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MODULE VI: Root Locus

Root Locus is defined as, the locus of the closed loop poles obtained when system gain 'K' is varied from -∞ to +∞ is called Root Locus. When 'K' is varied from 0 to +∞, the plot is called **Direct root locus.**

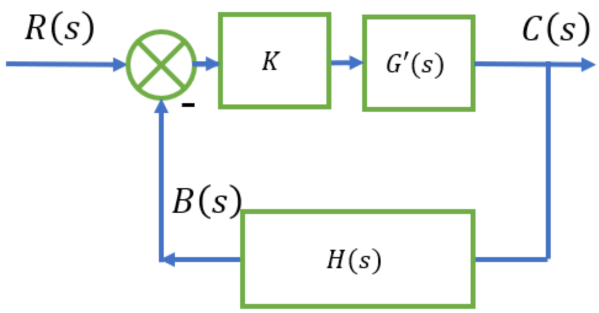

Fig: Variation of Roots due to Gain

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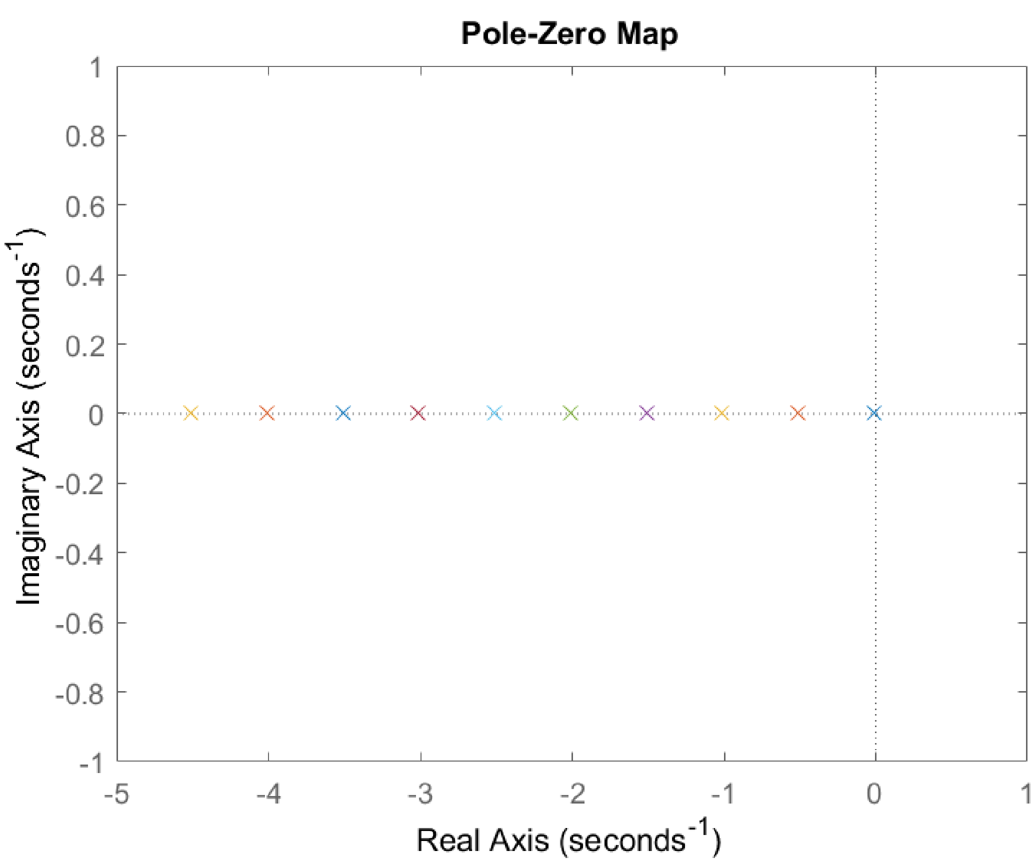
6.1 Construction of Root Locus by Manual Way

6.1.1 Root Locus for One Simple Open Loop Poles

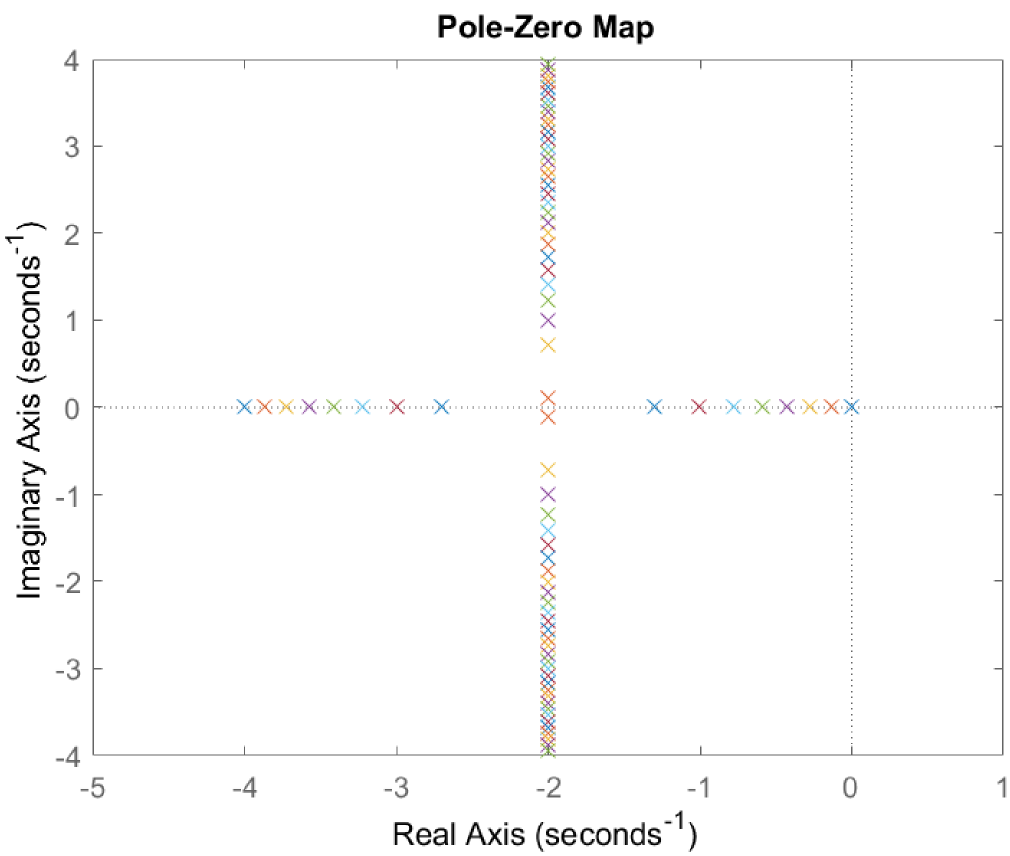
```
s=tf('s');
H=1;
figure;

hold on
for K=0.01:0.5:20
G=K/(s);
CLG=feedback(G,H);        % This is to create a closed loop TF
pzplot(CLG);              %Plot of CLosed Loop TF
xlim([-5 1]);
end
hold off
```




```
% rlocus(G);                %In rlocus you give Open Loop TF i.e GH
```

6.1.2 Root Locus for Two Simple Open Loop Poles

```
s=tf('s');
H=1;
figure;

hold on
for K=0.01:0.5:20
G=K/(s*(s+4));
CLG=feedback(G,H);
pzplot(CLG);
xlim([-5 1]);
end
hold off
```

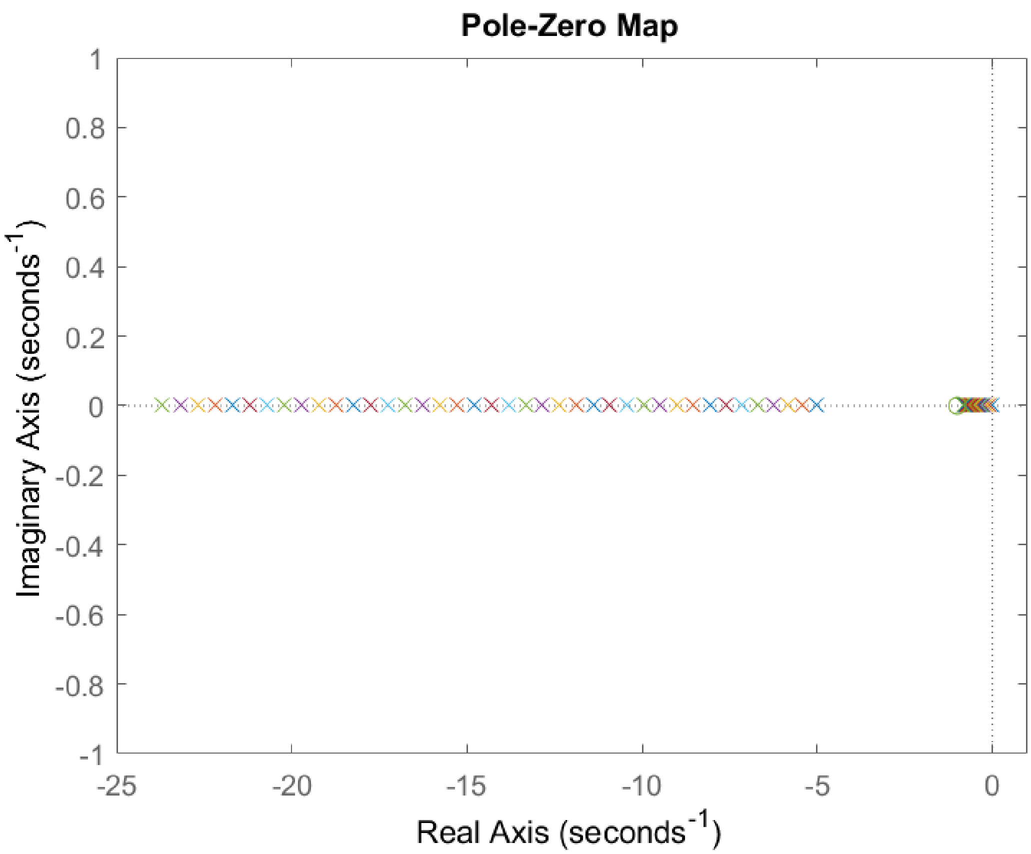



```
% rlocus(G);
```

6.1.3 Root Locus for Two Simple Open Loop Poles and a Zeros

```
s=tf('s');
H=1;
figure;
```

```
hold on
for K=0.01:0.5:20
G=K*(s+1)/(s*(s+5));
CLG=feedback(G,H);
pzplot(CLG);
xlim([-25 1]);
end
hold off
```



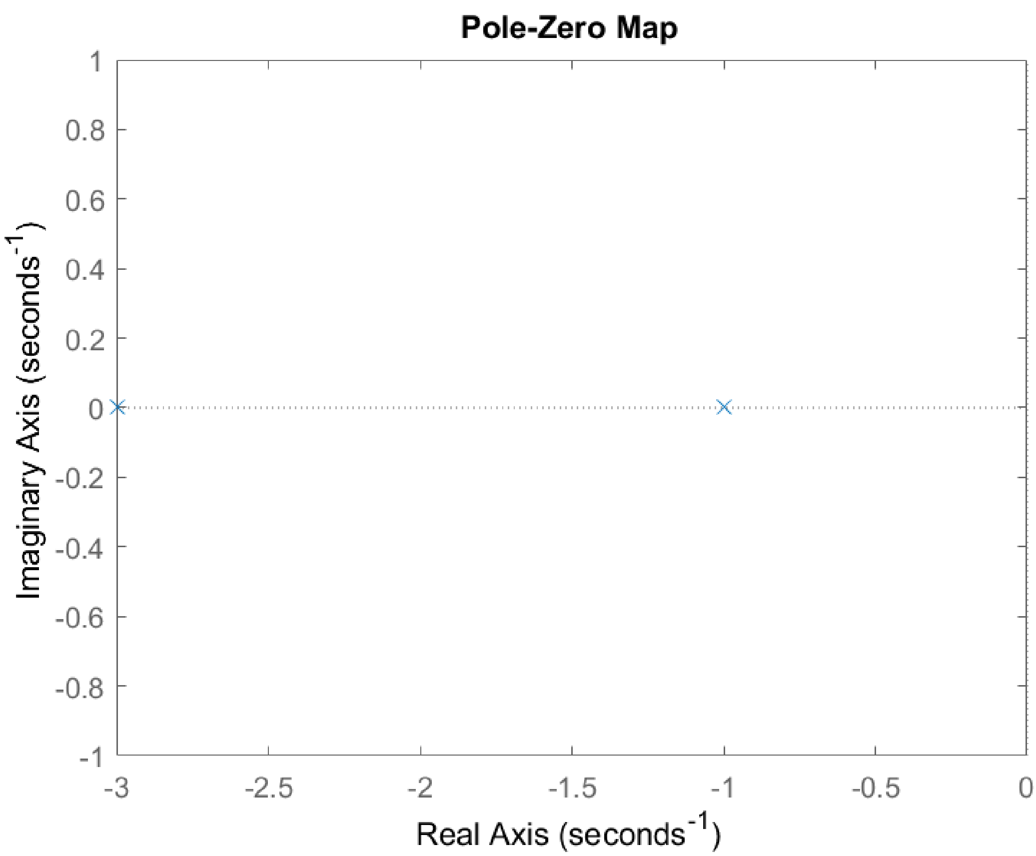

```
% rlocus(G);
```

6.2 Relation Between Root Locus and Time Response of System

4-cases	s_1, s_2	Roots(s_1, s_2)	System Response
$0 < \xi < 1$	$-\xi w_n \pm j\text{Constant}$	Complex conjugate	Underdamped response
$\xi = 0$	$0 \pm jw_n$	Imaginary	Undamped response
$\xi = 1$	$-w_n \pm j0$	Real and equal	Critically damped response
$\xi > 1$	$-\xi w_n \pm \text{Constant}$	Real and distinct	Over damped response

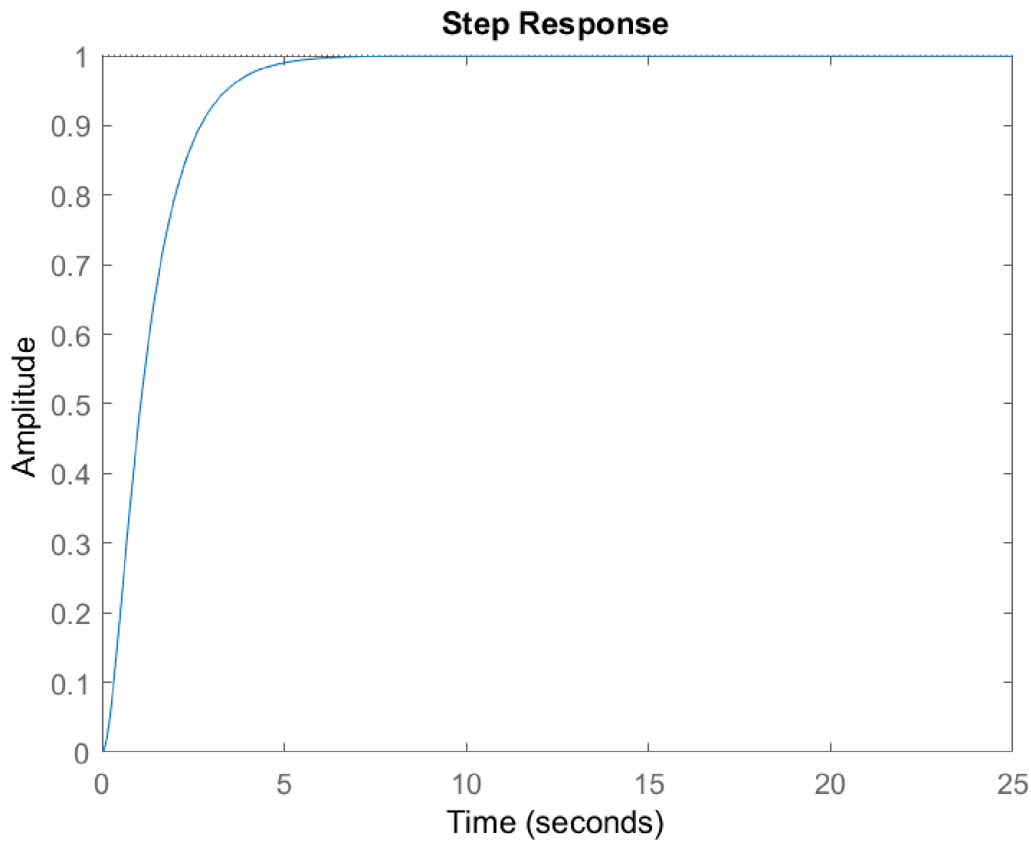
6.2.1 Variation of Gain by Slider

```
s=tf('s');
H=1;
figure;
K=3;   % Variation of Gain
G=K/(s*(s+4));
CLG=feedback(G,H);
pzmap(CLG);
```



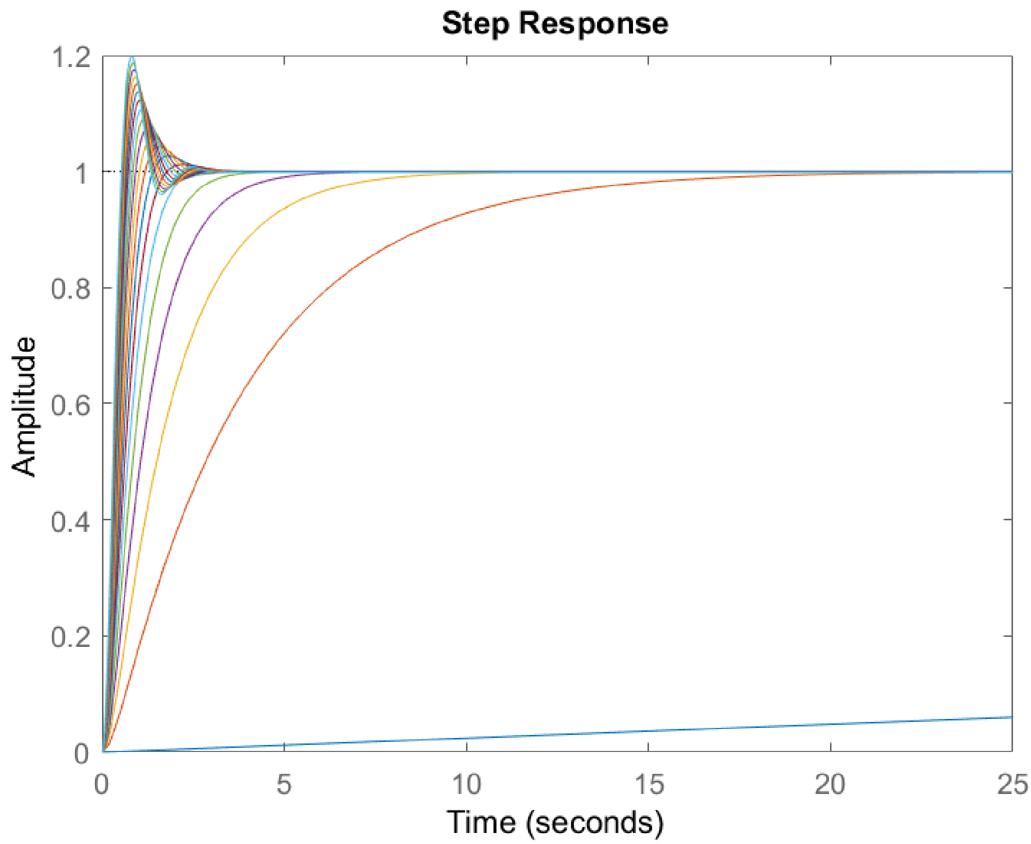

```
step(CLG);                    % Plot Step response of closed loop system
```

```
xlim([0,25]);
```



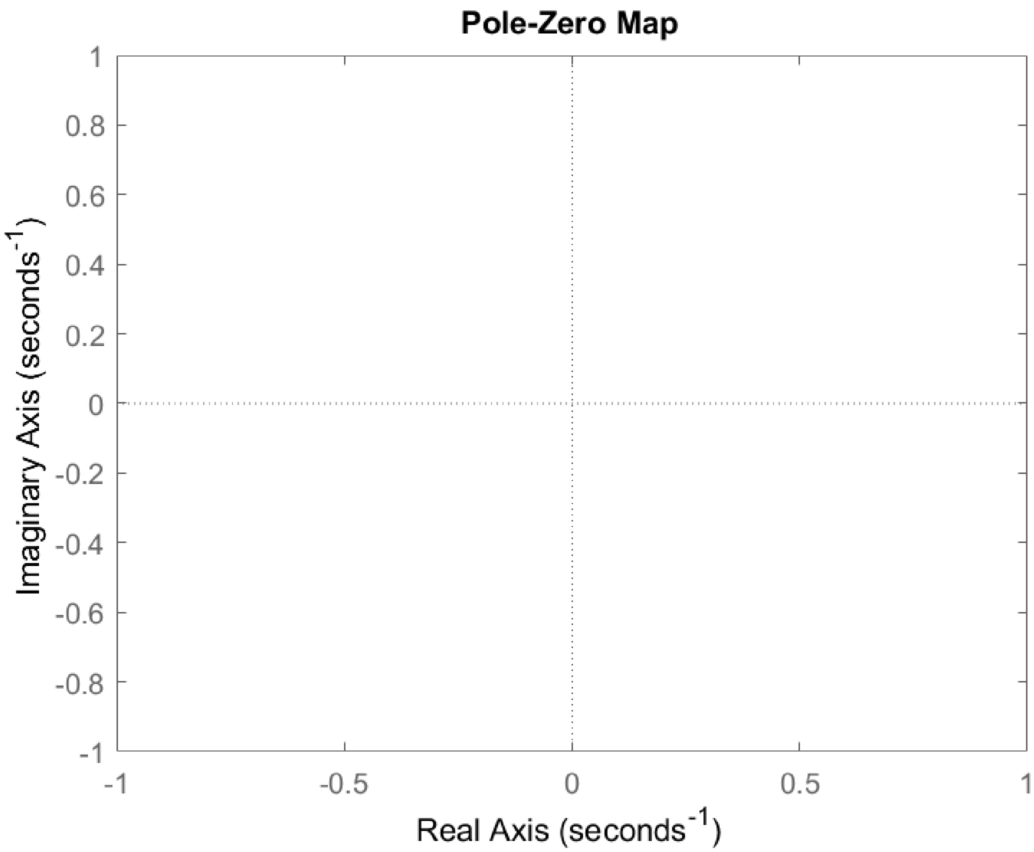
6.2.2 Variation of Gain by Loop

```
s=tf('s');
H=1;
figure;

hold on
for K=0.01:1:20   % Variation of Gain
G=K/(s*(s+4));
CLG=feedback(G,H);
step(CLG);                % Plot Step response of closed loop system
xlim([0,25]);
%legend(K);
end
hold off
```



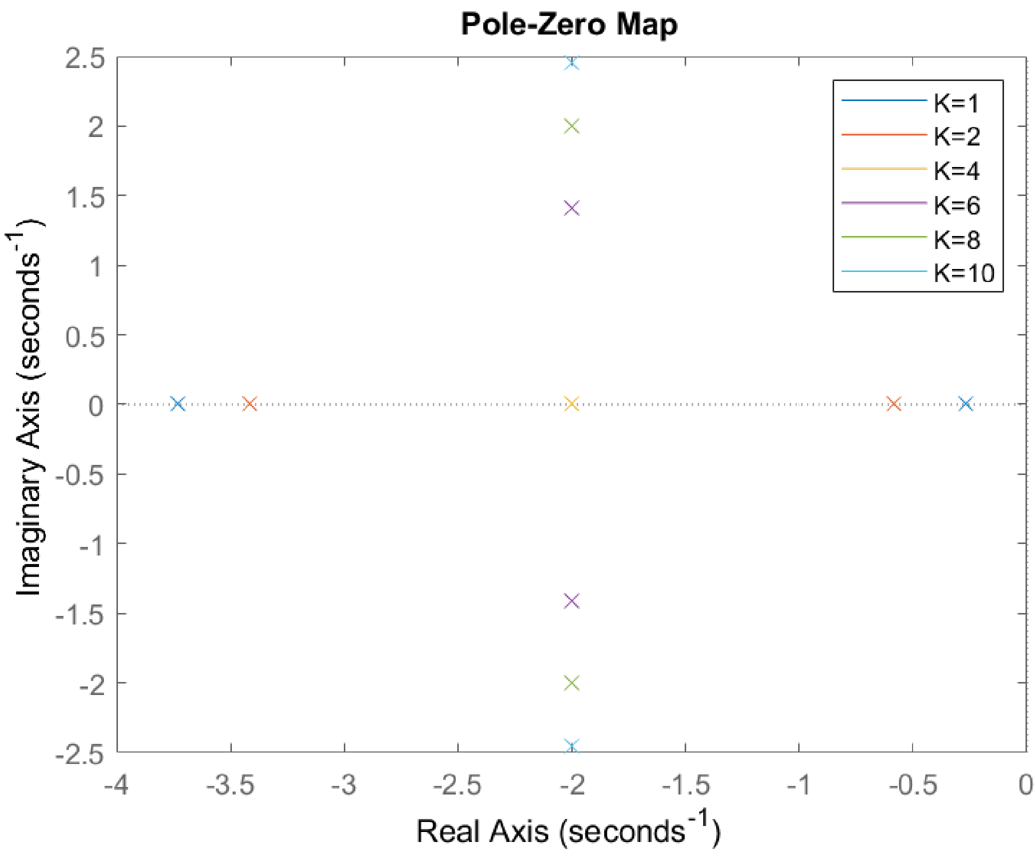
6.2.3 Variation of Gain by Defining Variable

```
clear
s=tf('s');
K=0;
G=K/(s*(s+4));
pzplot(G)
```

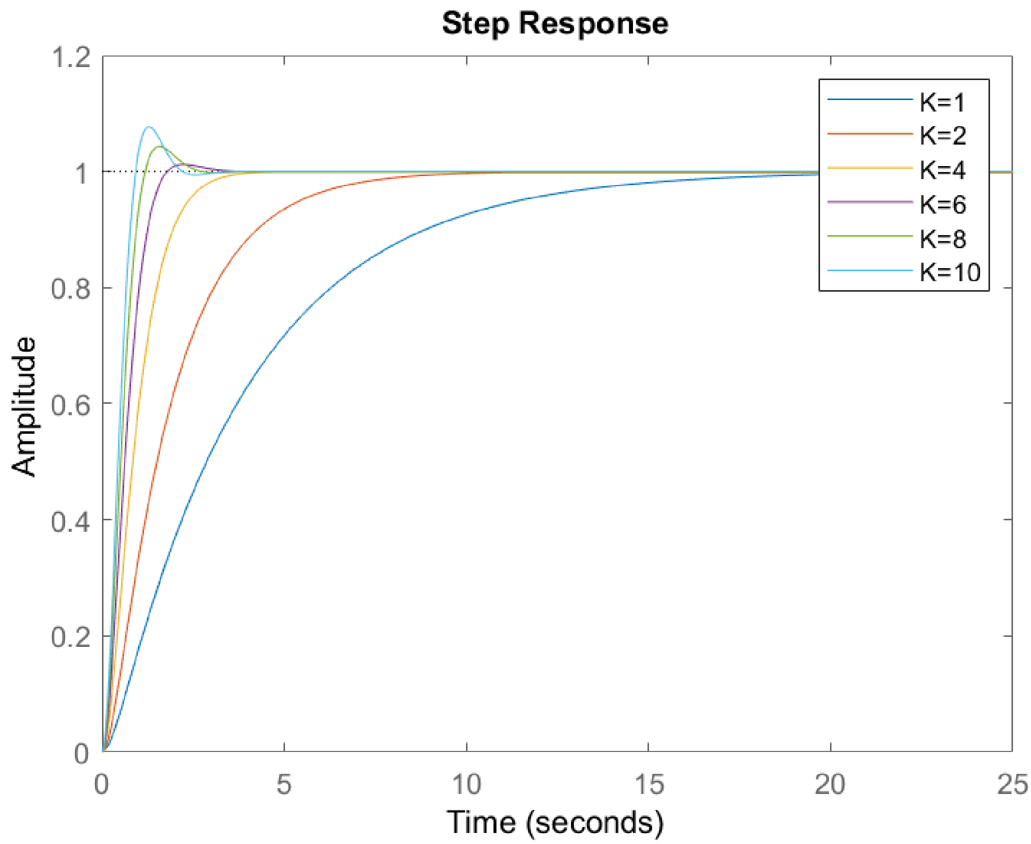



```
K=1;
G1=K/(s*(s+4));
K=2;
G2=K/(s*(s+4));
K=4;
G3=K/(s*(s+4));
K=5;
G4=K/(s*(s+4));
K=6;
G5=K/(s*(s+4));
K=8;
G6=K/(s*(s+4));
K=10;
G7=K/(s*(s+4));
H=1;
T1=feedback(G1,H);

T2=feedback(G2,H);

T3=feedback(G3,H);

T4=feedback(G4,H);

T5=feedback(G5,H);
```

```
T6=feedback(G6,H);

T7=feedback(G7,H);


pzmap(T1,T2,T3,T5,T6,T7);
legend K=1 K=2 K=4 K=6 K=8 K=10
```




```
step(T1,T2,T3,T5,T6,T7);
legend K=1 K=2 K=4 K=6 K=8 K=10
```




```
%rlocus(G1)
```

6.3 Sketching Root Locus by Matlab

It can be acieved by using rlocus() command.

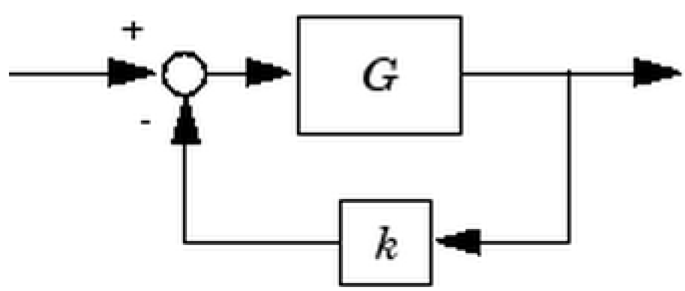
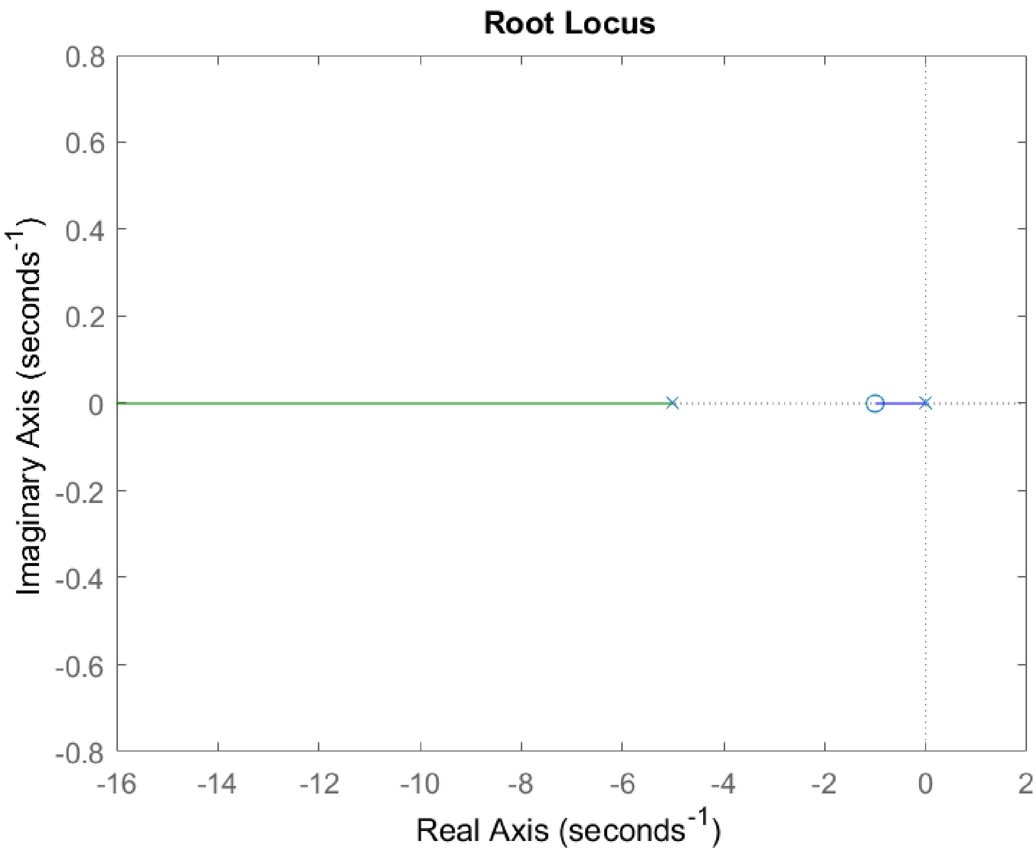


Fig: System in Matlab for rlocus()

```
G=K*(s+1)/(s*(s+5));  % Open Loop Gain
H=1;
GH=G*H;
rlocus(GH)  %rlocus() uses open loop transfer function to plot.
```

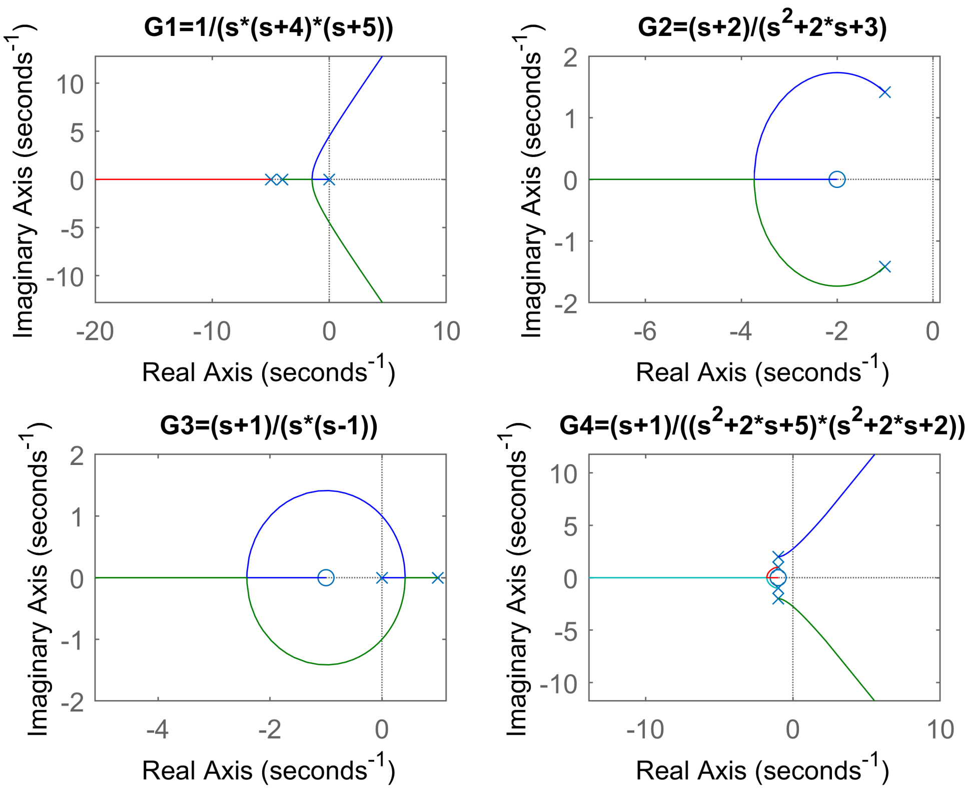



```
% Lets Plot some System
s=tf('s');
G1=1/(s*(s+4)*(s+5));
G2=(s+2)/(s^2+2*s+3);
G3=(s+1)/(s*(s-1));
G4=(s+1)/((s^2+2*s+5)*(s^2+2*s+2));
```

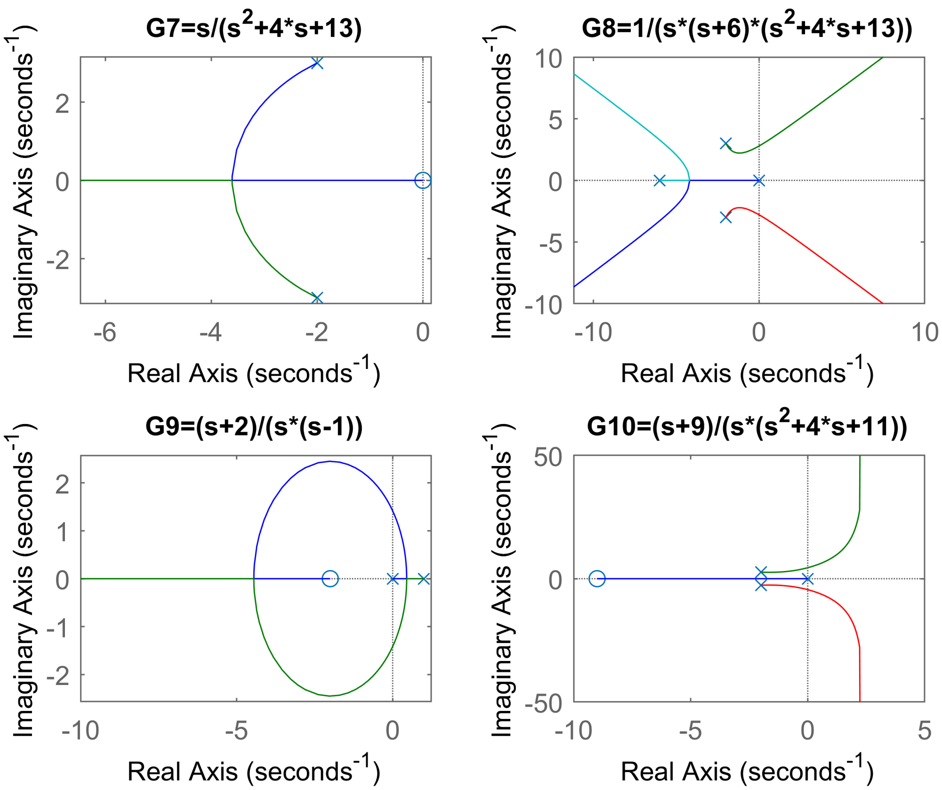
```
G4 =

              s + 1
  --------------------------------
  s^4 + 4 s^3 + 11 s^2 + 14 s + 10

Continuous-time transfer function.
```

```
figure
subplot(2,2,1);
rlocus(G1);
title('G1=1/(s*(s+4)*(s+5))')
subplot(2,2,2);
rlocus(G2);
title('G2=(s+2)/(s^2+2*s+3)');
subplot(2,2,3);
rlocus(G3);
title('G3=(s+1)/(s*(s-1))')
```

```
subplot(2,2,4);
rlocus(G4);
title('G4=(s+1)/((s^2+2*s+5)*(s^2+2*s+2))')
```




```
%% Some Example
G7=s/(s^2+4*s+13);
G8=1/(s*(s+6)*(s^2+4*s+13));
G9=(s+2)/(s*(s-1));
G10=(s+9)/(s*(s^2+4*s+11));
figure
subplot(2,2,1);
rlocus(G7);
title('G7=s/(s^2+4*s+13)');

subplot(2,2,2);
rlocus(G8);
title('G8=1/(s*(s+6)*(s^2+4*s+13))');

subplot(2,2,3);
rlocus(G9);
title('G9=(s+2)/(s*(s-1))');

subplot(2,2,4);
rlocus(G10);
title('G10=(s+9)/(s*(s^2+4*s+11))');
```




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Practice
% s=tf('s');
% G1=1/(s*(s+3));
% G2=1/(s*(s^2+5*s+6));
% G3=(s+3)/(s^2-s-2);
% G4=(s+1)/(s^3+4*s^2+6*s+4);
% G5=(s^2+2*s+2)/(s*(s^4+9*s^3+33*s^2+51*s+26));
% G6=(s^2+2*s+2)/(s^2*(s^4+9*s^3+33*s^2+51*s+26));
%rlocus(G2)
%rlocus(G3)
%rlocus(G4)
%rlocus(G5)
```

6.4 Finding Parameters of Root Locus by Matlab

```
%In case we require to find value of K and their root on Root Locus
[r,k] = rlocus(GH);
%In case when we want to find value of roots for certain gain
k1=0.5
```

```
k1 = 0.5000
```

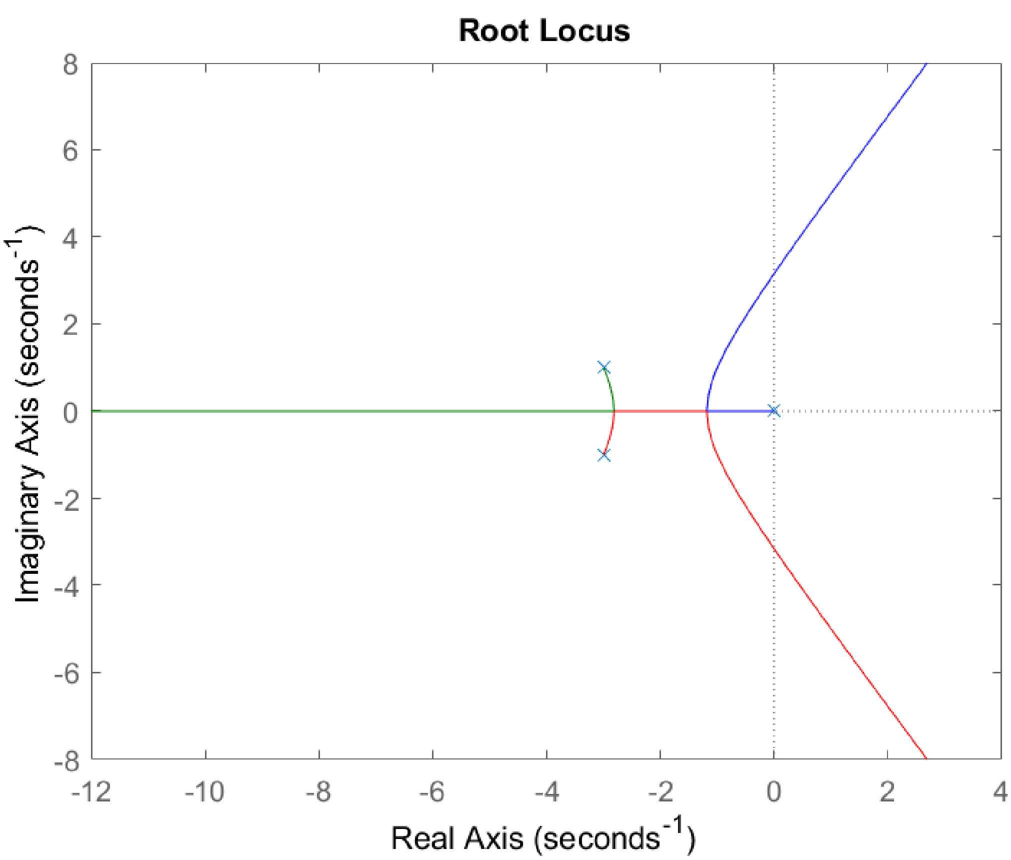
```
r1=rlocus(GH,k1)
```

```
r1 = 2×1
   -9.4721
   -0.5279
```

6.5 Choosing a Value of K from Root Locus

6.5.1 When we have zeta and Natural Frequency

```
%Generate s-plane grid of constant damping factors and natural frequencies
% To get the desired area for certain requirements
GH=K/(s^3+6*s^2+10*s);
rlocus(GH);
```

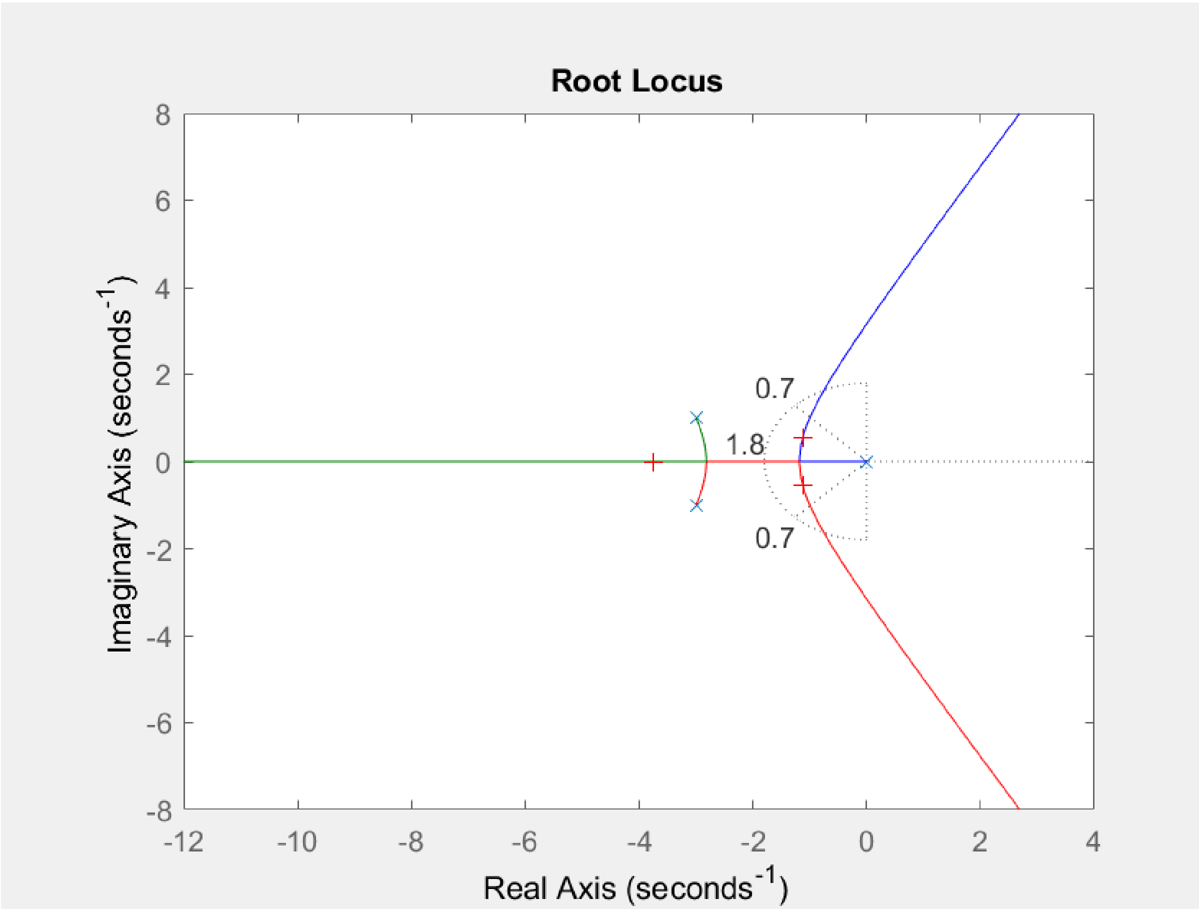


On the plot below, the two dotted lines at about a 45-degree angle indicate pole locations with ξ= 0.7; in between these lines, the poles will have ξ> 0.7 and outside of these lines ξ< 0.7. The semicircle indicates pole locations with a natural frequency ω_n= 1.8; inside of the circle, ω_n< 1.8 and outside of the circle ω_n > 1.8.

```
zeta = 0.7;
wn = 1.8;
```

```
sgrid(zeta,wn)

%To locate roots ans gain For your system
%Based on Requiement choose the location
[k,poles] = rlocfind(GH)
```

```
Select a point in the graphics window
```



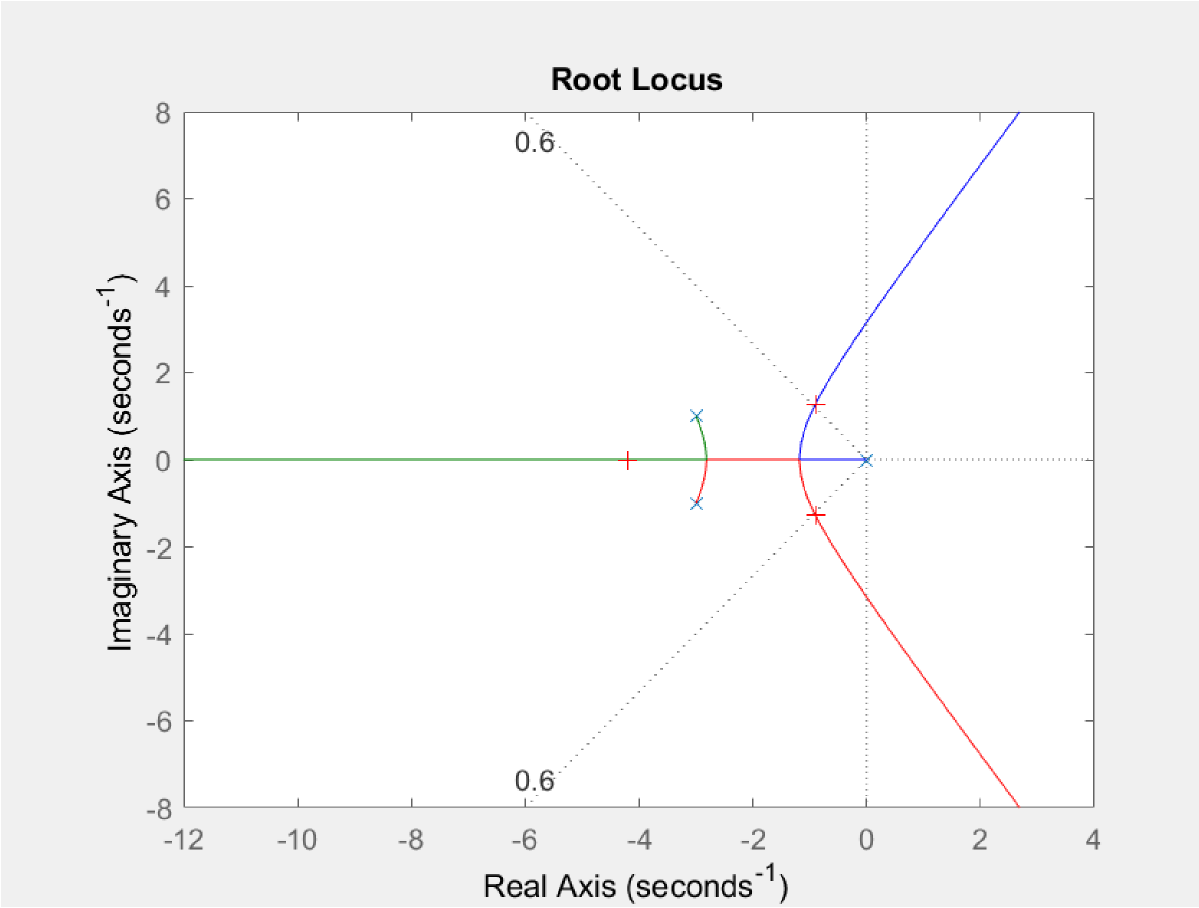

```
selected_point = -1.1438 + 0.5455i
k = 0.5849
poles = 3×1 complex
  -3.7486 + 0.0000i
  -1.1257 + 0.5414i
  -1.1257 - 0.5414i
```

6.5.2 When we have zeta Only

```
%Generate s-plane grid of constant damping factors and natural frequencies
% To get the desired area for certain requirements
GH=K/(s^3+6*s^2+10*s);
rlocus(GH);
zeta = 0.6;
wn = 0;          % Substitute Zero here
sgrid(zeta,wn)

%To locate roots ans gain For your system
%Based on Requiement choose the location
```

```
[k,poles] = rlocfind(GH)
```

```
Select a point in the graphics window
```



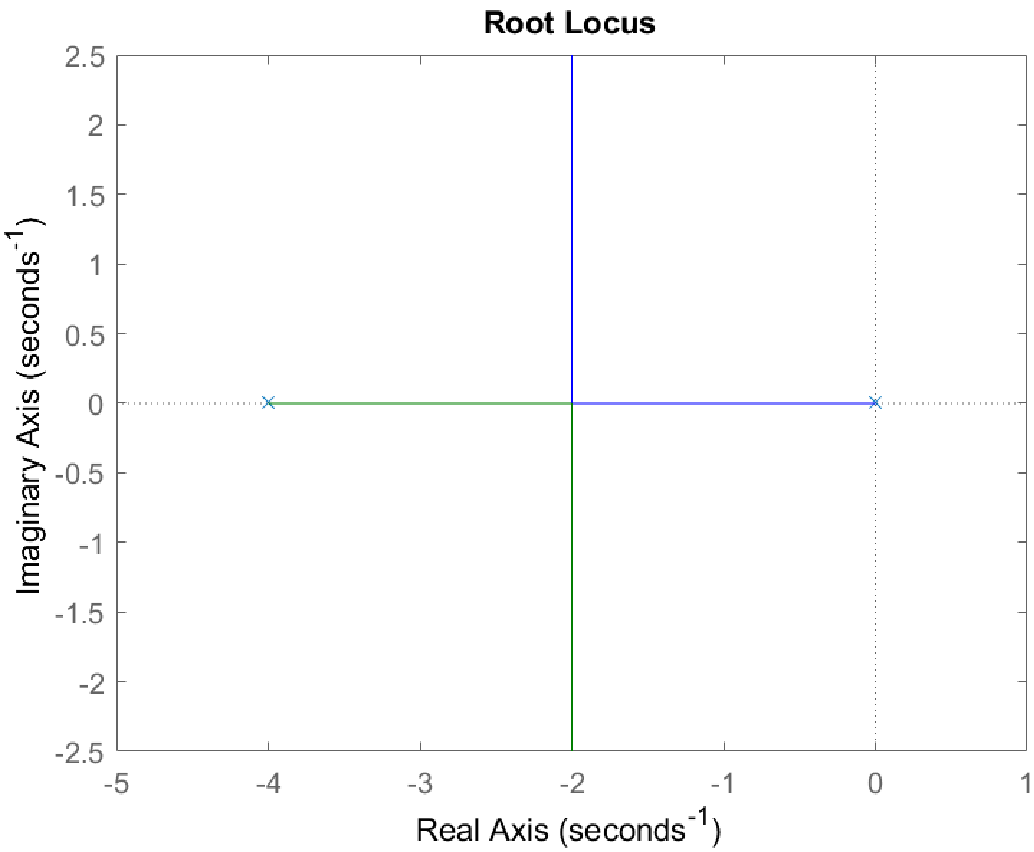

```
selected_point = -0.9163 + 1.2727i
k = 1.0162
poles = 3×1 complex
  -4.1931 + 0.0000i
  -0.9035 + 1.2678i
  -0.9035 - 1.2678i
```

6.6 Effect of Addition of Poles in Root Locus

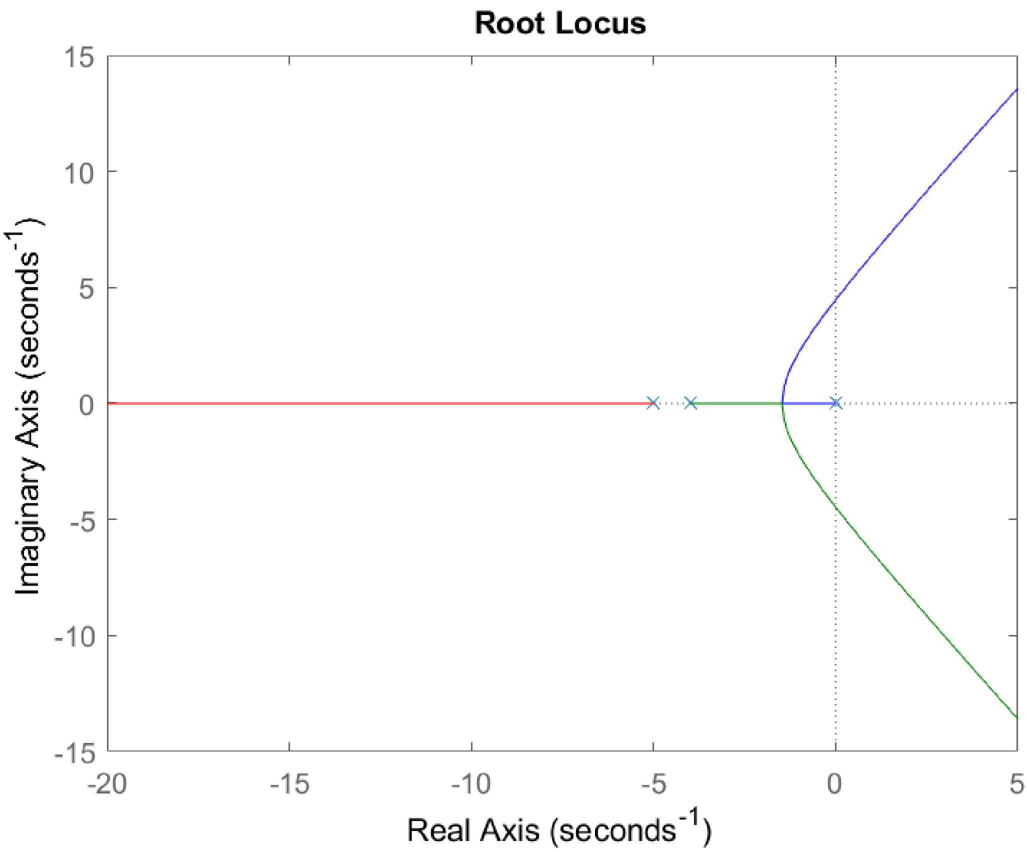
Effects of addition of open loop poles can be summarized as :

1. Root locus shifts towards imaginary axis.
2. System stability relatively decreases.
3. System becomes more oscillatory in nature.
4. Range of operating values of 'K' for stability of the system decreases.

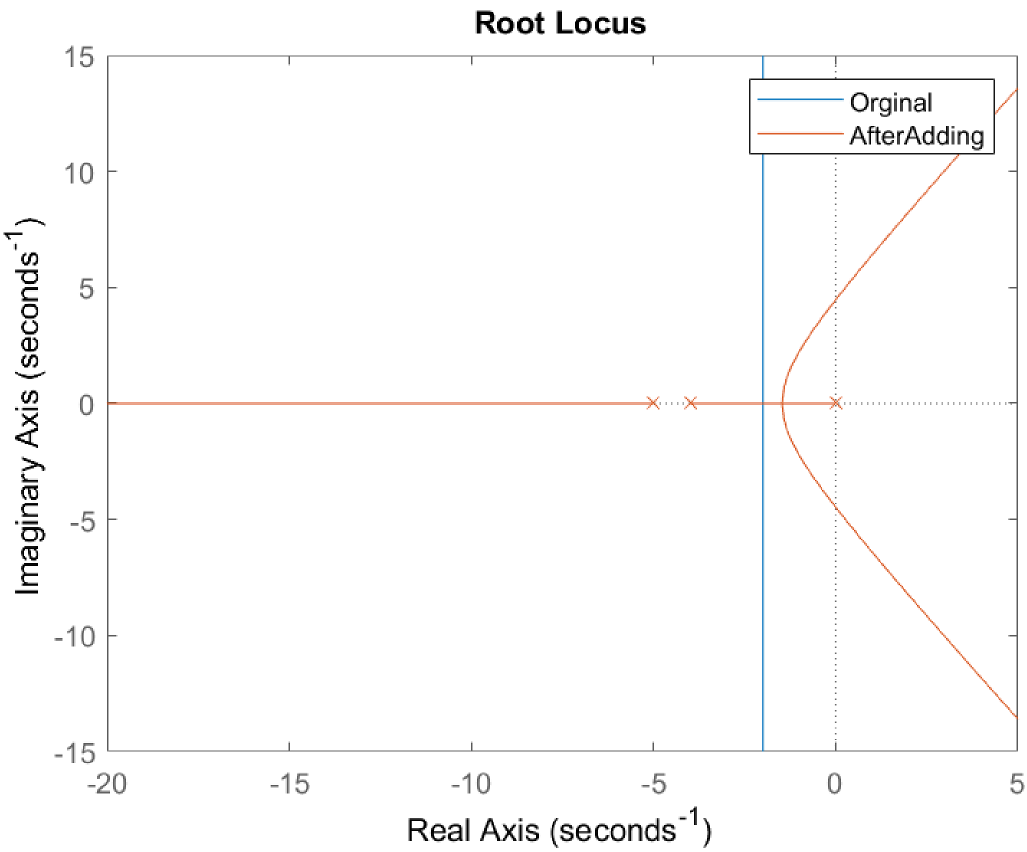
```
%A system with two poles
GH1=1/(s*(s+4));
rlocus(GH1);
```




```
%Adding one more Poles
GH2=1/(s*(s+4)*(s+5));
rlocus(GH2);
```




```
%Plotting Both in Combined Way
rlocus(GH1,GH2);
legend Orginal AfterAdding
```

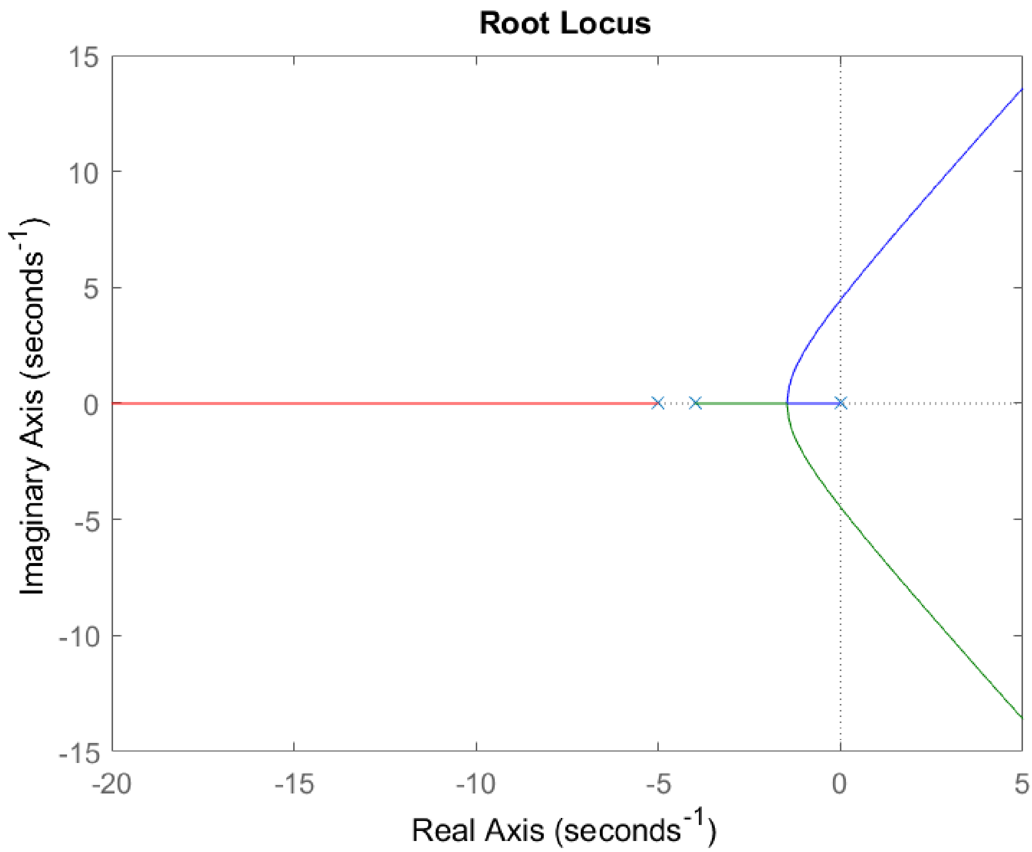


6.7 Effect of Addition of Zeros in Root Locus

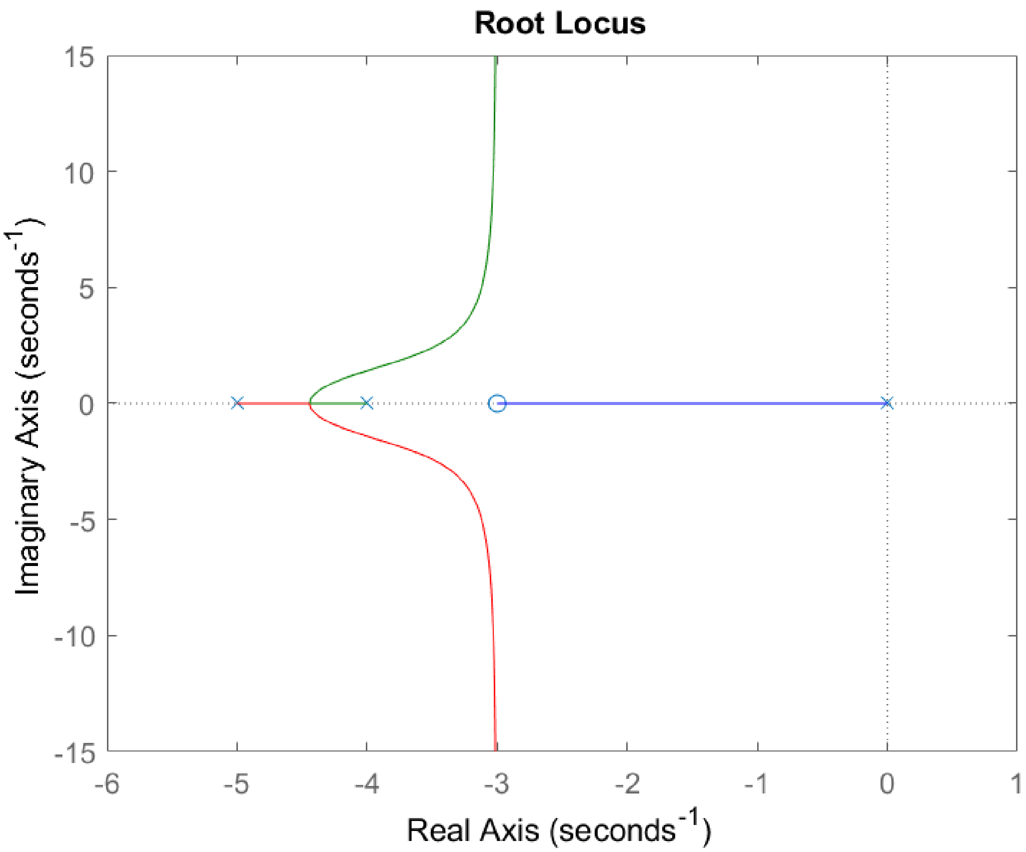
In short effect of addition of zeros are :

1. Root locus shifts to left away from imaginary axis.
2. Relative stability of the system increases.
3. System becomes less oscillatory.
4. Range of operating values of 'K' for system stability increases.

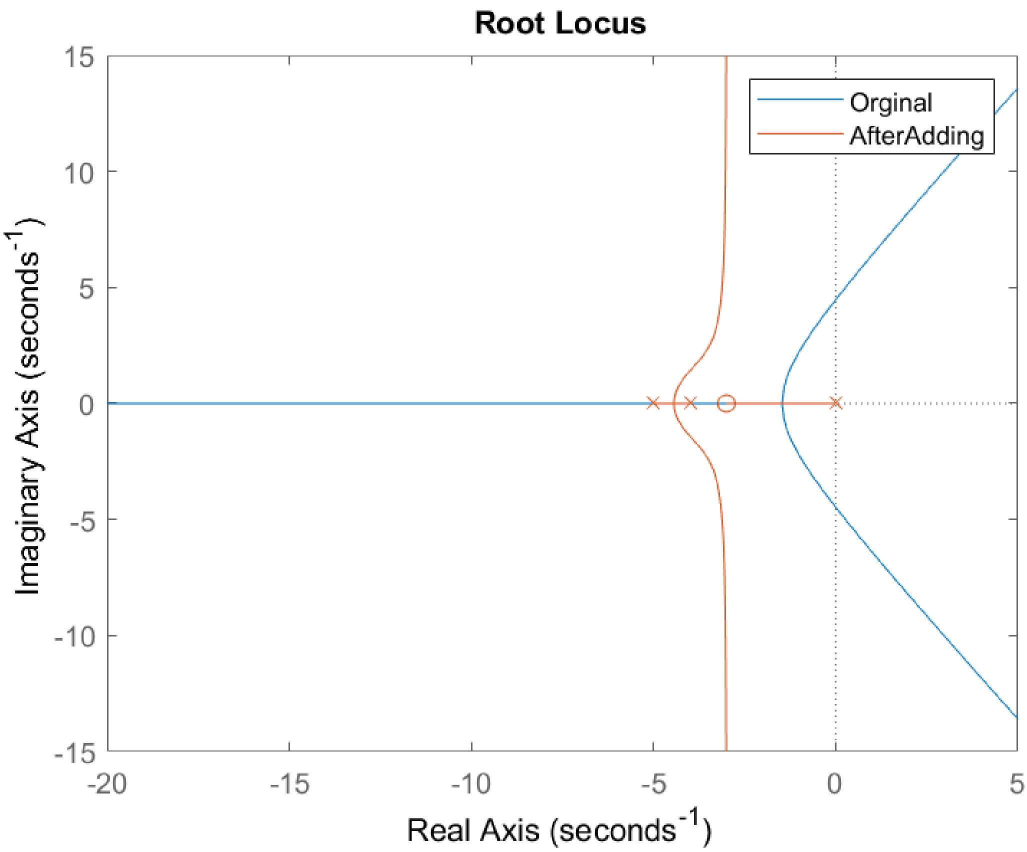
```
%A system with three poles
GH1=1/(s*(s+4)*(s+5));
rlocus(GH1);
```




```
%Adding one Zero
GH2=(s+3)/(s*(s+5)*(s+4));
rlocus(GH2);
```




```
%Plotting Both in Combined Way
rlocus(GH1,GH2);
legend Orginal AfterAdding
```



6.8 Cancellation of Poles by Zeros

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

$$H(s) = (s+2)$$

$$G(s)H(s) = K * \frac{(s+2)}{s(s+2)(s+4)}$$

```
G=K/(s*(s+2)*(s+4));
H=(s+2);
GH_yourself=K/(s*(s+4));
fprintf("The Combined Transfer Function by Yourself is");
```

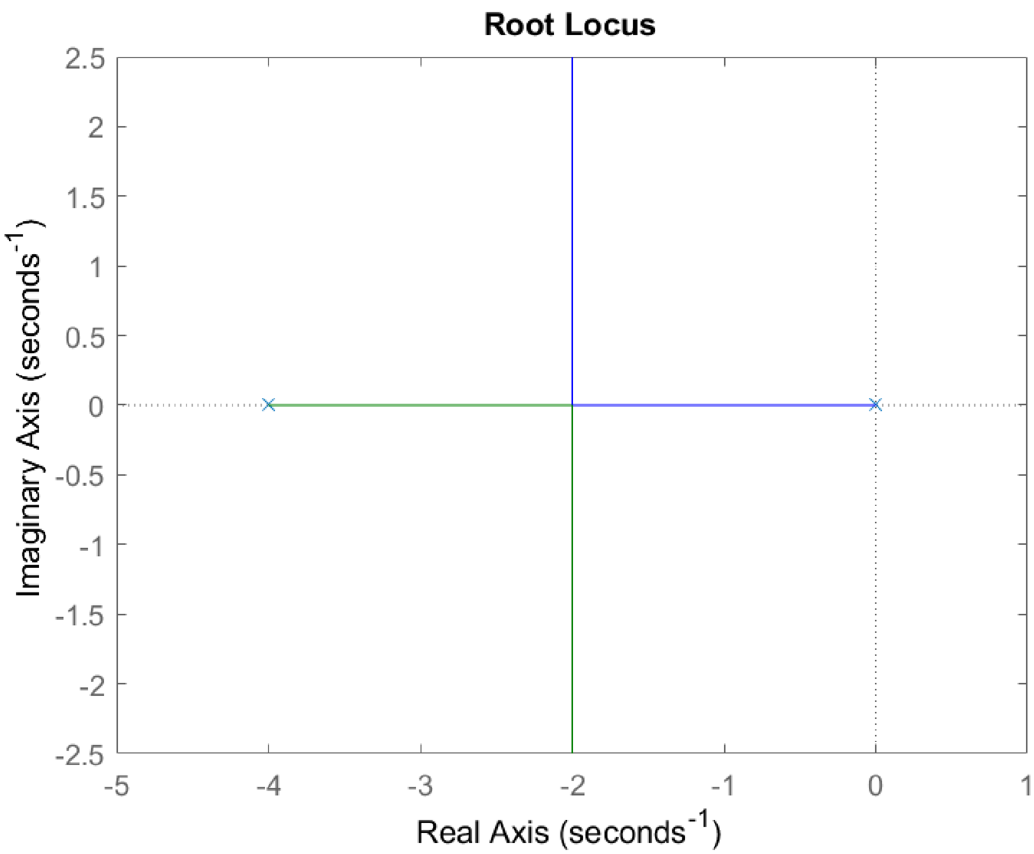
```
The Combined Transfer Function by Yourself is
```

```
GH_yourself
```

```
GH_yourself =

      10
  ---------
```

```
  s^2 + 4 s

Continuous-time transfer function.
```

```
rlocus(GH_yourself);
```



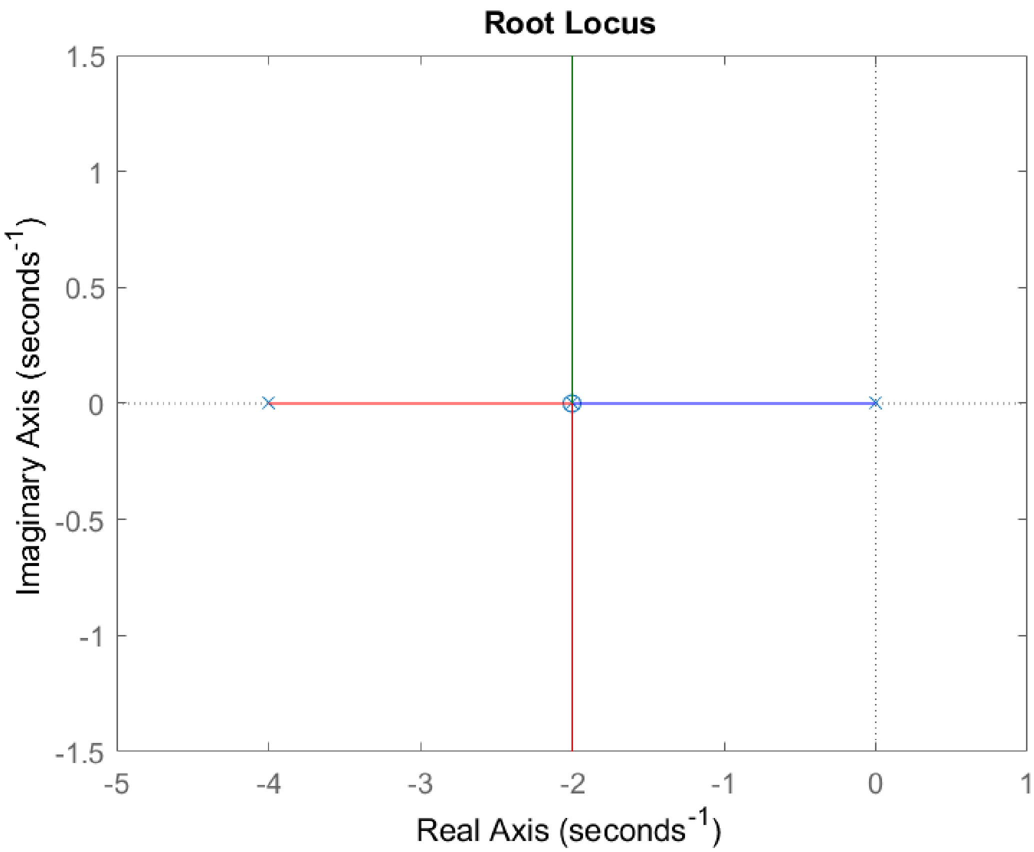

```
GH=G*H;
fprintf("The COmbined Transfer Function is");
```

```
The COmbined Transfer Function is
```

```
GH
```

```
GH =

      10 s + 20
  -----------------
  s^3 + 6 s^2 + 8 s

Continuous-time transfer function.
```

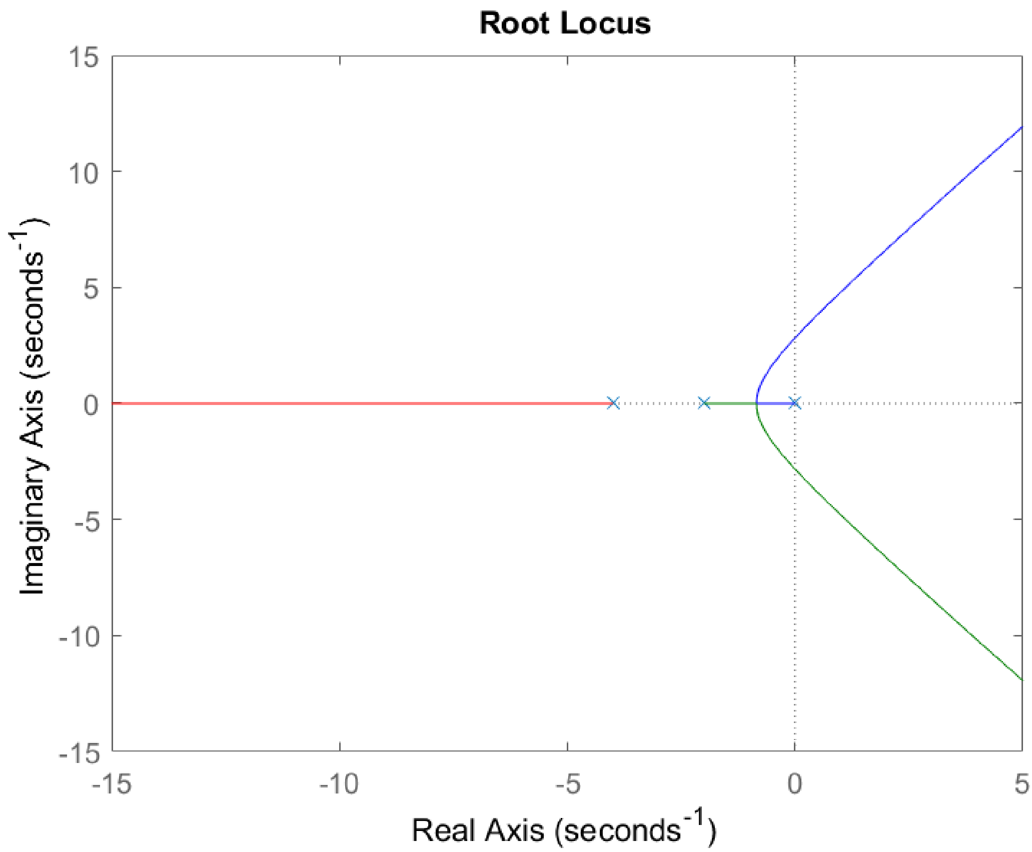
```
rlocus(GH);
```



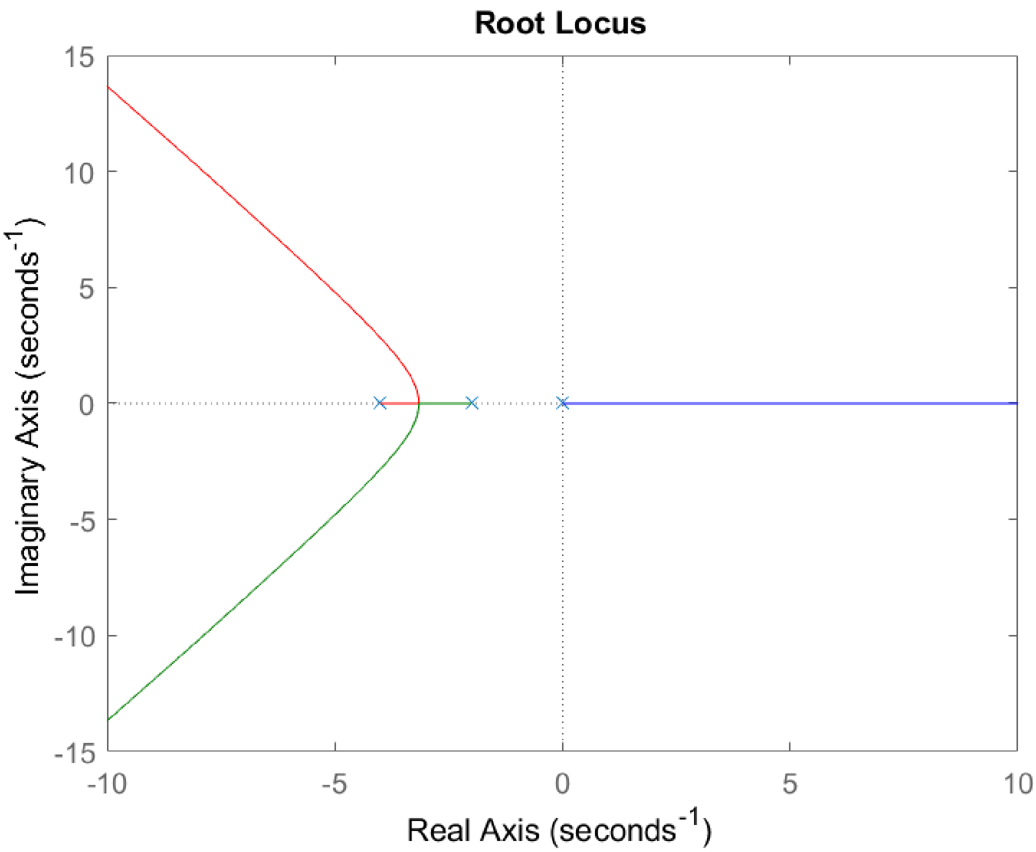
6.9 Inverse Root Locus

Root Locus is defined as, the locus of the closed loop poles obtained when system gain 'K' is varied from -∞ to +∞ is called Root Locus. When 'K' is varied from -∞ to 0, the plot obtained is called **Inverse root locus.**

```
G=K/(s*(s+2)*(s+4));
H=1;
GH=G*H;
rlocus(GH);
```




```
rlocus(-GH);    %Minus for Inverse Root Locus
```



References

1. Mathworks.inc
2. Nise, Norman S. "Control system engineering, John Wiley & Sons." *Inc, New York*(2011).
3. https://ctms.engin.umich.edu/CTMS/index.php?aux=Home
4. Bakshi, Uday A., and Varsha U. Bakshi. *Control system engineering*. Technical Publications, 2020.