On Song Popularity: An Analysis on Song Metadata Using the Million Song Dataset

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We were tasked with retrieving a unique dataset on which to perform statistical analyses of which there were multiple predictors and at least twenty observations. Using SQL, we chose a random sample of songs and their metadata from a public music database on which to perform our analyses and used SAS to visualize and manipulate and model our data. We use six predictors: artist hotness, familiarity, song duration, loudness, tempo, and release year to predict song hotness. We run SAS implementations for visualizing basic statistics of our data and running multilinear regressions and transformations on our data to determine an appropriate model for song hotness.

I. THE DATASET

The dataset was obtained from *The Million Song Dataset*¹, a public dataset of over a million songs used for music analysis and research purposes. Each song in the collection contains over 50 properties ranging from *danceability* and *energy* indices. For our purposes, we will look at *song hotness* as the response variable and use 6 other variables as predictors with 30 observations. A full list of the metadata attributes is also available online.² The variables are as follows:

hotIdx	Song hotness index, according to The Echo Nest, when downloaded (December 2010)
artistHotIdx	Artist hotness index, according to The Echo Nest, when downloaded (December 2010)
artistFamIdx	Artist familiar index, according to The Echo Nest, when downloaded (December 2010)
duration	Duration of the track in seconds
loudness	General loudness of the track
tempo	Tempo in beats per minute according to The Echo Nest
year	Year when this song was released, according to musicbrainz.org

A. Visualization

We begin by running procedures to visualize our data. We produce 30 song observations randomly sampled from the data subset. The full raw data is available in Fig 1.

```
data songs;
infile '\Client\H$\Dropbox\UNC\5\STOR
     455\Songs.DAT' dlm='09'x;
input hotIdx artistHotIdx artistFamIdx duration
     loudness tempo year;
run;
proc print data=songs;
run;
```

Next we wish to obtain some basic statistics on the data and investigate plots against the explanatory variables.

```
proc means data=songs;
var hotIdx artistHotIdx artistFamIdx duration
    loudness tempo year;
run;
proc gplot data=songs;
plot hotIdx*(artistHotIdx artistFamIdx duration
    loudness tempo year);
run;
proc univariate data=songs alpha=0.5;
var hotIdx;
histogram / endpoints=0 to 1 by 0.1;
run;
proc corr data=songs;
var hotIdx artistHotIdx artistFamIdx duration
    loudness tempo year;
run;
```

On inspection, we observe that hotIdx is a roughly bimodal distribution of indices varying generally between 0.2 and 0.9. There are no major outliers and data points for other variables are rather evenly distributed.

We also observed the correlation matrix between variables and there are very few outstanding collinearities except for the relationship between artistHotIdx and artistFamIdx. We will explore this in depth later.

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¹ Thierry Bertin-Mahieux, Daniel P.W. Ellis, Brian Whitman, and Paul Lamere. The Million Song Dataset. In Proceedings of the 12th International Society for Music Information Retrieval Conference (ISMIR 2011), 2011.

http://labrosa.ee.columbia.edu/millionsong/sites/default/files/poster.pdf

II. MODELING TRANSFORMATIONS

Next we attempt to model the multilinear regression with all variables and no linear transformations.

```
proc reg data=songs;
model hotIdx = artistHotIdx artistFamIdx duration
    loudness tempo year / ss1 ss2 VIF TOL r
    influence;
run;
```

We obtain [Rsqr, adjRsqr] as [0.3825, 0.2214]. We also observe that the variance inflation is moderately high for variables artistHotIdx and artistFamIdx, indicating that we may have to account for this in future transformations and selections.

Immediately, we attempt to transform the variables to improve the deficiencies in our plain model by log-transforming hotIdx.

```
data logSongs;
set songs;
logIndex = log(hotIdx);
run;
proc reg data=logSongs;
model logIndex = artistHotIdx artistFamIdx duration
        loudness tempo year / ss1 ss2 VIF TOL;
run;
```

The results yield an improvement in correlation. This configuration gives us [Rsqr, adjRsqr] as [0.4097, 0.2557]. Our variance inflation across the board looks roughly the same as before, implying little or no change in multicollinearity. We take these results into consideration and think of other ways to transform the data.

We attempt to identify an appropriate exponent λ for data Y^k by a *Box-Cox power transformation*. We use built-in SAS tools to accomplish this.

From these results, we obtain λ as -2 and transform our data accordingly.

```
*Select lambda as -3;
data songsK;
set songs;
hIndexRT = hotIdx**-3;
run;
proc reg data=songsK;
model hIndexRT = artistHotIdx artistFamIdx duration
    loudness tempo year / ss1 ss2 VIF TOL;
run;
```

Yet again, this transformation yields us a slight improvement in correlation to [Rsqr, adjRsqr] as

[0.4329, 0.2850] with no improvement in multicollinearity. We accept this as a final model of all variables and proceed to selection.

III. MODEL SELECTION

Before we proceed, we visualize a correlation matrix (Fig. 7) to observe any outstanding collinearities to consider when we perform model selection.

```
proc sgscatter data=songsK;
matrix hIndexRT artistHotIdx artistFamIdx duration
        loudness tempo year;
run;
```

Our beliefs that artistHotIdx and artistFamIdx are strongly correlated are more confirmed with this visualization as evidenced by plots (2, 3) or (3, 2). Thus we have a few options - we considered the brute option of completely omitting these two variables, however it is likely that they influence the response in some similar way. Formally, we could perform factor analysis to determine the true linear combination of these two variables, but I opt to simply take the difference (artistFamIdx - artistHotIdx) and use that as variable aIndex when we perform model selection.

```
data songsAdj;
set songsK;
aIndex = artistFamIdx - artistHotIdx;
run;
```

We are now ready to select a proper model. We will perform Forward Selection, Backward Elimination, Stepwise Selection on the revised dataset (with aIndex) and Stepwise Selection on the original dataset with respective variables artistHotIdx and artistFamIdx. We also perform Maximum Rsqr Improvement Selection, Adjusted Rsqr Selection, and Mallows' C_p Selection.

```
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=FORWARD VIF TOL slentry=0.1;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=B VIF TOL slstay=0.15;
run;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=STEPWISE VIF TOL slentry=0.15
       slstay=0.15;
run;
proc reg data=songsAdj;
model hIndexRT = artistHotIdx artistFamIdx duration
    loudness tempo year /
  selection=STEPWISE VIF TOL slentry=0.15
```

```
slstay=0.15;
run:
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=MAXR VIF TOL;
run:
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=adjrsq VIF TOL;
run;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx
    duration loudness tempo year /
  selection=cp VIF TOL;
run;
```

We obtain the following results for each selection algorithm in the format selection algorithm: (variables), [Rsqr, adjRsqr].

```
forward:
     (aIndex, duration), [0.2940, 0.2417]
backward:
     (aIndex, duration, loudness, year),
     [0.4091, 0.3146]
stepwise (aIndex):
     (aIndex, duration, loudness, year),
     [0.4091, 0.3146]
stepwise (w/o aIndex):
     (artistFamIdx), [0.1472, 0.1167]
maxR:
     (aIndex, artistFamIdx, duration, loudness,
     tempo, year), [0.4329, 0.2850]
adjRsqr:
     (aIndex, duration, loudness, year),
     [0.4091, 0.3146]
CP:
     (aIndex, duration, loudness, year),
     [0.4091, 0.3146]
```

The majority of selections yielded the variable configuration aIndex, duration, loudness, year with variance inflations corrected to < 1.6. Although our Rsqr coefficient decreased nominally, we can take solace in the fact that $Maximum\ Rsqr\ Improvement\ Selection$ only yielded a slightly higher statistic, however with a lower adjRsqr statistic in addition to wild variance inflation.

We can now apply a final multilinear regression onto this revised data and plot the residual v. response and normal quantile plots with studentized residuals (Fig. 15) for a final check on linearity.

```
proc reg data=songsAdj;
model hIndexRT = aIndex duration loudness year /
    ss1 ss2 VIF TOL collin;
plot student.*nqq.;
run;
```

IV. CONCLUSIVE REMARKS & IMPROVEMENTS

The final model we obtain for the data is:

```
songHotness = -1465.7 -
226.7*aggregateArtistIndex + 0.1622*duration -
0.9318*loudness + 0.475*year
```

Because of the relatively poor correlation coefficients observed throughout our analyses, we suspect that the relationship between song hotness v. artist popularity, song duration, loudness, and release year is not exactly linear, but our residual and normal quantile plots suggest that it is an appropriate fit. Thus it might be more prudent to suspect that there was not enough evidence or enough data to illustrate a good enough model (subset of 30 as opposed to a subset of 100,00+?). The variables aggregate artist index, song duration, loudness, and release year explain 40.91% of the variation in song hotness according to our model. Prior to our analyses, intuition held that song hotness would be most favorably predicted by tempo and loudness of the music - but even then in hindsight, that may very well not be a linear correlation seeing as that both extremely loud and soft songs are unfavorable in comparison to songs with a sweetspot timbre and tempo.

For future reference, it is vital to wisely choose the predictors for the response especially when choosing a larger amount of predictors in order to avoid multicollinearity. For our dataset, there were variables such as *danceability* and *key* that could have been explored as well, but we chose the more seemingly stable variables.

Because the song hotness, artist hotness, and artist familiarity indices were collected and assessed by one entity, there may be reason enough to suspect that bias and user anonymity play a role in the discord of our regression.

There were also other ways to transform the data in applying the Box-Cox method, it is possible that not choosing a sufficiently large enough range resulted in a skewed representation of our model. For our purposes, we chose a power transformation over the log transformation even though both were improvements from the original data.

Appendix

FIG. 1. Raw song dataset

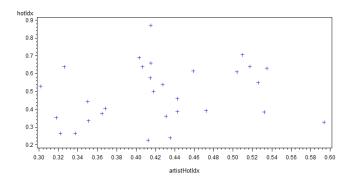
Observation	Song Title	Artist
1	Sea_ Subsurface	Aisha Duo
2	Nothing (Album Version)	unloco
3	Rowena	Loudon Wainwright III
4	The Forgotten Lands	David Arkenstone
5	Mais La Vie	Maurane / Lara Fabian
6	Between You And Me	Joy Enriquez
7	Standing On My Own	Agnostic Front
8	Nothing To You	Milk Inc.
9	Hope (Edited Album Version)	Twista FeaturingCee-Lo
10	True Nature	Jane's Addiction
11	Lord of Shadows and Golden Wood	
12	Looking In Your Eyes	Suicidal Tendencies
13	High Horse	Workhorse
14	Cumberland Gap	Danny Barnes
15	Ripped Pants	Spongebob Squarepants
16	Becoming (LP Version)	Lennie Tristano
17	Zap Zap	Dynamite Deluxe
18	Fortress	Tim Exile
19	Whiskey on the Fire	Aaron Watson
20	The Blinding Sun	Gustavo Santaolalla
21	Gossippo Perpetuo	Perrey And Kingsley
22	Erick Sermon	Erick Sermon
23	Sag Ihnen Bescheid	Headliners
24	This Is Not A Love Song	Public Image Ltd
25	Vitamina	Sonora Carruseles
26	Speechless	Mish Mash
27	Think About Me	Sister Hazel
28	Turn The Heat Up	Bobaflex
29	You Gotta Go	Fountains Of Wayne
30	All About Our Love	Sade

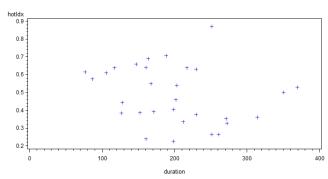
				-					
				The S	SAS System	11:21	Friday, Dec	ember 4, 201	5 87
0bs	hotIdx		artist HotIdx	artist FamIdx	duration	loudness	tempo	year	
1	0.50132		0.41797	0.63807	350.641	21.339	132.888	2005	
2	0.69029		0.40360	0.58828	163.866	-5.653	126.113	2001	
3	0.38703		0.44277	0.66108	152.137	-12.464	151.294	2009	
4	0.36037		0.43094	0.63940	314.592	14.881	141.975	1996	
5	0.22577		0.41261	0.58048	198.870	-13.637	107.087	2003	
6	0.26586		0.33739	0.59417	260.963	-3.941	96.087	2001	
7	0.61018		0.50431	0.70144	105.848	-5.152	112.292	2001	
8	0.46054		0.44310	0.65280	202.109	-7.597	140.017	2003	
9	0.32774		0.59381	0.88961	272.509	-5.126	106.967	2004	
10	0.63019		0.53516	0.71189	229.616	-3.782	92.099	2003	
11	0.54155		0.42748	0.59618	202.919	-18.650	130.955	1993	
12	0.55065		0.52618	0.73668	167.784	-9.966	166.090	1987	
13	0.26586		0.32222	0.47902	251.402	-7.200	85.209	2007	
14	0.23963		0.43525	0.47498	160.470	-13.579	189.782	2005	
15	0.61561		0.45894	0.62300	76.826	-8.892	122.887	2005	
16	0.35223		0.31785	0.51443	271.255	-15.324	200.876	1994	
17	0.57785		0.41483	0.63340	86.674	-10.182	93.298	2000	
18	0.53003		0.30185	0.62050	369.580	-6.555	110.000	2009	
19	0.39201		0.47250	0.65802	171.102	-8.810	130.985	2008	
20	0.63925		0.40663	0.64398	117.185	-13.505	90.463	2006	
21	0.44440		0.35033	0.53061	127.947	-12.717	135.780	2001	
22	0.40512		0.36834	0.57797	198.635	-9.384	97.812	1993	
23	0.63823		0.32624	0.53063	216.894	-5.893	160.158	2005	
24	0.87362		0.41552	0.68499	251.506	-10.347	127.369	1983	
25	0.37598		0.36503	0.56035	230.008	-6.564	108.292	1999	
26	0.33471		0.35121	0.50564	211.905	-7.963	124.043	2006	
27	0.70717		0.50988	0.73107	188.630	-5.150	138.581	1997	
28	0.66068		0.41541	0.58878	147.435	-3.124	93.033	2005	
29	0.38527		0.53227	0.71776	126.876	-3.594	85.047	2007	
30	0.64069		0.51734	0.77149	160.626	-8.211	161.991	2000	
00	0.04003		0.01704		SAS System		Friday, Dece		5 88
								,	
				The MEAR	NS Procedure				
Variab	le	N		Mean	Std Dev	Min	nimum	Maximum	
hotIdx		30	0.4	876606	0.1629004	0.225	7681	0.8736188	
artist		30		252308	0.0748211	0.301		0.5938084	
artist		30		278911	0.0920014	0.474		0.8896136	
durati		30		602540	72.3721651	76.825		9.5799500	
441401	~	-	100.0		. 2.0, 21001	10.020			

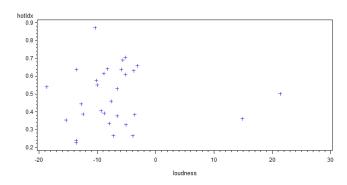
 loudness
 30
 -6.8914000
 7.8439325
 -18.6500000
 21.3390000

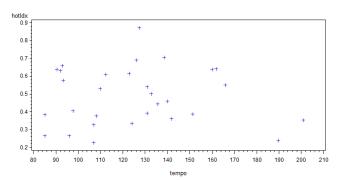
 tempo
 30
 125.3156667
 30.0939508
 85.0470000
 200.8760000

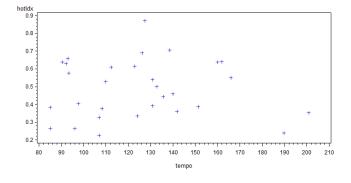
 year
 30
 2001.20
 6.2774913
 1983.00
 2009.00

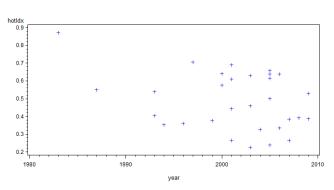












The SAS System

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The UNIVARIATE Procedure Variable: hotIdx

Moments

N	30	Sum Weights	30
Mean	0.48766057	Sum Observations	14.6298172
Std Deviation	0.16290038	Variance	0.02653653
Skewness	0.23773367	Kurtosis	-0.6075412
Uncorrected SS	7.90394455	Corrected SS	0.76955947
Coeff Variation	33.4044594	Std Error Mean	0.0297414

Basic Statistical Measures

Location

Variability

Mean	0.487661	Std Deviation	0.16290
Median	0.480928	Variance	0.02654
Mode	0.265861	Range	0.64785
		Interquartile Range	0.26982

Tests for Location: Mu0=0

Test -Statistic-			p Val	ue
Student's t Sign	t M	16.39669 15	Pr > t Pr >= M	<.0001 <.0001
Signed Rank	S	232.5	Pr >= S	<.0001

Quantiles (Definition 5)

Level	Quantile
100% Max 99%	0.873619 0.873619
95%	0.707172
90%	0.675486
75% Q3	0.630188
50% Median	0.480928
25% Q1	0.360371
10%	0.265861
5%	0.239629
1%	0.225768
0% Min	0.225768

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The UNIVARIATE Procedure Variable: hotIdx

Extreme Observations

Lowest		Highes	t
Value	0bs	Value	0bs
0.225768 0.239629	5 14	0.640688 0.660681	30 28
0.265861	13	0.690291	2
0.265861	6	0.707172	27
0.327737	9	0.873619	24

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The CORR Procedure

7 Variables:	hotIdx year	artistHotIdx	artistFamIdx	duration	loudness	tempo
		Simple	e Statistics			
Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
hotIdx artistHotIdx	30 30	0.48766 0.42523	0.16290 0.07482	14.62982 12.75692	0.22577 0.30185	0.87362 0.59381

FIG. 2. SAS Univariate Procedure

artistFamIdx	30	0.62789	0.09200	18.83673	0.47498	0.88961
					0.1.100	
duration	30	199.56025	72.37217	5987	76.82567	369.57995
loudness	30	-6.89140	7.84393	-206.74200	-18.65000	21.33900
tempo	30	125.31567	30.09395	3759	85.04700	200.87600
year	30	2001	6.27749	60036	1983	2009

Pearson Correlation Coefficients, N = 30 Prob > |r| under HO: Rho=0

	hotIdx	artist HotIdx	artist FamIdx	duration	loudness	tempo	year
hotIdx	1.00000	0.20728 0.2717	0.33787 0.0678	-0.22984 0.2218	0.01381 0.9423	-0.02899 0.8791	-0.33400 0.0712
artistHotIdx	0.20728 0.2717	1.00000	0.83957 <.0001	-0.31435 0.0907	0.10520 0.5801	-0.02217 0.9074	-0.06507 0.7326
artistFamIdx	0.33787 0.0678	0.83957 <.0001	1.00000	-0.04108 0.8294	0.18594 0.3252	-0.13965 0.4617	-0.13632 0.4726
duration	-0.22984 0.2218	-0.31435 0.0907	-0.04108 0.8294	1.00000	0.48934 0.0061	0.07897 0.6783	-0.07933 0.6769
loudness	0.01381 0.9423	0.10520 0.5801	0.18594 0.3252	0.48934 0.0061	1.00000	-0.12721 0.5029	0.12916 0.4963
tempo	-0.02899 0.8791	-0.02217 0.9074	-0.13965 0.4617	0.07897 0.6783	-0.12721 0.5029	1.00000	-0.26404 0.1586
year	-0.33400 0.0712	-0.06507 0.7326	-0.13632 0.4726 The SAS S	-0.07933 0.6769 System	0.12916 0.4963 11:21 Frida	-0.26404 0.1586 y, December	1.00000 4, 2015 95

The REG Procedure
Model: MODEL1
Dependent Variable: hotIdx

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error	6 23	0.29437 0.47519	0.04906 0.02066	2.37	0.0622
Corrected Total	29	0.76956	0.02000		

Root MSE 0.14374 R-Square 0.3825

Dependent Mean 0.48766 Adj R-Sq 0.2214 Coeff Var 29.47480

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Type II SS
Intercept	1	17.71117	9.30394	1.90	0.0695	7.13439	0.07487
artistHotIdx	1	-1.61522	0.82746	-1.95	0.0632	0.03306	0.07872
artistFamIdx	1	1.50343	0.63500	2.37	0.0267	0.07000	0.11581
duration	1	-0.00134	0.00053491	-2.50	0.0198	0.08733	0.12961
loudness	1	0.00575	0.00430	1.34	0.1946	0.01476	0.03688
tempo	1	0.00036694	0.00099115	0.37	0.7146	0.01700	0.00283
vear	1	-0.00860	0.00460	-1.87	0.0743	0.07222	0.07222

Parameter Estimates

			Variance		
Variable	DF	Tolerance	Inflation		
Intercept	1		0		
artistHotIdx	1	0.18586	5.38030		
artistFamIdx	1	0.20874	4.79069		
duration	1	0.47537	2.10364		
loudness	1	0.62601	1.59741		
tempo	1	0.80076	1.24881		
year	1	0.85355	1.17158		
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The REG Procedure Model: MODEL1 Dependent Variable: hotIdx

Output Statistics

	Dependent		Std Error		Std Error	Student			Cook's
0bs	Variable	Value	Mean Predict	Residual	Residual	Residual	-2-1 0	1 2	D
1	0.5013	0.4446	0.1026	0.0567	0.101	0.564	1 1	*	0.047
2	0.6903	0.5200	0.0371	0.1703	0.139	1.226	i i	**	0.015
3	0.3870	0.4832	0.0669	-0.0961	0.127	-0.756	*	į	0.023
4	0.3604	0.5176	0.0878	-0.1572	0.114	-1.382	**	į	0.162
5	0.2258	0.3768	0.0526	-0.1510	0.134	-1.129	**	i	0.028
6	0.2659	0.5045	0.0555	-0.2387	0.133	-1.800	***	į	0.081
7	0.6102	0.6030	0.0532	0.007182	0.134	0.0538	1 1	- 1	0.000
8	0.4605	0.4787	0.0341	-0.0181	0.140	-0.130	1 1	1	0.000
9	0.3277	0.4904	0.0976	-0.1627	0.105	-1.542	***	- 1	0.291
10	0.6302	0.3863	0.0760	0.2439	0.122	1.999	1 1	***	0.222
11	0.5416	0.4369	0.0701	0.1046	0.125	0.834	1 1	*	0.031
12	0.5506	0.6502	0.0771	-0.0996	0.121	-0.821	*	- 1	0.039
13	0.2659	0.2944	0.0742	-0.0285	0.123	-0.232	1 1		0.003
14	0.2396	0.2465	0.1043	-0.006859	0.0989	-0.0693	1 1		0.001
15	0.6156	0.5452	0.0569	0.0704	0.132	0.533	1 1	*	0.008
16	0.3522	0.4357	0.0866	-0.0835	0.115	-0.728	*		0.043
17	0.5778	0.6436	0.0670	-0.0658	0.127	-0.517	*		0.011
18	0.5300	0.3773	0.1017	0.1528	0.102	1.504	1 1	***	0.323
19	0.3920	0.4273	0.0471	-0.0353	0.136	-0.260	1 1		0.001
20	0.6392	0.5602	0.0630	0.0791	0.129	0.612	1 1	*	0.013
21	0.4444	0.5304	0.0551	-0.0860	0.133	-0.648	*		0.010
22	0.4051	0.5519	0.0620	-0.1468	0.130	-1.132	**		0.042
23	0.6382	0.4639	0.0636	0.1743	0.129	1.352	1 1	**	0.064
24	0.8736	0.6571	0.0874	0.2165	0.114	1.897	1 1	***	0.302
25	0.3760	0.4571	0.0430	-0.0812	0.137	-0.592	*		0.005
26	0.3347	0.3590	0.0482	-0.0243	0.135	-0.179	1 1		0.001
27	0.7072	0.5717	0.0451	0.1355	0.136	0.993	1 1	*	0.015
28	0.6607	0.4917	0.0517	0.1690	0.134	1.260		**	0.034
29	0.3853	0.5015	0.0635	-0.1163	0.129	-0.902	*		0.028
30	0.6407	0.6231	0.0701	0.0176	0.125	0.140	1 1		0.001

Output Statistics

0bs	RStudent	Hat Diag H	Cov Ratio	DFFITS
1	0.5551	0.5097	2.5259	0.5660
2	1.2405	0.0665	0.9109	0.3311
3	-0.7485	0.2167	1.4613	-0.3936
4	-1.4110	0.3732	1.1876	-1.0889
5	-1.1357	0.1338	1.0576	-0.4464

6 -1.8997 0.1493 0.5541 -0.7958 The SAS System 11:21 Friday, December 4, 2015 97

The REG Procedure
Model: MODEL1
Dependent Variable: hotIdx

Output Statistics

0bs	RStudent	Hat Diag H	Cov Ratio	DFFITS
7	0.0526	0.1369	1.5802	0.0210
8	-0.1271	0.0562	1.4389	-0.0310
9	-1.5932	0.4615	1.1803	-1.4749
10	2.1509	0.2796	0.4982	1.3400
11	0.8283	0.2381	1.4450	0.4630
12	-0.8145	0.2874	1.5559	-0.5173
13	-0.2268	0.2662	1.8300	-0.1366
14	-0.0678	0.5262	2.8765	-0.0714
15	0.5249	0.1569	1.4840	0.2265
16	-0.7201	0.3634	1.8215	-0.5440
17	-0.5089	0.2170	1.6061	-0.2679
18	1.5487	0.5003	1.3240	1.5497
19	-0.2544	0.1076	1.4984	-0.0883
20	0.6035	0.1920	1.5059	0.2941
21	-0.6395	0.1470	1.4067	-0.2655
22	-1.1390	0.1859	1.1227	-0.5443
23	1.3785	0.1960	0.9506	0.6805
24	2.0206	0.3698	0.6578	1.5478
25	-0.5831	0.0893	1.3463	-0.1826
26	-0.1753	0.1125	1.5231	-0.0624
27	0.9922	0.0984	1.1144	0.3278
28	1.2772	0.1292	0.9501	0.4920
29	-0.8980	0.1954	1.3187	-0.4426
30	0.1369	0.2379	1.7806	0.0765

Output Statistics

				DFBETAS				
		artist	artist					
Ubs	Intercept	HotIdx	FamIdx	duration	loudness	tempo	year	
1	-0.0388	-0.0034	-0.0243	0.0267	0.4188	0.0942	0.0405	
2	0.0615	-0.0748	-0.0049	-0.1943	0.1645	0.0301	-0.0561	
3	0.2809	0.1375	-0.1892	0.0949	0.0735	-0.2372	-0.2789	
4	-0.3647	-0.0781	0.1508	-0.0180	-0.8070	-0.1476	0.3635	
5	0.0294	-0.2383	0.2426	-0.2424	0.2883	0.2366	-0.0311	
6	-0.0913	0.4205	-0.2759	-0.0086	-0.0782	0.2872	0.0830	
7	0.0028	-0.0008	0.0029	-0.0134	0.0087	-0.0014	-0.0027	
8	0.0134	0.0025	-0.0077	-0.0015	0.0043	-0.0152	-0.0132	
9	0.4592	-0.2883	-0.3671	-0.7283	0.5759	0.1539	-0.4299	
10	-0.0152	1.0507	-0.7300	0.8199	-0.3659	-0.7508	0.0076	
11	0.2043	0.2317	-0.2182	0.2368	-0.3094	-0.1412	-0.2072	
12	-0.3155	-0.0665	-0.0248	0.0492	-0.0039	-0.1500	0.3233	
			The SA	S System	11:21 Fr	iday, Decembe	r 4, 2015	9

The REG Procedure Model: MODEL1 Dependent Variable: hotIdx

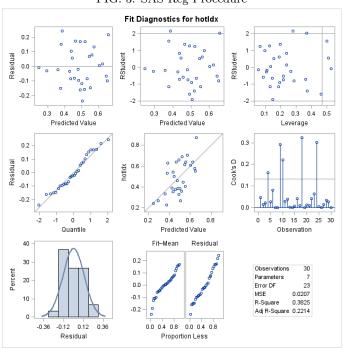
Output Statistics

				DFBETAS			
0bs	Intercept	artist HotIdx	artist FamIdx	duration	loudness	tempo	year
13	0.0135	-0.0437	0.0751	-0.0609	0.0311	0.0839	-0.0156
14	0.0132	-0.0461	0.0503	-0.0188	0.0145	-0.0228	-0.0132
15	-0.0364	-0.0220	0.0097	-0.1645	0.0712	0.0311	0.0392
16	-0.0539	0.0939	-0.0282	-0.1100	0.1515	-0.3321	0.0579
17	-0.0651	0.1289	-0.0954	0.2045	-0.0729	0.0657	0.0612
18	-0.7232	-0.6811	0.7126	0.6183	-0.5079	0.0254	0.7177
19	0.0623	-0.0212	0.0032	-0.0089	0.0223	-0.0203	-0.0620
20	-0.0772	-0.1559	0.1495	-0.1421	-0.0377	-0.0658	0.0802
21	-0.0190	0.1280	-0.0466	0.1708	-0.0391	-0.0662	0.0148
22	-0.4015	0.0450	0.0677	0.0330	0.0011	0.3103	0.3958
23	-0.2361	-0.3847	0.2277	-0.2211	0.1778	0.4503	0.2412
24	1.2452	-0.2369	0.2640	0.1500	-0.1798	-0.2727	-1.2576
25	-0.0753	-0.0118	0.0588	-0.0384	0.0039	0.0961	0.0728
26	0.0172	-0.0131	0.0308	-0.0129	0.0064	0.0067	-0.0182
27	0.0789	0.0367	0.0620	-0.0246	0.0393	0.0819	-0.0839

28	0.0112	0.0179	-0.1100	-0.2036	0.2181	-0.2211	-0.0005
29	0.0895	-0.1591	0.0742	0.0598	-0.0562	0.1979	-0.0916
30	-0.0161	-0.0271	0.0483	-0.0253	0.0041	0.0507	0.0149

Sum of Residuals 0 Sum of Squared Residuals 0.47519 Predicted Residual SS (PRESS) 0.91407

FIG. 3. SAS Reg Procedure



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The REG Procedure Model: MODEL1 Dependent Variable: logIndex

30 30 Number of Observations Read Number of Observations Used

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	6 23 29	1.49325 2.15125 3.64449	0.24887 0.09353	2.66	0.0414
De	ot MSE pendent Mean eff Var	0.30583 -0.77605 -39.40881	R-Square Adj R-Sq	0.4097 0.2557	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Type II SS
Intercept	1	31.31697	19.79609	1.58	0.1273	18.06746	0.23408
$\mathtt{artistHotIdx}$	1	-3.94591	1.76060	-2.24	0.0350	0.17340	0.46982
artistFamIdx	1	3.65513	1.35110	2.71	0.0126	0.38914	0.68453
duration	1	-0.00318	0.00114	-2.79	0.0103	0.45186	0.73028
loudness	1	0.01473	0.00915	1.61	0.1210	0.11989	0.24249
tempo	1	0.00125	0.00211	0.59	0.5602	0.10751	0.03268

year 1 -0.01606 0.00979 -1.64 0.1147 0.25145 0.25145

Variable	DF	Tolerance	Variance Inflation
Intercept	1		0
$\mathtt{artistHotIdx}$	1	0.18586	5.38030
artistFamIdx	1	0.20874	4.79069
duration	1	0.47537	2.10364
loudness	1	0.62601	1.59741
tempo	1	0.80076	1.24881
year	1	0.85355	1.17158

FIG. 4. SAS Reg Procedure

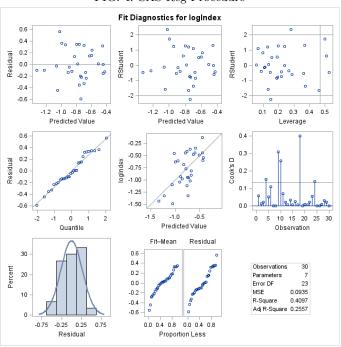
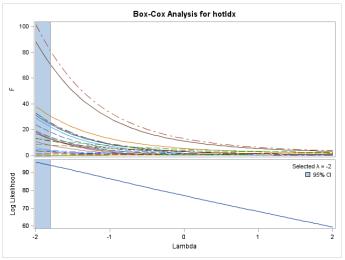


FIG. 5. SAS Transreg Procedure



The SAS System

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Tot	6 23 al 29	282.42099 369.90536 652.32635	47.07017 16.08284	2.93	0.0286
	Root MSE Dependent Mean Coeff Var	4.01034 6.07712 65.99088	R-Square Adj R-Sq	0.4329 0.2850	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Type II SS
Intercept	1	-307.66834	259.58501	-1.19	0.2480	1107.93985	22.59275
$\mathtt{artistHotIdx}$	1	61.03969	23.08670	2.64	0.0145	28.84248	112.42523
artistFamIdx	1	-55.99957	17.71686	-3.16	0.0044	94.78809	160.67893
duration	1	0.04383	0.01492	2.94	0.0074	70.42604	138.72475
loudness	1	-0.23531	0.11999	-1.96	0.0621	36.99989	61.85042
tempo	1	-0.02541	0.02765	-0.92	0.3677	27.07877	13.57824
year	1	0.15779	0.12840	1.23	0.2316	24.28572	24.28572

Variable	DF	Tolerance	Variance Inflation
Intercept	1		0
artistHotIdx	1	0.18586	5.38030
artistFamIdx	1	0.20874	4.79069
duration	1	0.47537	2.10364
loudness	1	0.62601	1.59741
tempo	1	0.80076	1.24881
year	1	0.85355	1.17158

FIG. 6. SAS Reg Procedure

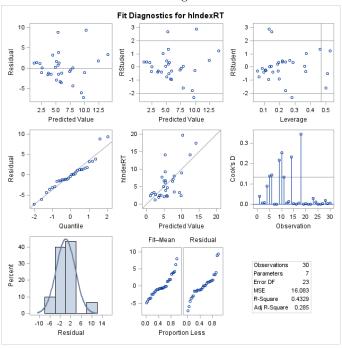
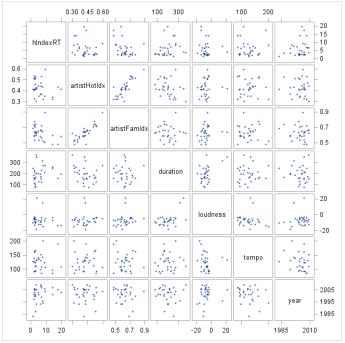


FIG. 7. Correlation Matrix



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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Forward Selection: Step 1

Variable aIndex Entered: R-Square = 0.1528 and C(p) = 8.3621

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue Pr > F
Model Error Corrected Total	1 28 29	99.68574 552.64062 652.32635	99.68574 19.73716	5.	0.0327
Variable	Parameter Estimate	Standard Error	Type II SS F	7 Value	Pr > F
Intercept aIndex	13.58603 -37.05174	3.43825 16.48673	308.17345 99.68574	15.61 5.05	0.0005 0.0327
	Bounds on	condition nu	ımber: 1, 1		

Forward Selection: Step 2

Variable duration Entered: R-Square = 0.2940 and C(p) = 4.6362

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	191.77541	95.88770	5.62	0.0091
Error	27	460.55095	17.05744		
Corrected Total	29	652.32635			

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Forward Selection: Step 2

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	11.33712	3.33966	196.56871		0.0021
aIndex	-52.34084	16.67951	167.96885		0.0041
duration	0.02680	0.01153	92.08967		0.0279

Bounds on condition number: 1.1843, 4.7373

No other variable met the 0.1000 significance level for entry into the model.

Summary of Forward Selection

Step	Variable Entered	Number Vars In	Partial R-Square	Model R-Square	C(p)	F Value	Pr > F
1	aIndex	1	0.1528	0.1528	8.3621	5.05	0.0327
2	duration	2	0.1412	0.2940	4.6362	5.40	0.0279
			The SAS	System	11:21 Friday	y, Decemb	er 4, 2015 105

The REG Procedure

Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

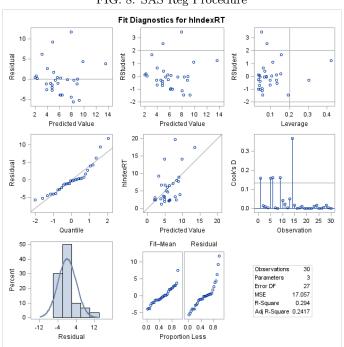
Analysis of Variance

Source		DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Tot	tal	2 27 29	191.77541 460.55095 652.32635	95.88770 17.05744	5.62	0.0091
	Root MSE Dependent M Coeff Var	ean	4.13007 6.07712 67.96096	R-Square Adj R-Sq	0.2940 0.2417	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	11.33712	3.33966	3.39	0.0021		0
aIndex	1	-52.34084	16.67951	-3.14	0.0041	0.84437	1.18432
duration	1	0.02680	0.01153	2.32	0.0279	0.84437	1.18432





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The REG Procedure
Model: MODEL1
Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.4329 and C(p) = 7.0000 The model is not of full rank. A subset of the model which is of full rank is chosen.

Analysis of Variance

		Sum of	Mean			
Source	DF	Squares	Square	F Val	ue Pr	> F
Model	6	282.42099	47.07017	2.	93 0.0	0286
Error	23	369.90536	16.08284			
Corrected Total	29	652.32635				
	Parameter	Standard				
Variable	Estimate	Error	Type II SS I	F Value	Pr > F	
Intercept	-307.66834	259.58501	22.59275	1.40	0.2480	
aIndex	-55.99957	17.71686	160.67893	9.99	0.0044	
$\mathtt{artistHotIdx}$	5.04011	11.47008	3.10535	0.19	0.6645	
duration	0.04383	0.01492	138.72475	8.63	0.0074	
loudness	-0.23531	0.11999	61.85042	3.85	0.0621	
tempo	-0.02541	0.02765	13.57824	0.84	0.3677	
year	0.15779	0.12840	24.28572	1.51	0.2316	

Bounds on condition number: 2.1036, 53.2

Backward Elimination: Step 1

Variable artistHotIdx Removed: R-Square = 0.4282 and C(p) = 5.1931

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Backward Elimination: Step 1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue Pr > F
Model Error Corrected Total	5 24 29	279.31564 373.01071 652.32635	55.86313 15.54211	3.	59 0.0144
Variable	Parameter Estimate	Standard Error	Type II SS I	F Value	Pr > F
Intercept aIndex duration loudness tempo year	-292.82758 -54.32265 0.04067 -0.21670 -0.02419 0.15158	253.01479 17.00763 0.01285 0.11036 0.02705 0.12546	20.81815 158.55667 155.68248 59.91985 12.42945 22.68623	1.34 10.20 10.02 3.86 0.80 1.46	0.2585 0.0039 0.0042 0.0613 0.3801 0.2388

Bounds on condition number: 1.6135, 33.784

Backward Elimination: Step 2

 $\label{eq:Variable} Variable \ artistFamIdx \ Entered: R-Square = 0.4329 \ and \ C(p) = 7.0000 \\ \text{NOTE: The variable which previously had small tolerance is now allowed to enter after removal of some variables from the model.}$

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	282.42099	47.07017	2.93	0.0286
Error	23	369.90536	16.08284		
Corrected Total	29	652.32635			

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Backward Elimination: Step 2

aIndex -61.03969 23.08670 112.42523 6.99 0.01	Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
duration 0.04383 0.01492 138.72475 8.63 0.00 loudness -0.23531 0.11999 61.85042 3.85 0.06 tempo -0.02541 0.02765 13.57824 0.84 0.36	aIndex artistFamIdx duration loudness tempo	-61.03969 5.04011 0.04383 -0.23531 -0.02541	23.08670 11.47008 0.01492 0.11999 0.02765	112.42523 3.10535 138.72475 61.85042 13.57824	6.99 0.19 8.63 3.85 0.84	0.2480 0.0145 0.6645 0.0074 0.0621 0.3677 0.2316

Bounds on condition number: 2.4064, 63.215

Backward Elimination: Step 3

Variable artistFamIdx Removed: R-Square = 0.4282 and C(p) = 5.1931

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue	Pr > F
Model Error Corrected Total	5 24 29	279.31564 373.01071 652.32635	55.86313 15.54211	3.	59	0.0144
Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr >	F
Intercept aIndex duration loudness tempo year	-292.82758 -54.32265 0.04067 -0.21670 -0.02419 0.15158	253.01479 17.00763 0.01285 0.11036 0.02705 0.12546	20.81815 158.55667 155.68248 59.91985 12.42945 22.68623	1.34 10.20 10.02 3.86 0.80 1.46	0.258 0.003 0.004 0.061 0.380 0.238	9 2 3 1

Bounds on condition number: 1.6135, 33.784

Backward Elimination: Step 4
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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Backward Elimination: Step 4

Variable tempo Removed: R-Square = 0.4091 and C(p) = 3.9659

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue	Pr > F
Model Error	4 25	266.88619 385.44016	66.72155 15.41761	4.	33	0.0085
Corrected Total	29	652.32635				
	Parameter	Standard				
Variable	Estimate	Error	Type II SS I	F Value	Pr >	F
Intercept	-359.86178	240.68605	34.46566	2.24	0.14	-
\mathtt{aIndex}	-49.33357	16.00210	146.53726	9.50	0.004	
duration	0.03794	0.01243	143.58644	9.31	0.00	53
loudness	-0.20173	0.10865	53.14988	3.45	0.07	52
year	0.18338	0.11983	36.10455	2.34	0.138	35

Bounds on condition number: 1.5224, 20.636

All variables left in the model are significant at the 0.1500 level.

Summary of Backward Elimination

Step	Variable Entered	Variable Removed	Number Vars In	Partial R-Square	Model R-Square	C(p)	F Value	Pr > F
1 2	artistFamIdx	artistHotIdx	5 6	0.0048 0.0048	0.4282 0.4329	5.1931 7.0000	0.19 0.19	0.6645 0.6645
3		artistFamIdx	5	0.0048	0.4282	5.1931	0.19	0.6645
4		tempo	4	0.0191	0.4091	3.9659	0.80	0.3801
		=	The S	SAS System	11:21	Friday.	December 4.	2015 110

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	4 266.88619 25 385.44016 29 652.32638		66.72155 15.41761	4.33	0.0085
Root M Depende Coeff	ent Mean	3.92653 6.07712 64 61168	R-Square Adj R-Sq	0.4091 0.3146	

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	-359.86178	240.68605	-1.50	0.1474		0
aIndex	1	-49.33357	16.00210	-3.08	0.0049	0.82918	1.20602
duration	1	0.03794	0.01243	3.05	0.0053	0.65684	1.52244
loudness	1	-0.20173	0.10865	-1.86	0.0752	0.73197	1.36617
vear	1	0.18338	0.11983	1.53	0.1385	0.93950	1.06439

Fit Diagnostics for hIndexRT 1 0 0.1 0.2 0.3 0.4 0.5 7.5 7.5 12.5 2.5 12.5 2.5 Predicted Value Predicted Value Leverage 0.5 10 0.4 Cook's D 0.3 0.2 0.1 o Quantile 10 15 10 15 20 25 30 Predicted Value Observation Residual Fit-Mean 30 Observations Percent 20 -Parameters Error DF 15.418 MSE Adj R-Square 0.3146 Residual Proportion Less

FIG. 9. SAS Reg Procedure

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Stepwise Selection: Step 1

Variable aIndex Entered: R-Square = 0.1528 and C(p) = 8.3621

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue Pr > F
Model Error Corrected Total	1 28 29	99.68574 552.64062 652.32635	99.68574 19.73716	5.	05 0.0327
Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept aIndex	13.58603 -37.05174	3.43825 16.48673	308.17345 99.68574	15.61 5.05	0.0005 0.0327
	Bounds on	condition nu	ımber: 1, 1		

Stepwise Selection: Step 2

Variable duration Entered: R-Square = 0.2940 and C(p) = 4.6362

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	191.77541	95.88770	5.62	0.0091

Error 27 460.55095 17.05744 Corrected Total 29 652.32635

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The REG Procedure
Model: MODEL1
Dependent Variable: hIndexRT

Stepwise Selection: Step 2

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	11.33712	3.33966	196.56871	11.52	0.0021
aIndex	-52.34084	16.67951	167.96885	9.85	0.0041
duration	0.02680	0.01153	92.08967	5.40	0.0279

Bounds on condition number: 1.1843, 4.7373

Stepwise Selection: Step 3

Variable loudness Entered: R-Square = 0.3538 and C(p) = 4.2108

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue	Pr > F
Model Error Corrected Total	3 26 29	230.78165 421.54471 652.32635	76.92722 16.21326	4.	74	0.0091
Variable	Parameter Estimate	Standard Error	Type II SS I	F Value	Pr >	F
Intercept aIndex duration loudness	8.41379 -52.60802 0.03586 -0.16955	3.76212 16.26244 0.01267 0.10931	81.09422 169.66905 129.85152 39.00624	5.00 10.46 8.01 2.41	0.034 0.003 0.003 0.133	33 39

Bounds on condition number: 1.5043, 12.011

Stepwise Selection: Step 4

Variable year Entered: R-Square = 0.4091 and C(p) = 3.9659

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The REG Procedure
Model: MODEL1
Dependent Variable: hIndexRT

Stepwise Selection: Step 4

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	e Pr > F
Model Error Corrected Total	4 25 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.3	3 0.0085
Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept aIndex duration loudness year	-359.86178 -49.33357 0.03794 -0.20173 0.18338	240.68605 16.00210 0.01243 0.10865 0.11983	34.46566 146.53726 143.58644 53.14988 36.10455	9.50 9.31 3.45	0.1474 0.0049 0.0053 0.0752 0.1385

Bounds on condition number: 1.5224, 20.636

All variables left in the model are significant at the 0.1500 level.

No other variable met the 0.1500 significance level for entry into the model.

Summary of Stepwise Selection

Step	Variable Entered	Variable Removed	Number Vars In	Partial R-Square	Model R-Square	C(p)	F Value	Pr > F
1	aIndex		1	0.1528	0.1528	8.3621	5.05	0.0327
2	duration		2	0.1412	0.2940	4.6362	5.40	0.0279
3	loudness		3	0.0598	0.3538	4.2108	2.41	0.1330
4	vear		4	0.0553	0.4091	3.9659	2.34	0.1385
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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	4 25 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.33	0.0085
Dep	t MSE endent Mean ff Var	3.92653 6.07712 64.61168	R-Square Adj R-Sq	0.4091 0.3146	

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	-359.86178	240.68605	-1.50	0.1474		0
aIndex	1	-49.33357	16.00210	-3.08	0.0049	0.82918	1.20602
duration	1	0.03794	0.01243	3.05	0.0053	0.65684	1.52244
loudness	1	-0.20173	0.10865	-1.86	0.0752	0.73197	1.36617
year	1	0.18338	0.11983	1.53	0.1385	0.93950	1.06439

Fit Diagnostics for hIndexRT RStudent 1 0 7.5 7.5 12.5 0.2 0.3 0.4 0.5 2.5 12.5 2.5 Predicted Value Predicted Value Leverage 10 0.5 0.4 Residual Cook's D 0.3 10 0 0.2 0.1 o 10 15 10 15 20 25 30 Quantile Predicted Value Observation Residual Fit-Mean 30 Observations Percent 20 -Parameters Error DF MSE 15 418 Adj R-Square 0.3146 Ó Residual Proportion Less

FIG. 10. SAS Reg Procedure

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Stepwise Selection: Step 1

Variable artistFamIdx Entered: R-Square = 0.1472 and C(p) = 8.5912

Analysis of Variance

DF	Sum of Squares	Mean Square	F Val	ue Pr > F
1 28 29	96.00114 556.32521 652.32635	96.00114 19.86876	4.	83 0.0364
Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
18.49447 -19.77628	5.70738 8.99688	208.63207 96.00114	10.50 4.83	0.0031 0.0364
	1 28 29 Parameter Estimate 18.49447 -19.77628	DF Squares 1 96.00114 28 556.32521 29 652.32635 Parameter Standard Estimate Error 18.49447 5.70738 -19.77628 8.99688	DF Squares Square 1 96.00114 96.00114 28 556.32521 19.86876 29 652.32635 Parameter Standard Estimate Error Type II SS 18.49447 5.70738 208.63207	DF Squares Square F Val 1 96.00114 96.00114 4. 28 556.32521 19.86876 29 652.32635 Parameter Standard Error Type II SS F Value 18.49447 5.70738 208.63207 10.50 -19.77628 8.99688 96.00114 4.83

All variables left in the model are significant at the 0.1500 level.

No other variable met the 0.1500 significance level for entry into the model.

Summary of Stepwise Selection

1 artistFamIdx

1 0.1472 0.1472 8.5912 4.83 0.0364 The SAS System 11:21 Friday, December 4, 2015 116

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

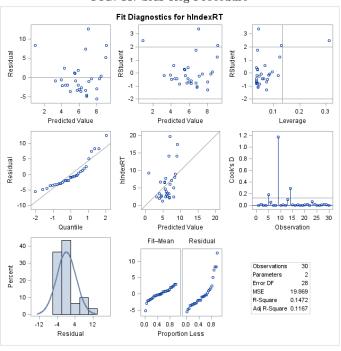
Analysis of Variance

Source		DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected To	otal	1 28 29	96.00114 556.32521 652.32635	96.00114 19.86876	4.83	0.0364
	Root MSE Dependent M Coeff Var	¶ean	4.45744 6.07712 73.34794	R-Square Adj R-Sq	0.1472 0.1167	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	18.49447	5.70738	3.24	0.0031	1.00000	0
artistFamIdx	1	-19.77628	8.99688	-2.20	0.0364		1.00000

FIG. 11. SAS Reg Procedure



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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Maximum R-Square Improvement: Step 1

Variable aIndex Entered: R-Square = 0.1528 and C(p) = 8.3621

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue	Pr > F
Model Error Corrected Total	1 28 29	99.68574 552.64062 652.32635	99.68574 19.73716	5.0	05	0.0327
Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr >	F
Intercept aIndex	13.58603 -37.05174	3.43825 16.48673	308.17345 99.68574	15.61 5.05	0.000 0.032	-

Bounds on condition number: 1, 1

The above model is the best 1-variable model found.

Maximum R-Square Improvement: Step 2

Variable duration Entered: R-Square = 0.2940 and C(p) = 4.6362

Analysis of Variance

Source	DF	Squares	Square	F Value	Pr > F	
Model Error	2 27	191.77541 460.55095	95.88770 17.05744	5.62	0.0091	
Corrected Total	29	652.32635 The SAS System		Fridav. I	December 4.	2015 118

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

 ${\tt Maximum}\ {\tt R-Square}\ {\tt Improvement:}\ {\tt Step}\ {\tt 2}$

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	11.33712	3.33966	196.56871		0.0021
aIndex	-52.34084	16.67951	167.96885		0.0041
duration	0.02680	0.01153	92.08967		0.0279

Bounds on condition number: 1.1843, 4.7373

The above model is the best 2-variable model found.

 ${\tt Maximum} \ {\tt R-Square} \ {\tt Improvement:} \ {\tt Step} \ {\tt 3}$

Variable loudness Entered: R-Square = 0.3538 and C(p) = 4.2108

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Val	ue	Pr > F
Model Error Corrected Total	3 26 29	230.78165 421.54471 652.32635	76.92722 16.21326	4.	74	0.0091
Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr >	F
Intercept aIndex duration	8.41379 -52.60802 0.03586	3.76212 16.26244 0.01267	81.09422 169.66905 129.85152	5.00 10.46 8.01	0.034 0.003 0.008	33

loudness -0.16955 0.10931 39.00624 2.41 0.1330

Bounds on condition number: 1.5043, 12.011

The above model is the best 3-variable model found.

The REG Procedure
Model: MODEL1
Dependent Variable: hIndexRT

Maximum R-Square Improvement: Step 4

Variable year Entered: R-Square = 0.4091 and C(p) = 3.9659

Analysis of Variance

		Sum of	Mean		
Source	DF	Squares	Square	F Valu	ie Pr > F
Model Error Corrected Total	4 25 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.3	0.0085
Variable	Parameter Estimate	Standard Error	Type II SS F	'Value	Pr > F
Intercept aIndex duration loudness year	-359.86178 -49.33357 0.03794 -0.20173 0.18338	240.68605 16.00210 0.01243 0.10865 0.11983	34.46566 146.53726 143.58644 53.14988 36.10455	9.50 9.31 3.45	0.1474 0.0049 0.0053 0.0752 0.1385
	Bounds on cond	dition number:	: 1.5224, 20.63	6	

The above model is the best 4-variable model found.

 ${\tt Maximum} \ {\tt R-Square} \ {\tt Improvement:} \ {\tt Step} \ {\tt 5}$

Variable tempo Entered: R-Square = 0.4282 and C(p) = 5.1931

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	279.31564	55.86313	3.59	0.0144
Error	24	373.01071	15.54211		
Corrected Total	29	652.32635 The SAS System	11:21	Friday,	December 4, 2015 120

The REG Procedure
Model: MODEL1
Dependent Variable: hIndexRT

 ${\tt Maximum} \ {\tt R-Square} \ {\tt Improvement:} \ {\tt Step} \ {\tt 5}$

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	-292.82758	253.01479	20.81815	1.34	0.2585
aIndex	-54.32265	17.00763	158.55667	10.20	0.0039
duration	0.04067	0.01285	155.68248	10.02	0.0042
loudness	-0.21670	0.11036	59.91985	3.86	0.0613
tempo	-0.02419	0.02705	12.42945	0.80	0.3801
year	0.15158	0.12546	22.68623	1.46	0.2388

Bounds on condition number: 1.6135, 33.784

The above model is the best 5-variable model found.

 ${\tt Maximum} \ {\tt R-Square} \ {\tt Improvement:} \ {\tt Step} \ {\tt 6}$

Variable artistFamIdx Entered: R-Square = 0.4329 and C(p) = 7.0000

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	6 23 29	282.42099 369.90536 652.32635	47.07017 16.08284	2.93	0.0286
Variable	Parameter Estimate	Standard Error	Type II SS I	F Value P	r > F
Intercept aIndex artistFamIdx duration loudness tempo year	-307.66834 -61.03969 5.04011 0.04383 -0.23531 -0.02541 0.15779	259.58501 23.08670 11.47008 0.01492 0.11999 0.02765 0.12840	22.59275 112.42523 3.10535 138.72475 61.85042 13.57824 24.28572	6.99 0 0.19 0 8.63 0 3.85 0 0.84 0	. 2480 . 0145 . 6645 . 0074 . 0621 . 3677 . 2316

Bounds on condition number: 2.4064, 63.215

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

 ${\tt Maximum} \ {\tt R-Square} \ {\tt Improvement:} \ {\tt Step} \ {\tt 6}$

The above model is the best 6-variable model found.

No further improvement in R-Square is possible. The SAS System $$11\!:\!21$$ Friday, December 4, 2015 122

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	6 23 29	282.42099 369.90536 652.32635	47.07017 16.08284	2.93	0.0286
De	oot MSE ependent Mean oeff Var	4.01034 6.07712 65.99088	R-Square Adj R-Sq	0.4329 0.2850	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	-307.66834	259.58501	-1.19	0.2480	•	0
aIndex	1	-61.03969	23.08670	-2.64	0.0145	0.41555	2.40645
artistFamIdx	1	5.04011	11.47008	0.44	0.6645	0.49802	2.00797
duration	1	0.04383	0.01492	2.94	0.0074	0.47537	2.10364
loudness	1	-0.23531	0.11999	-1.96	0.0621	0.62601	1.59741
tempo	1	-0.02541	0.02765	-0.92	0.3677	0.80076	1.24881
vear	1	0.15779	0.12840	1.23	0.2316	0.85355	1.17158

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Fit Diagnostics for hIndexRT Residual RStudent 0 -1 2.5 5.0 7.5 10.0 12.5 0.1 0.2 0.3 0.4 0.5 2.5 5.0 7.5 10.0 12.5 Predicted Value Predicted Value 0.3 Cook's D 0.2 10 0 0.1 5 10 15 Predicted Value 5 10 15 20 25 30 Observation 0 Quantile Residual Fit-Mean 40 10 -30 -20 -30 7 23 Observations 5 Parameters
Error DF
MSE
R-Square 0 16.083 0.4329 10 Adj R-Square 0.285 -10 -6 -2 2 6 10 14 0.0 0.4 0.8 0.0 0.4 0.8 Residual Proportion Less

FIG. 12. SAS Reg Procedure

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Adjusted R-Square Selection Method

Number of Observations Read 30 Number of Observations Used 30

Number in Model	Adjusted R-Square	R-Square	Variables in Model
Hodel	it bquare	it bquare	Valiables in Model
4	0.3146	0.4091	aIndex duration loudness year
5	0.3091	0.4282	aIndex duration loudness tempo year
4	0.2964	0.3934	aIndex duration loudness tempo
5	0.2897	0.4121	aIndex artistFamIdx duration loudness year
5	0.2897	0.4121	aIndex artistHotIdx duration loudness year
5	0.2897		artistHotIdx artistFamIdx duration loudness year
6	0.2850	0.4329	
6	0.2850	0.4329	aIndex artistHotIdx duration loudness tempo year
6	0.2850	0.4329	artistHotIdx artistFamIdx duration loudness tempo year
3	0.2792	0.3538	
5	0.2698	0.3957	
5	0.2698	0.3957	
5	0.2698	0.3957	artistHotIdx artistFamIdx duration loudness tempo
4	0.2509	0.3542	
4	0.2509	0.3542	
4	0.2509	0.3542	artistHotIdx artistFamIdx duration loudness
3	0.2501	0.3277	aIndex duration year
2	0.2417	0.2940	aIndex duration
3	0.2362	0.3152	aIndex duration tempo
4	0.2301	0.3363	aIndex duration tempo year
4	0.2226	0.3298	aIndex artistHotIdx duration year
4	0.2226	0.3298	aIndex artistFamIdx duration year
4	0.2226	0.3298	artistHotIdx artistFamIdx duration year
3	0.2164	0.2975	aIndex artistFamIdx duration
3	0.2164	0.2975	aIndex artistHotIdx duration
3	0.2164	0.2975	artistHotIdx artistFamIdx duration
4	0.2086	0.3177	aIndex artistFamIdx duration tempo
4	0.2086	0.3177	aIndex artistHotIdx duration tempo
4	0.2086	0.3177	artistHotIdx artistFamIdx duration tempo
5	0.2002	0.3381	aIndex artistHotIdx duration tempo year
5	0.2002	0.3381	aIndex artistFamIdx duration tempo year
5	0.2002	0.3381	artistHotIdx artistFamIdx duration tempo year

4	0.1420	0.2603	artistFamIdx duration loudness year	•
2	0.1295	0.1895	aIndex artistFamIdx	
2	0.1295	0.1895	aIndex artistHotIdx	
2	0.1295	0.1895	artistHotIdx artistFamIdx	
3	0.1271	0.2174	artistFamIdx duration year	
3	0.1263	0.2167	aIndex artistHotIdx year	
3	0.1263	0.2167	aIndex artistFamIdx year	
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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Adjusted R-Square Selection Method

Number in	Adjusted		
Model	R-Square	R-Square	Variables in Model
3	0.1263	0.2167	artistHotIdx artistFamIdx year
2	0.1232	0.1837	aIndex year
1	0.1226	0.1528	aIndex
2	0.1199	0.1806	artistFamIdx year
2	0.1170	0.1779	artistFamIdx duration
1	0.1167	0.1472	artistFamIdx
3	0.1097	0.2018	artistFamIdx duration loudness
5	0.1066	0.2606	artistFamIdx duration loudness tempo year
3	0.1023	0.1952	aIndex artistHotIdx tempo
3	0.1023	0.1952	aIndex artistFamIdx tempo
3	0.1023	0.1952	artistHotIdx artistFamIdx tempo
3	0.0967	0.1901	aIndex artistFamIdx loudness
3	0.0967	0.1901	aIndex artistHotIdx loudness
3	0.0967	0.1901	artistHotIdx artistFamIdx loudness
2	0.0958	0.1581	aIndex tempo
3	0.0954	0.1890	aIndex loudness year
4	0.0947	0.2195	aIndex artistHotIdx loudness year
4	0.0947	0.2195	aIndex_artistFamIdx_loudness_year
4	0.0947	0.2195	artistHotIdx artistFamIdx loudness year
3	0.0923	0.1862	artistFamIdx loudness year
4	0.0923	0.2175	artistFamIdx duration tempo year
2	0.0921	0.1547	aIndex loudness
4	0.0921	0.2173	aIndex artistHotIdx tempo year
4	0.0921	0.2173	aIndex artistFamIdx tempo year
4	0.0921	0.2173	artistHotIdx artistFamIdx tempo year
3	0.0904	0.1845	duration loudness year
3	0.0899	0.1840	aIndex tempo year
3	0.0864	0.1809	artistFamIdx tempo year
2 3	0.0861	0.1491	artistFamIdx loudness
2	0.0860	0.1805	artistFamIdx duration tempo
4	0.0855 0.0811	0.1486 0.2078	artistFamIdx tempo
4	0.0676	0.2078	artistFamIdx duration loudness tempo
4	0.0676	0.1962	aIndex artistFamIdx loudness tempo
4	0.0676	0.1962	aIndex artistHotIdx loudness tempo artistHotIdx artistFamIdx loudness tempo
3	0.0638	0.1902	aIndex loudness tempo
4	0.0599	0.1895	aIndex loudness tempo year
5	0.0578	0.2203	aIndex artistHotIdx loudness tempo year
5	0.0578	0.2203	aIndex artistFamIdx loudness tempo year
5	0.0578	0.2203	artistHotIdx artistFamIdx loudness tempo year
4	0.0561	0.1863	artistFamIdx loudness tempo year
4	0.0557	0.1859	artistHotIdx duration loudness year
4	0.0547	0.1851	duration loudness tempo year
3	0.0530	0.1510	artistFamIdx loudness tempo
J	0.0000	0.1010	The SAS System 11:21 Friday, December 4, 2015 125

The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Adjusted R-Square Selection Method

Number in Model	Adjusted R-Square	R-Square	Variables in Model
2	0.0320	0.0987	duration year
2	0.0265	0.0937	duration loudness
2	0.0255	0.0927	artistHotIdx year
1	0.0208	0.0545	year
5	0.0172	0.1866	artistHotIdx duration loudness tempo year
3	0.0151	0.1170	artistHotIdx duration year
1	0.0101	0.0442	artistHotIdx
2	0.0075	0.0760	loudness year
3	0.0054	0.1083	artistHotIdx loudness year
1	0.0021	0.0365	duration

3	0001	0.1034	duration tempo year
3	0023	0.1014	artistHotIdx duration loudness
3	0060	0.0981	artistHotIdx tempo year
2	0080	0.0615	artistHotIdx duration
2	0085	0.0611	tempo year
3	0086	0.0957	
2	0172	0.0530	
4	0194	0.1212	artistHotIdx duration tempo year
1	0220	0.0132	loudness
3	0256	0.0805	loudness tempo year
2	0264	0.0444	artistHotIdx tempo
4	0299	0.1121	artistHotIdx loudness tempo year
2	0349	0.0365	duration tempo
1	0354	0.0003	tempo
4	0406	0.1030	
3	0467	0.0615	artistHotIdx duration tempo
3	0563	0.0530	
2	0598	0.0132	loudness tempo

NOTE: Models of not full rank are not included.

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	4 25 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.33	0.0085
	MSE ndent Mean f Var	3.92653 6.07712 64.61168	R-Square Adj R-Sq	0.4091 0.3146	

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	-359.86178	240.68605	-1.50	0.1474		0
aIndex	1	-49.33357	16.00210	-3.08	0.0049	0.82918	1.20602
duration	1	0.03794	0.01243	3.05	0.0053	0.65684	1.52244
loudness	1	-0.20173	0.10865	-1.86	0.0752	0.73197	1.36617
wear	1	0 18338	0 11983	1 53	0 1385	0 93950	1 06439

Fit Diagnostics for hIndexRT RStudent 1 0 7.5 7.5 12.5 0.1 0.2 0.3 0.4 0.5 12.5 2.5 2.5 Predicted Value Predicted Value Leverage 20 0.5 10 0.4 Cook's D 0.3 10 0.2 0.1 0.0 0 Quantile 5 10 15 Predicted Value 10 15 20 25 30 Observation Residual Fit-Mean 30 30 5 25 15.418 Observations Percent 20 -Parameters
Error DF
MSE
R-Square R-Square 0.4091 Adj R-Square 0.3146 -12 Ó Residual Proportion Less

FIG. 13. SAS Reg Procedure

The SAS System

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

C(p) Selection Method

Number of Observations Read 30 Number of Observations Used 30

Number in			
Model	C(p)	R-Square	Variables in Model
4	0.0050	0 4004	
4	3.9659	0.4091	aIndex duration loudness year
3	4.2108		aIndex duration loudness
4	4.6037	0.3934	
2	4.6362		
5	5.1931		1 3
3	5.2707		
3	5.7757	0.3152	aIndex duration tempo
5	5.8443	0.4121	aIndex artistFamIdx duration loudness year
5	5.8443	0.4121	aIndex artistHotIdx duration loudness year
5	5.8443	0.4121	artistHotIdx artistFamIdx duration loudness year
4	6.1937	0.3542	aIndex artistHotIdx duration loudness
4	6.1937	0.3542	aIndex artistFamIdx duration loudness
4	6.1937	0.3542	artistHotIdx artistFamIdx duration loudness
3	6.4943	0.2975	aIndex artistFamIdx duration
3	6.4943	0.2975	aIndex artistHotIdx duration
3	6.4943	0.2975	artistHotIdx artistFamIdx duration
5	6.5100	0.3957	aIndex artistHotIdx duration loudness tempo
5	6.5100	0.3957	
5	6.5100	0.3957	artistHotIdx artistFamIdx duration loudness tempo
4	6.9188	0.3363	aIndex duration tempo year
6	7.0000	0.4329	aIndex artistFamIdx duration loudness tempo year
6	7.0000	0.4329	aIndex artistHotIdx duration loudness tempo year
6	7.0000	0.4329	
4	7.1839	0.3298	aIndex artistHotIdx duration year
4	7.1839		aIndex artistFamIdx duration year
4	7.1839		artistHotIdx artistFamIdx duration year
4	7.6731		aIndex artistFamIdx duration tempo
4	7.6731		
4	7.6731		artistHotIdx artistFamIdx duration tempo
1	8.3621	0.1528	aIndex

```
1
              8.5912
                          0.1472
                                    artistFamIdx
       5
              8.8457
                          0.3381
                                    aIndex artistHotIdx duration tempo year
       5
              8.8457
                          0.3381
                                    aIndex artistFamIdx duration tempo year
       5
              8.8457
                          0.3381
                                    artistHotIdx artistFamIdx duration tempo year
       2
              8.8733
                          0.1895
                                    aIndex artistFamIdx
       2
              8.8733
                          0.1895
                                    aIndex artistHotIdx
       2
              8.8733
                          0.1895
                                    artistHotIdx artistFamIdx
       2
              9.1107
                          0.1837
                                    aIndex year
              9.2340
                          0.1806
                                    artistFamIdx year
                                           The SAS System
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                                         The REG Procedure
                                           Model: MODEL1
                                   Dependent Variable: hIndexRT
                                       C(p) Selection Method
Number in
  Model
                        R-Square
                                   Variables in Model
                C(p)
              9.3430
                          0.1779
                                    artistFamIdx duration
              9.7414
                                    artistFamIdx duration year
                          0.2174
              9.7725
                          0.2167
                                    aIndex artistHotIdx year
              9.7725
                          0.2167
                                    aIndex artistFamIdx year
              9.7725
                          0.2167
                                    artistHotIdx artistFamIdx year
              10.0006
                          0.2603
                                    artistFamIdx duration loudness year
             10.1462
                          0.1581
                                    aIndex tempo
             10.2848
                          0.1547
                                    aIndex loudness
       3
             10.3736
                          0.2018
                                    artistFamIdx duration loudness
              10.5108
                          0.1491
                                    artistFamIdx loudness
             10.5338
                          0.1486
                                    artistFamIdx tempo
             10.6439
                          0.1952
                                    aIndex artistHotIdx tempo
       3
                                    aIndex artistFamIdx tempo
             10.6439
                          0.1952
       3
              10.6439
                          0.1952
                                    artistHotIdx artistFamIdx tempo
       3
             10.8479
                          0.1901
                                    aIndex artistFamIdx loudness
       3
             10.8479
                          0.1901
                                    aIndex artistHotIdx loudness
                                    artistHotIdx artistFamIdx loudness
             10.8479
                          0.1901
       3
       3
             10.8939
                          0.1890
                                    aIndex loudness year artistFamIdx loudness year
       3
             11,0088
                          0.1862
       3
             11.0773
                          0.1845
                                    duration loudness year
                          0.1840
       3
             11.0953
                                    aIndex tempo year
                                    artistFamIdx tempo year artistFamIdx duration tempo
                          0.1809
       3
             11.2238
       3
             11.2385
                          0.1805
             11.6559
       4
                          0.2195
                                    aIndex artistHotIdx loudness year
             11.6559
                          0.2195
                                    aIndex artistFamIdx loudness year
             11.6559
       4
                          0.2195
                                    artistHotIdx artistFamIdx loudness year
             11.7400
                          0.2175
                                    artistFamIdx duration tempo year
       4
             11.7467
                          0.2173
                                    \verb|aIndex| \verb|artistHotIdx| \verb|tempo| | \verb|year|
             11.7467
       4
                          0.2173
                                    aIndex artistFamIdx tempo year
             11.7467
                          0.2173
                                    artistHotIdx artistFamIdx tempo year
       5
             11.9904
                          0.2606
                                    artistFamIdx duration loudness tempo year
             12.0431
                          0.1607
       3
                                    aIndex loudness tempo
       4
             12.1315
                          0.2078
                                    artistFamIdx duration loudness tempo
       1
             12.3487
                          0.0545
                                    year
       3
             12.4376
                          0.1510
                                    artistFamIdx loudness tempo
             12.5562
                          0.0987
                                    duration year
       2
       4
             12.6029
                          0.1962
                                    aIndex artistFamIdx loudness tempo
       4
             12.6029
                          0.1962
                                    aIndex artistHotIdx loudness tempo
             12.6029
                          0.1962
                                    artistHotIdx artistFamIdx loudness tempo
       2
             12.7605
                          0.0937
                                    duration loudness
             12.7670
                          0.0442
                                    artistHotIdx
       2
             12.7985
                          0.0927
                                    artistHotIdx year
             12.8731
                          0.1895
                                    aIndex loudness tempo year
             13.0037
                          0.1863
                                    artistFamIdx loudness tempo year
                                           The SAS System
                                                                   11:21 Friday, December 4, 2015 129
                                         The REG Procedure
                                           Model: MODEL1
                                   Dependent Variable: hIndexRT
                                       C(p) Selection Method
Number in
  Model
                C(p)
                        R-Square
                                    Variables in Model
             13.0186
                          0.1859
                                    artistHotIdx duration loudness year
       4
              13.0518
                          0.1851
                                    duration loudness tempo year
              13.0801
                          0.0365
                                    duration
              13.4786
                          0.0760
                                    loudness year
             13.6256
                          0.2203
                                    aIndex artistHotIdx loudness tempo year
       5
             13.6256
                          0.2203
                                    aIndex artistFamIdx loudness tempo year
                                    artistHotIdx artistFamIdx loudness tempo year
       5
             13.6256
                          0.2203
             13.8155
                          0.1170
                                    artistHotIdx duration year
```

1	14.0233	0.0132	loudness
2	14.0645		
2	14.0837	0.0611	tempo year
3	14.1664	0.1083	
3	14.3672	0.1034	duration tempo year
2	14.4127	0.0530	artistHotIdx loudness
3	14.4476	0.1014	artistHotIdx duration loudness
1	14.5496	0.0003	tempo
3	14.5830	0.0981	artistHotIdx tempo year
3	14.6775	0.0957	duration loudness tempo
2	14.7615	0.0444	artistHotIdx tempo -
5	14.9907	0.1866	artistHotIdx duration loudness tempo year
2	15.0801	0.0365	duration tempo
3	15.2971	0.0805	loudness tempo year
4	15.6431	0.1212	artistHotIdx duration tempo year
4	16.0129	0.1121	artistHotIdx loudness tempo year
2	16.0232	0.0132	loudness tempo
3	16.0644	0.0615	artistHotIdx duration tempo
4	16.3846	0.1030	artistHotIdx duration loudness tempo
3	16.4127	0.0530	artistHotIdx loudness tempo

NOTE: Models of not full rank are not included.

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Tota	25 al 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.33	0.0085
I	Root MSE Dependent Mean Coeff Var	3.92653 6.07712 64.61168	R-Square Adj R-Sq	0.4091 0.3146	

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Tolerance	Variance Inflation
Intercept	1	-359.86178	240.68605	-1.50	0.1474		0
aIndex	1	-49.33357	16.00210	-3.08	0.0049	0.82918	1.20602
duration	1	0.03794	0.01243	3.05	0.0053	0.65684	1.52244
loudness	1	-0.20173	0.10865	-1.86	0.0752	0.73197	1.36617
year	1	0.18338	0.11983	1.53	0.1385	0.93950	1.06439

Fit Diagnostics for hIndexRT RStudent 1 0 7.5 0.1 0.2 0.3 0.4 0.5 7.5 12.5 2.5 12.5 2.5 Predicted Value Predicted Value Leverage 20 10 0.5 0.4 Cook's D 0.3 10 0.2 0.1 5 10 15 Predicted Value 10 15 20 25 30 Observation o Quantile Residual Fit-Mean 30 30 5 25 15.418 Observations Percent 20 -Parameters Error DF MSE R-Square R-Square 0.4091 Adj R-Square 0.3146 Ó Residual Proportion Less

FIG. 14. SAS Reg Procedure

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The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

Number of Observations Read 30 Number of Observations Used 30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model Error Corrected Total	4 25 29	266.88619 385.44016 652.32635	66.72155 15.41761	4.33	0.0085
Root M Depend Coeff	lent Mean	3.92653 6.07712 64.61168	R-Square Adj R-Sq	0.4091 0.3146	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Type II SS	Tolerance
Intercept	1	-359.86178	240.68605	-1.50	0.1474	1107.93985	34.46566	
aIndex	1	-49.33357	16.00210	-3.08	0.0049	99.68574	146.53726	0.82918
duration	1	0.03794	0.01243	3.05	0.0053	92.08967	143.58644	0.65684
loudness	1	-0.20173	0.10865	-1.86	0.0752	39.00624	53.14988	0.73197
year	1	0.18338	0.11983	1.53	0.1385	36.10455	36.10455	0.93950

Variable	DF	Variance Inflation
Intercept	1	0
aIndex	1	1.20602
duration	1	1.52244
loudness	1	1.36617
year	1	1.06439

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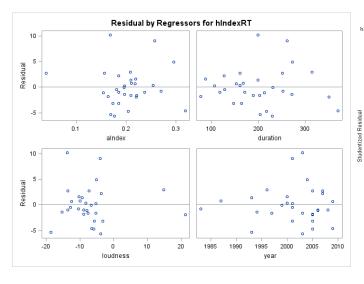
The REG Procedure Model: MODEL1 Dependent Variable: hIndexRT

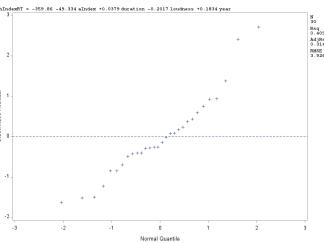
Collinearity Diagnostics

Number	Eigenvalue	Condition Index
1	4.33882	1.00000
2	0.57452	2.74810
3	0.05186	9.14715
4	0.03480	11.16570
5	0.00000445	987.19164

Collinearity Diagnostics

	Proportion of Variation						
Number	Intercept	aIndex	duration	loudness	year		
1	4.640779E-7	0.00235	0.00352	0.01119	4.673853E-7		
2	1.041306E-7	0.00223	0.01581	0.59225	1.058947E-7		
3	0.00002145	0.04168	0.94009	0.33724	0.00002186		
4	0.00003546	0.93455	0.02790	0.02039	0.00003627		
5	0.99994	0.01919	0.01267	0.03893	0.99994		





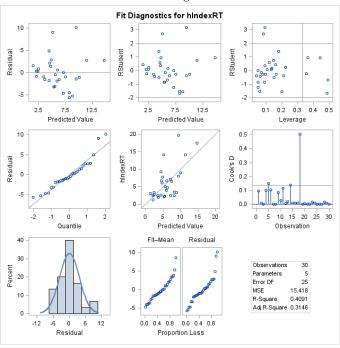


FIG. 15. SAS Reg Procedure

```
data songs;
infile '\\Client\H$\Dropbox\UNC\5\STOR 455\Songs.DAT' dlm='09'x;
input hotIdx artistHotIdx artistFamIdx duration loudness tempo year;
run;
proc print data=songs;
run;
proc means data=songs;
var hotIdx artistHotIdx artistFamIdx duration loudness tempo year;
proc gplot data=songs;
plot hotIdx*(artistHotIdx artistFamIdx duration loudness tempo year);
proc univariate data=songs alpha=0.5;
var hotIdx;
histogram / endpoints=0 to 1 by 0.1;
run:
proc corr data=songs;
var hotIdx artistHotIdx artistFamIdx duration loudness tempo year;
proc reg data=songs;
model hotIdx = artistHotIdx artistFamIdx duration loudness tempo year / ss1 ss2 VIF TOL r influence;
run;
data logSongs;
set songs;
logIndex = log(hotIdx);
run;
proc reg data=logSongs;
model logIndex = artistHotIdx artistFamIdx duration loudness tempo year / ss1 ss2 VIF TOL;
run;
proc transreg data=songs;
model BoxCox(hotIdx / convenient lambda=-2 to 2 by 0.05)
= qpoint(artistHotIdx artistFamIdx duration loudness tempo year);
run:
*Select lambda as -2:
data songsK;
set songs;
hIndexRT = hotIdx**-2;
run:
proc reg data=songsK;
model hIndexRT = artistHotIdx artistFamIdx duration loudness tempo year / ss1 ss2 VIF TOL;
run:
proc sgscatter data=songsK;
matrix hIndexRT artistHotIdx artistFamIdx duration loudness tempo year;
run;
data songsAdj;
set songsK;
aIndex = artistFamIdx - artistHotIdx;
run:
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year /
selection=FORWARD VIF TOL slentry=0.1;
run;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year / selection=B VIF TOL slstay=0.15;
run:
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year /
selection=STEPWISE VIF TOL slentry=0.15 slstay=0.15;
proc reg data=songsAdj;
model hIndexRT = artistHotIdx artistFamIdx duration loudness tempo year /
selection=STEPWISE VIF TOL slentry=0.15 slstay=0.15;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year /
selection=MAXR VIF TOL;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year /
selection=adjrsq VIF TOL;
proc reg data=songsAdj;
model hIndexRT = aIndex artistHotIdx artistFamIdx duration loudness tempo year / selection=cp VIF TOL;
proc reg data=songsAdj;
model hIndexRT = aIndex duration loudness year /
ss1 ss2 VIF TOL;
plot student.*nqq.;
run:
proc reg data=songsAdj;
model hIndexRT = aIndex duration loudness year /
```

ss1 ss2 VIF TOL collin;
run;