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# Arbitrary vectorial state conversion using liquid crystal spatial light modulators

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### ABSTRACT

There is significant interest in the generation of complex vector beams, where the combined polarisation and phase profile is chosen to have particular properties. These beams have often been implemented in an adaptable manner using liquid crystal spatial light modulators (SLMs). Existing methods have concentrated mainly on the generation of these vector beams from a fixed polarisation state, but there are also applications were conversion between vectorial states is necessary. We discuss the limitations of existing SLM-based modulation systems for conversion between arbitrary vectorial states. Three degrees of freedom are required in principle for transition between two arbitrary states. We show that a three-SLM system can provide conversion between two arbitrary polarisation states, but cannot necessarily provide a full  $2\pi$  radian phase modulation. Hence, we propose a design using four SLMs that removes these limitations on the phase modulation.

#### 1. Introduction

The optimum operation of many optical systems requires precise control of both polarisation and phase. In most cases, the requirement is for uniform polarisation states and constant (aberration free) phase. However, there are also many applications ranging through classical and quantum optics that require vectorial beam states, where spatial variations exist in both polarisation state and phase [1–10]. Certain beams can be implemented using fixed optical elements acting as spatially variant retarders, such as Q plates, metasurfaces, GRIN optics, segmented waveplate arrays or Fresnel cones [11–15]. However, more complex beam states have required liquid crystal spatial light modulators (SLMs), whose high pixel density provides fine spatial control of the beam profile. In order to provide the necessary degrees of freedom for full vectorial control – including phase and polarisation – multiple SLM passes are required.

In addition to the beam complexity afforded by these SLM devices, they are reconfigurable and hence have the potential to compensate for unwanted perturbations that affect the beam state. SLM based adaptive optics methods are commonly used to correct for phase perturbations – or aberrations – introduced into optical systems. However, appropriately configured multi-pass SLM systems can also be used for compensation of polarisation perturbations or combined vectorial aberrations, affecting both polarisation and phase [9,16–18]. Of course, the manipulation of polarisation in this way is inherently linked to phase through the Pancharatnam phase induced when the beam passed through a sequence of modulators.

### 2. Model system

In order to investigate the capabilities of SLMs in controlling the vectorial state (including polarisation and phase), we consider the system shown in Fig. 1. This consists of a sequence of transmissive SLMs positioned along the beam path at normal incidence, such that the distance between the SLMs is small enough so that diffraction effects between elements can be considered negligible. We note also that similar operation could be achieved if suitable 4f imaging systems had been used between each pair of SLMs, although they are omitted here for clarity. While many modern SLMs operate in a reflection rather than transmission configuration, which sometimes requires operation at a slight deviation from normal incidence. We make the approximation here that all beams are at normal incidence.

The illumination is assumed to be a collimated beam of monochromatic, fully polarised light, although the polarisation state may vary

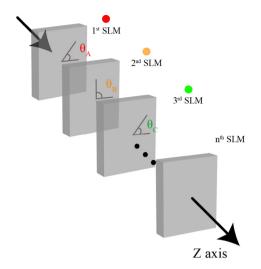
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Existing systems for vectorial beam control have mostly been concerned with the generation of a chosen beam profile from a fixed [19–23], known state, such as the linearly polarised output of a laser source. For broader implementation of vectorial beam control, a more general problem must be solved: the conversion between two arbitrary vectorial states. In this paper, we investigate the capabilities of existing systems, using two or three SLM passes, for such arbitrary state conversion and discuss their limitations. Furthermore, we propose a new configuration, which uses four SLM passes, that permits full conversion between arbitrary vectorial states.

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**Fig. 1.** Schematic of the multiple SLM system. The modulation axis of each SLM is set at a chosen angle  $\theta_{\Lambda}$ , etc.

across the beam profile. Phase variations may also be present in the incoming beam.

In order to model the operation of the most commonly used, parallel aligned nematic liquid crystal devices, the SLMs are assumed to operate as waveplates (linear retarders) with fixed axis, but variable retardance within a range of  $\pm\pi$  radians. The SLMs can be rotated around the optical axis, such that the equivalent waveplate axis of each SLM can be at any chosen orientation. In many practical systems of this type, the SLMs are not mechanically rotated, but additional waveplates are used instead to transform the polarisation state between SLM passes. We discuss the consequences of this configuration later in the paper.

The action of each of the SLM pixels is to introduce phase shifts along axis corresponding to the ordinary (fixed phase shift) and extraordinary (variable phase shift) axes of the variable waveplate. We refer to the polarisation components projected on the ordinary and extraordinary axes as the eigenmodes of the equivalent waveplate. We define an offset phase shift  $\Phi$ , which will be the same across the whole beam. The effects of each pixel are represented by  $\Phi$  absolute phase shift along the ordinary axis and the addition of  $\Phi + \delta$  radians for the extraordinary axis. As the offset  $\Phi$  will have and hence have no consequence for our analysis, we will neglect this term from further expressions. The action of the SLM is thus equivalent to the addition of a relative phase of  $\delta$  between the modes. The relative phase corresponds to the retardance  $\delta$  of the SLM pixel that is related to the change in polarisation state.

The change in polarisation state induced by the SLM pixel can be modelled as a transition on the Poincaré sphere. If the pixel can be represented by a Müller matrix with primary eigenvector given by the normalised Stokes 3-vector Q, corresponding to the extraordinary axis of the equivalent retarder, and retardance  $\psi$ , then the transition between the input state and the output state corresponds to a rotation through angle  $\psi$ , around the axis defined by **Q**. Hence, cascaded SLMs can be modelled geometrically by sequences of rotations about different axes on the Poincaré sphere, which correspond to a collection of circular arcs on the surface. We note the special case where the input polarisation is aligned with the primary eigenmode of the SLM and hence the input state lies on the axis of rotation; in this case there is no change in polarisation state, although there is a phase modulation. This corresponds to the normal operation of the SLM as a pure phase modulator. We can also note when the input is aligned with the secondary eigenmode, there is no polarisation or phase change.

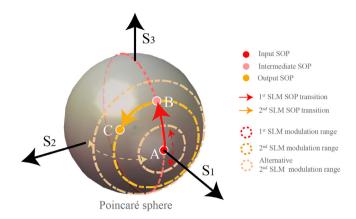


Fig. 2. State of polarisation (SOP) transitions on the Poincaré sphere for the two-SLM system.  $S_1$ ,  $S_2$  and  $S_3$  are the axes representing the Stokes vector components. Point A represents the linear polarised input state; point B is the state after the first SLM; point C shows the output state. Variation of the retardance of the two SLMs permits the translation of point C to any point on the surface of the sphere.

## 3. Generation of arbitrary polarisation and phase states from a defined input

Several methods have been developed for the generation of arbitrary states using combinations of SLMs [9,10,16,17,24–29]. The simplest of these methods consist of consecutive passes through multiple SLMs. In practice, this may be implemented in an equivalent way by passing the beam through different regions of the same SLM. We provide a brief review of operation of these systems, as the basis for later discussion.

The purpose of these systems is to generate a defined polarisation and/or phase state from a known input. The input beam is chosen to have a uniform, linear polarisation state. For generation of arbitrary polarisation states, a minimum of two SLM modulations is required, as the transition in polarisation state requires two free variables. In the optimal configuration, for full polarisation modulation capability, the extraordinary optical axis of the first SLM must be oriented at 45° to the polarisation of the input light. The second SLM is then oriented at 45° to the first SLM. On the Poincaré sphere, the action of the first SLM is to move the polarisation state along a great circle (see Fig. 2); the second SLM then moves the state along a circle in a perpendicular plane to the plane of the first SLM path. These two transitions can reach any point on the sphere and hence permit the generation of any polarisation state. Although we have assumed that all SLMs are capable of full  $\pm \pi$  radians retardance, this same modulation range could also be achieved if the second SLM were limited to  $\pm \pi/2$  radians. Note that if the orientation of either SLM is not perfectly aligned, the paths on the Poincaré sphere will not be in perpendicular planes, so there will be regions of the sphere that cannot be reached; hence, there will be polarisation states that cannot be generated by the system.

For generation of arbitrary polarisation and phase states, the dual SLM system mentioned above can be preceded with an initial SLM pass, where the SLM primary eigenmode is aligned with the input polarisation and hence provides pure phase modulation [16]. We note that a similar configuration could be used to convert from an arbitrary polarisation state to a predetermined fixed state.

### 4. General conversion between arbitrary polarisation and phase states

We now consider the use of a three-SLM system for the conversion between arbitrary polarisation and phase states. The SLMs are configured in the same way as described in the previous section. The current scenario is however different to that discussed in the previous section, where the input (or alternatively output) state was pre-defined

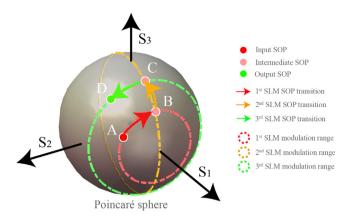


Fig. 3. State of polarisation (SOP) transitions on the Poincaré sphere for the three-SLM system.  $S_1$ ,  $S_2$  and  $S_3$  are the axes representing the Stokes vector components. Point A represents the input state; point B is the state after the first SLM; point C is the state after the second SLM; point D is the output state, after the third SLM.

to be a particular linear polarisation state. In general, three degrees of freedom are required to describe a transition between two arbitrary vectorial states. While in principle this should be achievable with three appropriate modulators, we explain here the limitations in the use of three such SLMs under consideration here to achieve full vectorial conversion.

The arbitrary input state and the desired output state are represented by two points located anywhere on the Poincaré sphere. The eigenmode axis of the first SLM is no longer aligned with the input polarisation (except for in a special case) so the action of the first SLM pass modifies the polarisation state; this corresponds to a rotation on the Poincaré sphere (see Fig. 3). The second SLM pass then generates a rotation in a perpendicular plane. The third SLM pass creates a further rotation following a path in a plane parallel to that of the first pass. It is clear from this geometrical construction that any two points on the sphere can be connected using three such rotations. Hence, it is possible to use these three SLM passes to convert any input polarisation into another polarisation state.

While this configuration permits conversion between two arbitrarily defined polarisation states, there are limitations on the corresponding phase modulation. As explained in the previous section, for input polarisation aligned with the first SLM's primary eigenmode, full adjustment of polarisation and phase is possible. However, for other input polarisations, the accessible phase states are limited. The extreme case is encountered when the input is aligned with the first SLM's second (non-modulating) eigenmode, for which the SLM introduces no phase shift. In the general case, therefore, full phase modulation is not achievable.

### 5. Limitations on phase modulation for three SLM system

The action of each of the three SLMs can be described by its Jones matrix  $\mathbf{J}_n$ , where n is the index of the SLM, which we will refer to by the subscripts A, B and C. The output field vector  $\overline{\mathbf{E}}_{out}$  is then calculated from the input field vector  $\overline{\mathbf{E}}_{in}$  using

$$\vec{\mathbf{E}}_{out} = \mathbf{J}_C \mathbf{J}_B \mathbf{J}_A \vec{\mathbf{E}}_{in} \tag{1}$$

The total phase induced by one of these Jones matrix operations can be described as [30]

$$\phi = \arg \left[ \mu_1 + \mu_2 + (\mu_1 - \mu_2) \, \overline{\mathbf{Q}} . \overline{\mathbf{A}} \right] \tag{2}$$

where the phase is defined according to the Pancharatnam connection between the states before and after the SLM [31,32].  $\overline{\mathbf{Q}}$  is the first eigenpolarization state of the polarising element, expressed as a normalised Stokes vector. Note that the other eigenpolarization state is identical to  $-\overline{\mathbf{Q}}$ . The parameters  $\mu_1$  and  $\mu_2$  are the corresponding

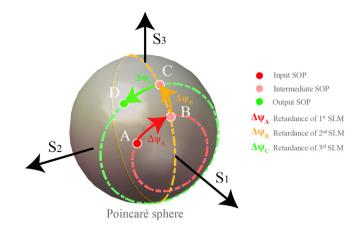


Fig. 4. State of polarisation (SOP) transitions on the Poincaré sphere for the three-SLM system.  $S_1$ ,  $S_2$  and  $S_3$  are the axes representing the Stokes vector components.

eigenvalues of these two states.  $\overline{A}$  is the Stokes vector of the input polarisation state. When describing a SLM, the vector  $\overline{\mathbf{Q}}$  corresponds to the modulation axis of the SLM and the corresponding eigenvalue is the complex transmittance of the mode

$$\mu_1 = \exp\left(i\psi_1\right) \tag{3}$$

where  $\psi_1$  is the induced varying phase, which depends upon the SLM pixel voltage. The other eigenvalue is given by

$$\mu_2 = \exp\left(i\psi_2\right) \tag{4}$$

where  $\psi_2$  is the induced fixed phase. Note that we can define  $\Delta\psi=\psi_1-\psi_2$  as the retardance of the SLM. Due to our choice of orientations of SLMs as 0°, 45° and 0°, the corresponding eigenpolarisations are given by

$$\vec{\mathbf{Q}}_A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \vec{\mathbf{Q}}_B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \vec{\mathbf{Q}}_C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

It follows that for a general input Stokes vector  $\vec{\mathbf{A}} = (S_1, S_2, S_3)^{\mathrm{T}}$ , the scalar products are given by  $\vec{\mathbf{Q}}_A . \vec{\mathbf{A}} = S_1$ ,  $\vec{\mathbf{Q}}_B . \vec{\mathbf{A}} = S_2$ , and  $\vec{\mathbf{Q}}_C . \vec{\mathbf{A}} = S_1$ . We can therefore calculate the phases induced by each SLM pass as

$$\phi_{A} = \arg \left[ e^{i\psi_{A1}} + e^{i\psi_{A2}} + \left( e^{i\psi_{A1}} - e^{i\psi_{A2}} \right) A_{1} \right] 
\phi_{B} = \arg \left[ e^{i\psi_{B1}} + e^{i\psi_{B2}} + \left( e^{i\psi_{B1}} - e^{i\psi_{B2}} \right) B_{2} \right] 
\phi_{C} = \arg \left[ e^{i\psi_{C1}} + e^{i\psi_{C2}} + \left( e^{i\psi_{C1}} - e^{i\psi_{C2}} \right) C_{1} \right]$$
(6)

where  $A_1$ ,  $B_2$ , and  $C_1$  are respectively the 1st, 2nd and 1st elements of the normalised Stokes vectors incident onto each of the three SLMs. We can note that the phases  $\psi_{A2}$ ,  $\psi_{B2}$  and  $\psi_{C2}$  have fixed values;  $A_1$  is determined by the input state;  $C_1$  is determined by the output state. The remaining four variables  $-\psi_{A1}$ ,  $\psi_{B1}$ ,  $\psi_{C1}$  and  $B_2$  – are interdependent, as the choice of e.g.  $\psi_{A1}$  will set the values of the other three. This interrelationship is clearly seen from Fig. 4. Therefore, for given input and output polarisation states, there is one degree of freedom available for manipulation of the phase.

Using the definition of the retardance of each SLM,  $\Delta\psi_A=\psi_{A1}-\psi_{A2}$  etc., we can write

$$\begin{aligned} \phi_{A} &= \psi_{A2} + \arg\left[ \left( 1 - A_{1} \right) + \left( 1 + A_{1} \right) e^{i\Delta\psi_{A}} \right] \\ \phi_{B} &= \psi_{B2} + \arg\left[ \left( 1 - B_{2} \right) + \left( 1 + B_{2} \right) e^{i\Delta\psi_{B}} \right] \\ \phi_{C} &= \psi_{C2} + \arg\left[ \left( 1 - C_{1} \right) + \left( 1 + C_{1} \right) e^{i\Delta\psi_{C}} \right] \end{aligned}$$
(7)

In each of the expressions for  $\phi_A$  and  $\phi_C$ , the only variable is the final retardance term  $\Delta \psi_A$  or  $\Delta \psi_C$ , respectively. In the expression for  $\phi_B$ , only the value of  $B_2$  is independently variable, although  $\Delta \psi_A$ ,  $\Delta \psi_B$ ,  $\Delta \psi_C$  and  $B_2$  are interdependent. For example, we can choose to set  $\Delta \psi_A$ 

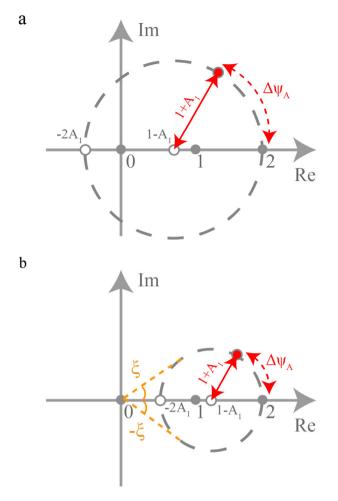


Fig. 5. Limitations on the output phase ranges for the modulation terms describing the operation of one SLM, as expressed in Eq. (8).

and then  $\Delta \psi_B$ ,  $\Delta \psi_C$  and  $B_2$  are uniquely determined by geometrical relationships, which are easily derived, but omitted here for brevity.

By considering the argument term in the expression for  $\phi_A$  (see Fig. 5),

$$\phi'_{A} = \arg \left[ (1 - A_{1}) + (1 + A_{1}) e^{i\Delta\psi_{A}} \right]$$
 (8)

we can deduce that for  $0 < A_1 \le 1$  then the phase can access the full range  $-\pi < \phi'_A \le \pi$ ; for  $-1 \le A_1 < 0$ , is limited to a range  $-\xi < \phi'_A < \xi$ , where  $\xi < \frac{\pi}{2}$ . In the limiting case where  $A_1 = -1$ , then  $\phi'_A = 0$ ; this corresponds to the case where the illuminating polarisation is aligned with the non-modulating axis of the SLM. Similar relationships and limitations can be found for  $\phi_C$ . Geometric considerations also show there are in general constraints on the range of phase values attainable by  $\phi_B$ .

This geometrical analysis shows that for each of the three SLM passes, when considered separately, there exist combinations of states at the input and output for which full phase modulation cannot be achieved.

Following the analysis in [30], the total phase introduced by the three SLM passes equals to the sum of phase introduced individually by each pass ( $\phi_A + \phi_B + \phi_C$ ) and the phase equivalent to half the area of the quadrilateral enclosed by the three passes on the Poincaré sphere ( $\phi'$ ), such that

$$\phi_{Total} = \phi_A + \phi_b + \phi_c + \phi' \tag{9}$$

where  $\phi'$  is equal to half of the area enclosed by the path DCBA, as shown in Fig. 4.

A conclusion that can readily be drawn from this is that there exist combinations of polarisation states at the input and output of the whole three-SLM system, for which it is not possible to achieve an arbitrary phase transition. Numerical modelling of random combinations of input and output states confirms that this configuration of SLMs is not versatile enough to perform such arbitrary transformation. This system cannot therefore be considered a universal transformer for vectorial states.

### 6. Configurations for fully arbitrary polarisation and phase conversion

A limitation of the three SLM system for arbitrary conversion of the vectorial state is that the initial SLM pass only provides pure phase modulation for one input polarisation state and limited modulation for a range of other input states. An alternative configuration that allows sufficient phase control would include an additional polarisation insensitive device, such as a deformable mirror or mirror-based SLM, which would introduce a polarisation independent phase shift. This device could be placed at any point in the optical system, before, between or after the three SLM passes.

It is interesting to consider, however, whether an additional SLM pass could be used to achieve the equivalent goal. We propose to do this using an additional SLM pass with its modulation axis orthogonal to the first pass. The sequence of the four SLM orientations would be  $0^{\circ}$ ,  $90^{\circ}$ ,  $45^{\circ}$  then  $0^{\circ}$ , where we have inserted the additional pass at  $90^{\circ}$  in the second position. For this new case, the eigenpolarization axes are given by

$$\vec{\mathbf{Q}}_A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \vec{\mathbf{Q}}_B = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \ \vec{\mathbf{Q}}_C = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \vec{\mathbf{Q}}_D = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$
 (10)

Considering the action of the first two SLM passes, we see that the locus of the possible polarisation state outputs from each pass follows the same circle on the Poincaré sphere. Hence, the accessible polarisation states are the same as would be possible for a single pass with SLM orientation at 0°. However, the phases introduced in these two passes are given by

$$\begin{split} \phi_{A} &= \psi_{A2} + \arg \left[ \left( 1 - A_{1} \right) + \left( 1 + A_{1} \right) e^{\mathrm{i}\Delta\psi_{A}} \right] \\ \phi_{B} &= \psi_{B2} + \arg \left[ \left( 1 + A_{1} \right) + \left( 1 - A_{1} \right) e^{\mathrm{i}\Delta\psi_{B}} \right] \end{split} \tag{11}$$

where in the latter expression we have noted that the first element of the Stokes vector is not changed by the action of the first SLM, so  $B_1=A_1$ . According to the geometrical arguments in the previous section, we can see that when the range of phase modulation in the first pass is limited by the input polarisation state (that is when  $A_1<0$ ), then full modulation range is achievable in the second pass. Similarly, when the phase modulation due to the second pass is limited (for  $B_1=A_1>0$ ), full phase modulation is possible through the first pass. This also applies for the extreme cases of linear input polarisations at  $0^\circ$  or  $90^\circ$ . Again, numerical simulation has confirmed that the limitations of the three-SLM system can be overcome by this four-SLM system.

We conclude therefore that the proposed four pass SLM system can act as a full vectorial state transformer. We note that this particular arrangement of SLMs is not unique and the same performance can be achieved with certain other angular arrangements of SLMs. One obvious configuration would be  $0^{\circ}$ ,  $45^{\circ}$ ,  $0^{\circ}$  then  $90^{\circ}$  orientations.

### 7. Conclusion

We have illustrated the capabilities and limitations of multiple pass SLM based systems for the conversion between two arbitrary vectorial states. While the three-pass system permits the generation of arbitrary phase and polarisation states from a fixed input state, it cannot perform all possible transformations between two arbitrary vectorial states. In particular, while it can achieve transition to any other polarisation

state, it cannot necessarily provide sufficient range of phase modulation. The four pass system overcomes the limitations of the three pass system by filling in the missing phase, by virtue of complementarity of the phase modulation produced by the two SLMs at  $0^{\circ}$  and  $90^{\circ}$  orientation.

For simplicity, the modelling here was explained for transmissive SLMs placed in close sequence and rotated about the optical axis. Practical systems have used waveplates between the SLM passes, to transform the polarisation states, rather than rotating the SLM axes. As these waveplates create additional paths on the Poincaré sphere, the overall phases will change. However, this does not affect the main conclusions of this paper, as the modulatable range of phase is determined only by the action of the SLMs.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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