3.0 Is Not An Integer

In the following document, we will show that 3.0 is not an integer, or $3.0 \notin \mathbb{Z}$. This result stems from the fundametal idea, which we will prove, that $0.\overline{99} = 1.0$. The notation $0.\overline{99}$ is a symbolic way to represent 0.99999... where the 9 repeats **infinitely**. It is clear that we can accept $0.\overline{99}$ as being an element of the real numbers $\mathbb{R}$ and not an element of the set of integers $\mathbb{Z}$ or rationals $\mathbb{Q}$. We will now show that $0.\overline{99} = 1.0$.

Theorem: $0.\overline{99} = 1.0$.

Proof: We can represent the number $0.\overline{99}$ as a geometric series represented as

$$\begin{aligned} 0.\overline{99} &= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10000} + \cdots \\ &= \frac{9}{10} + \left(\frac{9}{10}\right)\left(\frac{1}{10}\right) + \left(\frac{9}{10}\right)\left(\frac{1}{100}\right) + \left(\frac{9}{10}\right)\left(\frac{1}{1000}\right) + \cdots \\ &= \frac{9}{10} + \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^1 + \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^2 + \left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^3 + \cdots \\ &= \sum_{n=1}^{\infty}\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^{n-1}. \end{aligned}$$

The general form for a geometric series is given by

$$\sum_{n=1}^{\infty} ar^{n-1},$$

Which leads us to assign the common ratio $r = \frac{1}{10}$ and $a = \frac{9}{10}$. By definition, a geometric series converges to $\frac{a}{1-r}$ if $|r| < 1$. This leads us to

$$\begin{aligned} 0.\overline{99} &= \frac{\frac{9}{10}}{1 - \frac{1}{10}} \\ &= \frac{9}{10 - 1} \\ &= \frac{9}{9} \\ &= 1. \end{aligned}$$

△

Now we can extend our argument to the original claim, that $3.0 \notin \mathbb{Z}$.

Theorem: $3.0 \notin \mathbb{Z}$.

Proof: Assume that $3.0 \in \mathbb{Z}$. Then $2.0 + 1 \in \mathbb{Z}$. We know as a fundamental truth that if $x \in \mathbb{R}$ and $y \notin \mathbb{Z}$, then $x + y \notin \mathbb{Z}$. We have already shown that $1 = 0.\overline{99}$, so it follows that $2.0 + 1 = 2.0 + 0.\overline{99}$. $0.\overline{99}$ is not an integer, and therefore $3.0 = 2.0 + 0.\overline{99} \notin \mathbb{Z}$.

△

Before we conclude, it should also be noted that from a practical standpoint, 3.0 cannot be considered an integer. In this day and age, the study of programming is becoming a more and more necessary skill for people to learn. Programming languages such as Java, C++, Ruby, and countless other languages that are widely used would never assert that $3.0 \in \mathbb{Z}$. For example, consider the following Ruby code:

```ruby
def main
    if 3.0 == 3 then
        puts "3.0 = 3"
    else
        puts "3.0 != 3"
end
```

This code will execute the else block of code. In the following Java code, the compiler would automatically cast the value 3.0 to 3, since it was explicitly declared as an integer value:

```java
public class ThreePointOh {
    public static void main(String[] args){
        int tpo = 3.0;
        System.out.println(tpo); //Outputs 3, not 3.0
    }
}
```

It is simply not practical to continue under the assumption that $3.0 \in \mathbb{Z}$.