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# MODULE 4: Time Response of Control System

Time Response Analysis is the response given by the system which is function of the time, to the applied excitation is called time response of a control system. The **time response** represents how the state of a dynamic system changes in time when subjected to a particular input. The time response of a linear dynamic system consists of the sum of the **transient response** which depends on the initial conditions and the **steady-state response** which depends on the system input.

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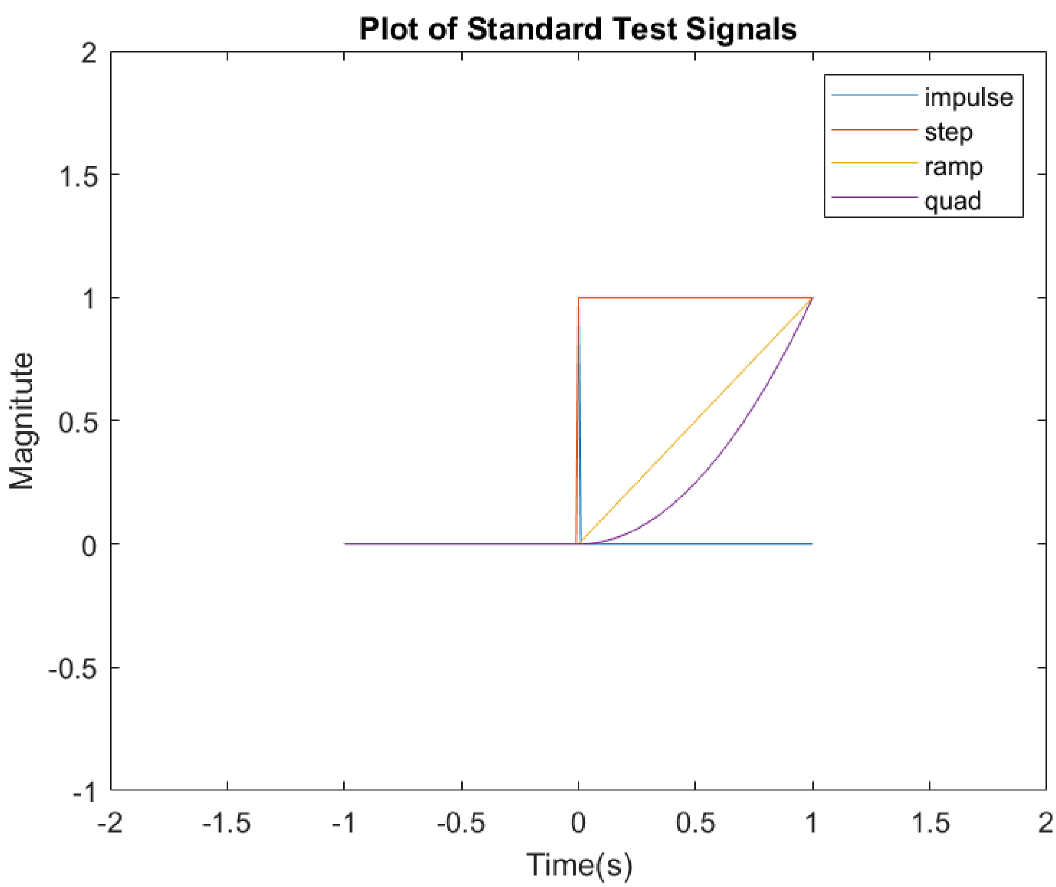
## 4.1 Standard Test Signal

Those signals which are most commonly used as reference inputs to the system are defined as **Standard Test Inputs.** They are:

- A sudden change **[Step Signal]**
- A momentary shock **[Impulse Signal]**
- A constant velocity **[Ramp Signal]**
- A constant acceleration **[Parabolic Signal]**

```
% Standard Test Signal in MATLAB
%time
dt=0.01;
t = (-1:dt:1)';

% Defining the signal
impulse = t==0;          % If t==0, we have 1
unitstep = t>=0;         % If t>0, We have 1
ramp = t.*unitstep;      % ramp= t *. unit [Scalar Product]
quad = t.^2.*unitstep;   % quad=t *.ramp
```

```
%Plotting the signal
plot(t,[impulse unitstep ramp quad])
% xlim([-2 2])
% ylim([-1 2])
legend impulse step ramp quad
```




```
title('Plot of Standard Test Signals')
```

```
xlabel('Time(s)')
ylabel('Magnitute')
```

## 4.2 Laplace Transform of Test Signal

```
syms A t
unitstep=t/t;
ramp=A*t;
quad=A*t^2/2;

u=A*laplace(unitstep);
r=laplace(ramp);
p=laplace(quad);
u
```

u =

$$\frac{A}{s}$$

```
r
```

r =

$$\frac{A}{s^2}$$

```
p
```

p =

$$\frac{A}{s^3}$$

## 4.3 Time Response of First Order System with different Sources

$$G(s) = \frac{10}{s + 10}$$

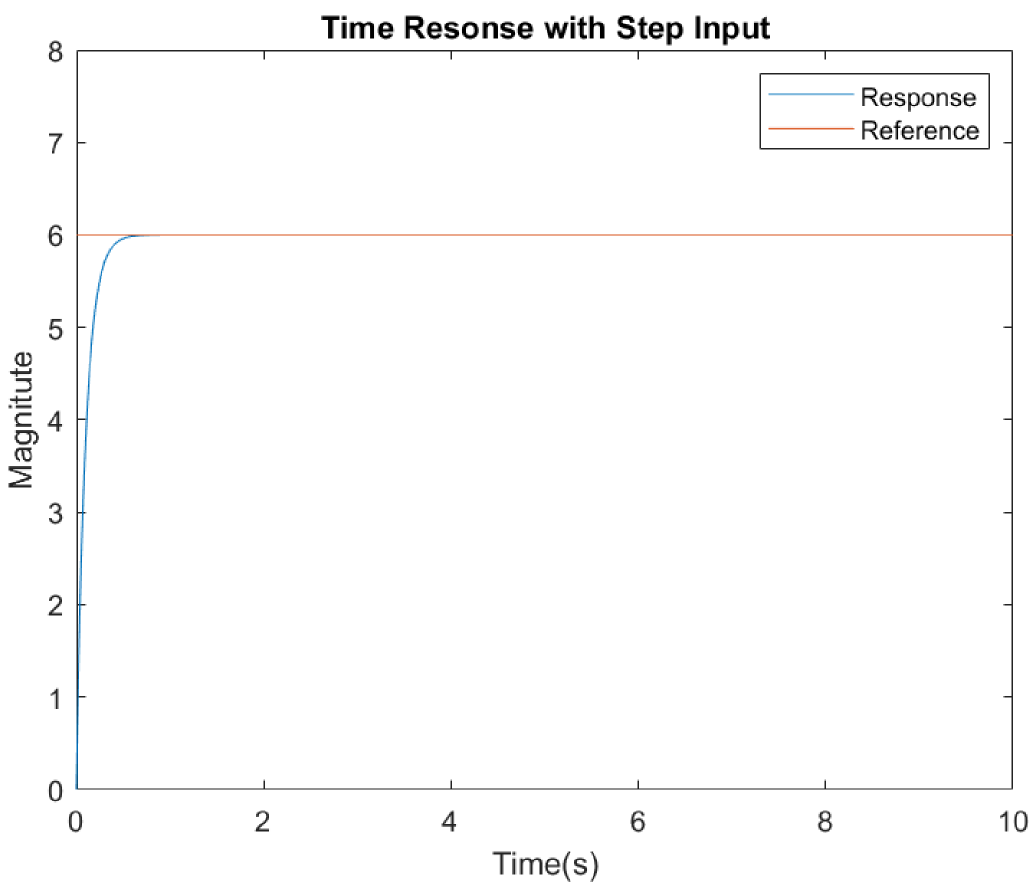
### 4.3.1 With Step Input

```
% Defining the System
syms s;
G1=10/(s+10);

% For Step Signal
A=6;
R=A/s;
r=ilaplace(R);
```

```
% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 = $6 - 6\,e^{-10\,t}$

```
%Time Response of System
figure
fplot(c1,[0,10]);
hold on
fplot(r,[0,10]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Step Input');
legend Response Reference;
ylim([0,8])
```




### 4.3.2 With Ramp Input

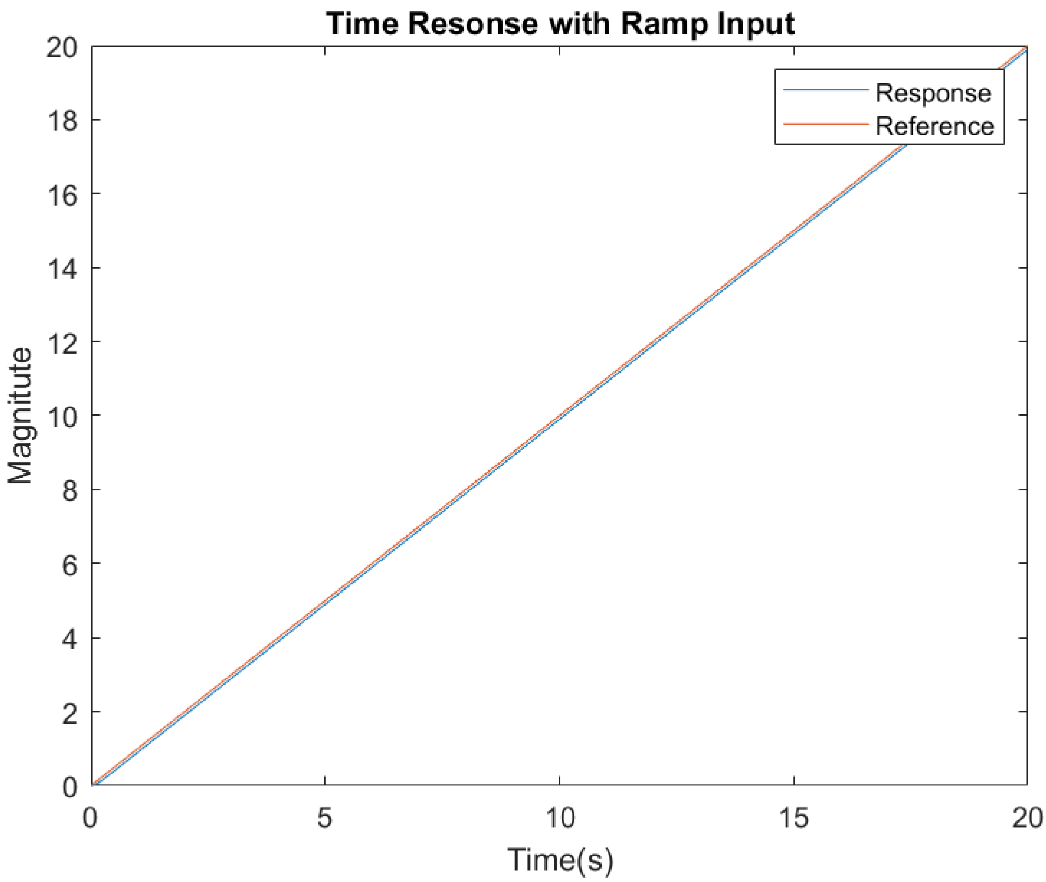
```
% Defining the System
```

```
syms s;
G1=10/(s+10);

% For Ramp Signal
A=1;
R=A/s^2;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$t + \frac{e^{-10\,t}}{10} - \frac{1}{10}$$

```
%Time Response of System
figure
fplot(c1,[0,20]);
hold on
fplot(r,[0,20]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Ramp Input');
legend Response Reference;
```




### 4.3.3 With Parabolic Input

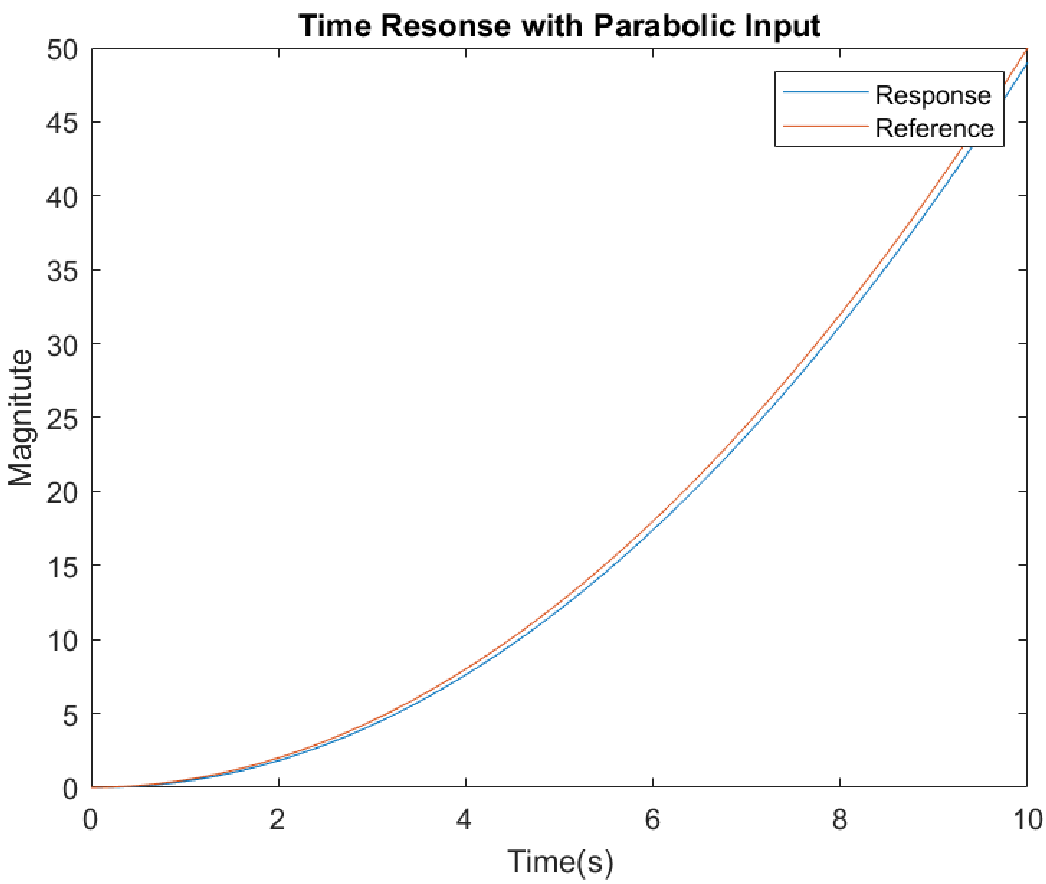
```
% Defining the System
syms s;
G1=10/(s+10);

% For Parabolic Signal
A=1;
R=A/s^3;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$\frac{t^2}{2} - \frac{e^{-10\,t}}{100} - \frac{t}{10} + \frac{1}{100}$$

```
%Time Response of System
figure
```

```
fplot(c1,[0,10]);
hold on
fplot(r,[0,10]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Parabolic Input');
legend Response Reference;
```




## 4.4 Effect of Variation of Time Constant in First Order System

$$G(s) = \frac{1}{1 + T\,s}$$

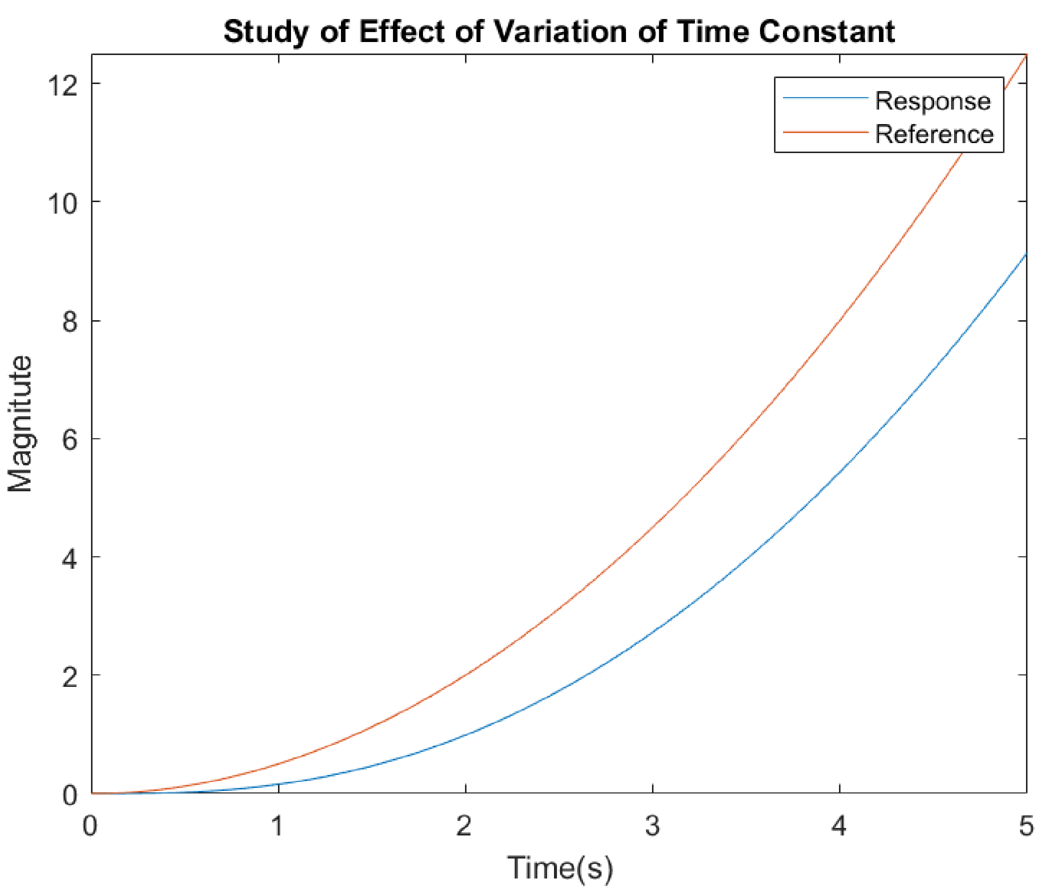
```
%Defining the System Parametera
T=0.8;

% Defining the System
syms s;
G1=1/(1+T*s);

% For Different Inputs Signal
A=1;
j=3;
```

```
R=A/s^j;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$\frac{t^2}{2} - \frac{16\,e^{-\frac{5\,t}{4}}}{25} - \frac{4\,t}{5} + \frac{16}{25}$$

```
%Time Response of System
figure
fplot(c1,[0,5]);
hold on
fplot(r,[0,5]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Study of Effect of Variation of Time Constant');
legend Response Reference;
```

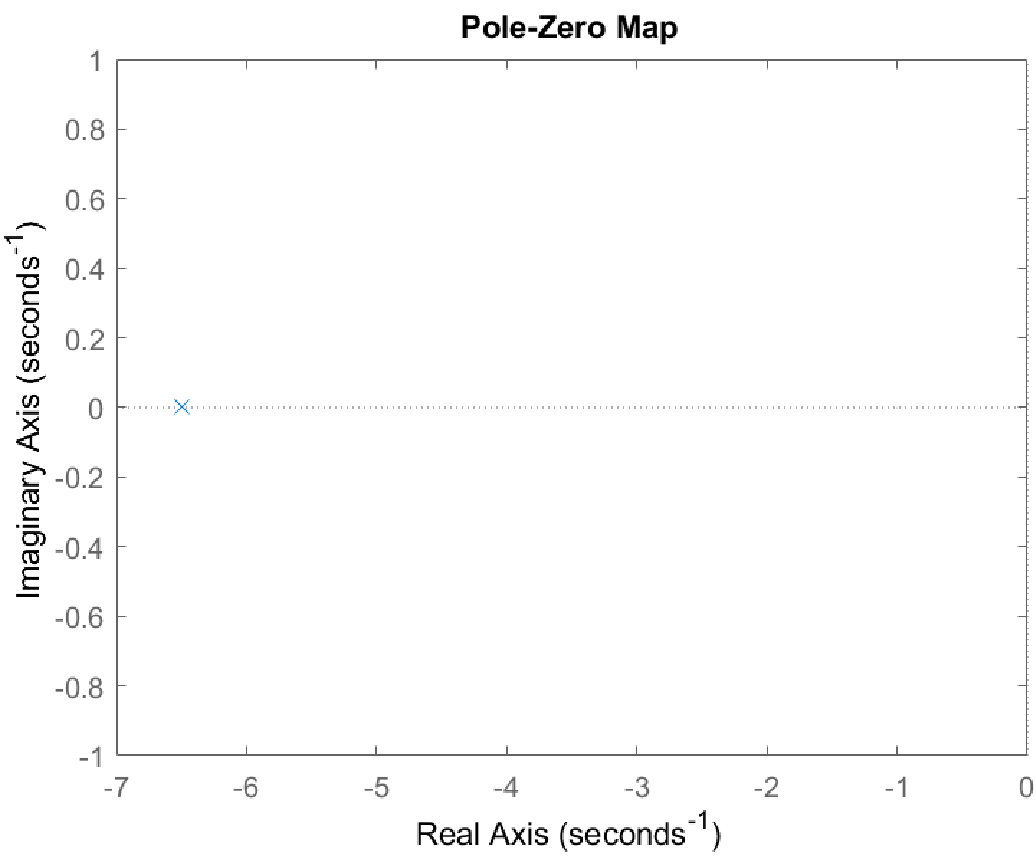


```
%axis([2.1,2.3,16.8,17.1]);   % Zoom it to see
```

## 4.5 Effect of Variation of Pole's Location on Time Response

$$G(s) = \frac{1}{s+a}$$

```
%Defining the System Parametera
a=6.5;
s=tf('s');
G=10/(s+a);
pzplot(G)
```



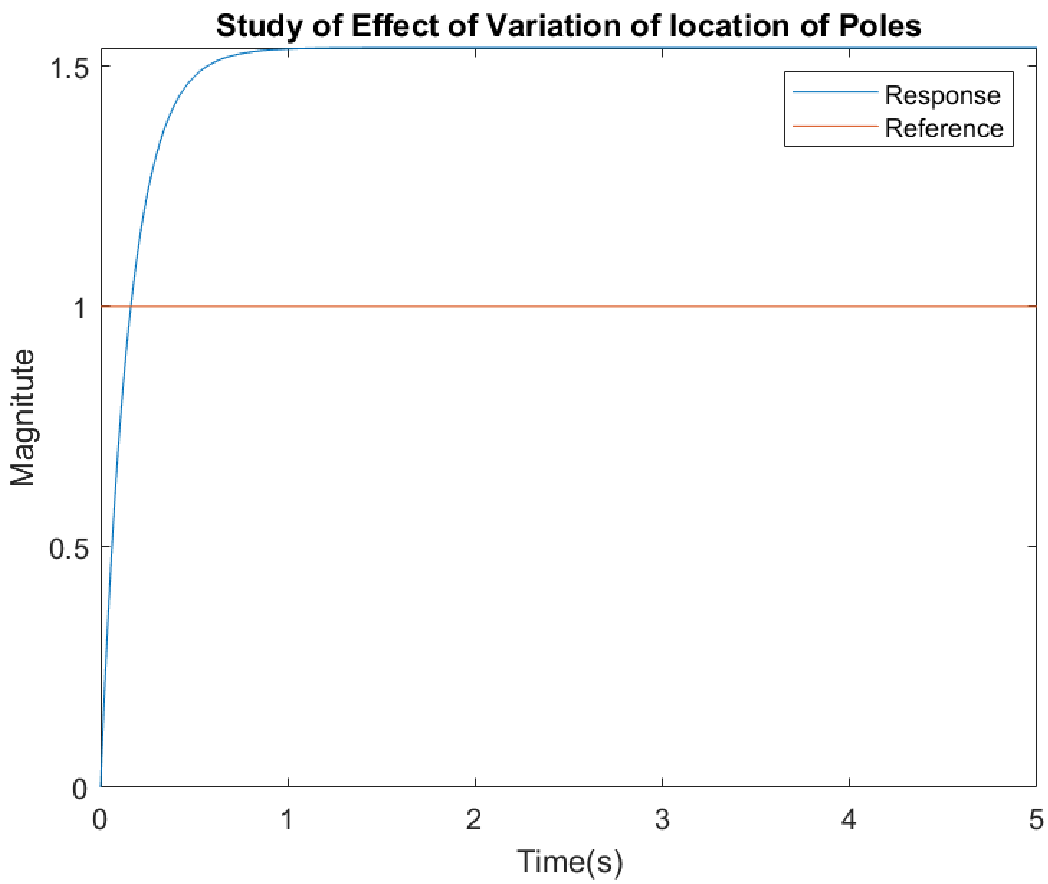

```
% Defining the System
syms s t;
G1=10/(s+a);

% For Different Inputs Signal
A=1;
j=1;
R=A/s^j;
r=ilaplace(R);

% Output in s-Domain
```

```
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$\frac{20}{13} - \frac{20\,e^{-\frac{13\,t}{2}}}{13}$$

```
%Time Response of System
figure
fplot(c1,[0,5]);
hold on
fplot(r,[0,5]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Study of Effect of Variation of location of Poles');
legend Response Reference;
```




## 4.6 Time Response of Second Order with different sources

$$G(s) = \frac{24}{s^2 + 5s + 24}$$

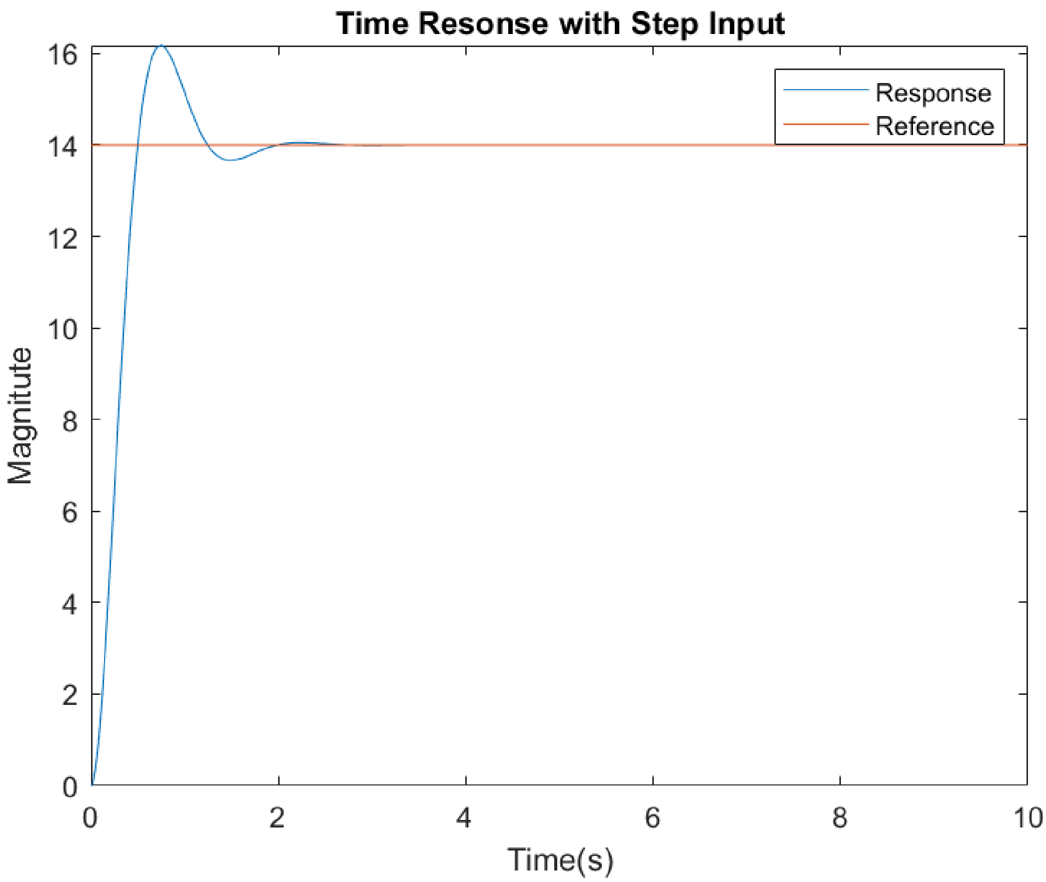
### 4.6.1 With Step Input

```
% Defining the System
syms s;
G1=24/(s^2+5*s+24);    % Refer Example 2 of Lecture 3.4

% For Step Signal
A=14;
R=A/s;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$14 - 14\,e^{-\frac{5\,t}{2}} \left( \cos\left(\frac{\sqrt{71}\ t}{2}\right) + \frac{5\ \sqrt{71}\ \sin\left(\frac{\sqrt{71}\ t}{2}\right)}{71} \right)$$

```

%Time Response of System
figure
fplot(c1,[0,10]);
hold on
fplot(r,[0,10]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Step Input');
legend Response Reference;
```



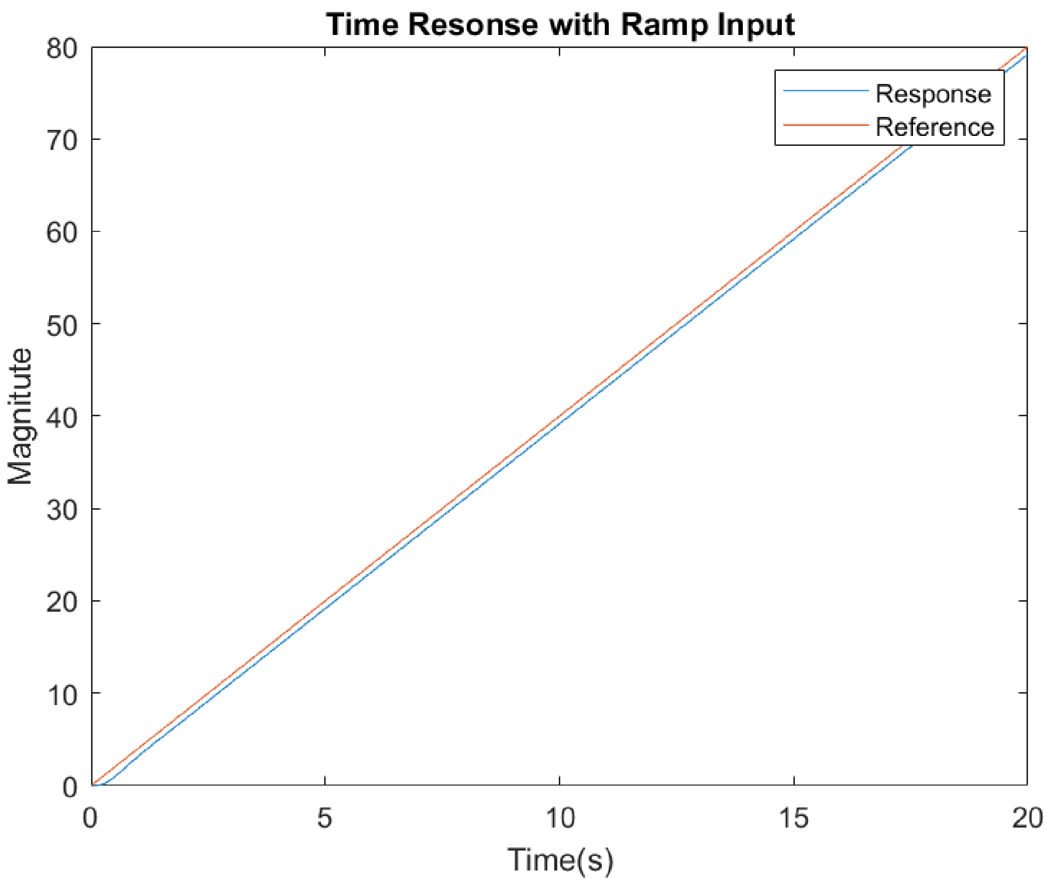

### 4.6.2 With Ramp Input

```
% Defining the System
syms s;
G1=24/(s^2+5*s+24);     % Refer Example 2 of Lecture 3.4

% For Ramp Signal
A=4;
R=A/s^2;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

$$4\,t + \frac{5\,e^{-\frac{5\,t}{2}} \left(\cos\left(\frac{\sqrt{71}\,t}{2}\right) - \frac{23\,\sqrt{71}\,\sin\left(\frac{\sqrt{71}\,t}{2}\right)}{355}\right)}{6} - \frac{5}{6}$$

```
%Time Response of System
figure
fplot(c1,[0,20]);
hold on
fplot(r,[0,20]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Ramp Input');
legend Response Reference;
```




### 4.6.3 With Parabolic Input

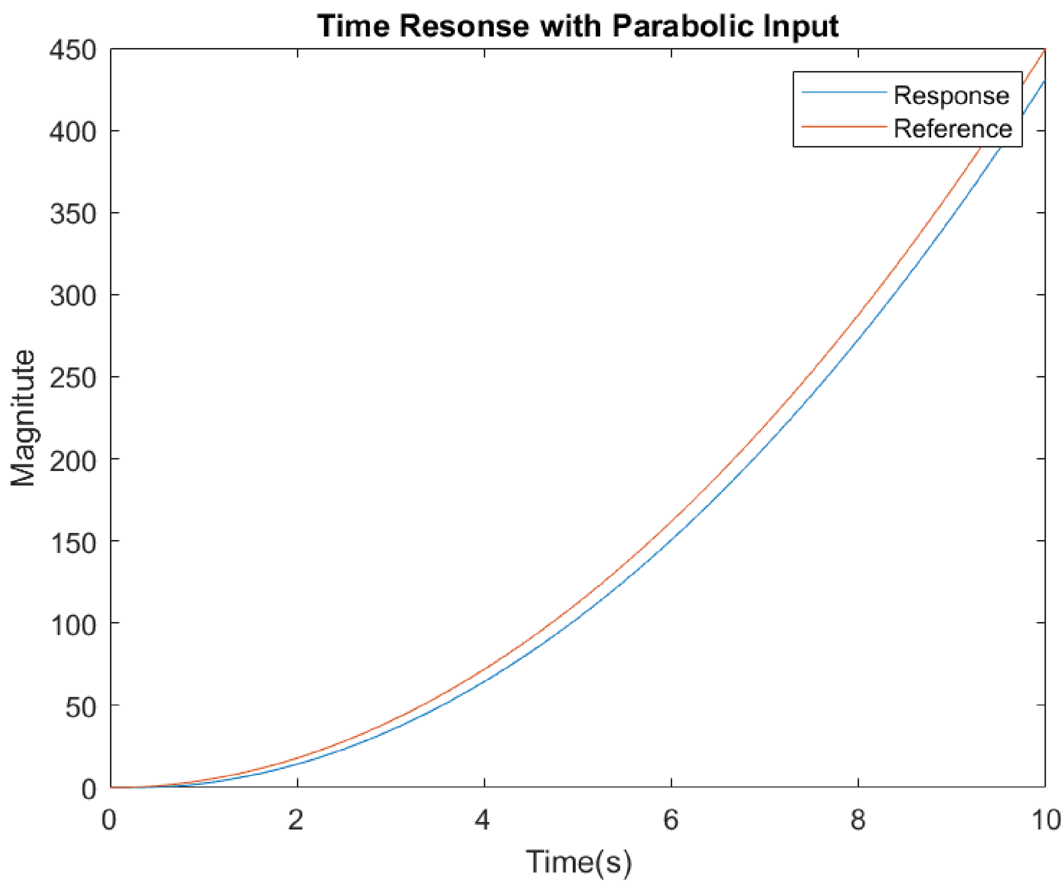
```
% Defining the System
syms s;
G1=24/(s^2+5*s+24);     % Refer Example 2 of Lecture 3.4

% For Parabolic Signal
A=9;
R=A/s^3;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
% Output in Time Domain
```

```
c1=ilaplace(C1)
```

c1 =

$$\frac{9\,t^2}{2}-\frac{15\,t}{8}-\frac{e^{-\frac{5\,t}{2}}\left(\cos\left(\frac{\sqrt{71}\,t}{2}\right)-\frac{235\,\sqrt{71}\,\sin\left(\frac{\sqrt{71}\,t}{2}\right)}{71}\right)}{64}+\frac{1}{64}$$

```
%Time Response of System
figure
fplot(c1,[0,10]);
hold on
fplot(r,[0,10]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Time Resonse with Parabolic Input');
legend Response Reference;
```




## 4.7 Effect of Variation of Damping Ratio

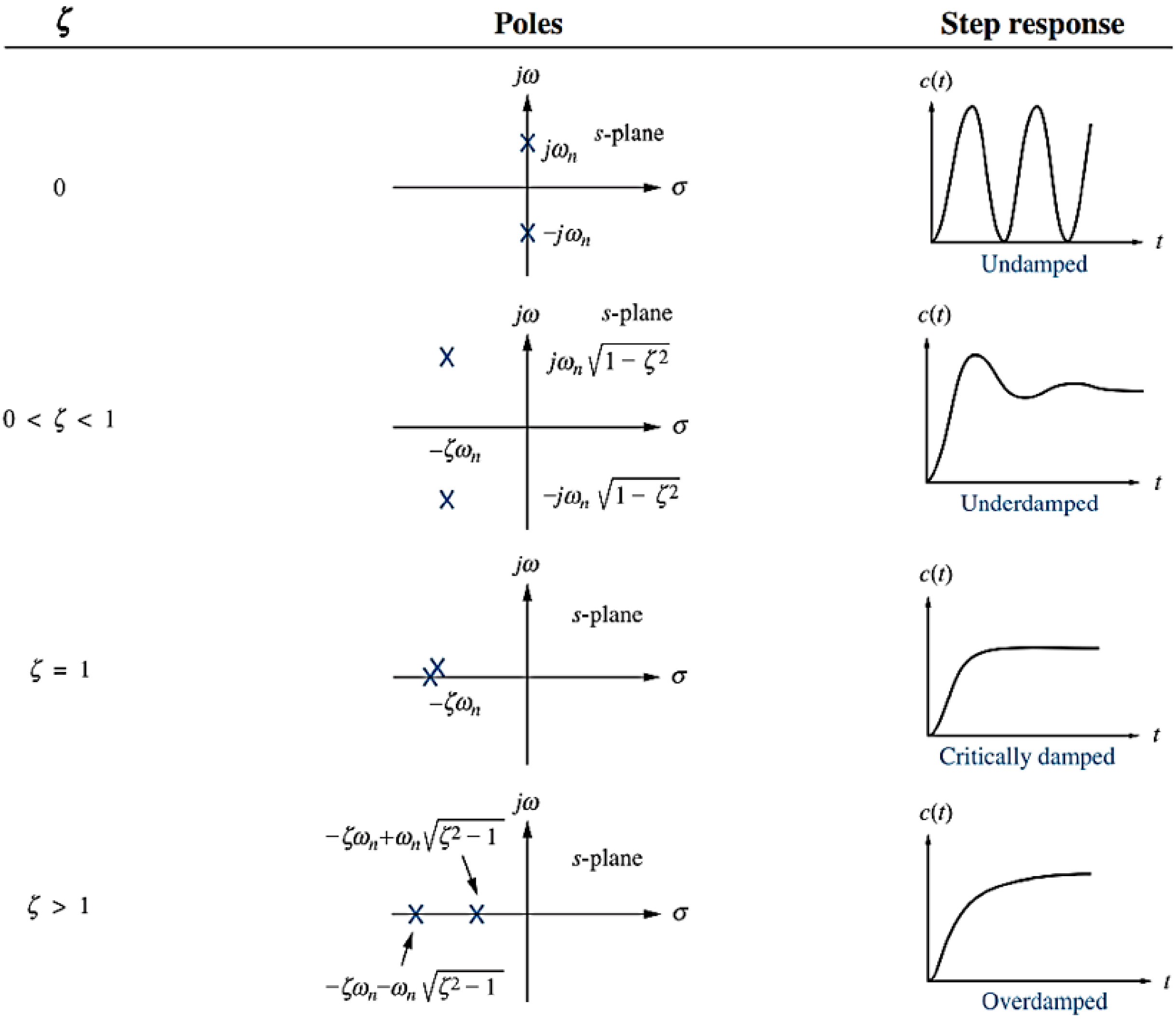



Fig: Effect of Variation of Damping Ratio [2]

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

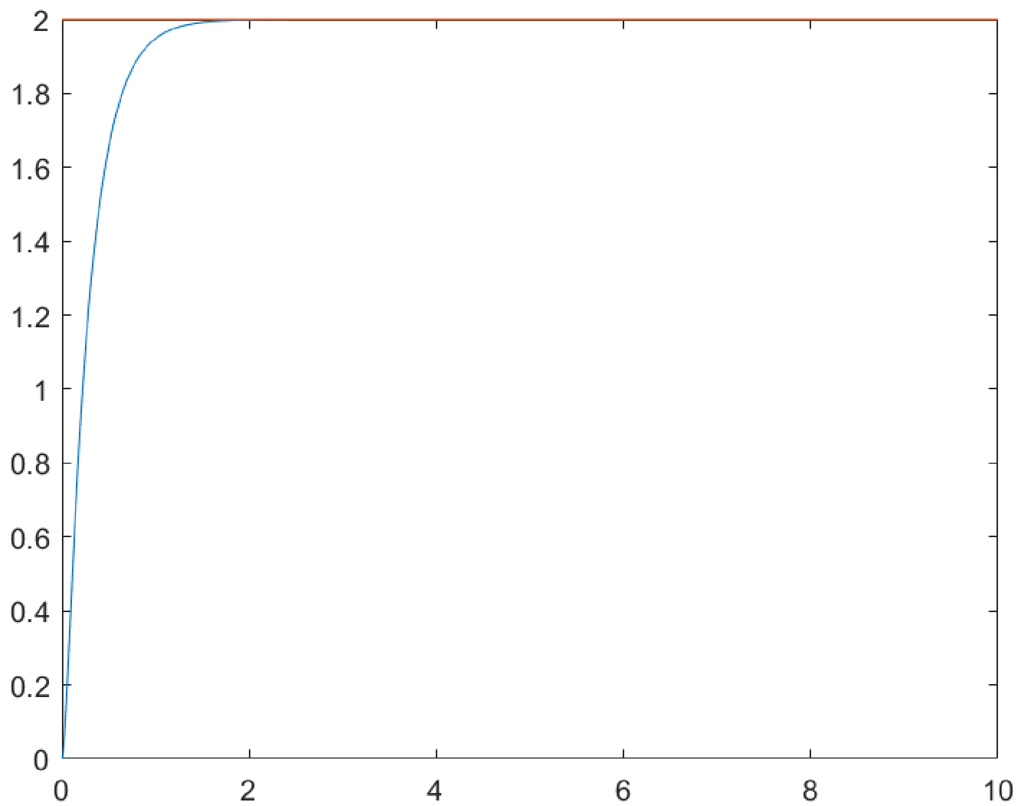
```
% Natural Frequency
w=10;   % Also Check for variation of Wn
% Damping Ratio
Z=1.5;

% Defining the System
syms s;
G2=w^2/(s^2+2*Z*w*s+w^2);


% For Step Signal
A=2;
R=A/s;

%Response in Time Domain
r=ilaplace(R);
```

```
% Output in s-Domain
C2=G2*R;
% Output in Time Domain
c2=ilaplace(C2)
```

c2 =

$$2 - 2\,e^{-15\,t}\left(\cosh(5\,\sqrt{5}\,t) + \frac{3\,\sqrt{5}\,\sinh(5\,\sqrt{5}\,t)}{5}\right)$$

```
%Time Response of System
figure
fplot(c2,[0,10]);
hold on
fplot(r,[0,10]);
hold off
```




## 4.8 Effect of Addition of Zeros on Time Response

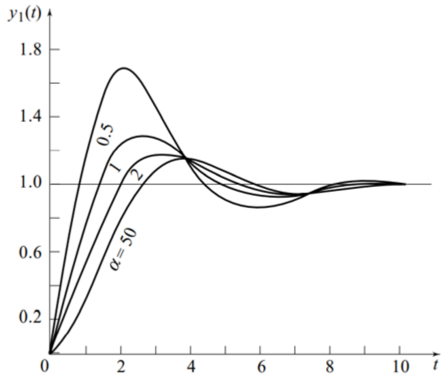
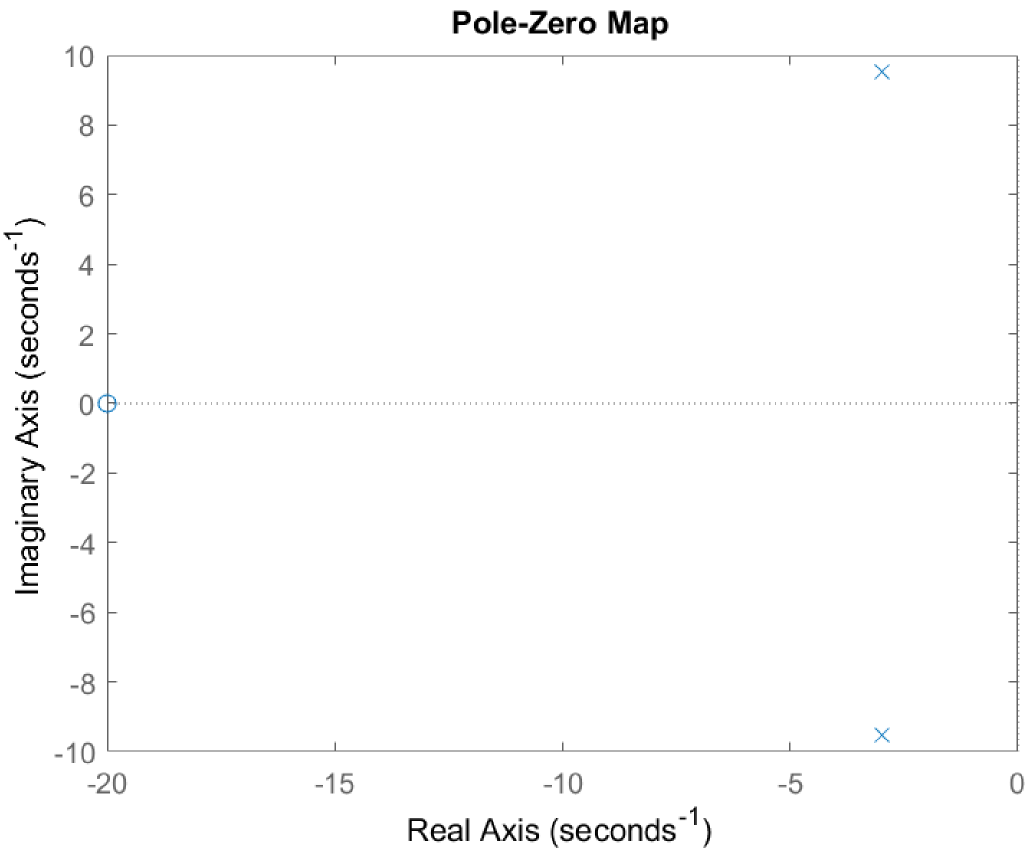



Fig: Effect of Addition of Zeros[2]

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$y_1(t) = y(t) + \frac{1}{\alpha}\frac{dy(t)}{dt}$$

```
% Natural Frequency
w=10;
% Damping Ratio
Z=0.3;
% Variable for Variation of Zeros
a=20;

%FOr Plotting in P_Z Plane
s=tf('s');
G2S=w^2/(s^2+2*Z*w*s+w^2);
Za=(s+a)/a;
G=G2S*Za;
pzplot(G)
```




```
% Defining the System
syms s;
G1=w^2/(s^2+2*Z*w*s+w^2);

% Assign Zeros to the System
G2S=w^2/(s^2+2*Z*w*s+w^2);
Za=(s+a)/a;    % Noramlized Zero
G2=G2S*Za;      % Zero Addition

% For Different Inputs Signal
A=1;
j=1;
R=A/s^j;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
C2=G2*R;
D=s*C1;
% Output in Time Domain
c1=ilaplace(C1)
```

```
c1 =
```

$$1 - e^{-3t}\left(\cos(\sqrt{91}\ t) + \frac{3\sqrt{91}\ \sin(\sqrt{91}\ t)}{91}\right)$$

```
c2=ilaplace(C2)
```

c2 =

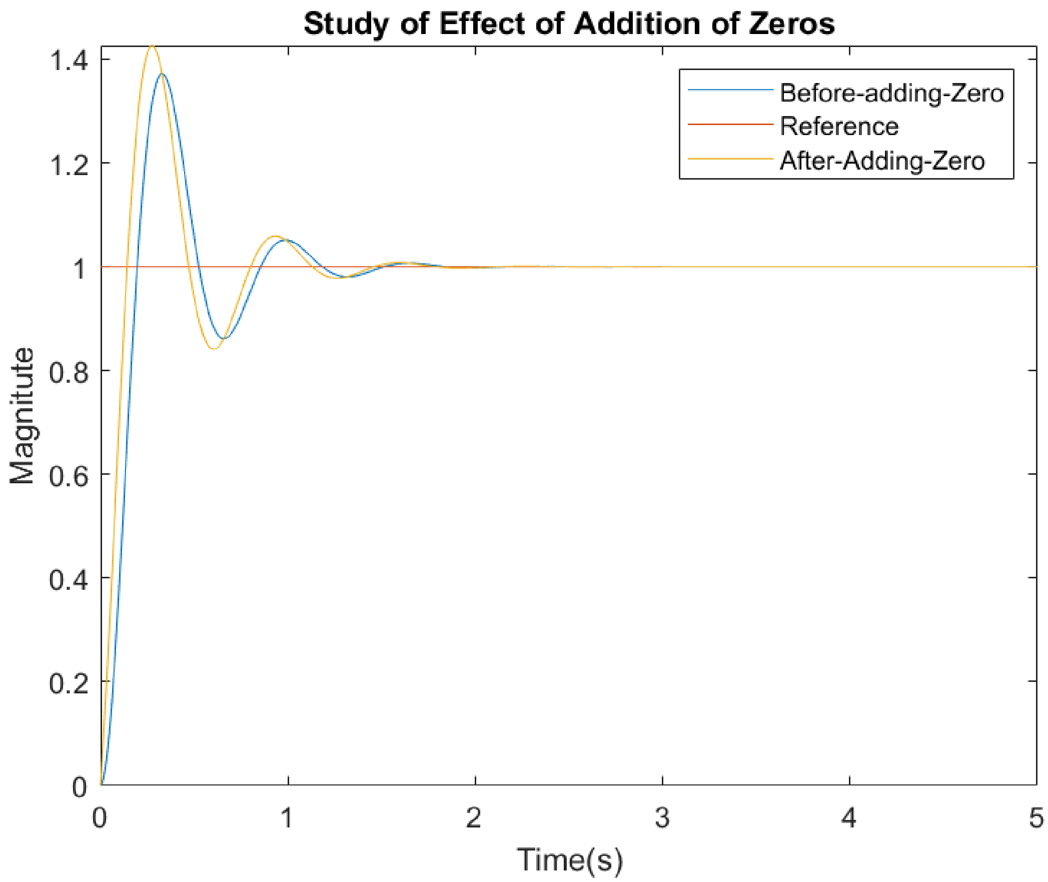
$$1 - e^{-3t}\left(\cos(\sqrt{91}\ t) - \frac{2\sqrt{91}\ \sin(\sqrt{91}\ t)}{91}\right)$$

```
d=ilaplace(D)
```

d =

$$\frac{100\sqrt{91}\ e^{-3t}\sin(\sqrt{91}\ t)}{91}$$

```
%Time Response of System
figure
fplot(c1,[0,5]);
hold on
fplot(r,[0,5]);
fplot(c2,[0,5]);
%fplot(d,[0,5]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Study of Effect of Addition of Zeros');
legend Before-adding-Zero Reference After-Adding-Zero ;
```




## 4.9 Effect of Addition of Poles on Time Response

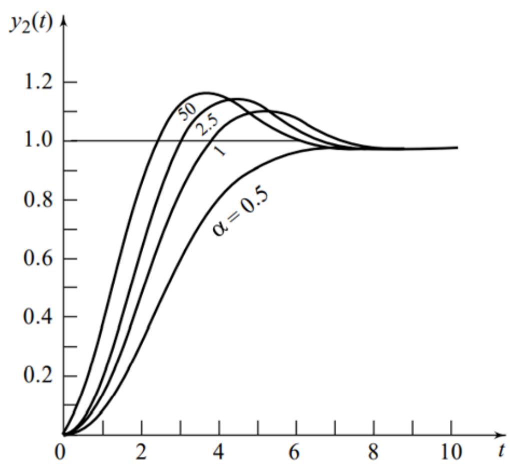


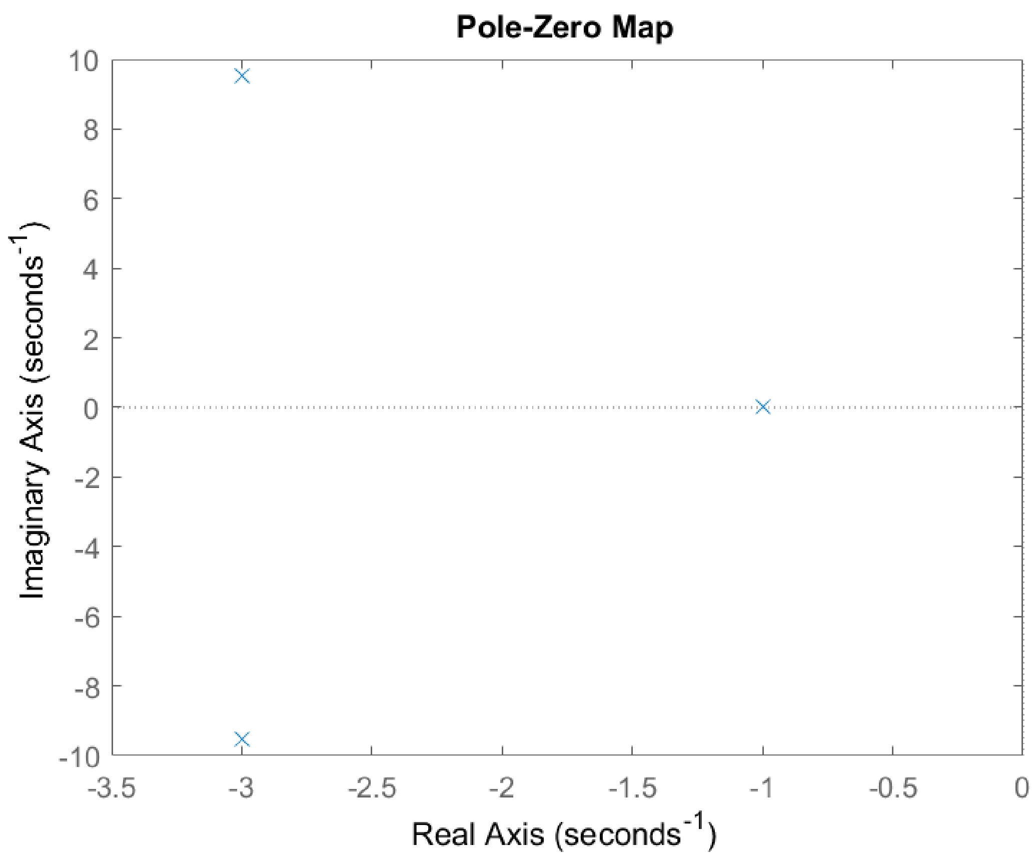

**Fig. 6.22** Unit-step response of a standard second-order system ($\zeta = 0.5$, $\omega_n = 1$) for several locations of an additional

Fig: Effect of Addition of Poles[2]

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} * \frac{a}{s+a}$$

$$Y_2(s) = Ae^{-\alpha t} + y(t)$$

```
% Natural Frequency
w=10;
% Damping Ratio
Z=0.3;
% Variable for Variation of Zeros
a=1;

%FOr Plotting in P_Z Plane
s=tf('s');
G2S=w^2/(s^2+2*Z*w*s+w^2);
Za=a/(s+a);
G=G2S*Za;
pzplot(G)
```




```
% Defining the System
syms s;
G1=w^2/(s^2+2*Z*w*s+w^2);
```

```
% Assign Zeros to the System
G2S=w^2/(s^2+2*Z*w*s+w^2);
Za=a/(s+a);   % Noramlized Pole
G2=G2S*Za;     % Pole Addition

% For Different Inputs Signal
A=1;
j=1;
R=A/s^j;
r=ilaplace(R);

% Output in s-Domain
C1=G1*R;
C2=G2*R;
% Output in Time Domain
c1=ilaplace(C1)
```

c1 =

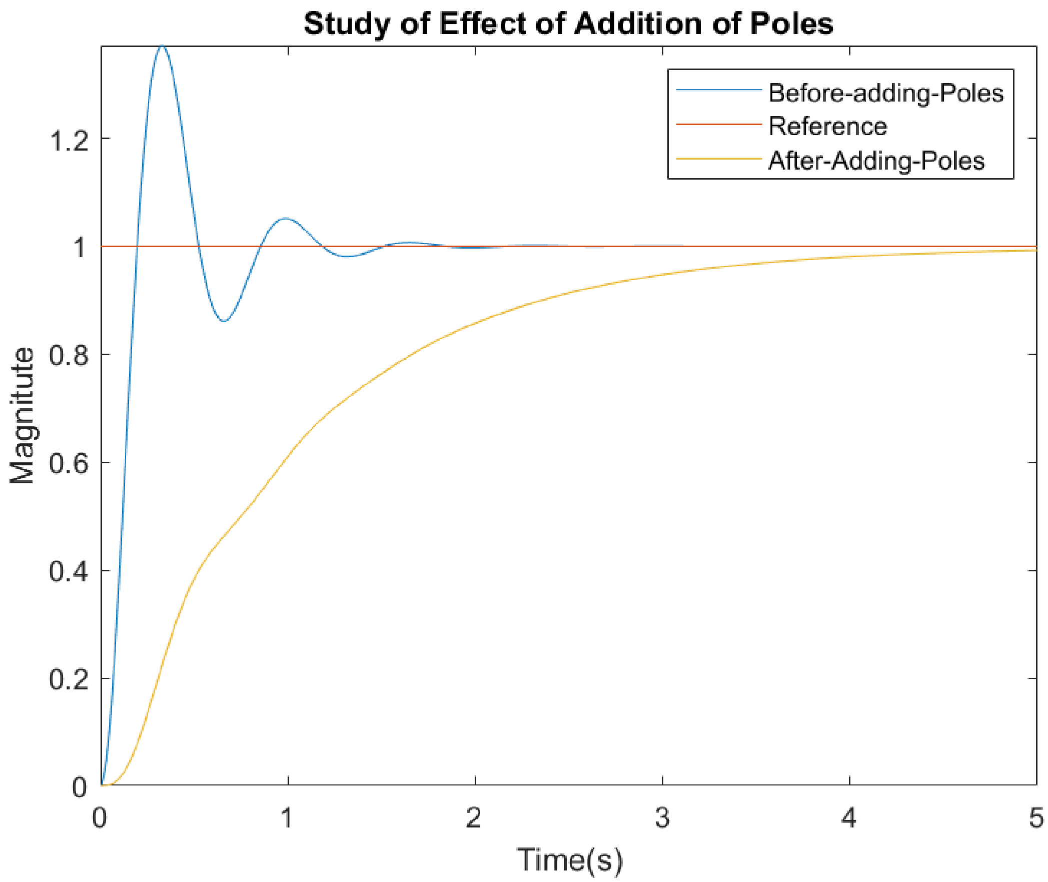
$$1-e^{-3t}\left(\cos(\sqrt{91}\ t)+\frac{3\ \sqrt{91}\ \sin(\sqrt{91}\ t)}{91}\right)$$

```
c2=ilaplace(C2)
```

c2 =

$$\frac{e^{-3t}\left(\cos(\sqrt{91}\ t)-\frac{17\ \sqrt{91}\ \sin(\sqrt{91}\ t)}{91}\right)}{19}-\frac{20\,e^{-t}}{19}+1$$

```
%Time Response of System
figure
fplot(c1,[0,5]);
hold on
fplot(r,[0,5]);
fplot(c2,[0,5]);
hold off
xlabel('Time(s)');
ylabel('Magnitute');
title('Study of Effect of Addition of Poles');
legend Before-adding-Poles Reference After-Adding-Poles ;
```




## 4.10 What about Dominating Poles?

If ratio of real part of one pole is greater than 5 times the other than we can say the former one is dominant poles. SImilar case is with Poles and Zeros. They are nearer to the Origin.

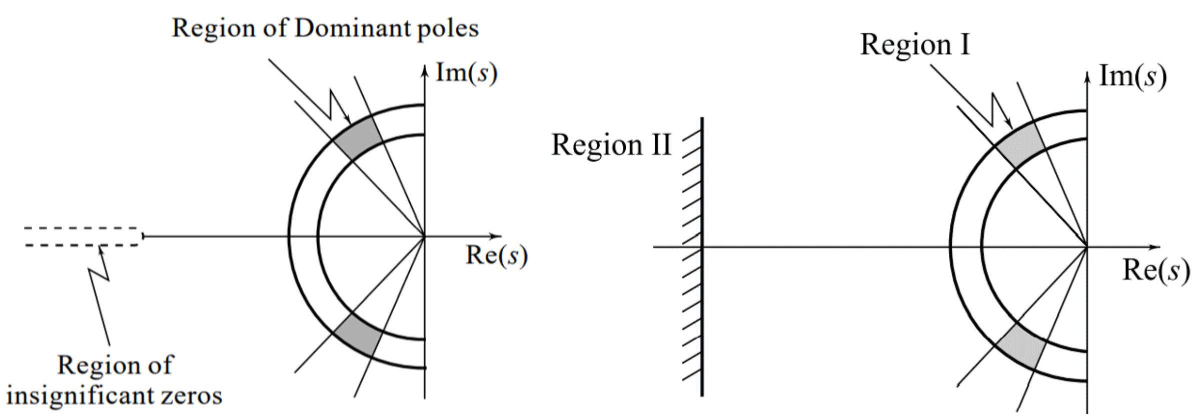
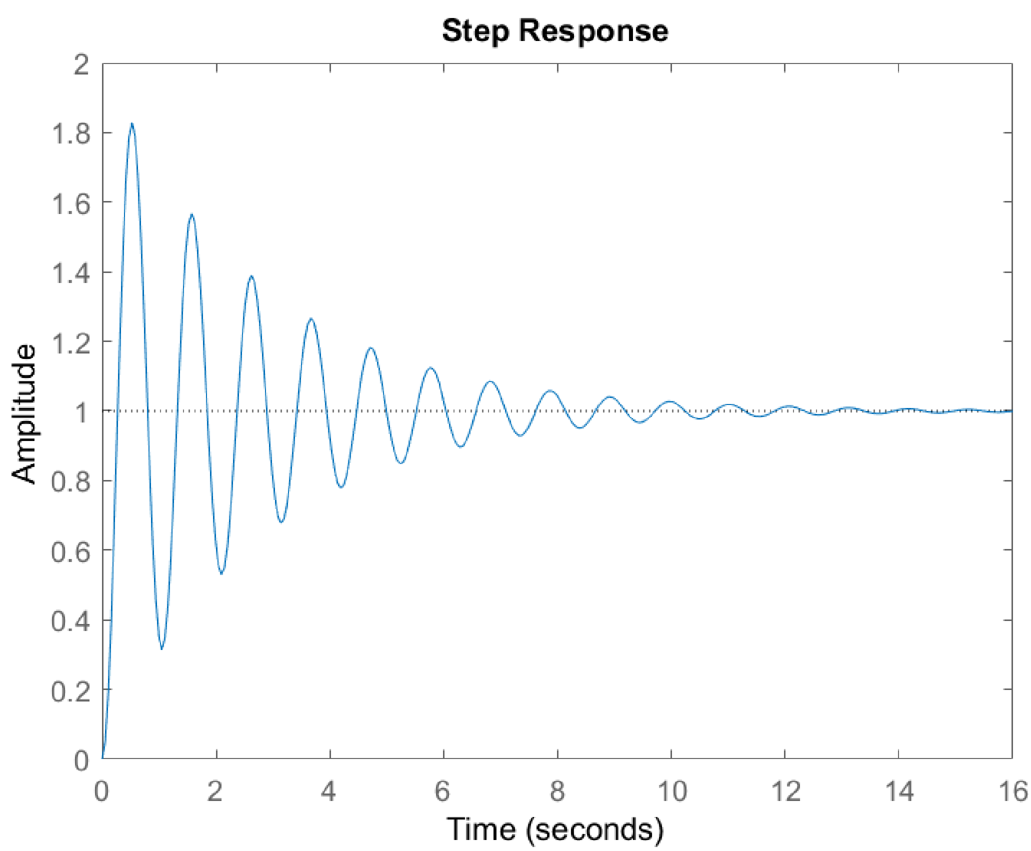



Fig: Dominant Poles with Zeros and Other Poles [2]

## 4.11 Transient Response Specfications

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

```
% Natural Frequency
w=6;
% Damping Ratio
Z=0.06;

% Defining the System
num=w^2;
den=[1 2*Z*w w^2];
G=tf(num,den);
% To Plot Step Function Response
step(G)
```




```
%To Get Transient Response Data
S=stepinfo(G)
```

```
S = struct with fields:

        RiseTime: 0.1820
    SettlingTime: 10.5765
     SettlingMin: 0.3146
     SettlingMax: 1.8279
       Overshoot: 82.7909
      Undershoot: 0
            Peak: 1.8279
```

```
        PeakTime: 0.5236
```

```
S.RiseTime
```

```
ans = 0.1820
```

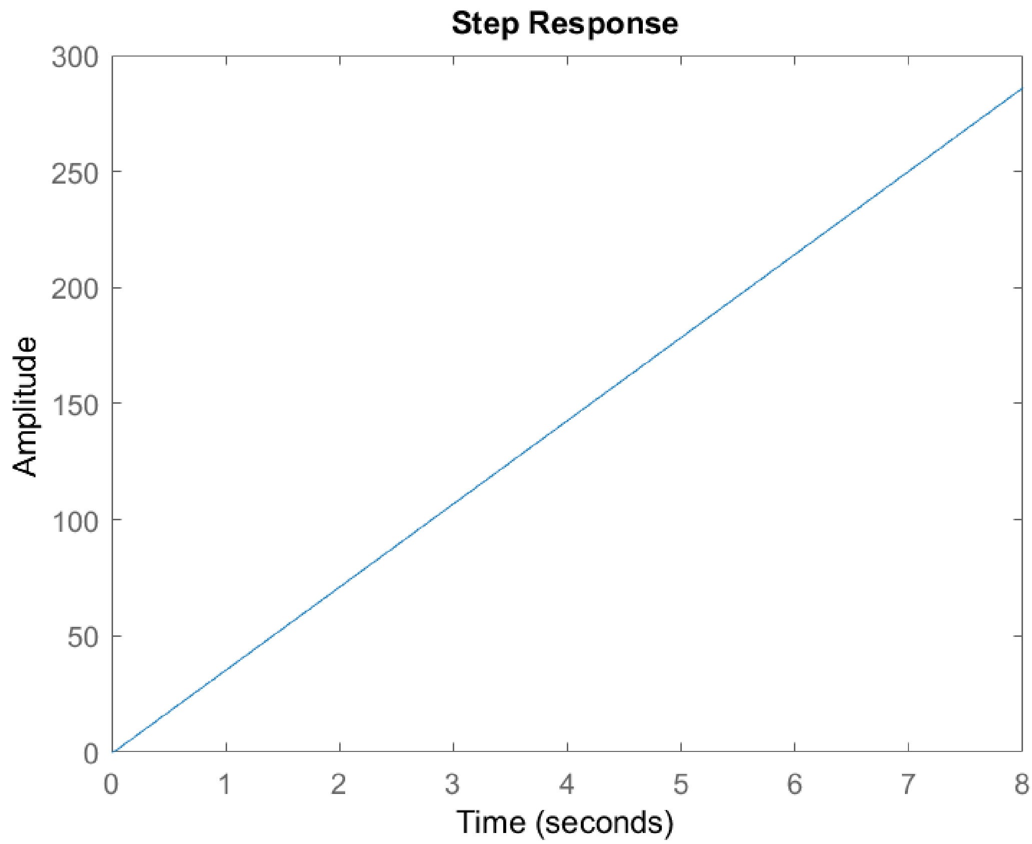
```
% Defining Settling Threshods
S1 = stepinfo(G,'SettlingTimeThreshold',0.005);
st1 = S1.SettlingTime
```

```
st1 = 14.2632
```

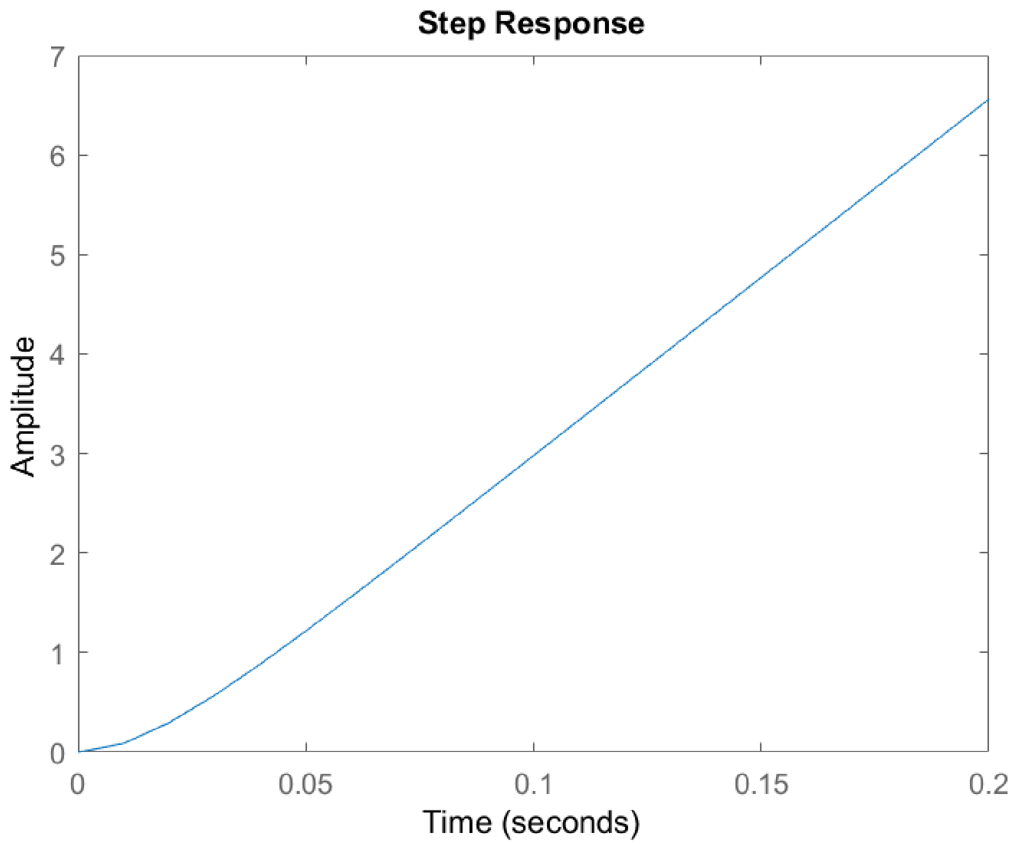
## 4.12 Working with DC Motor

### 4.12.1 DC Motor Position Control in Script

```
J = 3.2284E-6;
b = 3.5077E-6;
K = 0.0274;
R = 4;
L = 2.75E-6;
s = tf('s');
Pos_motor = K/(s*((J*s+b)*(L*s+R)+K^2));

step(Pos_motor)
```




```
t=0:0.01:0.2;
step(Pos_motor,t)
```




```
ans = logical
   0
ans = 3×1
10^6 ×
         0
   -1.4545
   -0.0001
```

### 4.12.2 DC Motor Position Control in Simulink

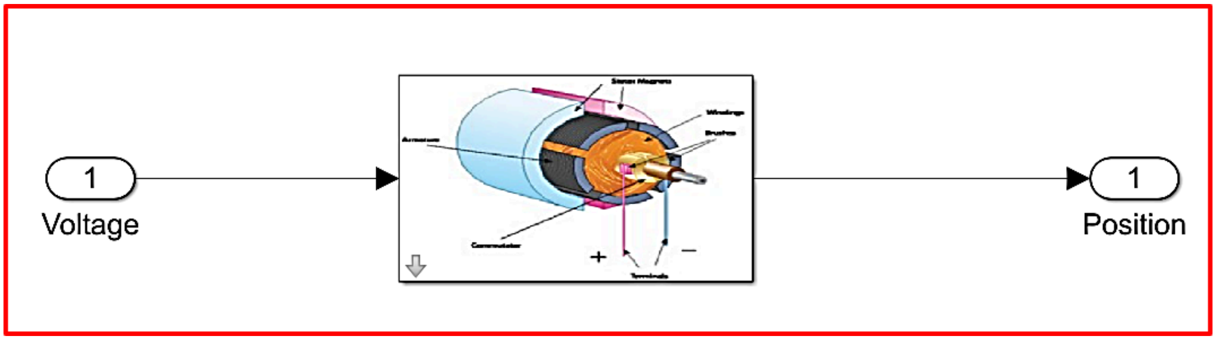



Fig: Simulink Model of DC Motor

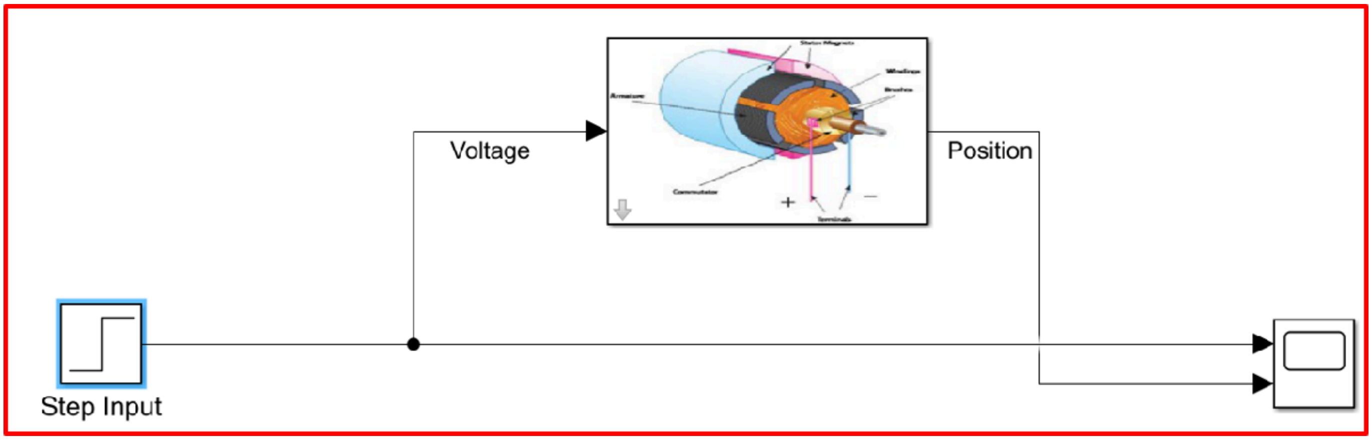



Fig: DC Motor with Step Input

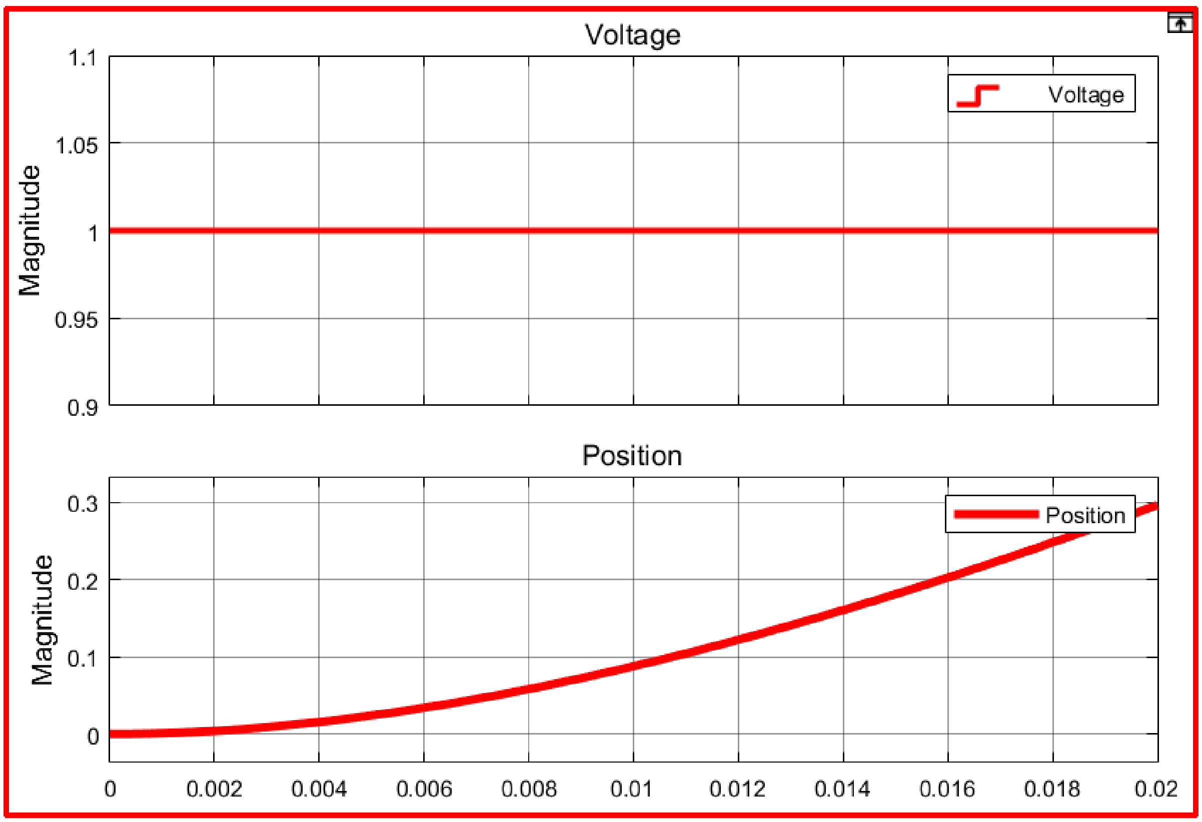



Fig: Time Response of Unit Step Input of Position Control DC Motor

# References


1. Mathworks.inc
2. Nise, Norman S. "Control system engineering, John Wiley & Sons." *Inc, New York*(2011).
3. https://ctms.engin.umich.edu/CTMS/index.php?aux=Home
4. Bakshi, Uday A., and Varsha U. Bakshi. *Control system engineering*. Technical Publications, 2020.