

Steven Rosendahl
Homework 1.7

1. Find the Cartesian coordinates of the point whose polar coordinates are given.

$$\begin{aligned}(\sqrt{2}, \frac{\pi}{4}) &= (\sqrt{2} \cos(\frac{\pi}{4}), \sqrt{2} \sin(\frac{\pi}{4})) \\ &= (1, 1)\end{aligned}$$

2. Give a set of polar coordinates for the point whose Cartesian coordinates are given.

$$\begin{aligned}(-2, 2) &= (\sqrt{(-2)^2 + 2^2}, \arctan(-1)) \\ &= (\sqrt{4}, -\frac{\pi}{4}) \\ &= (2, -\frac{\pi}{4})\end{aligned}$$

3. Find the Cartesian coordinates of the point whose cylindrical coordinates are given.

$$\begin{aligned}(1, \frac{2\pi}{3}, -2) &= (\cos(\frac{2\pi}{3}), \sin(\frac{2\pi}{3}), -2) \\ &= (-\frac{1}{2}, \frac{\sqrt{3}}{2}, -2)\end{aligned}$$

4. Find the rectangular coordinates of the point whose spherical coordinates are given.

$$\begin{aligned}(1, \frac{3\pi}{4}, \frac{2\pi}{3}) &= (\cos \frac{3\pi}{4} \sin \frac{2\pi}{3}, \sin \frac{3\pi}{4} \sin \frac{2\pi}{3}, \cos \frac{3\pi}{4}) \\ &= (\frac{-\sqrt{3}/2}{2}, \frac{\sqrt{3}/2}{2}, \frac{-1}{\sqrt{2}})\end{aligned}$$

5. Find a set of cylindrical coordinates of the point whose Cartesian coordinate is given.

$$\begin{aligned}(-1, \sqrt{3}, 13) &= (\sqrt{(-1)^2 + (\sqrt{3})^2}, \arctan -\frac{1}{\sqrt{3}}, 13) \\ &= (2, -\frac{\pi}{6}, 13)\end{aligned}$$

6. Convert the provided equation into the other two coordinate systems.

$$\rho \sin \phi \sin \theta = 2$$

In cylindrical coordinates:

$$r \sin \theta = 2$$

In cartesian coordinates:

$$y = 2$$

7. Convert the provided equation into the other two coordinate systems.

$$z^2 = 2x^2 + 2y^2$$

In cylindrical coordinates:

$$\begin{aligned} z^2 &= r^2 \cos^2 \theta + r^2 \sin^2 \theta \\ z^2 &= r^2 \end{aligned}$$

In spherical coordinates:

$$\begin{aligned} \rho^2 \cos^2 \phi &= \rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta \\ \cos^2 \phi &= \sin^2 \phi (\cos^2 \theta + \sin^2 \theta) \\ \cos^2 \phi &= \sin^2 \phi \end{aligned}$$

8. Sketch the solid whose cylindrical coordinates (r, θ, z) satisfy the given inequalities.

$$0 \leq r \leq 3, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad -1 \leq z \leq 2$$

9. Sketch the solid whose cylindrical coordinates (r, θ, z) satisfy the given inequalities.

$$r \leq z \leq 5, \quad 0 \leq \theta \leq \pi$$

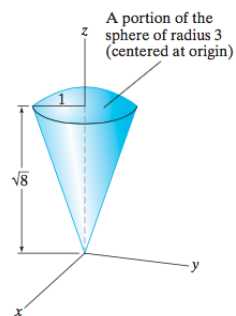
10. Sketch the solid whose spherical coordinates satisfy the given inequalities.

$$0 \leq \rho \leq 1, \quad 0 \leq \phi \leq \frac{\pi}{2}$$

11. Sketch the solid whose spherical coordinates satisfy the given inequalities.

$$0 \leq \phi \leq \frac{\pi}{4}, \quad 0 \leq \rho \leq 2$$

12. Consider the solid shown below:



(a) Describe the solid, using spherical coordinates.

$$0 \leq \rho \leq 3, \quad 0 \leq \phi \leq \arctan \frac{1}{\sqrt{8}}, \quad 0 \leq \theta \leq 2\pi$$

(b) Describe the solid, using cylindrical coordinates.

$$\sqrt{8} r \leq z \leq \sqrt{9 - r^2}, \quad 0 \leq r \leq 1, \quad 0 \leq \theta \leq 2\pi$$