1. Why does l'Hôpital's rule apply to $\lim_{x\to 0} \frac{V^x-1}{x}$, and precisely why is this limit equal to $\lim_{x\to 0} \frac{V^x \ln V - 0}{1}$?

   l'Hôpital's rule applies since

   $$\lim_{x\to 0} \frac{V^x-1}{x} = \frac{V^0-1}{0} = \frac{1-1}{0} = \frac{0}{0}.$$

   Using l'Hôpital's rule yields

   $$\lim_{x\to 0} \frac{V^x \ln V - 0}{1}.$$

2. Use the Gompertz equation in the form $\frac{dV}{dt} = (a - b\ln V)V$ to explain why $lim_{t\to\infty} V(t) = e^{a/b}$.

   We can solve $\frac{dV}{dt} = (a - b\ln V)V$ for $V(t)$. We get

   $$V(t) = e^{\frac{a}{b} - \left(\frac{a}{b} - \ln V_0\right)e^{-bt}}.$$

   If we take the limit as $t \to \infty$, we get

   $$\begin{aligned} \lim_{t\to\infty} V(t) &= e^{\frac{a}{b} - \left(\frac{a}{b} - \ln V_0\right)(0)} \\ &= e^{\frac{a}{b}}. \end{aligned}$$

3. In solving the Gompertz differential equation, we assumed that $\int \frac{1}{u}\, du = \ln u + C$ rather than the more formally correct answer $\int \frac{1}{u}\, du = \ln|u| + C$. Were we safe in ignoring the absolute value signs?

   It was okay to ignore the absolute value signs since $u$ will always be positive. When solving the Gompertz equation, we had that $u = Ce^{-bt}$. The term $e^{-bt}$ is always positive, and our constant $C$ was determined by $V_0$, which is always positive. Therefore, $u > 0$, and $|u| = u$.

4. Show that the Gompertz curve has a single point of inflection at the time when $\ln V = \frac{a}{b} - 1$.

   The Gompertz equation has the form

   $$\frac{dV}{dt} = aV - bV\ln V.$$

   If we differentiate this again, we get

   $$\frac{d^2V}{dt^2} = a - b - b\ln V.$$

   Setting this equal to 0 yields

   $$\begin{aligned} 0 &= a - b - b\ln V \\ a &= b + b\ln V \\ \frac{a}{b} &= 1 + \ln V \\ \ln V &= \frac{a}{b} - 1. \end{aligned}$$

5. With the estimated parameter values for the Gompertz model of chicken growth, show that the predicted long-range limit to the size is 4.476.

   From the model, we know that $g(t) = 4.47e^{-3.41e^{-0.251t}}$. If we take the limit as $t \to \infty$, we have

   $$\lim_{t\to\infty} g(t) = 4.47e^{-3.41e^{0}} = 4.47.$$

6. Carry out the details of fitting the logistic model to the chicken weight data to show that $d = 3.155450907$ and $a = 0.4124054532$.