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Homework 3

1. Compute $a \cdot b$, $||a||$ and $||b||$ where $\vec{a} = 4i - 3j + k$ and $\vec{b} = i + j + k$.

$$\begin{aligned} a \cdot b &= 4 - 3 + 1 \\ &= 2 \\ ||a|| &= \sqrt{16 + 9 + 1} \\ &= \sqrt{26} \\ ||b|| &= \sqrt{1 + 1 + 1} \\ &= \sqrt{3} \end{aligned}$$

2. Find the angle between $\vec{a} = \sqrt{3}\ i + j$ and $\vec{a} = -\sqrt{3}\ i + j$.

$$\begin{aligned} \theta &= \arccos\left(\frac{a \cdot b}{||a||\ ||b||}\right) \\ &= \arccos\left(\frac{-2}{4}\right) \\ &= \arccos\left(-\frac{1}{2}\right) \\ &= \frac{2\pi}{3} \end{aligned}$$

3. Find the angle between $\vec{a} = i + j$ and $\vec{b} = i + j + k$.

$$\begin{aligned} \theta &= \arccos\left(\frac{a \cdot b}{||a||\ ||b||}\right) \\ &= \arccos\left(\frac{2}{\sqrt{6}}\right) \end{aligned}$$

4. Find the angle between $\vec{a} = (1, -2, 3)$ and $\vec{b} = (3, -6, -5)$.

$$\begin{aligned} \theta &= \arccos\left(\frac{a \cdot b}{||a||\ ||b||}\right) \\ &= \arccos(0) \\ &= \frac{\pi}{2} \end{aligned}$$

5. Calculate $proj_a b$ where $\vec{a} = i + j$ and $\vec{b} = 2i + 3j - k$.

$$\begin{aligned} proj_a b &= \frac{a \cdot b}{||a||^2}\vec{a} \\ &= \frac{4}{2}(i + j) \\ &= 2i + 2j \end{aligned}$$

6. Calculate $proj_a b$ where $\vec{a} = i + j + 2k$ and $\vec{b} = 2i - 4j + k$.

$$\begin{aligned} proj_a b &= \frac{a \cdot b}{||a||^2}\vec{a} \\ &= \frac{0}{4}(i + j + 2k) \\ &= 0 \end{aligned}$$

7. Give a vector of length 3 that points in the same direction as the vector $i + j - k$.

   The length is just a scalar value multiplied by the vector. Therefore we have the unit vector $i + j - k$, with length 3 will be

   $$3i + 3j - 3k.$$

8. Is there ever a case where $proj_a b = proj_b a$.