

1 Modeling Free Fall

1. Measure the height of the object above ground
2. Record time in seconds that it takes for object to hit the ground
3. Determine the quantitative relationship between the height and the time (a mathematical model)

Galileo makes some assumptions before experimenting:

1. $F = ma$: we already know the mathematical expression for force
2. Gravity is the only force acting on the object
3. Acceleration due to gravity is $-32\frac{m}{s^2}$.

$$\begin{aligned}mg &= my'' \\g &= y'' \\ \iint g &= \iint y'' \\ -16t^2 &= y + Ct + D \\ y &= -16t^2 + v_0t + y_0\end{aligned}$$

Galileo assumed that gravity was the only force, but what if air resistance is added? We know that air resistance acts opposite to the velocity. We now have

$$\begin{aligned}\kappa v - 32m &= mv' \\ v' &= 32 - \kappa_0 v\end{aligned}$$

2 Discrete Modeling

We can now consider a discrete case for modeling.

2.1 Credit Card Balancing

We can work with the equation

$$B_1 = B_0 - P$$

as a simple model for a credit card balance. We will eventually find that this model is very simplistic, and we will need to expand on it later. We have a recurrence relation where

$$\begin{aligned}B_2 &= B_1 - P \\ B_3 &= B_2 - P \\ &\dots\end{aligned}$$

Substituting into the recurrence relation tells us that

$$B_n = B_0 - nP.$$

We can reform the model to help use come up with a better representation of the situation. In the real world, credit cards often come with interest rates. Taking this into account gives us

$$B_{\text{new}} = (1 + r)B_{\text{old}} - P.$$

We again have a recurrence relation. We will let $s = (1 + r)$:

$$\begin{aligned} B_1 &= sB_0 - P \\ B_2 &= sB_1 - P = s^2B_0 - p(1 + s) \\ B_3 &= sB_2 - P = s^3B_0 - p(1 + s + s^2) \\ B_4 &= sB_3 - P = s^4B_0 - p(1 + s + s^2 + s^3) \\ &\dots \end{aligned}$$

We can derive a general solution of the form

$$\begin{aligned} B_n &= s^n B_0 - p(1 + s + s^2 + s^3 + \dots + s^{n-1}) \\ &= (1 + r)^n B_0 - p \left(\frac{(1 + r)^n - 1}{r} \right) \end{aligned}$$

2.1.1 Solving With Excel

We can use Excel to represent the model. We will consider the case where $r = 1.5\%$ with an initial balance of \$1000, and \$10 per month on payments. In the A1 cell, we want to have

$$=(1.015)^{A1} * 1000 - 10*((1.015)^{A1}-1)/0.015$$