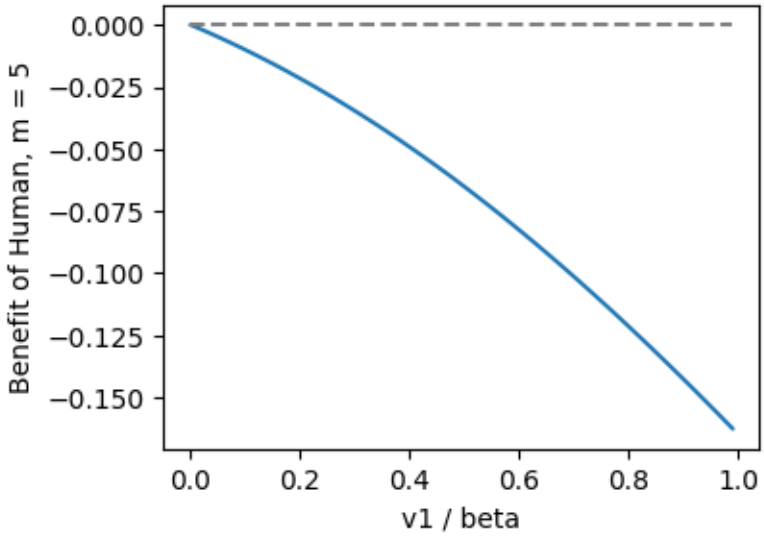
Consider the algorithm and the human having different super stars. For example, in the following experiment,

- the superstar of the algorithm is x_1 while the superstar of the human is x_2.
- $m = 10$ and $k = 2$.

The following results show collaboration will always cause the human make worse decision.

[49]:
```
m = 10
k = 2
algorthm_super_star = 1
human_super_star = 2

## v1 / beta, benefits
v1_betas, benefits = [], []
for ratio in np.arange(0, 1, 0.01):
    D_a = gRUMBel(m, algorthm_super_star, ratio, 1) #
    D_h = gRUMBel(m, human_super_star, ratio, 1)
    joint_system = HumanAI(m, D_a, D_h)

    benefit = joint_system.benefit_of_human_single_best(k)
    v1_betas.append(ratio)
    benefits.append(benefit)

plt.figure(figsize=(4, 3))
plt.plot(v1_betas, benefits)
```

```
plt.plot(v1_betas, [0 for _ in range(len(v1_betas))], linestyle='--',␣
 ↪color='gray')
plt.xlabel('v1 / beta')
plt.ylabel(f'Benefit of Human, m = {m}')
plt.show()
```



3 Experiment 2: Aligned Human v.s. Misaligned Human

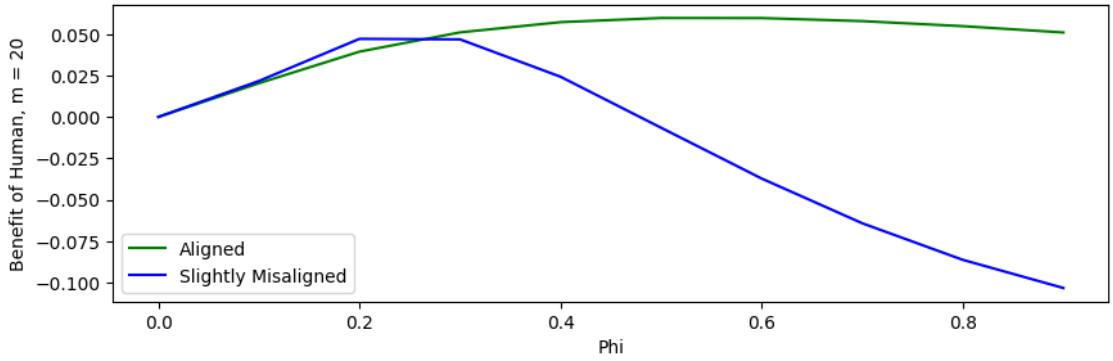
Question: Does there exist cases where misalignment may be helpful for accuracy. * e.g.: example where a misaligned human achieves complementarity, an aligned human doesn't.

Answer: Yes, consider the following case, where

- The algorithm and the human have the same accuracy parameter ϕ
- The misaligned human receives better benefit in some cases.

[54]:
```
m = 20
k = 2
aligned_pi_h_star = [i for i in range(1, m+1)]
slight_misaligned_pi_h_star = [2, 1] + [i for i in range(m, 2, -1)]

## Accuracy from 0 to 1
phis, aligned_benefits, misaligned_benefits = [], [], []
for phi in np.arange(0, 1, 0.1):
    D_a = Mallows(m, phi, list(range(1, m+1)))
    aligned_D_h = Mallows(m, phi, aligned_pi_h_star)
    misaligned_D_h = Mallows(m, phi, slight_misaligned_pi_h_star)

    aligned_joint_system = HumanAI(m, D_a, aligned_D_h)
```

```
    misaligned_joint_system = HumanAI(m, D_a, misaligned_D_h)

    aligned_benefit = aligned_joint_system.benefit_of_human_single_best(k)
    misaligned_benefit = misaligned_joint_system.benefit_of_human_single_best(k)

    phis.append(phi)
    aligned_benefits.append(aligned_benefit)
    misaligned_benefits.append(misaligned_benefit)

plt.figure(figsize=(10, 3))
plt.plot(phis, aligned_benefits, color='green', label='Aligned')
plt.plot(phis, misaligned_benefits, color='blue', label='Slightly Misaligned')
plt.xlabel('Phi')
plt.ylabel(f'Benefit of Human, m = {m}')
plt.legend()
plt.show()
```



4 Experiment 3: Comparion between Type-i and Type-(i+1)

4.1 Type-i Worst v.s Type-(i+1) Best

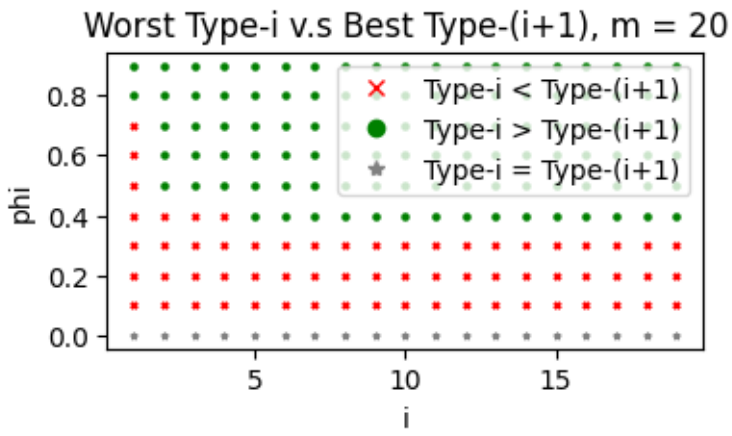
The following experiment suggests that, a type-i human may not always receives better benefit than a type-(i+1) human.

Consider the benefit of following types of humans, i.e.,

- π_i^{least} : $\{x_i, x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_m\}$
- π_{i+1}^{best} : $\{x_{i+1}, x_m, ..., x_{i+2}, x_i, ..., x_1\}$

[76]:
```
m = 20
k = 2
results = []

def compare_type_i_and_i_plus_one(i, m, phi, results):
```

```
    pi_a = list(range(1, m+1))
    pi_i_least = [i] + [j for j in range(1, m+1) if j != i]
    pi_plus_one_best = [i+1] + [j for j in range(m, 0, -1) if j != i+1]

    D_a = Mallows(m, phi, pi_a)
    D_h = Mallows(m, phi, pi_i_least)
    D_h_one = Mallows(m, phi, pi_plus_one_best)

    joint_system_i = HumanAI(m, D_a, D_h)
    joint_system_i_plus_one = HumanAI(m, D_a, D_h_one)

    benefit_of_i = joint_system_i.benefit_of_human_single_best(k)
    benefit_of_i_1 = joint_system_i_plus_one.benefit_of_human_single_best(k)

    if benefit_of_i < benefit_of_i_1:
        results.append((i, phi, 'red', 'x'))
    elif benefit_of_i > benefit_of_i_1:
        results.append((i, phi, 'green', 'o'))
    else:
        results.append((i, phi, 'gray', '*'))


for phi in np.arange(0, 1, 0.1):
    for i in range(1, m):
        compare_type_i_and_i_plus_one(i, m, phi, results)

plt.figure(figsize=(4, 2))
for i, phi, color, marker in results:
    plt.scatter(i, phi, color=color, marker=marker, s=5)
plt.xlabel("i")
plt.ylabel("phi")
plt.title(f"Worst Type-i v.s Best Type-(i+1), m = {m}")
plt.legend(handles=[
    plt.Line2D([0], [0], color='red', marker='x', linestyle='None',␣
 ↪label='Type-i < Type-(i+1)'),
    plt.Line2D([0], [0], color='green', marker='o', linestyle='None',␣
 ↪label='Type-i > Type-(i+1)'),
    plt.Line2D([0], [0], color='gray', marker='*', linestyle='None',␣
 ↪label='Type-i = Type-(i+1)')
], loc=1)
plt.show()
```



4.2 Representative Permutations of Type-i and Type-(i+1)

The above simulation demostrates that, a type-i human may not always get higher benefit than type-(i+1). Thus, a natural question arises: *how do their received benefit overlap?*

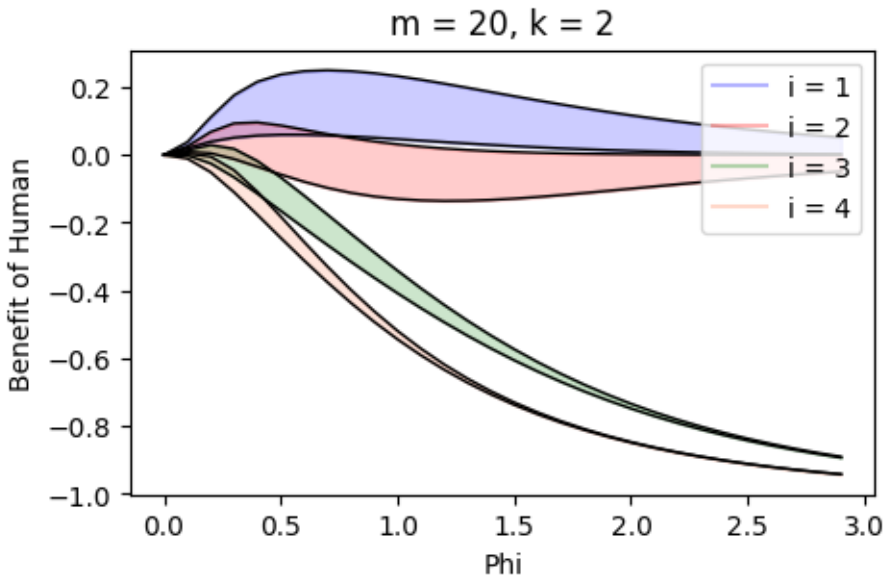
In the next simulation,

- We assume the human and the algorithm have the same accuracy.
- Then we plot the lines of highest and lowest benefit of every type of human and fill the regions between the two lines for every type of humans.

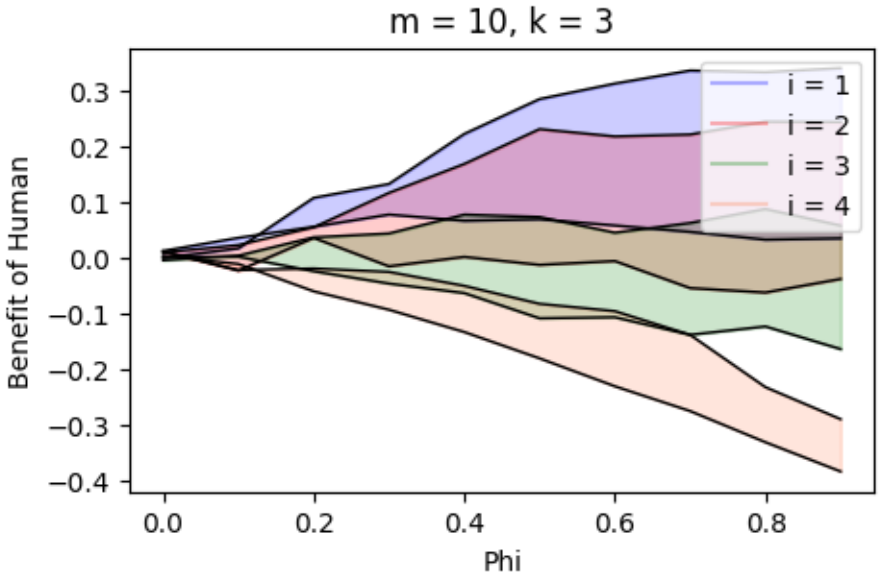
[11]:
```
def type_i_worst_and_best(m, k, i, phis):
    """
        return the benefits of the worst type-i and the best type-(i+1) for all␣
↪phi
    """
    pi_a_star = [j + 1 for j in range(m)]
    pi_h_star_worst = [i] + [j for j in range(1, m+1) if j != i]
    pi_h_star_best = [i] + [j for j in range(m, 0, -1) if j != i]
    worst_benefits = []
    best_benefits = []

    for phi in phis:
        D_a = Mallows(m, phi, pi_a_star)
        D_h_worst = Mallows(m, phi, pi_h_star_worst)
        D_h_best = Mallows(m, phi, pi_h_star_best)

        joint_system_worst = HumanAI(m, D_a, D_h_worst)
        joint_system_best = HumanAI(m, D_a, D_h_best)

        worst_benefits.append(joint_system_worst.
↪benefit_of_human_single_best(k))
```

```
        best_benefits.append(joint_system_best.benefit_of_human_single_best(k))

    return worst_benefits, best_benefits

def plot_type_i_worst_and_best(m, k, i, phis, verbose=False):
    plt.figure(figsize=(5, 3))
    all_i = list(range(1, 5))

    shadow_colors = ['blue', 'red', 'green', 'coral']

    for idx, i in enumerate(all_i):
        if verbose:
            print("i = ", i)
        worst_benefits, best_benefits = type_i_worst_and_best(m, k, i, phis)
        plt.plot(phis, worst_benefits, color='black', linewidth=1)
        plt.plot(phis, best_benefits, color='black', linewidth=1)
        plt.fill_between(phis, worst_benefits, best_benefits,␣
 ↪color=shadow_colors[idx], alpha=0.2)

    legends = []
    for idx, i in enumerate(all_i):
        legends.append(mlines.Line2D([], [], color=shadow_colors[idx], alpha =␣
 ↪0.3, label=f"i = {i}"))
    plt.legend(handles=legends, loc='upper right')

    plt.xlabel('Phi')
    plt.ylabel('Benefit of Human')
    plt.title(f"m = {m}, k = {k}")

    plt.show()

m, k = 20, 2
plot_type_i_worst_and_best(m, k, 1, np.arange(0, 3, 0.1))
m, k = 10, 3
plot_type_i_worst_and_best(m, k, 1, np.arange(0, 1, 0.1), verbose=True)
```

```
i =  1
i =  2
i =  3
i =  4
```


m = 20, k = 2
Benefit of Human
Phi
i = 1
i = 2
i = 3
i = 4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

```
i =  1
i =  2
i =  3
i =  4
```


m = 10, k = 3
Benefit of Human
Phi
i = 1
i = 2
i = 3
i = 4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.0
0.2
0.4
0.6
0.8

4.3 Experiment 4: Misaligned Setting with Different Accuracy

In the above experiments, we focus on the case where the algorithm and the human have the same accurcy parameters.

So what if they can be **differently accurate**? When does complementarity hold?

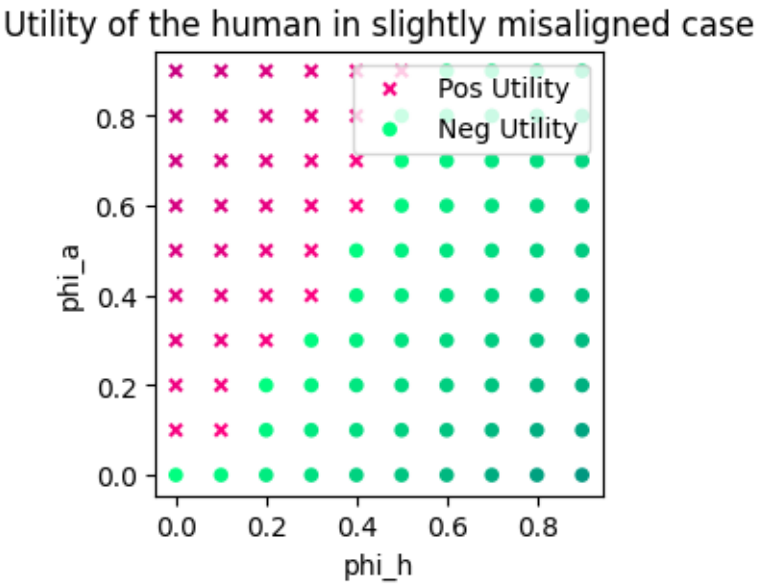
In the following experiment, we vary ϕ_a and ϕ_h to see the benefits of the human. In particular,

- We fix $\pi_a^* = [x_1, \dots, x_m]$, and
- consider the following three misaligned ground-truth:
  - $\pi_h^* = [x_2, x_1, x_3, \dots, x_m]$;
  - $\pi_h^* = [x_2, x_1, x_m, \dots, x_3]$;
  - $\pi_h^* = [x_m, x_{m-1}, \dots, x_1]$.

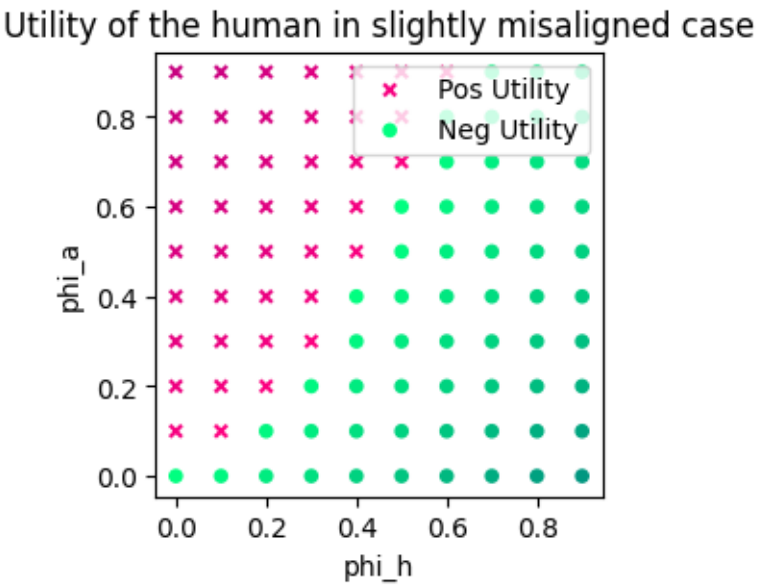
[19]:
```
def diff_accurate_misaligned(phi_h_star, k):
    pos_phi_as, neg_phi_as = [], []
    pos_phi_hs, neg_phi_hs = [], []
    pos_colors, neg_colors = [], []

    print("phi_h_star: ", phi_h_star)
    for phi_a in np.arange(0, 1, 0.1):
        for phi_h in np.arange(0, 1, 0.1):
            D_a = Mallows(m, phi_a, list(range(1, m+1)))
            D_h = Mallows(m, phi_h, phi_h_star)
            joint_system = HumanAI(m, D_a, D_h)

            utility = joint_system.benefit_of_human_single_best(k)
            intensity = abs(utility)
            color = (1 - intensity , 0, 0.5) if utility > 0 else (0, 1-␣
↪intensity, 0.5)

            if utility > 0:
                pos_phi_as.append(phi_a)
                pos_phi_hs.append(phi_h)
                pos_colors.append(color)
            else:
                neg_phi_as.append(phi_a)
                neg_phi_hs.append(phi_h)
                neg_colors.append(color)

    fig = plt.figure(figsize=(3, 3))
    plt.scatter(pos_phi_hs, pos_phi_as, c=pos_colors, s=20, marker="x",␣
↪label="Pos Utility")
    plt.scatter(neg_phi_hs, neg_phi_as, c=neg_colors, s=20, marker="o",␣
↪label="Neg Utility")
    plt.xlabel("phi_h")
    plt.ylabel("phi_a")
    plt.legend(loc=1)
```

```
    plt.title("Utility of the human in slightly misaligned case")
    plt.show()


misaligned_pi_h_star1 = [2, 1] + [i for i in range(3, m+1, 1)]
diff_accurate_misaligned(misaligned_pi_h_star1, 2)

misaligned_pi_h_star2 = [2, 1] + [i for i in range(m, 2, -1)]
diff_accurate_misaligned(misaligned_pi_h_star2, 2)

misaligned_pi_h_star3 = [i for i in range(m, 0, -1)]
diff_accurate_misaligned(misaligned_pi_h_star3, 2)
```

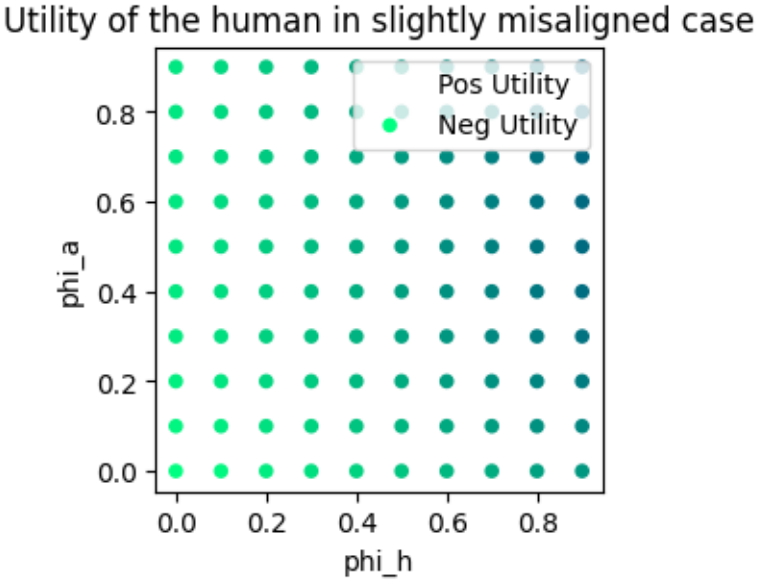
```
phi_h_star:  [2, 1, 3, 4, 5, 6, 7, 8, 9, 10]
```




```
phi_h_star:  [2, 1, 10, 9, 8, 7, 6, 5, 4, 3]
```




```
phi_h_star:  [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
```



5 Experiment 5: A Possible Comparison with Other Pluralistic Framework

Most pluralistic alignment work is implicitly studying the case where human has perfect accuracy (e.g. I know my political opinions perfectly).

This experiment aims to extend our setting to the scenario studied in other fluralistic framework.

To be more specifc, we investigate the case where the human has **perfect accuracy**. However, as humans' energy and attention are limited, they are still suffers from the scalability issue. For example, she can only handle a bounded-size of items or has a probability of missing some items.

Consider the following scenario:

- A professor is reviewing hundreds of PhD applications. Due to the heavy teaching and research workload, the professor has a probability of p_i of missing an application i.
- The professor decides to meet with some students that she has noticed.
- The professor has a keen eye for talent. After interviewing with the students, she is able to select the best applicant. However, among the emails she misses, some outstanding applicants may have been overlooked.
- Suppose there is an AI-tool that helps with reviewing the applicants. It can handle all the applicants in seconds. However, the tool is not very accurate.
- So will it help the professor recruit the best PhD student by using AI to screen the resumes to get a shorter list and then reviews the students in the list?

[9]:
```
m = 100  # Assume there are 100 students

## Expected index of the recruited student
def expect_index(m, p):
    expect_index = 0
    for i in range(m):
        i_being_picked = 1
        for j in range(i):
            i_being_picked *= p[j]
        i_being_picked *= (1 - p[i])
        expect_index += i_being_picked * (i + 1)

    return expect_index

# Professor A: carefully review the resumes
# The probability of missing an applicant increases when the quality of student␣
 ↪decreases
p_prof_A = [min(0.1 + 0.1*(i+1), 1) for i in range(m)]

# Professor B: randomly review the resumes
# The probability of missing an applicant is the same
p_prof_B = [0.9 for _ in range(m)]

E_id_A = expect_index(m, p_prof_A)
E_id_B = expect_index(m, p_prof_B)
print("Expected index of the recruited student for Professor A: {}".
 ↪format(E_id_A))
```

```
print("Expected index of the recruited student for Professor B: {}".
 ↪format(E_id_B))

## Estimate the expected benefit with AI assitance
def expect_index_with_AI_assistance(k, phi_a, verbose=False):
    D_AI = Mallows(m, phi_a, list(range(1, m+1)))
    number_of_iter = 100
    expected_index_with_AI_assistance = 0

    for _ in range(number_of_iter):
        sampled_perm = D_AI.sample()
        short_list = sampled_perm[:k]
        if verbose:
            print("short list: ", short_list)
        expected_index_with_AI_assistance += min(short_list)
    expected_index_with_AI_assistance /= number_of_iter

    return expected_index_with_AI_assistance

## An highly accurate AI assistant
phi_a = 1
E_AI = expect_index_with_AI_assistance(10, phi_a)
print("Expected index with highly-accurate AI assistance: {}".format(E_AI))

## An mediumly accurate AI assistant
phi_a = 0.2
E_AI = expect_index_with_AI_assistance(10, phi_a)
print("Expected index with mediumly-accurate AI assistance: {}".format(E_AI))

## An zero accurate AI assistant
phi_a = 0
E_AI = expect_index_with_AI_assistance(10, phi_a)
print("Expected index with zero-accurate AI assistance: {}".format(E_AI))
```

```
Expected index of the recruited student for Professor A: 1.2832416000000002
Expected index of the recruited student for Professor B: 9.997078246122367
Expected index with highly-accurate AI assistance: 1.0
Expected index with mediumly-accurate AI assistance: 1.11
Expected index with zero-accurate AI assistance: 10.63
```

6 Experiment 6: Misaligned Setting Beyond Single Best Item

(Ongoing...)

[9]:
```
import itertools
print(list(itertools.permutations([1, 2, 3])))
```

```
pi_h_stars = list()
```

```
[(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)]
```

7 Experiment 7: When does Collaboration Helps for Misaligned Human?

After classifying the misaligned humants into groups, we raise the following questions:

- *for what value of i, a misaligned human of type i can get benefit/hurt?*

Observe that, the probability of a human of type i picking her best item x_i is bounded by the following inequalities:

$$\Pr[x_i \text{ is included in the first } k \text{ items of } \pi_a\ ] \cdot \frac{1}{Z_k(\phi_h)} \leq \Pr[x_J = x_i],$$
$$\Pr[x_i \text{ is included in the first } k \text{ items of } \pi_a\ ] \geq \Pr[x_J = x_i]\,.$$

Since we can effiently compute the probability of x_i being included in the first k items of π_a in $O(m \cdot k)$ time using dynamic programming, we can then figure out the following things using simulation:

- `max_index_of_benefit`: any misaligned human with index no more than `max_index_of_benefit` will benefit from the collaboration;
- `min_index_of_hurt`: any misaligned human with index no less than `min_index_of_hurt` only get hurt from the collaboration.

The following simulation results show that, when ϕ increases, the two numbers tend to be close and even close to the number of presented items k.

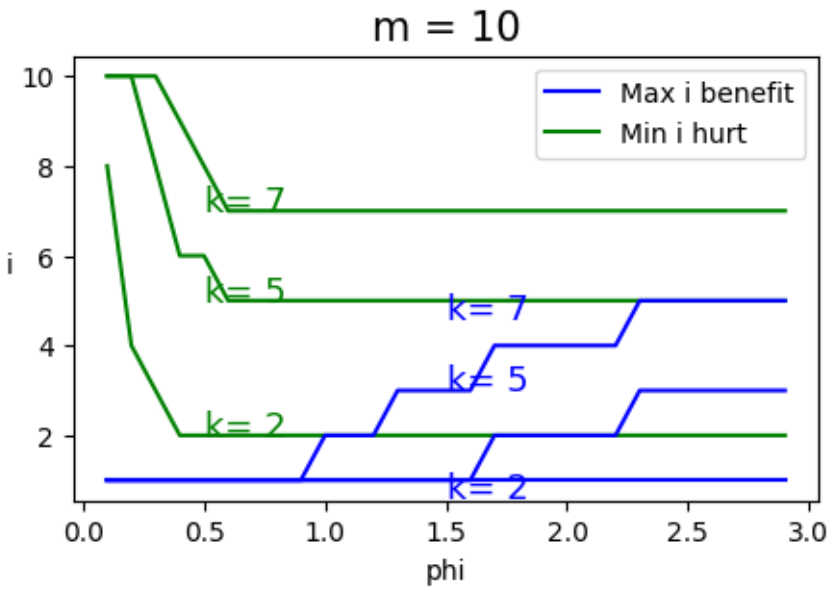
[27]:
```
def index_of_misaligned_get_benefit_hurt(m, ks, verbose=False):
    pi_a_star = [i + 1 for i in range(m)]
    max_indices_benefit = []
    min_indices_hurt = []
    for k in ks:
        tmp_max_indices_benefit = []
        tmp_min_indices_hurt = []
        for phi in np.arange(0.1, 3, 0.1):
            D_a = Mallows(m, phi, pi_a_star)
            D_h = Mallows(m, phi, pi_a_star) # pi_a_star does not really use␣
↪for D_h below

            ## The following codes could be improved by binary search
            prob_of_first_k = np.array([D_a.prob_of_xi_being_first_k(i+1, k)␣
↪for i in range(m)])
            lower_bound_of_xi_first = 1 / D_h.Z_lam(k)
            lower_bounds_of_system_picking_xi = lower_bound_of_xi_first *␣
↪prob_of_first_k
            upper_bounds_of_system_picking_xi = prob_of_first_k
```

```
            prob_of_human_picking_best_item = 1 / D_h.Z_lam(m)

            if verbose:
                print(prob_of_human_picking_best_item)
                print(lower_bounds_of_system_picking_xi)
                print(upper_bounds_of_system_picking_xi)
            max_index_benefit = 0
            min_index_hurt = 0
            for i in range(m):
                if lower_bounds_of_system_picking_xi[i] >=␣
 ↪prob_of_human_picking_best_item - 1e-3:
                    max_index_benefit = i + 1

                if upper_bounds_of_system_picking_xi[i] >=␣
 ↪prob_of_human_picking_best_item:
                    min_index_hurt = i + 1
            tmp_max_indices_benefit.append(max_index_benefit)
            tmp_min_indices_hurt.append(min_index_hurt)

        max_indices_benefit.append(tmp_max_indices_benefit)
        min_indices_hurt.append(tmp_min_indices_hurt)
        print(tmp_max_indices_benefit)
        print(tmp_min_indices_hurt)

    return max_indices_benefit, min_indices_hurt

def plot_min_max(m, ks, max_indices_benefit, min_indices_hurt, blue_scale = 1,␣
 ↪blue_distance = 2):
    phis = np.arange(0.1, 3, 0.1)
    plt.figure(figsize=(5, 3))
    for i in range(len(ks)):
        plt.plot(phis, max_indices_benefit[i], label=f'k = {ks[i]}',␣
 ↪color='blue')
        plt.plot(phis, min_indices_hurt[i], label=f'k = {ks[i]}', color='green')

    blue_ys = [ks[i] for i in range(len(ks))]
    green_ys = [ks[- i -1] for i in range(len(ks))]
    for i in range(len(ks)):
        text(1.5, blue_ys[i] * blue_scale - blue_distance, "k= %d" % ks[i],␣
 ↪color="blue", fontsize=13)
        text(0.5, green_ys[i], "k= %d" % ks[-i-1], color="green", fontsize=13)

    upper_bound_legend = mlines.Line2D([], [], color='blue', label='Max i␣
 ↪benefit')
```

```
    lower_bound_legend = mlines.Line2D([], [], color='green', label='Min i␣
↪hurt')
    plt.legend(handles=[upper_bound_legend, lower_bound_legend], loc='upper␣
↪right')
    plt.title("m = {}".format(m), fontsize=15)
    plt.xlabel("phi")
    plt.ylabel("i", rotation=0)
    plt.savefig("figs/misalign/{}.png".format(m), dpi=800)
```

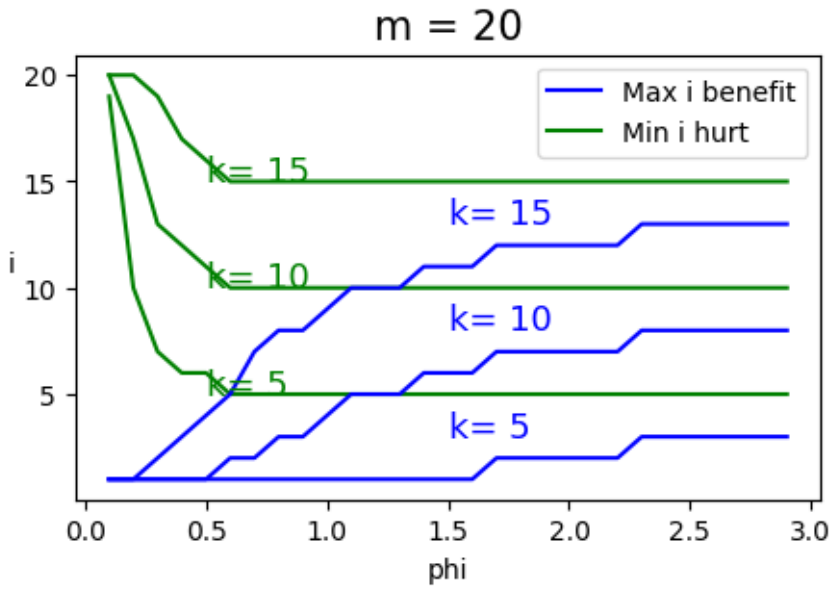
[28]:
```
m = 10
ks = [2, 5, 7]
max_indices_benefit, min_indices_hurt = index_of_misaligned_get_benefit_hurt(m,␣
↪ks)
plot_min_max(m, ks, max_indices_benefit, min_indices_hurt, blue_scale=0.8,␣
↪blue_distance=1)
```

```
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1]
[8, 4, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2]
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3,
3, 3, 3]
[10, 10, 8, 6, 6, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
5, 5, 5]
[1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5,
5, 5, 5]
[10, 10, 10, 9, 8, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
7, 7, 7]
```



[29]:
```
m = 20
ks = [5, 10, 15]
max_indices_benefit, min_indices_hurt = index_of_misaligned_get_benefit_hurt(m,␣
  ↪ks)
plot_min_max(m, ks, max_indices_benefit, min_indices_hurt)
```

```
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3,
3, 3, 3]
[19, 10, 7, 6, 6, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
5, 5, 5]
[1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8,
8, 8, 8]
[20, 17, 13, 12, 11, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10,
10, 10, 10, 10, 10, 10, 10, 10, 10, 10]
[1, 1, 2, 3, 4, 5, 7, 8, 8, 9, 10, 10, 10, 11, 11, 11, 12, 12, 12, 12, 12, 12,
13, 13, 13, 13, 13, 13, 13]
[20, 20, 19, 17, 16, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15,
15, 15, 15, 15, 15, 15, 15, 15, 15, 15]
```



[31]:
```
m = 100
ks = [5, 25, 50, 75]
max_indices_benefit, min_indices_hurt = index_of_misaligned_get_benefit_hurt(m,␣
  ↪ks)
plot_min_max(m, ks, max_indices_benefit, min_indices_hurt)
```

```
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3,
```

```
3, 3, 3]
[18, 10, 7, 6, 6, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5]
[2, 3, 7, 11, 14, 15, 17, 18, 18, 19, 20, 20, 20, 21, 21, 21, 22, 22, 22, 22, 22, 22, 23, 23, 23, 23, 23, 23, 23]
[46, 32, 28, 27, 26, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25]
[10, 25, 32, 36, 39, 40, 42, 43, 43, 44, 45, 45, 45, 46, 46, 46, 47, 47, 47, 47, 47, 47, 48, 48, 48, 48, 48, 48, 48]
[72, 57, 53, 52, 51, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50]
[31, 50, 57, 61, 64, 65, 67, 68, 68, 69, 70, 70, 70, 71, 71, 71, 72, 72, 72, 72, 72, 72, 73, 73, 73, 73, 73, 73, 73]
[98, 82, 78, 77, 76, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75, 75]
```