Formal Solutions To ODE's

1 Homogeneous Solutions

1.1 First Order

We have the following differential equation

$$y' + ay = 0.$$

Then

$$y = ce^{-at},$$

where c is determined by the initial condition.

1.2 Second Order

We have the following differential equation

$$y'' + ay' + by = 0. \tag{1}$$

Then we solve the following equation for r,

$$r^2 + ar + b = 0.$$

If $r = \alpha \pm i\beta$, then the general solution is

$$y = c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x)$$

where c_1 and c_2 are determined by the initial conditions.

If Equation 1 has two real solutions, r_1 and r_2, then

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

where c_1 and c_2 are determined by the initial conditions.

If Equation 1 has a real repeated root, r, then

$$y = c_1 e^{rx} + c_2 x e^{rx}$$

where c_1 and c_2 are determined by the initial conditions.