1. Simulate 36 rolls of a fair die. Give the relative frequency and the corresponding theoretical probability of each of the outcomes and compare them.

   With 36 rolls, the following numbers are produced:

   $$\{2,6,3,3,4,5,2,1,1,2,1,1,5,5,4,3,5,4,1,1,4,3,1,1,3,3,2,6,3,1,2,5,5,2,4,4\}$$

Dice Number	Frequency	Theoretical Probability
1	9	1/4
2	6	1/6
3	7	7/36
4	6	1/6
5	6	1/6
6	2	1/18

2. Simulate 96 rolls of a pair of dice where the sum is observed. Give the relative frequency and the corresponding theoretical probability of each of the outcomes. Make a table showing your results. Repeat the experiment 6 times and consider the total as if 576 rolls were simulated.

   Rolling two die 96 times produces the matrix

$$\begin{bmatrix} 7 & 3 & 6 & 12 & 9 & 7 & 2 & 6 \\ 10 & 7 & 9 & 6 & 10 & 9 & 7 & 7 \\ 5 & 8 & 4 & 7 & 3 & 11 & 7 & 11 \\ 10 & 6 & 10 & 8 & 4 & 8 & 4 & 9 \\ 10 & 9 & 6 & 11 & 7 & 7 & 9 & 7 \\ 5 & 8 & 5 & 10 & 7 & 4 & 12 & 6 \\ 5 & 10 & 6 & 8 & 9 & 9 & 12 & 4 \\ 2 & 7 & 5 & 9 & 6 & 4 & 7 & 5 \\ 9 & 6 & 4 & 10 & 6 & 6 & 6 & 11 \\ 8 & 7 & 6 & 8 & 9 & 9 & 8 & 9 \\ 5 & 6 & 9 & 12 & 5 & 8 & 7 & 7 \\ 8 & 6 & 12 & 3 & 10 & 8 & 6 & 6 \end{bmatrix} \Longrightarrow$$

Value	Frequency	Probability
1	0	0
2	2	1/48
3	3	1/32
4	7	7/96
5	8	1/12
6	17	17/96
7	16	1/6
8	11	11/96
9	14	7/48
10	9	3/32
11	4	1/24
12	5	5/96

   Running the experiment 6 times produces

Value	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Total
1	0	0	0	0	0	0	0
2	1.5	1	1	1	1	2.5	8
3	2.33	3	1.66	2.33	3.33	1.66	14.33
4	2.5	1.75	1.75	3	1.5	1.75	12.25
5	1.6	2.2	2.6	1.6	2	2.2	12.2
6	2.5	3.16	2.33	2	1	1.5	12.5
7	2	2.286	2.142	2	2.571	2.571	13.571
8	2.5	1.25	1.5	1.75	2	1.625	10.625
9	0.33	1.33	0.88	1.44	1.11	1.11	6.22
10	1	0.4	1.1	0.7	0.8	1	5
11	0.27	0.18	0.54	0.54	0.63	0.45	2.63
12	0.25	0.33	0.25	0.0833	0.25	0.25	1.4166

3. Simulate 10 free-throws for Kobe Bryant, whose free-throw average in 2008 was 81%. How many of the shots were successful.

   We can let 1 represent a successful free-throw, and 2 represent a miss. Running one test yields

Shot	Result
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	2
9	2
10	1

   From the table, we can see that he made 8 shots and missed two.

4. A baseball player is a 0.331 hitter. Simulate 10 at-bats for this player and tell how many hits he gets.

   We can let 0 represent a miss, and 1 represent a hit.

Shot	Result
1	1
2	0
3	1
4	1
5	0
6	0
7	1
8	0
9	0
10	1

   Based on this run, the batter hits 50% of the time.

5. A student who has not studied for a 10 question multiple choice test with 4 choices among the answers (a, b, c, d) for each question, decides to simulate such a test and answer the questions according to a simulation in which each choice has the same probability. Assume the correct answers are

   (a, b, b, c, d, d, a, c, b, a).

   What is the student's score?

Student	Answer	Correct
a	a	Yes
d	b	No
a	b	No
a	c	No
b	d	No
c	d	No
a	a	Yes
d	c	No
d	b	No
a	a	No
		Score: 20%

6. In sampling 4 balls at random from an urn containing 30 balls, *without replacing* after each draw, we consider the balls as numbered 1 to 30. In selecting random whole number from 1 to 30, we ignore any number that has already been selected and continue the selection until we obtain a sample size of 4. Assume there are 20 red balls and 10 green balls in the urn. Draw 10 samples of size 4 and tabulate the number of red balls in each sample. Compare your results with the theoretical probability.

7. Students are queued up at the registrars office with the registration windows open at 8. There are four open windows; students approach the first open window as they advance to the front of the queue. Assume that 10% of the students require 5 minutes of service time, 30% require 7 minutes, 40% require 10 minutes, and 20% require 15 minutes. Simulate the service of the first 20 students in a random queue. Show the schedule of service at the four windows (A, B, C, D), and determine how long it takes to process these students and give the average time from 8 to leaving the service window.

   One result of running the simulation yields

   $$5, 10, 7, 7, 10, 10, 5, 15, 15, 7, 15, 15, 10, 10, 7, 7, 10, 7, 15, 15$$

   as the queue. Assuming that students prefer A to B to C to D, we have the following process

A	B	C	D
5	10	7	7
10	15	10	5
7	15	15	15
15	7	10	10
7	15	7	10
44	62	49	47

   The average time is 8.8 minutes for A, 12.4 for B, 9.8 for C, and 9.4 for D. The overall average time is 10.1 minutes.

8. Simulate the following bank queue, and give the average time needed to process each customer. There are four bank tellers, and 40% of customers need 3 minutes, 50% need 5, and 10% need 8. Assume there are 20 customers in line.

   One run of the simulation yields

   $$3, 5, 3, 3, 5, 5, 3, 5, 5, 3, 5, 5, 5, 5, 3, 3, 5, 3, 5, 8$$

   as the queue. On possible outcome of the tellers is

Teller 1	Teller 2	Teller 3	Teller 4
3	5	3	3
5	3	5	5
5	3	5	5
5	5	3	3
8	5	3	5
26	21	19	21

   The average time is 5.2 for Teller 1, 4.2 for Teller 2, 3.8 for Teller 3, and 4.2 for Teller 4. The overall average is 4.35 minutes for each customer.

9. A gas station with four self-serve pumps has determined that 80% of all customers completely fill their gas tanks and the remaining 20% fill their tank with a fixed dollar amount's worth of fuel. Suppose that it takes an average of 5 minutes for a complete fill up and 3 minutes for a partial fill up, and that for an hour customers arrive steadily. Simulate this process for 30 customers.

   One outcome of the simulation is

   $$5, 5, 5, 5, 5, 5, 5, 5, 3, 3, 5, 3, 3, 5, 5, 5, 5, 5, 5, 3, 3, 5, 5, 3, 3, 5, 5, 5, 5, 5, 3.$$

   This yields the setup:

Pump 1	Pump 2	Pump 3	Pump 4
5	5	5	5
5	5	5	3
5	3	3	3
5	3	3	5
3	5	5	3
5	5	5	5
	5	3	
28	31	29	26

   The overall average time is 4.4 minutes per customer.

10. Simulate 108 rolls of three dice and show the frequency of each possible sum of the faces.

    Here is the outcome of one such run

$$\begin{bmatrix} 12 & 6 & 12 & 7 & 10 & 13 & 13 & 5 & 13 \\ 8 & 10 & 10 & 14 & 11 & 5 & 8 & 14 & 9 \\ 12 & 5 & 10 & 6 & 14 & 5 & 6 & 11 & 16 \\ 14 & 13 & 12 & 7 & 9 & 11 & 13 & 12 & 7 \\ 15 & 8 & 11 & 13 & 8 & 10 & 7 & 14 & 13 \\ 12 & 8 & 12 & 10 & 7 & 15 & 13 & 12 & 13 \\ 15 & 12 & 11 & 9 & 7 & 8 & 11 & 4 & 14 \\ 7 & 12 & 14 & 10 & 16 & 6 & 12 & 12 & 7 \\ 14 & 9 & 14 & 3 & 14 & 14 & 8 & 10 & 10 \\ 8 & 9 & 12 & 5 & 9 & 8 & 13 & 9 & 9 \\ 5 & 18 & 13 & 14 & 9 & 11 & 7 & 12 & 15 \\ 11 & 16 & 14 & 16 & 8 & 6 & 12 & 10 & 10 \end{bmatrix} \Longrightarrow$$

Dice Number	Frequency
3	1
4	1
5	6
6	5
7	9
8	10
9	9
10	11
11	8
12	15
13	11
14	13
15	4
16	4
17	0
18	1