

Integral Cheat Sheet

$$\int_a^b x \sin\left(\frac{n\pi x}{\ell}\right) dx = \left[-\frac{\ell x \cos\left(\frac{n\pi x}{\ell}\right)}{n\pi} + \frac{\ell^2 \sin\left(\frac{n\pi x}{\ell}\right)}{n^2 \pi^2} \right] \Big|_a^b$$

$$\int_a^b x^2 \sin\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^3 \pi^3} \left[(2\ell^2 - n^2 \pi^2 x^2) \cos\left(\frac{n\pi x}{\ell}\right) + 2\ell n \pi x \sin\left(\frac{n\pi x}{\ell}\right) \right] \Big|_a^b$$

$$\int_a^b x^3 \sin\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^4 \pi^4} \left[(6\ell^2 n \pi x - n^3 \pi^3 x^3) \cos\left(\frac{n\pi x}{\ell}\right) + 3\ell(-2\ell^2 + n^2 \pi^2 x^2) \sin\left(\frac{n\pi x}{\ell}\right) \right] \Big|_a^b$$

$$\int_a^b x \cos\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{\ell^2 \cos\left(\frac{n\pi x}{\ell}\right)}{n^2 \pi^2} + \frac{\ell x \sin\left(\frac{n\pi x}{\ell}\right)}{n\pi} \right] \Big|_a^b$$

$$\int_a^b x^2 \cos\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^3 \pi^3} \left[2\ell n \pi x \cos\left(\frac{n\pi x}{\ell}\right) + (-2\ell^2 + n^2 \pi^2 x^2) \sin\left(\frac{n\pi x}{\ell}\right) \right] \Big|_a^b$$

$$\int_a^b x^3 \cos\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^4 \pi^4} \left[(-6\ell^3 + 3\ell n^2 \pi^2 x^2) \cos\left(\frac{n\pi x}{\ell}\right) + n \pi x (-6\ell^2 + n^2 \pi^2 x^2) \sin\left(\frac{n\pi x}{\ell}\right) \right] \Big|_a^b$$

$$\int_a^b e^x \sin\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{e^x \ell \left(-n\pi \cos\left(\frac{n\pi x}{\ell}\right) + \ell \sin\left(\frac{n\pi x}{\ell}\right) \right)}{\ell^2 + n^2 \pi^2} \right] \Big|_a^b$$

$$\int_a^b e^x \cos\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{e^x \ell \left(\ell \cos\left(\frac{n\pi x}{\ell}\right) + n\pi \sin\left(\frac{n\pi x}{\ell}\right) \right)}{\ell^2 + n^2 \pi^2} \right] \Big|_a^b$$

$$\int_0^\infty e^{\alpha x^2} \sin(\beta x) dx = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} e^{-\frac{\beta^2}{4\alpha}}$$

$$\int_0^\infty e^{\alpha x^2} \cos(\beta x) dx = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} e^{-\frac{\beta^2}{4\alpha}}$$