

Proofs

Axiom 1 *The number $0.99999\dots = 1.00000\dots$.*

Theorem 1 *There is no bijection between the natural numbers \mathbb{N} and the set $(0, 1)$.*

Proof: Suppose $|\mathbb{N}| = |(0, 1)|$. Let $f : \mathbb{N} \rightarrow (0, 1)$ be a bijection. We will let $a_i = f(i)$ and a_{ij} be the j^{th} decimal value of a_i . We can construct the bijection by

$$a_1 = .a_{11}a_{12}a_{13}a_{14}\dots = f(a_1)$$

$$a_2 = .a_{21}a_{22}a_{23}a_{24}\dots = f(a_2)$$

$$a_3 = .a_{31}a_{32}a_{33}a_{34}\dots = f(a_3)$$

$$a_4 = .a_{41}a_{42}a_{43}a_{44}\dots = f(a_4)$$

\dots

Now we can take an element $b \in (0, 1)$ such that b_i is the i^{th} digit of b . We want to construct the digits of b such that each space along the a_{ii} diagonal is not equal to a digit in b . We can express this by

$$b_i = \begin{cases} n, & a_i \neq n \\ n + 1, & a_i = n \\ 8, & a_i = 9 \end{cases}$$