

Determining Methods of Integration

For the following problems, do not perform the integration. Rather, indicate the method you would use to determine the antiderivative. The following methods are available:

- Nothing: nothing needs to be done, so perform the integration.
- Algebraic manipulation: specify what must be done and simplify the integrand.
- u -substitution: write out the substitution.
- Integration by Parts: identify u , dv , and how many cycles need to be performed.
- Trig Combination: identify the substitution and the necessary trig identities.
- Trig Substitution: identify the substitution.
- Partial Fractions: specify the correct form of the decomposition.

1. $\int \frac{dx}{\sqrt{12-6x-x^2}}$

10. $\int \tan^4 x \sec x \, dx$

2. $\int \sin^3 x \cos^2 x \, dx$

11. $\int \frac{2x+1}{x(x-3)^3(2x^2+6x+25)} \, dx$

3. $\int \sqrt{4x^2-1} \, dx$

12. $\int \sqrt{x^2+4x+13} \, dx$

4. $\int x \sec^2 x \, dx$

13. $\int \frac{x-3}{x^2+4x+7} \, dx$

5. $\int \csc x \cot x \, dx$

14. $\int \frac{x+1}{2x^2+4x+21} \, dx$

6. $\int \frac{3}{\sqrt{(x-4)^2+9}} \, dx$

15. $\int \frac{2x+5}{(x^2+5x+13)^3} \, dx$

7. $\int \frac{e^{2x}}{1+e^{4x}} \, dx$

16. $\int \frac{2x+1}{(x^2-6x+18)^5} \, dx$

8. $\int \ln x \, dx$

17. $\int \frac{1}{\sqrt{4-x^2}} \, dx$

9. $\int x \arcsin x \, dx$

18. $\int x^2 - 1 - \sqrt{x} \, dx$

25. $\int x^2 (\ln x)^3 \, dx$

19. $\int \arctan x \, dx$

26. $\int 5^x \, dx$

20. $\int \frac{x^3}{\sqrt{4-x^2}} \, dx$

27. $\int \tan^4 x \sin^6 x \, dx$

21. $\int \frac{2x+3}{x^2+3x+6} \, dx$

28. $\int (x^3 + 1)(\sqrt{x} - x^{1/3}) \, dx$

22. $\int \csc x \, dx$

29. $\int \cos^2 x \sin^6 x \, dx$

23. $\int x \cos x \, dx$

30. $\int e^{x/3} \sin(\pi x) \, dx$

24. $\int \frac{dx}{x^2+3x+6}$

31. $\int \sec x \, dx$

32. $\int (1 + e^{-x}) \sec^2(x - e^{-x}) \, dx$

Answers

1. Complete the Square or Trig Sub.
2. Trig Combination: $u = \cos x$, $du = -\sin x \, dx$.
3. Trig Substitution: $u = 2x = \sec \theta$.
4. Integration by Parts: $u = x$, $dv = \sec^2 x \, dx$.
5. Nothing: $-\csc x + c$.
6. Trig Substitution: $u = x - 4 = 3 \tan \theta$.
7. u -sub: $u = e^{2x}$.
8. Integration by Parts: $u = \ln x$, $dv = 1 \, dx$.
9. Integration by Parts: $u = \arcsin x$, $dv = x \, dx$.
10. Trig Combination and Integration by Parts: convert $\tan^4 x$ to powers of $\sec x$.
11. Partial Fractions: $\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)^3} + \frac{Ex+F}{2x^2+6x+25}$.
12. Complete the Square and Trig Substitution: $(x+2)^2$ and then $u = x+2 = 3 \tan \theta$.
13. Complete the Square and u -sub: $u = 2+x$.
14. u -sub: $u = 2x^2 + 4x + 21$.
15. u -sub: $u = x^2 + 5x + 13$.
16. Complete the Square, u -sub, and Trig Substitution: $(x-3)^2 + 9$, $u = x-3$, $w = 3 \tan \theta$.
17. Trig Substitution
18. Algebraic Manipulation: break up into three different parts.
19. Integration by Parts: $u = \arctan \theta$, $du = dx$.
20. u -sub: $u = 4 - x^2$.
21. u -sub: $u = x^2 + 3x + 6$.
22. Nothing: follow the formula $\int \csc x \, dx = \ln |\csc x - \cot x| + c$.
23. Integration by Parts: $u = x$, $dv = \cos x \, dx$.
24. Complete the Square
25. Integration by Parts: $u = (\ln x)^3$, $dv = x^2 \, dx$.
26. Basic Formula: $\frac{5^x}{\ln 5} + c$.
27. Trig Combination: $u = \tan x$.
28. Algebraic Manipulation: foil the polynomials.
29. Trig Combination and half-angle formula
30. Integration by Parts (twice): $u = \sin \pi x$, $dv = e^{x/3} \, dx$.
31. Basic Formula: $\ln |\sec x + \tan x| + c$.
32. u -sub: $u = x - e^{-x}$.