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Homework 1

1. Let A be the point with coordinates $(1, 0, 2)$, let B be the point with coordinates $(3, 3, 1)$, and let C be the point with coordinates $(2, 1, 5)$.

   (a) Describe the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$.

   $\overrightarrow{AB}$ will be the vector from $(1, 0, 2)$ to $(3, 3, 1)$, which is the same as the vector $(-4, 3, -1)$.

   (b) Describe the vectors $\overrightarrow{AC}, \overrightarrow{BC}$, and $\overrightarrow{AC} + \overrightarrow{CB}$.

   $\overrightarrow{AC}$ will be the vector from $(1, 0, 2)$ to $(2, 1, 5)$, which is the same as the vector $(1, 1, 3)$.
   $\overrightarrow{BC}$ will be the vector from $(-3, 3, 1)$ to $(2, 1, 5)$, which is the same as the vector $(5, -2, 4)$.
   $\overrightarrow{CB} = (1, 2, -4)$, so $\overrightarrow{AC} + \overrightarrow{CB} = (2, 3, -1)$.

   (c) Explain with pictures why $\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$.

2. If $(-12, 9, z) + (x, 7, -3) = (2, y, 5)$, what are x, y, and z?

$$-12 + x = 2 \qquad 9 + 7 = y \qquad z - 3 = 5$$
$$x = 14 \qquad y = 16 \qquad z = 8$$

3. What is the length of the vector $< 3, 1 >$?

   The length of the vector is $\sqrt{3^2 + 1^2}$, or $\sqrt{10}$.

4. Find the displacement vectors from P_1 to P_2, and sketch P_1, P_2, and $\overrightarrow{P_1P_2}$.

   (a) $P_1(1, 0, 2)$, $P_2(2, 1, 7)$ (b) $P_1(1, 6, 1), P_2(0, 4, 2)$ (c) $P_1(0, 4, 2), P_2(1, 6, 1)$ (d) $P_1(3, 1), P_2(2, 1)$

   $\overrightarrow{P_1P_2} = (1, 1, 5)$ $\quad$ $\overrightarrow{P_1P_2} = (-1, -2, 3)$ $\quad$ $\overrightarrow{P_1P_2} = (1, 2, -3)$ $\quad$ $\overrightarrow{P_1P_2} = (-1, -2)$

5. If A is the point in $\mathbb{R}^3$ with coordinates $(2, 5, -6)$ and the displacement vector from A to B is $(12, -3, 7)$, what are the coordinates of B?

   The vector $\overrightarrow{AB} = (12, -3, 7) = (x_B - 2, y_B - 5, z_B + 6)$. Setting up a system of equations yields

$$12 = x_B - 2 \qquad -3 = y_B - 5 \qquad 7 = z_B + 6$$
$$x_B = 14 \qquad y_B = 2 \qquad z_B = 1$$

   Therefore $B = (14, 2, 1)$.

6. Suppose that you and your friend are in New York talking on cellular phones. You inform each other of your own displacement vectors from the Empire State Building to your current position. Explain how you can use this information to determine the displacement vector from you to your friend.

   The Empire State Building acts as the origin. If you know your point (A) and your friend's point, (B), you can calculate the displacement vector $\overrightarrow{AB}$, which will provide you with the displacement vector from you to your friend by $\overrightarrow{AB} = (x_b - x_a, y_b - y_a)$.

7. (a) Let $\vec{a} = (2,0)$ and $\vec{b} = (1,1)$. For $0 \leq s \leq 1$ and $0 \leq t \leq 1$, consider the vector $\vec{x} = s\vec{a} + t\vec{b}$. Explain why the vector $\vec{x}$ lies in the parallelogram determined by $\vec{a}$ and $\vec{b}$.

   For $\vec{x}$ to be in the parallelogram $\vec{a}\,\vec{b}$, $\vec{x}$ cannot be greater than $(3,1)$ and cannot be less than $(0,0)$. $\vec{x}$ is defined as

$$\begin{aligned}\vec{x} &= s\vec{a} + t\vec{b} \\ &= (2s, 0) + (t, t) \\ &= (2s + t, t),\end{aligned}$$

   which at $s = 0$ and $t = 0$ still lies within the bound, and at $s = 1$ and $t = 1$ still lies within the bound.

   (b) Now suppose that $\vec{a} = (2,2,1)$ and $\vec{b} = (0,3,2)$. Describe the set of vectors $\{\vec{x} = s\vec{a} + t\vec{b} | 0 \leq s \leq 1, 0 \leq t \leq 1\}$.

   The set of vectors will be in the form of

$$(2s, 2s + 3t, s + 2t)$$

8. A flea falls onto marked graph paper at the point $(3,2)$. She begins moving from that point with velocity vector $\vec{v} = (1,2)$ (i.e., she moves 1 graph paper unit per minute in the negative x-direction and 2 graph paper units per minute in the negative y-direction).

   (a) What is the speed of the flea?

$$\begin{aligned}\text{speed} &= \sqrt{1+4} \\ &= \sqrt{5}\end{aligned}$$

   (b) Where is the flea after 3 minutes?

   The flea's position can be expressed as $f(t) = (3 - t, 2 - 2t)$. Plugging in 3 for t yields $(0,4)$.

   (c) How long does it take the flea to get to the point $(-4,-12)$?

   We have the equation $f(t) = (3 - t, 2 - 2t)$. We setting it equal to $(-4,-12)$ and solving for t yields $t = 7$.

   (d) Does the flea reach the point $(13,27)$? Why or why not?

   Using the equation above, we can solve again for t. Doing so yields $t = 16$ and $t = 14.5$, which is impossible. Therefore the flea does not reach that point.