

1 Executive Summary

There is a tradeoff between bias and variance. Reducing variance increases bias in our model and vice versa. We have to find an optimal point while fitting a model so that we make sure the variance is not too high on the training set and the error on the test set also does not increase. We can use Regularization techniques for reducing variance. We have some techniques of regularization:

- 1) Ridge regularization (L2),
- 2) Lasso regularization (L1),
- 3) Elastic Net.

Lasso, Ridge, and Elastic Net are modifications of ordinary least squares linear regression, which use additional penalty terms in the cost function to keep coefficient values small and simplify the model.

The results of our analysis indicate how regularization techniques can be used to improve the predictive power of a model.

The need for regularization

Regularization can sometimes lead to better model performance. Regularization can be used to avoid overfitting, a more generic model may be preferred over a very specific one

The foundations of a regularizer

Regularizers are attached to the loss values of a machine learning model, and they are thus included in the optimization step. Combining the original loss value with the regularization component, the model will become simpler with likely losing not much of their predictive abilities.

2 Introduction

I plan to build a linear regression model in python to estimate the sales price of houses. The dataset contains 244 features which can lead to making an overfitted model. I will test 3 types of regularization here to see the effect on the performance of the model on both training as well as test set.

3 Loading and Exploring Data

3.1 Loading libraries

```
In [2]: 1 import numpy as np
2 import pandas as pd
3 import seaborn as sns
4 import matplotlib.pyplot as plt
5 from pandas import Series, DataFrame
6 from math import sqrt
7 import warnings
8 warnings.filterwarnings('ignore')
9 from sklearn.model_selection import (train_test_split, cross_val_score, RepeatedKFold)
10 from sklearn.ensemble import GradientBoostingRegressor
11 from sklearn import metrics
```

3.2 Loading Data

```
In [419]: 1 data = pd.read_csv('AmesHousing_Processed.csv')
```

```
1 The data has already been processed. It is checked for the existence of outliers and missing values. We only need to perform feature scaling.
```

3.3 Data size and structure

```
In [420]: 1 data.shape
```

```
Out[420]: (2925, 244)
```

Our dataframe consists of 10 predictors and our response variable is "AverageTemperature".

```
In [505]: 1 # Let us look at some records of our data
          2 data.head()
```

Out[505]:

	Unnamed: 0	MS SubClass	Lot Frontage	Lot Area	Land Slope	Overall Qual	Overall Cond	Year Built	Year Remod/Add	Mas Vnr Area	...	Sale Type_New	Sale Type_Oth	Sale Type_VWD	Sale Type_WD
0	0	20	141.0	31770	0	6	5	1960	1960	112.0	...	0	0	0	1
1	1	20	80.0	11622	0	5	6	1961	1961	0.0	...	0	0	0	1
2	2	20	81.0	14267	0	6	6	1958	1958	108.0	...	0	0	0	1
3	3	20	93.0	11160	0	7	5	1968	1968	0.0	...	0	0	0	1
4	4	60	74.0	13830	0	5	5	1997	1998	0.0	...	0	0	0	1

5 rows × 244 columns

4 EDA

```
In [422]: 1 data.describe()
```

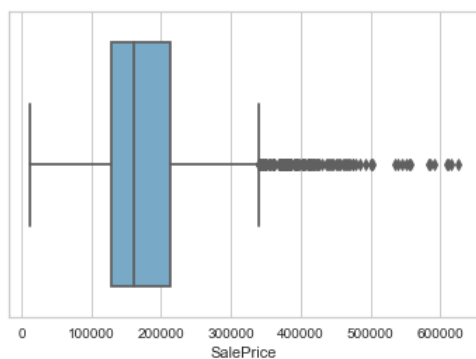
Out[422]:

	Unnamed: 0	MS SubClass	Lot Frontage	Lot Area	Land Slope	Overall Qual	Overall Cond	Year Built	Year Remod/Add	Mas Vn Are
count	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000	2925.000000
mean	1462.000000	57.396581	57.460855	10103.583590	0.053675	6.088205	5.563761	1971.302906	1984.234188	99.91863
std	844.519094	42.668752	33.075613	7781.999124	0.248506	1.402953	1.112262	30.242474	20.861774	175.56615
min	0.000000	20.000000	0.000000	1300.000000	0.000000	1.000000	1.000000	1872.000000	1950.000000	0.000000
25%	731.000000	20.000000	43.000000	7438.000000	0.000000	5.000000	5.000000	1954.000000	1965.000000	0.000000
50%	1462.000000	50.000000	63.000000	9428.000000	0.000000	6.000000	5.000000	1973.000000	1993.000000	0.000000
75%	2193.000000	70.000000	78.000000	11515.000000	0.000000	7.000000	6.000000	2001.000000	2004.000000	162.000000
max	2924.000000	190.000000	313.000000	215245.000000	2.000000	10.000000	9.000000	2010.000000	2010.000000	1600.000000

8 rows × 244 columns

```
In [506]: 1 sns.boxplot( data["SalePrice"], palette="Blues")
```

Out[506]: <AxesSubplot: xlabel='SalePrice'>



As we saw in our previous analysis the average temperature has been around 17 degrees

```
In [507]: 1 data.isna().sum().sum()
```

Out[507]: 0

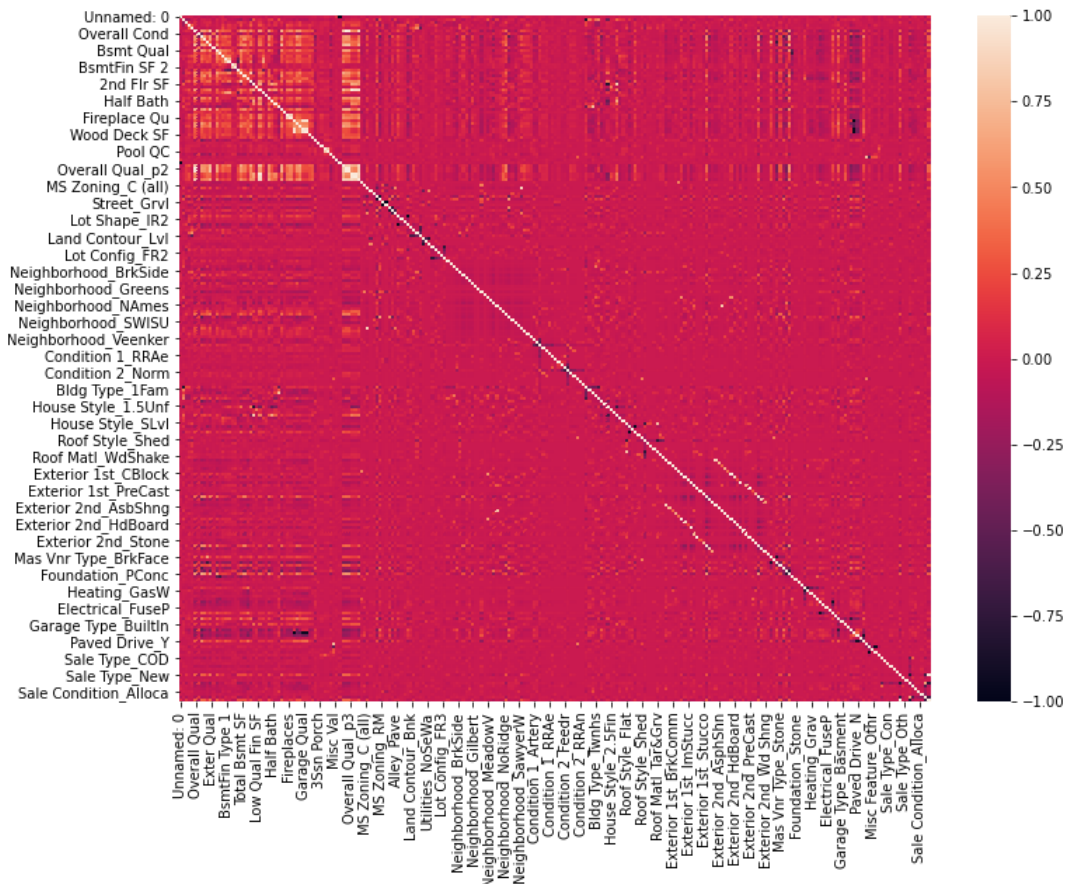
No missing values

```
In [508]: 1 # Let us keep a copy of our dataset
          2 dataset = data.copy()
```

4.1 The Correlations Plot

In [426]:

```
1 fig, ax = plt.subplots(figsize=(12,9))
2 sns.heatmap(dataset.corr(), ax=ax);
```



4.2 Feature Scaling

In [427]:

```
1 from sklearn.preprocessing import StandardScaler
2
3 scaler = StandardScaler()
4 # We need to fit the scaler to our data before transformation
5 dataset.loc[:, dataset.columns != 'SalePrice'] = scaler.fit_transform(
6     dataset.loc[:, dataset.columns != 'SalePrice'])
```

In [428]:

```
1 dataset
```

Out[428]:

	Unnamed: 0	MS SubClass	Lot Frontage	Lot Area	Land Slope	Overall Qual	Overall Cond	Year Built	Year Remod/Add	Mas Vnr Area	...	Sale Type_New	Sale Type_Oth
0	-1.731459	-0.876589	2.526134	2.784647	-0.216028	-0.062882	-0.506946	-0.373807	-1.161854	0.068826	...	-0.296252	-0.048971
1	-1.730274	-0.876589	0.681560	0.195152	-0.216028	-0.775786	0.392276	-0.340735	-1.113911	-0.569220	...	-0.296252	-0.048971
2	-1.729090	-0.876589	0.711798	0.535098	-0.216028	-0.062882	0.392276	-0.439950	-1.257739	0.046038	...	-0.296252	-0.048971
3	-1.727906	-0.876589	1.074666	0.135775	-0.216028	0.650022	-0.506946	-0.109233	-0.778312	-0.569220	...	-0.296252	-0.048971
4	-1.726722	0.061025	0.500126	0.478933	-0.216028	-0.775786	-0.506946	0.849847	0.659971	-0.569220	...	-0.296252	-0.048971
...
2920	1.726722	0.529832	-0.618714	-0.278457	-0.216028	-0.062882	0.392276	0.419915	-0.011228	-0.569220	...	-0.296252	-0.048971
2921	1.727906	-0.876589	-1.737554	-0.156617	3.808700	-0.775786	-0.506946	0.386843	-0.059170	-0.569220	...	-0.296252	-0.048971
2922	1.729090	0.647034	0.137259	0.043366	-0.216028	-0.775786	-0.506946	0.684489	0.372314	-0.569220	...	-0.296252	-0.048971
2923	1.730274	-0.876589	0.590843	-0.012028	3.808700	-0.775786	-0.506946	0.089198	-0.442712	-0.569220	...	-0.296252	-0.048971
2924	1.731459	0.061025	0.500126	-0.061252	3.808700	0.650022	-0.506946	0.717560	0.468200	-0.033717	...	-0.296252	-0.048971

2925 rows × 244 columns

5 Building the Model

```
In [429]: 1 from sklearn.linear_model import LinearRegression
2 from sklearn.model_selection import cross_val_score
3 from sklearn import metrics
4 Model = LinearRegression()
```

```
In [430]: 1 x_train, x_test, y_train, y_test = train_test_split(
2     dataset.drop('SalePrice', axis=1), dataset[['SalePrice']],
3     test_size=0.3, random_state=101)
```

```
In [431]: 1 Model.fit(x_train, y_train)
```

Out[431]: LinearRegression()

5.1 Model Score on Training and Test Set

```
In [432]: 1 Model.score(x_train,y_train)
```

Out[432]: 0.9481149995701218

```
In [433]: 1 Model.score(x_test,y_test)
```

Out[433]: -1.4468301664039589e+20

The model score is not good on the test set.

5.2 Model Prediction

```
In [434]: 1 pred = Model.predict(x_test)
```

```
In [435]: 1 # Let us define a function for caculating RMSE
2 def rmse(predictions, targets):
3     return np.sqrt(((predictions - targets) ** 2).mean())
```

```
In [436]: 1 rmse_val = rmse(np.array(pred), np.array(y_test))
2 results = pd.DataFrame()
3 results["Method"] = ["Linear Regression-All Features"]
4 results["RMSE"] = rmse_val
5 results
```

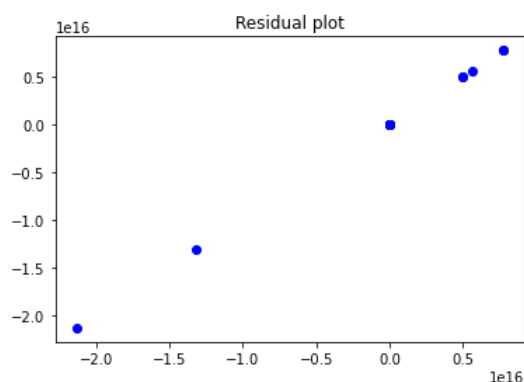
Out[436]:

	Method	RMSE
0	Linear Regression-All Features	9.717902e+14

When you overfit the training data, the test MSE will be very large because the supposed patterns that the method found in the training data simply don't exist in the test data

```
In [440]: 1 # Let us plot the residuls of our model
2 plt.scatter(pred, (pred-y_test), c='b')
3 #plt.hlines(y=0,xmin= 0, xmax=100)
4 plt.title('Residual plot')
```

Out[440]: Text(0.5, 1.0, 'Residual plot')



If we can detect a clear pattern or trend in our residuals, then our model has room for improvement.

5.3 Intercept and Coefficients

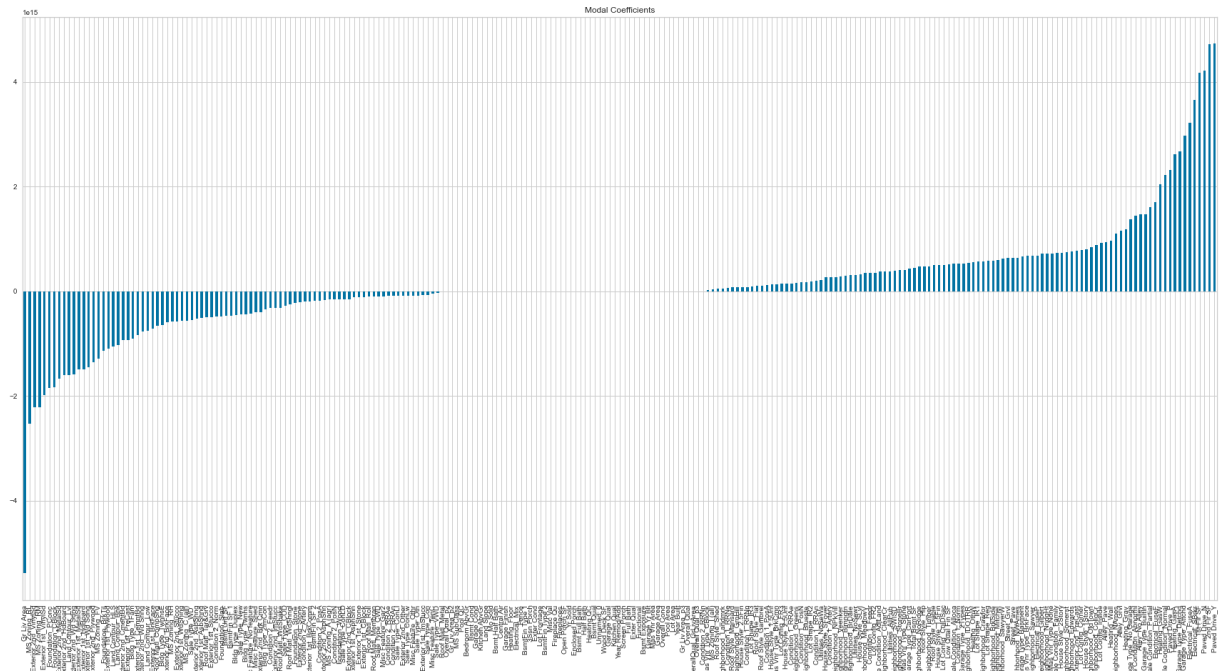
In [297]: `#print (Model.coef_)`

In [448]: `print (Model.intercept_)`

`[-1.15373485e+12]`

In [449]: `plt.figure(figsize=(30,15))
predictors = x_train.columns
coef = Series(Model.coef_[0],predictors).sort_values()
coef.plot(kind='bar', title='Modal Coefficients')`

Out[449]: `<AxesSubplot:title={'center':'Modal Coefficients'}>`



5.4 Ridge regularization (L2)

In [450]: `from sklearn.linear_model import Ridge
from sklearn.model_selection import GridSearchCV
ridge = Ridge()`

5.4.1 Hyper parameter tuning

In [451]: `parameters = {'alpha': [1e-15, 1e-10, 1e-8, 1e-4, 1e-3, 1e-2, 1, 5, 10, 20, 25, 50, 60, 100, 200]}
ridgeReg = GridSearchCV(ridge, parameters, scoring='neg_mean_squared_error', cv=5)`

In [452]: `ridgeReg.fit(x_train, y_train)`

Out[452]: `GridSearchCV(cv=5, estimator=Ridge(),
param_grid={'alpha': [1e-15, 1e-10, 1e-08, 0.0001, 0.001, 0.01, 1,
5, 10, 20, 25, 50, 60, 100, 200]},
scoring='neg_mean_squared_error')`

In [453]: `ridgeReg.best_params_`

Out[453]: `{'alpha': 20}`

In [454]: `ridgeReg.best_score_`

Out[454]: `-453794162.24202394`

In [455]: `ridgeReg = Ridge(alpha=20, normalize=True)
ridgeReg.fit(x_train, y_train)`

Out[455]: `Ridge(alpha=20, normalize=True)`

5.5 Model Score on Training and Test Set

```
In [456]: 1 ridgeReg.score(x_train, y_train)
```

Out[456]: 0.6079077829943409

```
In [457]: 1 ridgeReg.score(x_test, y_test)
```

Out[457]: -0.7235885835714739

5.6 Model Prediction

```
In [458]: 1 pred = ridgeReg.predict(x_test)
```

```
In [459]: 1 rmse_val = rmse(np.array(pred), np.array(y_test))
          2 results.loc[1] = ["RidgeReg", rmse_val]
          3 results
```

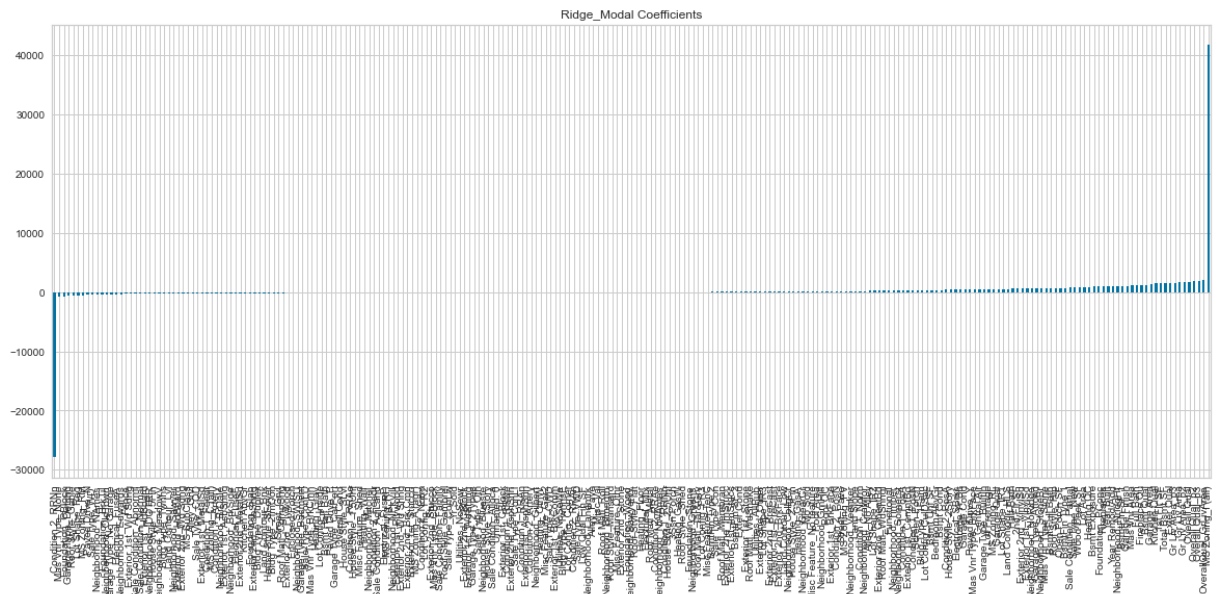
Out[459]:

	Method	RMSE
0	Linear Regression-All Features	9.717902e+14
1	RidgeReg	1.060671e+05

5.6.1 Ridge_Modal Coefficients

```
In [460]: 1 %matplotlib inline
2 plt.figure(figsize=(20,8))
3 predictors = x_train.columns
4 coef = Series(ridgeReg.coef_[0],predictors).sort_values()
5 coef.plot(kind='bar', title='Ridge Modal Coefficients')
```

```
Out[460]: <AxesSubplot:title={'center':'Ridge_Modal Coefficients'}>
```



5.6.2 Coefficient Magnitude & Coefficient Index

```
In [461]: 1 ridgeReg2 = Ridge(alpha=60, normalize=True)
          2 ridgeReg2.fit(x_train,y_train)
```

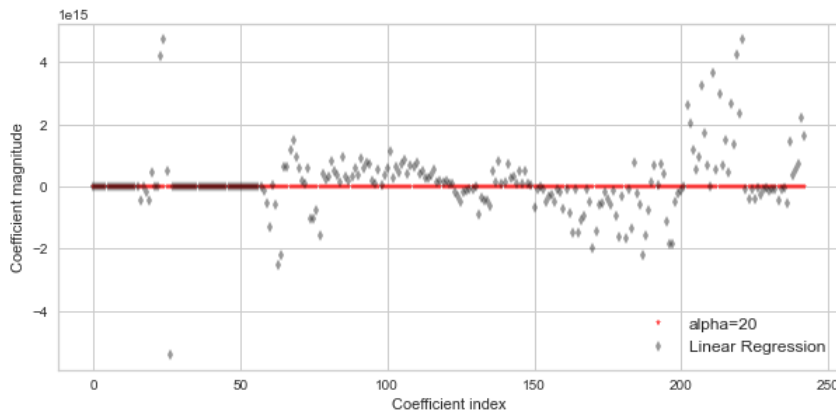
```
Out[461]: Ridge(alpha=60, normalize=True)
```

```
In [462]: 1 ridgeReg3 = Ridge(alpha=1e-15, normalize=True)
          2 ridgeReg3.fit(x_train,y_train)
```

```
Out[462]: Ridge(alpha=1e-15, normalize=True)
```

5.6.3 Comparison with Linear Regression

In [465]: `# Let us plot the Coefficient magnitude of our modes with respect to the alpha values ↔`



This is an example of shrinking coefficient magnitude using Ridge regression. For $\alpha = 60$, we can see the coefficient is nearly zero, which is not the case for $\alpha = 1e-15$. For small values of α ($1e-15$) in which the coefficient is less restricted, the magnitudes of the coefficient is almost the same as of linear regression.

5.7 Lasso regularization (L1)

In [466]: `1 from sklearn.linear_model import Lasso
2 lasso = Lasso()`

5.7.1 Hyper parameter tuning

In [467]: `1 parameters = {'alpha': [1e-15, 1, 5, 10, 20, 25, 50, 60, 100, 500, 750, 1000]}
2 lassoReg = GridSearchCV(lasso, parameters, scoring='neg_mean_squared_error', cv = 5)
3 lassoReg.fit(x_train, y_train)`

Out[467]: `GridSearchCV(cv=5, estimator=Lasso(),
param_grid={'alpha': [1e-15, 1, 5, 10, 20, 25, 50, 60, 100, 500,
750, 1000]},
scoring='neg_mean_squared_error')`

In [468]: `1 lassoReg.best_params_`

Out[468]: `{'alpha': 100}`

In [469]: `1 lassoReg.best_score_`

Out[469]: `-435972196.32547045`

In [470]: `1 lassoReg = Lasso(alpha=100, normalize=True)
2 lassoReg.fit(x_train, y_train)`

Out[470]: `Lasso(alpha=100, normalize=True)`

5.8 Model Score on Training and Test Set

In [471]: `1 lassoReg.score(x_train, y_train)`

Out[471]: `0.8962518805250732`

In [472]: `1 lassoReg.score(x_test, y_test)`

Out[472]: `0.8799222725945898`

5.9 Model Prediction

In [473]: `1 predic = lassoReg.predict(x_test)`

In [474]: `1 pred = predic.reshape(-1,1)`

In [475]: `1 lassoReg.coef_[0]`

Out[475]: 0.0

In [476]: `1 rmse_val = rmse(np.array(pred), np.array(y_test))
2 results.loc[2] = ["lassoReg", rmse_val]
3 results`

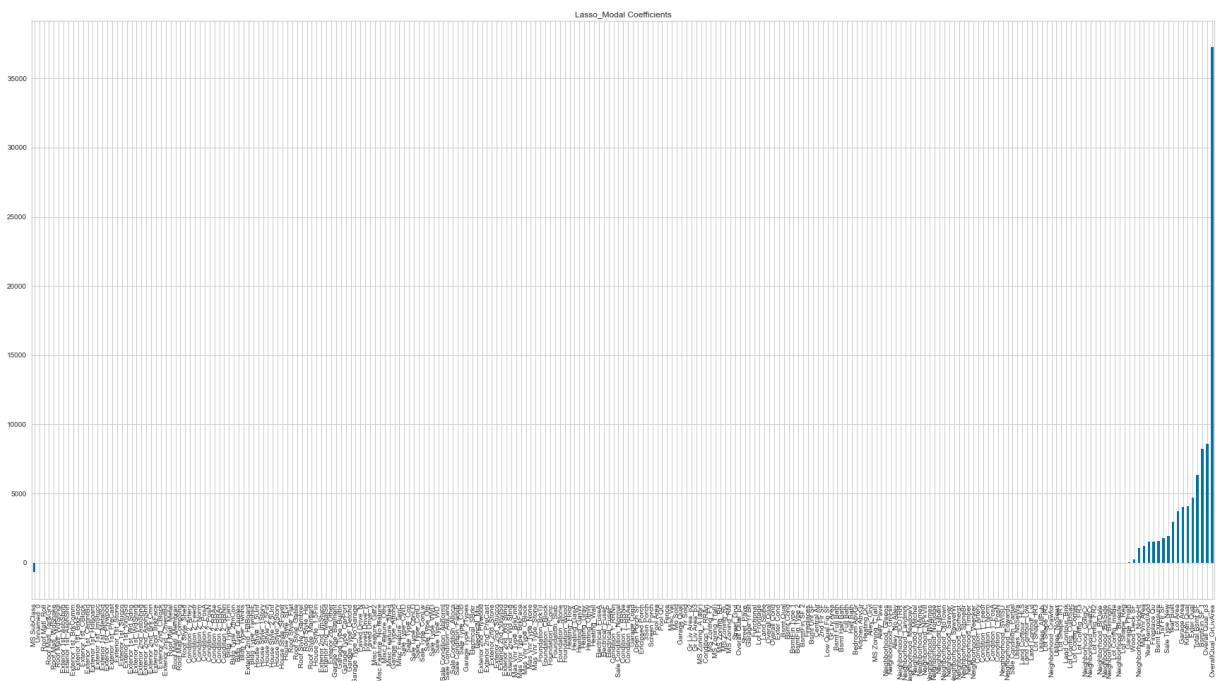
Out[476]:

	Method	RMSE
0	Linear Regression-All Features	9.717902e+14
1	RidgeReg	1.060671e+05
2	lassoReg	1.060671e+05

5.9.1 Lasso_Modal Coefficients

In [477]: `1 plt.figure(figsize=(30,15))
2 predictors = x_train.columns
3 coef = Series(lassoReg.coef_,predictors).sort_values()
4 coef.plot(kind='bar', title='Lasso_Modal Coefficients')`

Out[477]: <AxesSubplot:title={'center':'Lasso_Modal Coefficients'}>



5.9.2 Coefficient Magnitude & Coefficient Index

In [478]: `1 lassoReg2 = Lasso(alpha=500, normalize=True)
2 lassoReg2.fit(x_train,y_train)`

Out[478]: Lasso(alpha=500, normalize=True)

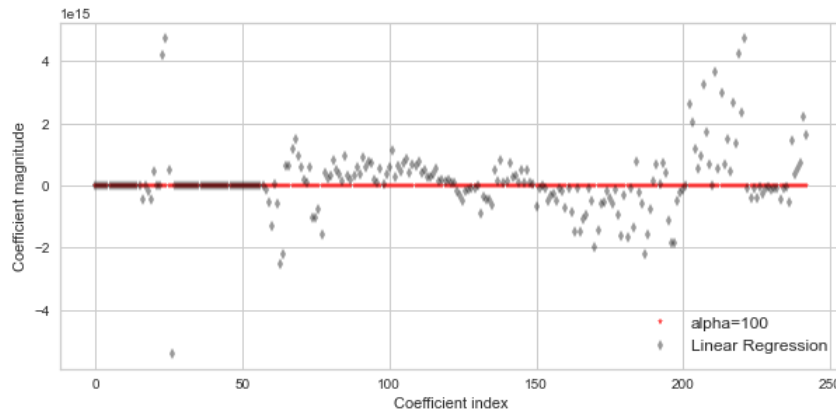
In [479]: `1 lassoReg3 = Lasso(alpha=1000, normalize=True)
2 lassoReg3.fit(x_train,y_train)`

Out[479]: Lasso(alpha=1000, normalize=True)

5.9.3 Comparison with Linear Regression


```
In [503]: 1 # Let us plot the Coefficient magnitude of our modes with respect to the alpha values ↔
```

No handles with labels found to put in legend.



For larger values of alpha [20, 60], we can see most of the coefficients are zero or nearly zero.

5.10 Elastic Net

```
In [486]: 1 from sklearn.linear_model import ElasticNet
```

5.10.1 Hyper parameter tuning

```
In [489]: 1 import numpy as np
2 from sklearn.model_selection import GridSearchCV
3 parametersGrid = {"max_iter": [1, 5, 10],
4                  "alpha": [0.0001, 0.001, 0.01, 0.1, 1, 10, 100],
5                  "l1_ratio": np.arange(0.0, 1.0, 0.1)}
6 eNet = ElasticNet()
7 grid = GridSearchCV(eNet, parametersGrid, scoring='r2', cv=10)
```

```
In [490]: 1 grid.fit(x_train, y_train)
```

```
Out[490]: GridSearchCV(cv=10, estimator=ElasticNet(),
    param_grid={'alpha': [0.0001, 0.001, 0.01, 0.1, 1, 10, 100],
    'l1_ratio': array([0. , 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]),
    'max_iter': [1, 5, 10]},
    scoring='r2')
```

```
In [491]: 1 grid.best_params_
```

```
Out[491]: {'alpha': 1, 'l1_ratio': 0.4, 'max_iter': 10}
```

```
In [492]: 1 grid.best_score_
```

```
Out[492]: 0.9179132390772669
```

```
In [495]: 1 ENreg = ElasticNet(alpha=1, l1_ratio=0.4, max_iter=10, normalize=False)
2 ENreg.fit(x_train, y_train)
```

```
Out[495]: ElasticNet(alpha=1, l1_ratio=0.4, max_iter=10)
```

5.11 Model Score on Training and Test Set

```
In [496]: 1 ENreg.score(x_train, y_train)
```

```
Out[496]: 0.9300815903071482
```

In [497]:

```
1 ENreg.score(x_test,y_test)
```

Out[497]: 0.9080795970022025

5.12 Model Prediction

In [498]:

```
1 pred = ENreg.predict(x_test)
```

In [499]:

```
1 pred=pred.reshape(-1,1)
```

In [500]:

```
1 rmse_val = rmse(np.array(pred), np.array(y_test))
2 results.loc[3] = ["ElasticNet", rmse_val]
3 results
```

Out[500]:

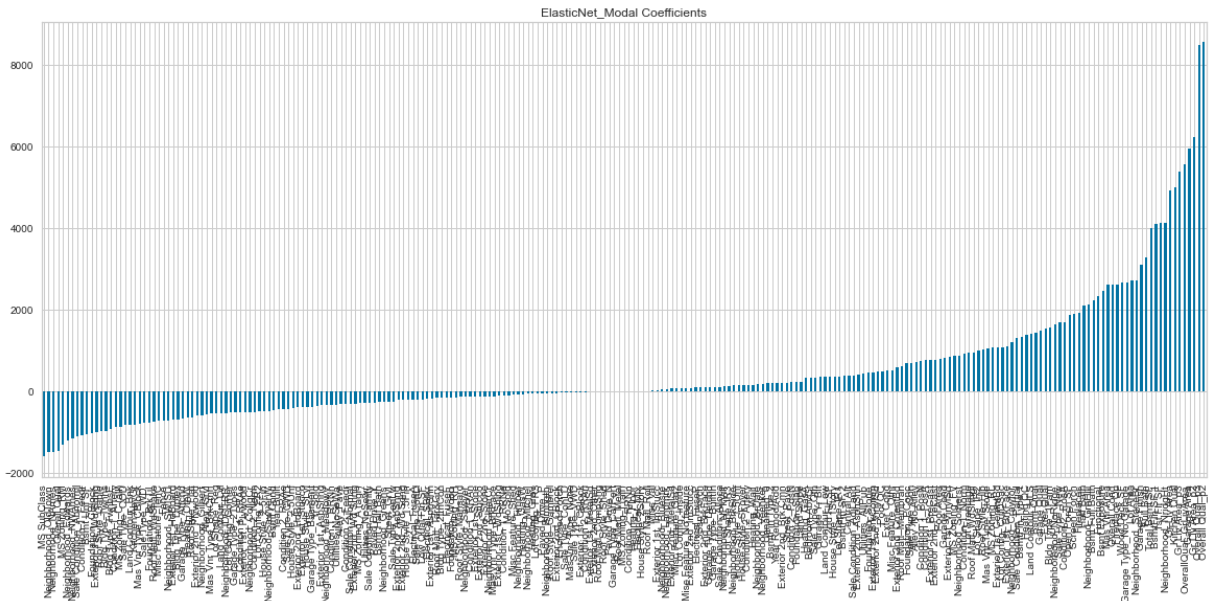
	Method	RMSE
0	Linear Regression-All Features	9.717902e+14
1	RidgeReg	1.060671e+05
2	lassoReg	1.060671e+05
3	ElasticNet	2.449456e+04

5.12.1 ElasticNet_Modal Coefficients

In [501]:

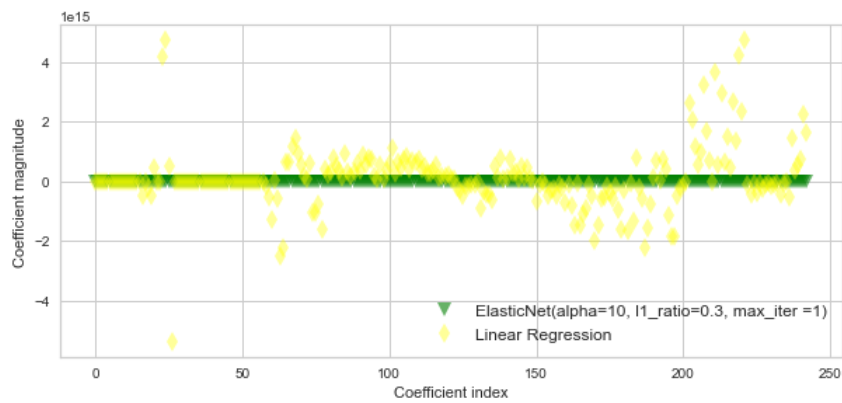
```
1 %matplotlib inline
2 plt.figure(figsize=(20,8))
3 predictors = x_train.columns
4 coef = Series(ENreg.coef_,predictors).sort_values()
5 coef.plot(kind='bar', title='ElasticNet_Modal Coefficients')
```

Out[501]: <AxesSubplot:title={'center': 'ElasticNet_Modal Coefficients'}>



5.12.2 Comparison with Linear Regression

```
In [502]: ▶ 1 # Let us plot the Coefficient magnetitude of oue modes with respect to the alpha values ↔
```



6 Conclusion

The results of our analysis indicate how regularization techniques can be used to improve the predictive power of a model. In this case study we observed that lots of the predictors are not associated with our response variable therefore, their coefficients better be omitted from the model. We also showed the importance of parameter tuning for having an optimized version of the model.