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Homework 2

1. Order these functions from slowest to fastest order of growth, indicating which are the same order. Logarithms are all base 2.

$$\begin{array}{ll} 5n^3 - n^2 + n & O(n^3) \\ 6 & O(1) \\ 2^{n-1} & O(2^n) \\ log(log(n)) & O(log(n)) \\ 2^n & O(2^n) \\ 3^n & O(3^n) \\ (log(n))^2 & O(n\ log(n)) \\ n\ log(n) & O(n\ log(n)) \end{array}$$

$$6 \leq log(log(n)) \leq (log(n))^2 \leq n\ log(n) \leq 5n^3 - n^2 + n \leq 2^{n-1} \text{ and } 2^n \leq 3^n$$

2. Find two constants c, c' such that, for large enough n, we have

$$c\ log(n) \leq log(2n^2) \leq c'\ log(n)$$

We can simplify the middle term so that the n is linear for each log.

$$c\ log(n) \leq 2log(2n) \leq c'\ log(n)$$
$$c\ log(n) \leq 2(log(n) + log(2)) \leq c'\ log(n)$$
$$c\ log(n) \leq 2 + 2log(n) \leq c'\ log(n)$$

If we let $c = 1$ and $c' = 4$, then for large enough n, the inequality will hold.

3. How large must n be to have $n > 13\sqrt{n}$?

We can solve the inequality for n.

$$n > 13\sqrt{n}$$
$$n - 13\sqrt{n} > 0$$
$$\sqrt{n}(\sqrt{n} - 13) > 0$$

We know that $n > 0$ is not a valid, since $n = 1$ breaks the inequality. However, we also have that $\sqrt{n} - 13 > 0$, which tells us that n must be greater than 13^2.