Steven Rosendahl
Homework 6

1. For each of the following linear congruences of the form $ax \equiv c \mod n$, determine whether a solution exists. If so, find a formula for all solutions and determine how many solutions there are in $\mathbb{Z}_n$.

   (a) $3x \equiv 5 \mod 7$

   The GCD of a and n is 1, so there is a solution since $1 \mid 5$, and there is only one solution. If we solve the Diophantine equation $3x+7y = 5$, we get a general solution for x as $x = 3+7n$. The solution we want is 3, since all other n give a solution equal to 3 in $\mathbb{Z}_7$.

   (b) $4x \equiv 9 \mod 12$

   This congruence has no solution since $gcd(4, 12) = 4$, but $4 \nmid 9$.

   (c) $18x \equiv 27 \mod 45$

   The $gcd(18, 45) = 9$, and $9 \mid 27$, so there is a solution, and in fact there are 9 solutions. If we solve the Diophantine equation $18x_0 + 45y_0 = 27$, we get that $x_0 = -1 + 5n$. The nine solutions can be found by starting with $n = 1$ to $n = 9$, and are $4, 9, 14, 19, 24, 29, 34, 39, 44$.

2. Let S denote the number of solution to the linear congruence $ax \equiv c \mod 20$. Prove that $S \in \{0, 1, 2, 3, 4, 5, 10, 20\}$.