

# Formal Solutions To ODE's

## 1 Homogeneous Solutions

### 1.1 First Order

We have the following differential equation

$$y' + ay = 0.$$

Then

$$y = ce^{-at},$$

where  $c$  is determined by the initial condition.

### 1.2 Second Order

We have the following differential equation

$$y'' + ay' + by = 0. \tag{1}$$

Then we solve the following equation for  $r$ ,

$$r^2 + ar + b = 0.$$

If  $r = \alpha \pm i\beta$ , then the general solution is

$$y = c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x)$$

where  $c_1$  and  $c_2$  are determined by the initial conditions.

If Equation 1 has two real solutions,  $r_1$  and  $r_2$ , then

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

where  $c_1$  and  $c_2$  are determined by the initial conditions.

If Equation 1 has a real repeated root,  $r$ , then

$$y = c_1 e^{rx} + c_2 x e^{rx}$$

where  $c_1$  and  $c_2$  are determined by the initial conditions.

## 2 Nonhomogeneous

Suppose we have the following differential equation.

$$ay'' + by' + cy = f(x) \tag{2}$$

We begin by finding the solution to the homogeneous equation,

$$ay'' + by' + cy = 0.$$

Call this solution the general solution or  $y_g$ . Note, do **NOT** apply any initial conditions for this solution.

We now have a table for  $f(x)$  and the form of the particular solution.

$f$	$y_p$
$k$	$A$
$x$	$Ax + B$
$x^2$	$Ax^2 + Bx + C$
$e^x$	$Ae^x$
$\cos(n\pi x)$ or $\sin(n\pi x)$	$A \cos(n\pi x) + B \sin(n\pi x)$
$x \cos(n\pi x)$ or $x \sin(n\pi x)$	$(Ax + B) \cos(n\pi x) + (Cx + D) \sin(n\pi x)$

If your  $f$  is a linear combination of the above, then your  $y_p$  is a linear combination of the applicable entries above. To find the  $A, B, C, D$ , plug  $y_p$  back into Equation 2, and match terms with  $f(x)$ . The solution to Equation 2 is  $y = y_g + y_p$ . Now apply your initial conditions to determine your as of now undetermined coefficients.