

# Modeling Zombies and Infection

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Ricky Marske and Steven Rosendahl

## A Simple Model

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  - Left with the system

$$\begin{cases} S' = -\beta(t)SI \\ I' = \beta(t)SI - \nu I \\ R' = \nu I \end{cases}$$

## **Adding Complexity**

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We will look at each one of these additional features individually

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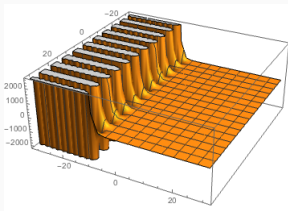
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$$Z(x, t) = \frac{Z_0}{L} + \sum_{n=1}^{\infty} \frac{2Z_0}{n\pi} \sin\left(\frac{n\pi}{L}\right) \cos\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 Dt}$$

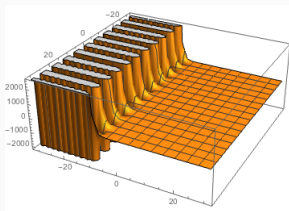


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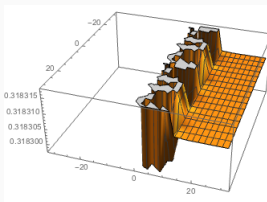


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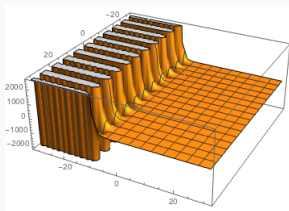


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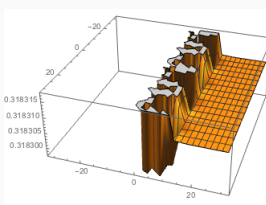


$n = 100$

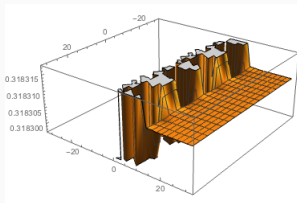
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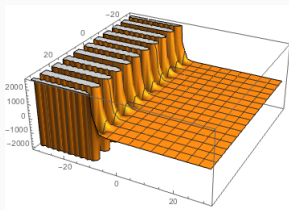


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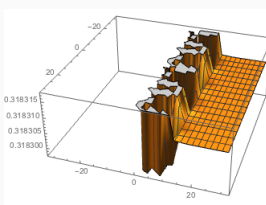


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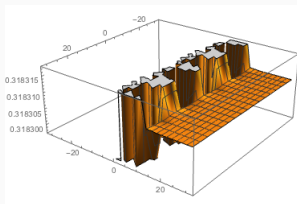
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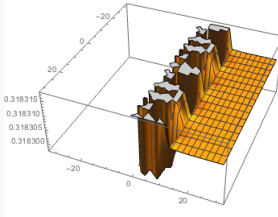
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- This behavior can be modeled using NetLogo

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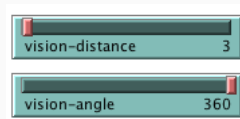
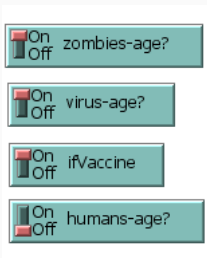
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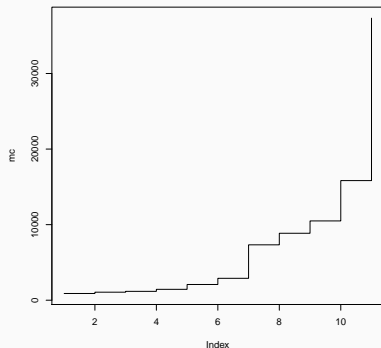
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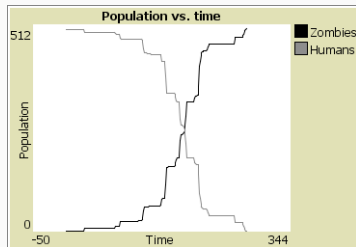
$$y = 3122.33 + 0.580e^x$$

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Data From Twitch



Data From NetLogo



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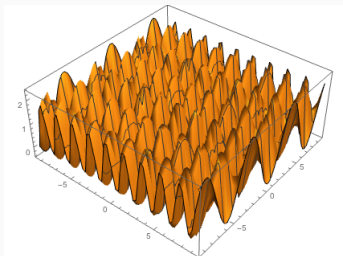
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- This yields the solution

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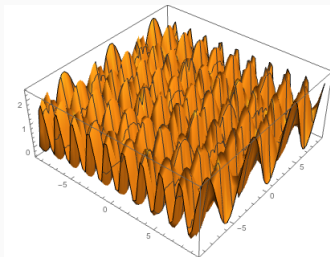
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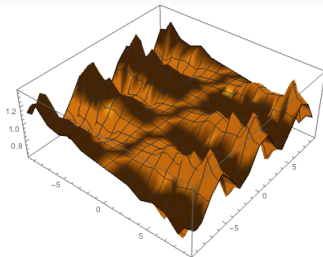
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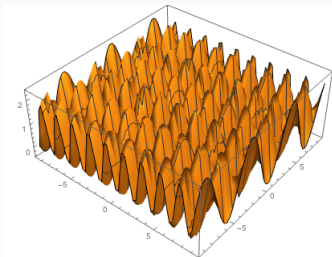


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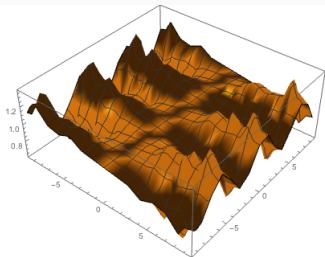


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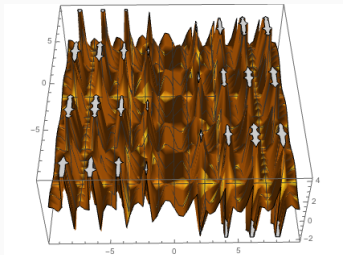
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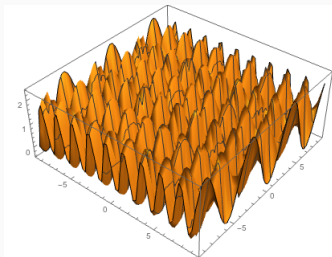


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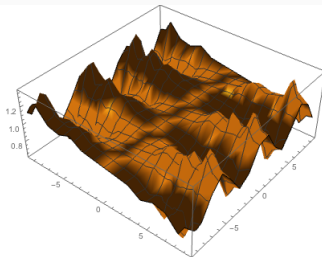


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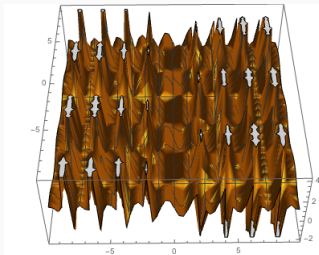
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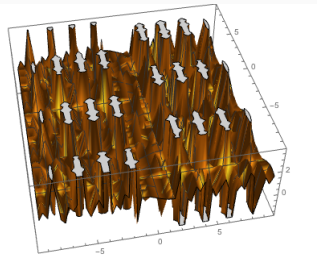
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- There is a saddle regardless of  $\zeta$
- The system will move towards the saddle no matter what

