Proofs Definitions

1. ***Set:*** A set is a collection of objects. The items in a set are called elements.
2. ***The Empty Set:*** The empty set, given by $\emptyset$, is the set with no elements.
3. ***Subset:*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. Then $\mathbb{A} \subset \mathbb{B}$ if $\forall x \in \mathbb{A}$, $x \in \mathbb{B}$.
4. ***Power Set:*** Let $\mathbb{A}$ be a set. Then the Power Set of $\mathbb{A}$, noted as $\mathcal{P}(\mathbb{A})$, is the set of all subsets of $\mathbb{A}$.
5. ***The Universal Set:*** The universal set $\mathcal{U}$ is the set of which all sets are subsets.
6. ***Intersection:*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. The intersection of $\mathbb{A}$ and $\mathbb{B}$, given by $\mathbb{A} \cap \mathbb{B}$ is $\{x | x \in \mathbb{A} \wedge x \in \mathbb{B}\}$.
7. ***Union:*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. The union of $\mathbb{A}$ and $\mathbb{B}$, given by $\mathbb{A} \cup \mathbb{B}$ is $\{x | x \in \mathbb{A} \vee x \in \mathbb{B}\}$.
8. ***Trivial Intersection (Disjoint):*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. If $\mathbb{A}$ and $\mathbb{B}$ have a trivial intersection, then $\mathbb{A} \cap \mathbb{B} = \emptyset$.
9. ***Set Difference:*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. The set difference of $\mathbb{A}$ and $\mathbb{B}$, noted as $\mathbb{A} - \mathbb{B}$, is $\{x | x \in \mathbb{A} \wedge x \notin \mathbb{B}\}$.
10. ***Cartesian Product:*** Let $\mathbb{A}$ and $\mathbb{B}$ be sets. Then the product of $\mathbb{A}$ and $\mathbb{B}$, given by $\mathbb{A} \times \mathbb{B}$, is $\{(a, b) | a \in \mathbb{A} \wedge b \in \mathbb{B}\}$.
11. ***Well Ordered:*** A set $\mathbb{A}$ is well ordered if for every non-empty set $\mathbb{B} \subset \mathbb{A}$, $\mathbb{B}$ has a least element.
12. ***Compliment:*** The compliment of a set $\mathbb{A}$ in regards to $\mathcal{U}$ is the set difference of $\mathcal{U}$ and $\mathbb{A}$.
13. ***Negation:*** The negation of a statement ρ, given by $\neg\rho$, is the statement that has the opposite truth values of ρ.
14. ***Disjunction:*** The disjunction of statements ρ and φ is the statement where either ρ and φ is true, or both are true, given by $\rho \vee \varphi$.
15. ***Conjunction:*** The conjunction of statements ρ and φ is the statement where ρ and φ are both true, given by $\rho \wedge \varphi$.
16. ***Implication:*** The implication of ρ and φ, noted as $\rho \implies \varphi$, is the statement where if ρ then φ is true.

ρ	φ	$\rho \wedge \varphi$	$\rho \vee \varphi$	$\rho \implies \varphi$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	1

17. ***Logically Equivalent:*** Two statements ρ and φ are logically equivalent, noted by $\rho \equiv \varphi$, if they have the same truth values.
18. ***Tautology:*** A statement ρ is a tautology if it is true for all possible truth values.
19. ***Contradiction:*** A statement ρ is a contradiction if it is false for all possible truth values.
20. ***Universal Quantifier:*** The universal quantifier, $\forall$, is a quantifier that asserts that a given statement ρ holds for all elements in the specified domain.

21. ***Existential Quantifier:*** The existential quantifier, $\exists$, is a quantifier that assets that a given statement ρ holds for at least one element in the specified domain.

22. ***Axiom:*** An axiom is a statement that is accepted as true without proof.

23. ***Theorem:*** A theorem is a statement that is proved to be true.

24. ***Lemma:*** A lemma is a statement that serves as an intermediate step in a proof.

25. ***Corollary:*** A corollary is a statement that follows from an eariler result.

26. ***Vacuous Statement:*** A vacuous statement is a statement in which the assumption is always false. The statement is always true.

27. ***Division:*** Let $a, b \in \mathbb{R}$. To say a divides b, noted as $a|b$, implies that $b = ax$ for some $x \in \mathbb{Z}$.

28. ***Induction:*** For all $n \in \mathbb{N}$, let $\rho(n)$ be a statement. If ρ_0 is true and $\rho(n) \implies \rho(n+1)$, then $\rho(n)$ is true for all n.

29. ***Strong Induction:*** For all $n \in \mathbb{N}$, let $\rho(n)$ be a statement. If ρ_0 is true and $\rho(i) \implies \rho(n+1)$ for all $i \in \mathbb{N}$, then $\rho(n)$ is true for all $n \in \mathbb{N}$.

30. ***Relation:*** A relation $\mathcal{R}$ from set $\mathbb{A}$ to set $\mathbb{B}$ is a subset of $\mathbb{A} \times \mathbb{B}$. Set $\mathbb{A}$ is related to set $\mathbb{B}$, noted as $\mathbb{A}\mathcal{R}\mathbb{B}$ if $(a, b) \in \mathcal{R}$ for $a \in \mathbb{A}$ and $b \in \mathbb{B}$.

31. ***Inverse Relation:*** Given a relation $\mathcal{R}$ from $\mathbb{A}$ to $\mathbb{B}$, $\mathcal{R}$ inverse, noted as $\mathcal{R}^{-1}$ or $\mathcal{R}^{\text{opp}}$, is $\{(b, a)|(a, b) \in \mathcal{R}\}$.

32. ***Reflixivity:*** A relation $\mathcal{R}$ is reflexive on a set $\mathbb{A}$ if $a\mathcal{R}a$ for all $a \in \mathbb{A}$.

33. ***Symmetry:*** A relation $\mathcal{R}$ is symmetric on a set $\mathbb{A}$ if $a\mathcal{R}\eta \implies \eta\mathcal{R}a$ for all $a, \eta \in \mathcal{R}$.

34. ***Transitivity:*** A relation $\mathcal{R}$ is transitive on a set $\mathbb{A}$ if $a\mathcal{R}\eta$ and $\eta\mathcal{R}\delta \implies a\mathcal{R}\delta$ for $a, \eta, \delta \in \mathbb{A}$.

35. ***Equivalence Relation:*** A relation $\mathcal{R}$ is an equivalence relation if it is reflexive, symmetric, and transitive.

36. ***Equivalence Class:*** Let $\mathbb{A}$ be a non-empty set with elements a and η. The equivalence class of a, noted as $[a]$, is $\{\eta|\eta\mathcal{R}a\}$.

37. ***Partition:*** Let $\mathbb{A}$ be a non-empty set. A patition of $\mathbb{A}$, given by $\mathbb{P}$, is a set of subsets of $\mathbb{A}$ where $\cup_{i\in\mathbb{I}}\mathbb{A} = \mathbb{P}$ and $\mathbb{A}_i \cap \mathbb{A}_j \neq \emptyset$ for all $i \neq j$.

38. ***Function:*** A function f from $\mathbb{A} \to \mathbb{B}$ is a relation from $\mathbb{A} \to \mathbb{B}$ where if (a, b) and (a, c) are in f, then $b = c$, and $\forall x \in \mathbb{A}, \exists y \in \mathbb{B}$ such that $(x, y) \in f$.

39. ***Image:*** The image of a function $f : \mathbb{A} \to \mathbb{B}$ is $\{f(a)|a \in \mathbb{A}\}$.

40. ***Preimage:*** Let f be a function such that $f : \mathbb{A} \to \mathbb{B}$ and $\mathbb{D} \subset \mathbb{B}$. Then the preimage of $\mathbb{D}$, noted as $f^{-1}(\mathbb{D})$, is $\{a \in \mathbb{A}|f(a) \in \mathbb{D}\}$.

41. ***Injection:*** A function $f : \mathbb{A} \to \mathbb{B}$ is injective if $f(a) = f(b) \implies a = b$ for all $a, b \in \mathbb{A}$.

42. ***Surjection:*** A function $f : \mathbb{A} \to \mathbb{B}$ is surjective if $\forall b \in \mathbb{B}, \exists a \in \mathbb{A}$ such that $f(a) = b$.

43. ***Bijection:*** A function $f : \mathbb{A} \to \mathbb{B}$ is bijective if it is injective and surjective.

44. ***Composition:*** Let $f : \mathbb{A} \to \mathbb{B}$ and $g : \mathbb{B} \to \mathbb{D}$ be functions. Then the composition of g and f, given by $g \circ f$, is defined as $(g \circ f)(a) = g(f(a))$.