Bayes Theorem

1. Suppose there are 2 cookie jars. Jar A has 10 chocolate chip and 30 plain cookies while jar B has 20 of each type. We pick a cookie and it turns out to be plain. What is the probability that we picked out of the jar A?

   We are trying to find the probability that you chose a cookie from jar A given that you picked a plain cookie.

   $$P(\text{A}|\text{plain}).$$

   We can find this value by

   $$P(\text{A}|\text{plain}) = \frac{P(\text{A and plain})}{P(\text{plain})} = \frac{(1/2)(3/4)}{(1/2)(3/4) + (1/2)(20/40)}.$$

2. The blue M&M was introduced in 1995. Before 1995, the ratios of colors was

30% brown	20% yellow	20% red
10% green	10% orange	10% tan

   After 1995, the ratios were

24% blue	20% green	16% orange
14% yellow	13% red	13% brown

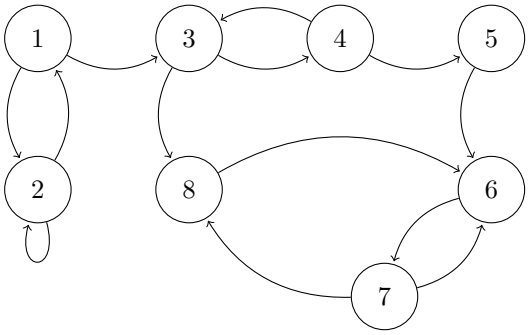
   If we pick an M&M that turns out to be yellow, what is the probability that the bag came before 1995?

   We want to find $P(1994|\text{yellow})$. By Bayes theorem, we can find this value by

   $$P(1994|\text{yellow}) = \frac{P(1994 \text{ and yellow})}{P(\text{yellow})} = \frac{(.5)(.2)}{(.5)(.2) + (.5)(.25)}$$

Communication Classes

1. What are the communication classes of

   We can find the communication classes by determining if we can move from state i to j and from state j to i.

2. Consider the matrix

   $$\begin{pmatrix} .6 & .4 \\ .3 & .7 \end{pmatrix}$$

   (a) How many communication classes are there?

   There is 1 communication class, since every state is accessible from ever other state.

   (b) List the communication class(es).

Since there are two states, with one communication class, the communication class is $\{A, B\}$.

(c) Find π.

To find the π vector, we can solve the system

$$\begin{cases} .6\pi_1 + .3\pi_2 = \pi_1 \\ \pi_1 + \pi_2 = 1 \end{cases}$$

3. Consider the model of moving from the city to the suburbs with an initial state.

	Sub	City
Sub	.95	.05
City	.03	.97

(a) What is the probability that a person in 3 years transfers from city to suburb?

(b) What number of people in the city will be in the suburbs next year?

(c) What is the ultimate number of people in the suburbs? The cities?