CS 461 : Computer Graphics
Assignment 2

Name : Shubham Kumar
Roll No. : 17010064

Bezier curves (Cubic Bezier curves)

Curve can be written as

$$P(t) = (1-t)^3 * P_0 + 3t(1-t)^2 * P_1 + 3t^2(1-t) * P_2 + t^3 * P_3$$

$$X(t) = (1-t)^3 * x_0 + 3t(1-t)^2 * x_1 + 3t^2(1-t) * x_2 + t^3 * x_3$$

$$y(t) = (1-t)^3 * y_0 + 3t(1-t)^2 * y_1 + 3t^2(1-t) * y_2 + t^3 * y_3$$

where t goes from 0 to 1
(0 and 1 included)

Here we are assumming there are 7 Points ~~est~~ having Positional Continuity

There are three types of Continuity
- Positional Continuity
- Tangential Continuity
- Curvature Continuity

Since there are only 7 Points so they satisfy Positional Continuity

~~Let, there are two Curves with Control Point P_0, P_1, P~~

Let there are two curves P and Q with Control Points P_0, P_1, P_2, P_3 and Q_0, Q_1, Q_2, Q_3

they satisfy Positional Continuity so,

$Q_0 = P_3$ or $P_3 = Q_0$ —— (1)

for satisfying Positional continuity end Point of first curve should be the starting Point of second curve.

Tangential Continuity

for satisfying this Continuity velocity at the Joint should be same for both the curves.

for having same velocity, the derivative for both the curves at Join should be same. This is also called C^1 Continuity.

If the derivative of 2 curves are some multiple of each other, means they are facing the same direction but have different magnitudes of velocity (this is called G^1 Continuity).

So for achieving Tangential Continuity derivatives at the join for two curves should be equal.

Conditions for tangential Continuity
- should have Positional Continuity
- should have equal derivative at Join Point for both the curves. we can derive a formula showing Relations between Points of ~~the~~ both the curves to achieve this continuity. let's assume we have two curves P and Q

As we know that

$$P(t) = (1-t)^3 * P_0 + 3t(1-t)^2 * P_1 + 3t^2(1-t) * P_2 + t^3 * P_3$$

similarly

$$Q(t) = (1-t)^3 Q_0 + 3t(1-t)^2 * Q_1 + 3t^2(1-t) * Q_3 + t^3 Q_3$$

$$P'(t) = -3(1-t)^2 P_0 + 3[3t^2-4t+1] * P_1 + 3[2t-3t^2) * P_2 + 3t^2 P_3$$

$$Q'(t) = -3(1-t)^2 * Q_0 + 3(3t^2-4t+1) * Q_1 + 3(2t-3t^2) * Q_2 + 3t^2 Q_3$$

Now we have to equate derivative at end Point of P. so at P(1) and starting Point of Q at Q(0) so

$$P'(1) = Q'(0)$$

$$\Rightarrow 0 + 0 + (-3) * P_2 + 3P_3 = -3Q_0 + 3Q_1 + 0 + 0$$

$$P_3 - P_2 = Q_1 - Q_0$$

[since $Q_0 = P_3$ from Positional continuity]

so $\boxed{Q_1 = 2P_3 - P_2}$ (2)

for tangential continuity

Curvature Continuity :- for achieving this Continuity Acceleration for both curves at Join must be equal.

Conditions for achieving curvature continuity
- Should have Positional Continuity.
- Should have tangential Continuity
- Should have equal double derivative at Join for both the curves.

we can derive double derivative of both curves in terms of control Points.

$P''(t) = 6(1-t)*P_0 + 3(6t-4)*P_1 + 3(2-6t)*P_2 + 6t*P_3$

$Q''(t) = 6(1-t)*Q_0 + 3(6t-4)*Q_1 + 3(2-6t)*Q_2 + 6t*Q_3$

Here we have to equat

$Q''(0) = P''(3)$

$\Rightarrow 6Q_0 - 12Q_1 + 6Q_2 = 6P_3 - 12P_2 + 6P_1$

$\Rightarrow Q_0 - 2Q_1 + Q_2 = P_3 - 2P_2 + P_1$

$\Rightarrow$ by Putting $Q_0 = P_3$ from Positional continuity and $Q_1 = 2P_3 - P_2$ from tangential Continuity.

we get

$Q_2 = P_1 + 4[P_3 - P_2]$

so we get

$$\boxed{Q_2 = P_1 + 4(P_3 - P_2)} \quad \text{------} \quad 3$$

from this discussion we get.

Let's assume we have two curves P and Q with control points P_0, P_1, P_2, P_3 and Q_0, Q_1, Q_2, Q_3 in order.
So we have established some relations between these control points for different continuities and are as follows -

① Positional Continuity
for cubic → $\boxed{Q_0 = P_3}$
or, $\boxed{Q_0 = P_n}$ ← for generalisation

② Tangential Continuity
for cubic → $Q_0 = P_3$ and $Q_1 = 2P_3 - P_2$
or in general $Q_0 = P_n$ and $Q_1 = 2P_n - P_{n-1}$

③ Curvature Continuity (smooth curve)

for cubic ~~bezier~~ bezier curve
$Q_0 = P_3$; $Q_1 = 2P_3 - P_2$ and $Q_2 = P_1 + 4(P_3 - P_2)$
or in general
$Q_0 = P_n$; $Q_1 = 2P_n - P_{n-1}$ and $Q_2 = P_{n-2} + 4(P_n - P_{n-1})$

Here n is the degree of Bernstein Polynomial.