Integral Cheat Sheet

$$\int_a^b x \sin\left(\frac{n\pi x}{\ell}\right) dx = \left[-\frac{\ell x \cos\left(\frac{n\pi x}{\ell}\right)}{n\pi} + \frac{\ell^2 \sin\left(\frac{n\pi x}{\ell}\right)}{n^2\pi^2}\right]\Bigg|_a^b$$

$$\int_a^b x^2 \sin\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^3\pi^3}\left[(2\ell^2 - n^2\pi^2x^2)\cos\left(\frac{n\pi x}{\ell}\right) + 2\ell n\pi x \sin\left(\frac{n\pi x}{\ell}\right)\right]\Bigg|_a^b$$

$$\int_a^b x^3 \sin\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^4\pi^4}\left[(6\ell^2 n\pi x - n^3\pi^3x^3)\cos\left(\frac{n\pi x}{\ell}\right) + 3\ell(-2\ell^2 + n^2\pi^2x^2 \sin\left(\frac{n\pi x}{\ell}\right))\right]\Bigg|_a^b$$

$$\int_a^b x \cos\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{\ell^2 \cos\left(\frac{n\pi x}{\ell}\right)}{n^2\pi^2} + \frac{\ell x \sin\left(\frac{n\pi x}{\ell}\right)}{n\pi}\right]\Bigg|_a^b$$

$$\int_a^b x^2 \cos\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^3\pi^3}\left[2\ell n\pi x \cos\left(\frac{n\pi x}{\ell}\right) + \left(-2\ell^2 + n^2\pi^2x^2\right)\sin\left(\frac{n\pi x}{\ell}\right)\right]\Bigg|_a^b$$

$$\int_a^b x^3 \cos\left(\frac{n\pi x}{\ell}\right) dx = \frac{\ell}{n^4\pi^4}\left[(-6\ell^3 + 3\ell n^2\pi^2x^2)\cos\left(\frac{n\pi x}{\ell}\right) + n\pi x(-6\ell^2 + n^2\pi^2x^2)\sin\left(\frac{n\pi x}{\ell}\right)\right]\Bigg|_a^b$$

$$\int_a^b e^x \sin\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{e^x \ell\left(-n\pi\cos\left(\frac{n\pi x}{\ell}\right) + \ell\sin\left(\frac{n\pi x}{\ell}\right)\right)}{\ell^2 + n^2\pi^2}\right]\Bigg|_a^b$$

$$\int_a^b e^x \cos\left(\frac{n\pi x}{\ell}\right) dx = \left[\frac{e^x \ell\left(\ell\cos\left(\frac{n\pi x}{\ell}\right) + n\pi\sin\left(\frac{n\pi x}{\ell}\right)\right)}{\ell^2 + n^2\pi^2}\right]\Bigg|_a^b$$

$$\int_a^b \sin(x)\cos(nx)\, dx = \left[\frac{\cos(x)\cos(nx) + n\sin(x)\sin(nx)}{n^2 - 1}\right]\Bigg|_a^b$$

$$\int_a^b \cos(x)\cos(nx)\, dx = \left[\frac{-\sin(x)\cos(nx) + n\cos(x)\sin(nx)}{n^2 - 1}\right]\Bigg|_a^b$$

$$\int_a^b \sin(x)\sin(nx)\, dx = \left[\frac{-n\sin(x)\cos(nx) + \cos(x)\sin(nx)}{n^2 - 1}\right]\Bigg|_a^b$$

$$\int_a^b \cos(x)\sin(nx)\, dx = \left[\frac{n\cos(x)\cos(nx) + \sin(x)\sin(nx)}{1 - n^2}\right]\Bigg|_a^b$$

$$\int_a^b \sin^2(x)\, dx = \frac{b - a + \cos(a)\sin(a) - \cos(b)\sin(b)}{2}$$

$$\int_a^b \cos^2(x)\, dx = \frac{b - a - \cos(a)\sin(a) + \cos(b)\sin(b)}{2}$$

$$\int_0^\infty e^{\alpha x^2}\sin(\beta x)\, dx = 0$$

$$\int_0^\infty e^{\alpha x^2}\cos(\beta x)\, dx = \frac{\sqrt{\pi}}{2\sqrt{\alpha}}e^{-\frac{\beta^2}{4\alpha}}$$

$$\int_0^\infty e^{-u^2}\, du = \frac{\sqrt{\pi}}{2}$$