$$\{ u_t - k u_{xx} = 0$$
$$u(0,t) = u(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\sum_{n=1}^{\infty} b_n e^{-k\left(\frac{n\pi}{\ell}\right)^2 t} \sin\left(\frac{n\pi x}{\ell}\right)$$
$$b_n = \frac{2}{\ell}\int_0^\ell f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$\{ u_t - k u_{xx} = 0$$
$$u_x(0,t) = u_x(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\frac{0}{2+\sum_{n=1}^{\infty} a_n e^{-k\left(\frac{n\pi}{\ell}\right)^2 t} \cos\left(\frac{n\pi x}{\ell}\right) a_n=\frac{2}{\ell}\int_0^\ell f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx \; a_0=\frac{2}{\ell}\int_0^\ell f(x)\,dx}$$

$$\{ u_t - k u_{xx} = 0$$
$$u(0,t) = u_x(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\sum_{n=1}^{\infty} a_n e^{-k\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)^2 t} \sin\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)$$
$$a_n = \frac{2}{\ell}\int_0^\ell f(x)\sin\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)dx$$

$$\{ u_t - k u_{xx} = 0$$
$$u_x(0,t) = u(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\sum_{n=1}^{\infty} a_n e^{-k\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)^2 t} \cos\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)$$
$$a_n = \frac{2}{\ell}\int_0^\ell f(x)\cos\left(\left[n+\frac{1}{2}\right]\frac{\pi}{\ell}x\right)dx$$

$$\{ u_{tt} - c^2 u_{xx} = 0$$
$$u(0,t) = u(\ell,t) = 0$$
$$u(x,0) = f(x),\, u_t(x,0) = g(x)$$
$$\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi ct}{\ell}\right) + b_n\sin\left(\frac{n\pi ct}{\ell}\right)\right]\sin\left(\frac{n\pi x}{\ell}\right)$$
$$a_n = \frac{2}{\ell}\int_0^\ell f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$
$$b_n = \frac{2}{n\pi c}\int_0^\ell g(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$\{ u_{tt} - c^2 u_{xx} = 0$$
$$u_x(0,t) = u_x(\ell,t) = 0$$
$$u(x,0) = f(x),\, u_t(x,0) = g(x)$$
$$\frac{0}{2+\frac{b_0}{2}t+\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi ct}{\ell}\right)+b_n\sin\left(\frac{n\pi ct}{\ell}\right)\right]\cos\left(\frac{n\pi x}{\ell}\right) a_n=\frac{2}{\ell}\int_0^\ell f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx \; a_0=\frac{2}{\ell}\int_0^\ell f(x)\,dx \; b_n=\frac{2}{n\pi c}\int_0^\ell g(x)\cos\left(\frac{n\pi x}{\ell}\right)dx \; b_0=\frac{2}{n\pi c}\int_0^\ell g(x)\,dx}$$

$$\{ u_{tt} - c^2 u_{xx} = 0$$
$$u(0,t) = u_x(\ell,t) = 0$$
$$u(x,0) = f(x),\, u_t(x,0) = g(x)$$
$$0 + b_0 t + \sum_{n=1}^{\infty}\left[a_n\cos\left(\left[n+\frac{1}{2}\right]\frac{\pi t}{\ell}\right) + b_n\sin\left(\left[n+\frac{1}{2}\right]\frac{\pi t}{\ell}\right)\right]\sin\left(\left[n+\frac{1}{2}\right]\frac{\pi x}{\ell}\right)$$
$$a_n = \frac{2}{\ell}\int_0^\ell f(x)\sin\left(\left[n+\frac{1}{2}\right]\frac{\pi x}{\ell}\right)dx \; a_0 = \frac{2}{\ell}\int_0^\ell f(x)\,dx$$
$$b_n = \frac{2}{n\pi c}\int_0^\ell g(x)\sin\left(\left[n+\frac{1}{2}\right]\frac{\pi x}{\ell}\right)dx \; b_0 =$$

$u_x(0,y) =$
$u_x(\ell,y) =$
0
$0 +$
$b_0 y +$
$\sum_{n=1}^{\infty} \left[b_n \sinh\left(\frac{n\pi y}{\ell}\right) + a_n \cosh\left(\frac{n\pi y}{\ell}\right)\right] \cos\left(\frac{n\pi x}{\ell}\right)$
$\{ u_{xx} +$
$u_{yy} =$
0
$u(0,y) =$
$u_x(\ell,y) =$
0
$\sum_{n=1}^{\infty} \left[b_n \sinh\left(\left[n+\frac{1}{2}\right]\frac{\pi y}{\ell}\right) + a_n \cosh\left(\left[n+\frac{1}{2}\right]\frac{\pi y}{\ell}\right)\right] \sin\left(\left[n+\frac{1}{2}\right]\frac{\pi x}{\ell}\right)$
$\{ u_{xx} +$
$u_{yy} =$
0
$u_x(0,y) =$
$u(\ell,y) =$
0
$\sum_{n=1}^{\infty} \left[b_n \sinh\left(\left[n+\frac{1}{2}\right]\frac{\pi y}{\ell}\right) + a_n \cosh\left(\left[n+\frac{1}{2}\right]\frac{\pi y}{\ell}\right)\right] \cos\left(\left[n+\frac{1}{2}\right]\frac{\pi x}{\ell}\right)$
$\{ u_{rr} +$
$\frac{1}{r} u_r +$
$\frac{1}{r^2} u_{\theta\theta} =$
0
$u(h,\theta) =$
$f(\theta)$
$\theta) =$
$\frac{a_0}{2} +$
$\sum_{n=1}^{\infty} a_n \frac{r^n}{a^n} \cos(n\theta) +$
$b_n \frac{r^n}{a^n} \sin(n\theta)$
$a^n a_n =$
$\int_{-\pi}^{\pi} f(\theta) \cos(n\theta)\, d\theta$
$a^n b_n =$
$\int_{-\pi}^{\pi} f(\theta) \sin(n\theta)\, d\theta$

$\{ u_{\ t} - ku_{xx} = h(x,t)$
$u(0,t) = u(\ell,t) = 0$
$u(x,0) = f(x)$
$\sum_{n=1}^{\infty} b_n(t) \sin\left(\frac{n\pi x}{\ell}\right)$
$\sum_{n=1}^{\infty} \left[b_n^{'}(t) + k\left(\frac{n\pi}{\ell}\right)^2 b_n(t)\right] \sin\left(\frac{n\pi x}{\ell}\right) = h(x,t)$
$\{ u_{\ t} - ku_{xx} = h(x,t)$
$u_x(0,t) = u_x(\ell,t) = 0$
$u(x,0) = f(x)$
$_0(t)\frac{}{2+\sum_{n=1}^{\infty} a_n(t)\cos\left(\frac{n\pi x}{\ell}\right)\frac{a_0'(t)}{2}+\sum_{n=1}^{\infty}\left[a_n'(t)+k\left(\frac{n\pi}{\ell}\right)^2 a_n\right]\cos\left(\frac{n\pi x}{\ell}\right)=h(x,t)}$
$\{ u_{\ tt} - c^2 u_{xx} = h(x,t)$
$u(0,t) = u(\ell,t) = 0$
$u(x,t) = f(x), u_t(x,0) = g(x)$
$\sum_{n=1}^{\infty} b_n(t) \sin\left(\frac{n\pi x}{\ell}\right)$
$\sum_{n=1}^{\infty} \left[b_n''(t) + \left(\frac{n\pi}{\ell}\right)^2 b_n(t)\right] \sin\left(\frac{n\pi x}{\ell}\right) = h(x,t)$
$\{ u_{\ tt} - c^2 u_{xx} = h(x,t)$
$u_x(0,t) = u_x(\ell,t) = 0$
$u(x,t) = f(x), u_t(x,0) = g(x)$
$_0(t)\frac{}{2+\sum_{n=1}^{\infty} b_n(t)\cos\left(\frac{n\pi x}{\ell}\right)\frac{b_0''(t)}{2}+\sum_{n=1}^{\infty}\left[b_n''(t)+\left(\frac{n\pi}{\ell}\right)^2 b_n(t)\right]\cos\left(\frac{n\pi x}{\ell}\right)=h(x,t)}$
$\{ u_{\ t} -$
$ku_{xx} =$
$h(x,t)$
$u(0,t) =$
$L(t), u(\ell,t) =$
$R(t)$
$u(x,0) =$
$f(x)$
$\{ u_{\ t} -$
$ku_{xx} =$
$h(x,t)$
$u(0,t) =$
$L(t), u(\ell,t) =$
$R(t)$
$u(x,0) =$
$f(x)$
$\delta(x,t) =$
$L +$
$\frac{x}{\ell}(R -$
$L)$
$\gamma(x,t) =$
$u(x,t) -$
$\delta(x,t)$

$\{u_t - ku_{xx} = 0$
$u(x,0) = f(x)$

$$\frac{}{\sqrt{4k\pi t}\int_{-\infty}^{\infty} e^{\frac{-(x-y)^2}{4kt}} f(y)\,dy}$$

$\{u_t - ku_{xx} = h(x,t)$
$u(x,0) = f(x)$

$$\frac{}{\sqrt{4k\pi t}\int_{-\infty}^{\infty} e^{\frac{-(x-y)^2}{4kt}} f(y)\,dy + \int_0^t \int_{-\infty}^{\infty} \frac{1}{\sqrt{4k\pi(t-s)}} e^{\frac{-(x-y)^2}{4k(t-s)}} h(y,s)\,dy\,ds}$$

$\{u_{tt} - c^2u_{xx} = 0$
$u(x,0) = f(x), u_t(x,0) = g(x)$

$$\frac{}{2[f(x+ct)+f(x-ct)]+\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds}$$

$\{u_{tt} - c^2u_{xx} = h(x,t)$
$u(x,0) = f(x), u_t(x,0) = g(x)$

$$\frac{}{2[f(x+ct)+f(x-ct)]+\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds + \frac{1}{2c}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} h(y,s)\,dy\,ds}$$