The Rössler System

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Introduction

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- System constructed to display simplest possible *Strange Attractor*

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- Strange attractors are often associated with chaotic systems.

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\begin{aligned}
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\dot{y} &= x + ay \\
\dot{z} &= b + z(x - c)
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  - Is the system globally stable? Locally?
  - When does chaotic behavior occur?

First Glance

- Finding fixed points

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```
Solve[f[x, y, z] == 0 && g[x, y, z] == 0 && h[x, y, z] == 0, {x, y, z}]
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$$\begin{cases} x = \frac{1}{2}\left(c - \sqrt{c^2 - 4ab}\right) \\ y = \frac{1}{2}\left(\frac{\sqrt{c^2-4ab}}{a} - \frac{c}{a}\right) \\ z = \frac{c-\sqrt{c^2-4ab}}{2a} \end{cases} \qquad \begin{cases} x = \frac{1}{2}\left(c + \sqrt{c^2 - 4ab}\right) \\ y = \frac{1}{2}\left(-\frac{\sqrt{c^2-4ab}}{a} - \frac{c}{a}\right) \\ z = \frac{c+\sqrt{c^2-4ab}}{2a} \end{cases}$$

Simplification

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- Much easier to analyze stability of system

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- Other stability exists, but are often named after the bifurcations that produce them

Linearization

- Jacobian of system will tell us about behavior of fixed points