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***Homework 2***

1. Calculate $gcd(5,7)$ and find two pairs of integers $(u,v)$ such that $gcd(5,7) = 5u + 7v$.

   By the definition of the $gcd$, we have that $gcd(5,7) = 1$.

$$5(3) + 7(-2) = 1$$
$$5(10) + 7(-7) = 1$$

2. Show that $c|a$ and $c|b$ if and only if $c|gcd(a,b)$.

   Assume $c|a$ and $c|b$. Then $c$ is a common factor of $a$ and $b$. We know that $gcd(a,b)$ is also a common factor of both $a$ and $b$. Since both $gcd(a,b)$ and $c$ are common factors of $a$ and $b$, then they must also be common factors of each other. Therefore, $c|gcd(a,b)$.

   Let $d = gcb(a,b)$, and assume that $c|d$. Then $d = cn$ for some $n \in \mathbb{Z}$. By the definition of $gcd$, we know that $d|a$. Therefore, $cn|a$, which means that $a = cnm$ for some $m \in \mathbb{Z}$. Then $a = cj$, where $j = nm \in \mathbb{Z}$. Therefore $c|a$. We also know that $d|b$ by the definition of $gcd$. By the same argument, we know that $c|b$ as well. Therefore $c|b$ and $c|a$.

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3. Suppose $a_1, a_2, \ldots, a_n$ are integers not all equal to 0. We define $gcd(a_1, a_2, ..., a_n)$ to be the largest integer which divides $a_k$ for all $1 \leq k \leq n$. Prove that $gcd(a_1, a_2, \ldots, a_n) = gcd(gcd(a_1, a_2), a_3, ..., a_n)$.

   Let $gcd(a_1, a_2, \ldots, a_n) = d$ and $gcd(gcd(a_1, a_2), a_3, \ldots, a_n) = \alpha$. We know that, by the definition of $gcd$, that $d|a_k$ for $1 \leq k \leq n$. We know that $d|a_1$ and $d|a_2$, so it must also divide $gcd(a_1, a_2)$. Therefore, $d|gcd(gcd(a_1, a_2), a_3, \ldots, a_n)$, so $d|\alpha$. We also know that $\alpha|gcd(a_1, a_2)$, and that $\alpha|a_k$ for $2 < k \leq n$. Since $\alpha$ divides $gcd(a_1, a_2)$, it must divide $a_1$ and $a_2$. Therefore $\alpha|a_k$ for $1 \leq k \leq n$, so $\alpha|gcd(a_1, a_2, \ldots, a_n)$, or $\alpha|d$. We know that if $d|\alpha$ and $\alpha|d$, then $|d| = |\alpha|$. In this case, both $\alpha$ and $d$ are positive, so we know that $\alpha = d$.

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4. Use your answer to the previous problem, along with the Euclidian Algorithm, to determine $gcd(1092, 1155, 2002)$ (It's possible that you'll need a calculator to do the arithmetic on this problem).