The Count Distinct Problem

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- Image we have a set called $\mathbb{V}$ that contains a billion elements of the same type.
- How many *unique* elements are in $\mathbb{V}$?

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We will use $\mathbb{S}$ to represent the set of all the data, and $\mathbb{V}$ to represent the set of unique elements.

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- We can ignore the duplicate values in $\mathbb{V}$.

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  4. Results:
     - On average: 461 unique encounters

Implementation

```
1  array.each do |pokemon|
2        hash_table[pokemon.hash] = pokemon.hash
3  end
4  total = 0
5  hash_table.each do |val|
6        if val != -1
7              total = total + 1
8        end
9  end
10 puts "The total number of unique encounters is #{total}"
```

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  - Twitter Problem deals with $\mathbb{S}$ of size 200,000,000.
  - Collisions and collision policies also add to the amount of memory required.

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4. Take the harmonic average of all the totals in the bitmap.

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- m is the number of spaces in the bitmap $(\mathbb{V})$.

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- We can lower the sample size and apply a best fit line to the data.
  1. For 24 hours, gather 2000 tweets containing "#" every 2 minutes.
  2. Using the HyperLogLog, determine the unique number of total hashtags every time a new sample is gathered.

Implementation

```
1  mhll = Hyperll::HyperLogLog.new(10)
2  File.open("twitter_data.txt","r") do |file|
3        file.each_line do |line|
4              mhll.offer line
5        end
6  end
7  str = "Unique Elements: #{mhll.cardinality}"
8  puts str
```

Results

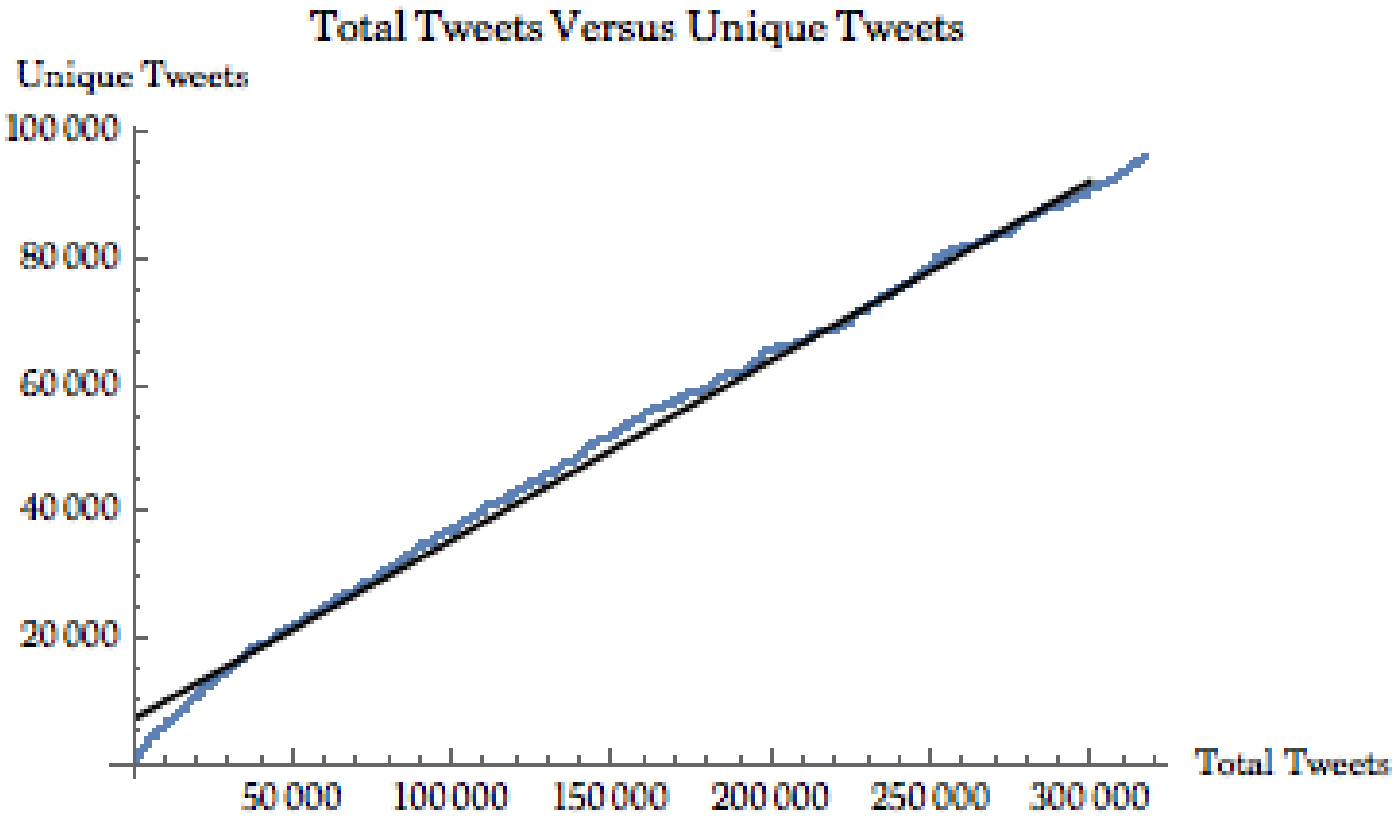

Figure: $0.284356x + 7361.39$

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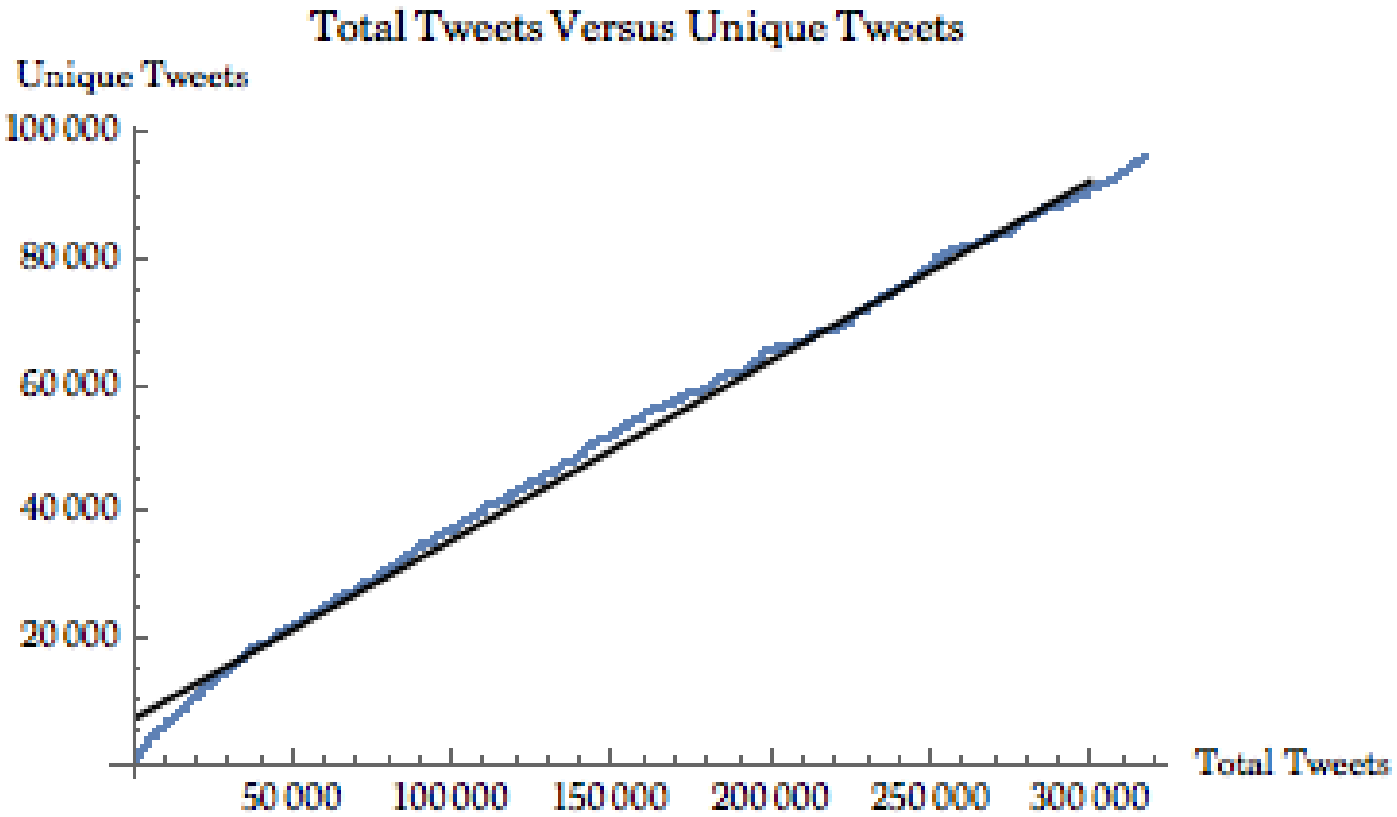

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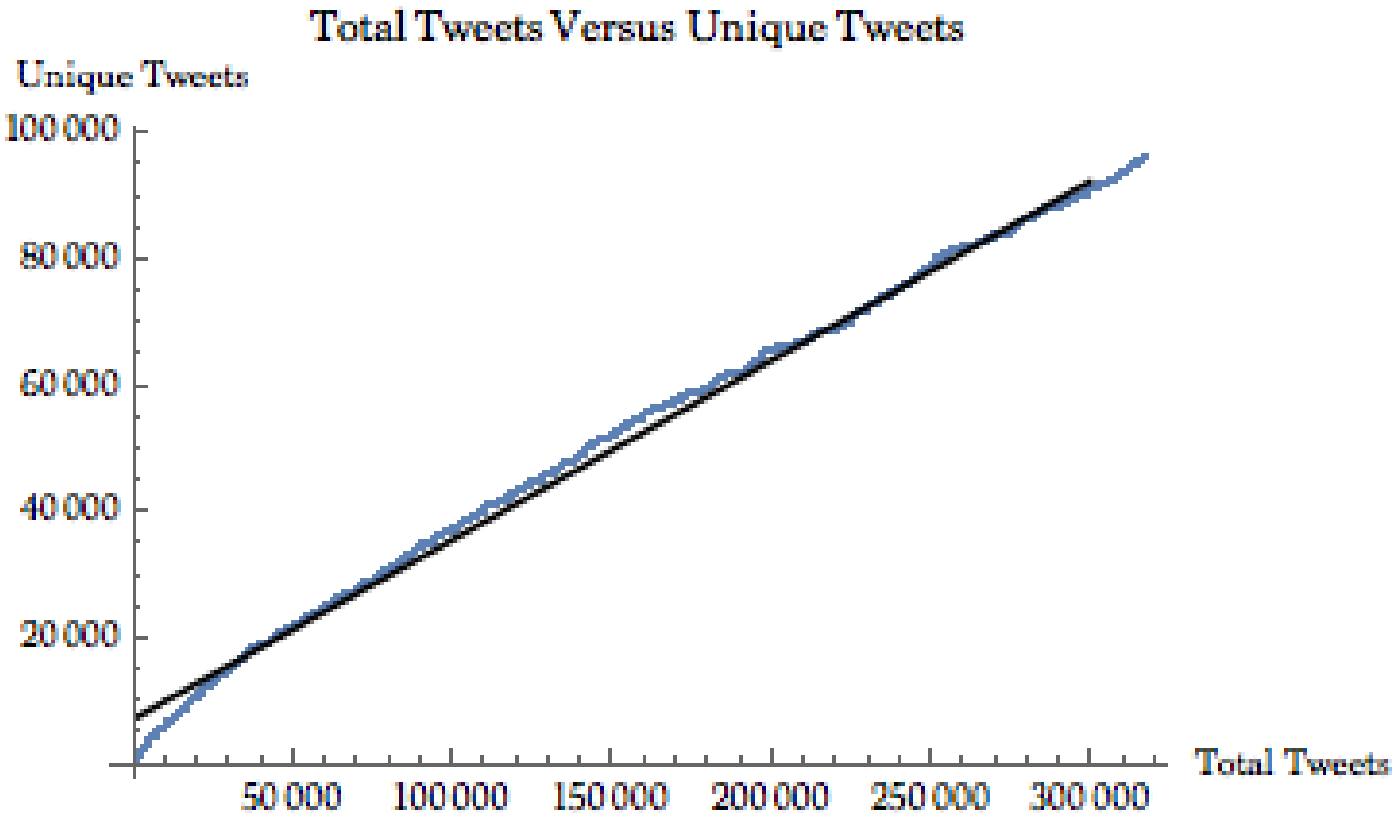

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