Definition 0.1. A Binary Operation on a set $\mathcal{S}$ is a mapping $*$ from $\mathcal{S} \times \mathcal{S} \to \mathcal{S}$ that associates elements in $\mathcal{S} \times \mathcal{S}$ to an element in $\mathcal{S}$.

Definition 0.2. A Group is a pairing of a set $\mathcal{G}$ and a binary operator $*$ $(\mathcal{G}, *)$ such that

1. $*$ is associative, i.e. $a * (b * c) = (a * b) * c$.
2. $\exists e \in \mathcal{G} \ni x * e = e * x = x$. This is called the identity.
3. $\forall x \in \mathcal{G}, \exists y \in \mathcal{G} \ni x * y = y * x = e$. y is called the inverse of x.

If $*$ is commutative, then $\mathcal{G}$ is called abelian.

Theorem 0.1. Let $*$ be a binary operation on $\mathcal{S}$. If there is an identity, it is unique.

Proof. Let e, f be identities in $(\mathcal{S}, *)$. Then $e * f = e$ and $e * f = f$, so $e = f$. $\square$