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    - "Nonparaxial Propagation Properties of Specially Correlated Radially Polarized Beams in Free Space"
    - "Closed-form bases for the description of monochromatic, strongly focused, electromagnetic fields"
    - "Measuring the nonseparability of vector vortex beams"
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    - "A practical algorithm for the determination of phase from image and diffraction plane pictures"
    - "Kinoform design with an optimal-rotation-angle method"
    - "New iterative algorithm for the design of phaseonly gratings"
    - "Continuous-relief diffractive optical elements for two-dimensional array generation"
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    - "Cylindrical vector beams: from mathematical concepts to applications"
    - "Vector Beams for Fundamental Physics and Applications"
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    - "Orbital angular momentum: origins, behavior and applications"
  - SLM
    - "Creation and detection of optical modes with spatial light modulators"

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$$ \boldsymbol{E} = \begin{bmatrix} E_{x0} e^{i\varphi_x} \\ E_{y0} e^{i\varphi_y} \end{bmatrix} $$

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$$ \varphi_y = \varphi_x $$

$$ E_{y0} = E_{x0} $$

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$$ \boldsymbol{E} = E_{x0} e^{i \varphi_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix} $$

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$$ Re(\boldsymbol{E}) = E_{x0} \cos{\varphi_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix} $$

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$$ \begin{aligned} \varphi_y &= \varphi_x - \frac{\pi}{2} \\ \varphi_y - \varphi_x &= -\frac{\pi}{2}\\ \therefore \delta &= -\frac{\pi}{2} \end{aligned} $$

※□□□□□$\delta$□□□□□

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$$ \begin{aligned} \boldsymbol{E} &= E_{x0} \begin{bmatrix} e^{i \varphi_x} \\ e^{i (\varphi_x - \frac{\pi}{2})} \end{bmatrix} \\ &= E_{x0} e^{i\varphi_x} \begin{bmatrix} 1 \\ e^{-i\pi} \end{bmatrix} \\ &= E_{x0} e^{i\varphi_x} \begin{bmatrix} 1 \\ -i \end{bmatrix} \\ \end{aligned} $$

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$$ Re(\boldsymbol{E}) = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos{\varphi_x} \\ \sin{\varphi_x} \end{bmatrix} $$

$\varphi_x$ $x-y$

z x y $z$

***x,y***

$y$ $x$ $\frac{\pi}{2}$

$$ \boldsymbol{E} = E_{x0} e^{i\varphi_x} \begin{bmatrix} 1 \\ i \end{bmatrix} $$

$$ Re(\boldsymbol{E}) = E_{x0} \begin{bmatrix} \cos{\varphi_x} \\ -\sin{\varphi_x} \end{bmatrix} $$

Alt text

$z$

Alt text

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$$ E_{x0}^2 e^{2i\varphi_x} \begin{aligned} \begin{bmatrix} 1 & -i \end{bmatrix}^* \begin{bmatrix} 1 \\ i \end{bmatrix} \end{aligned} = 0 $$

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**$y$□□□□□□□□□□□□□($x$□□□□□□□□□□□□□□)□□□□□□(y□□□□□□□□)**

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$$ \begin{aligned} J &= \begin{bmatrix} e^{i\varphi_x} & 0 \\ 0 & e^{i\varphi_y} \end{bmatrix} \\ &= e^{i\varphi_x} \begin{bmatrix} 1 & 0 \\ 0 & e^{i(\varphi_y - \varphi_x)} \end{bmatrix} \\ &= e^{i\varphi_x} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \\ \end{aligned} $$

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$$ J = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} $$

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$$ \begin{aligned} J &= \begin{bmatrix} \cos{\theta} & -\sin{\theta} \\ \sin{\theta} & \cos{\theta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} \cos{\theta} & \sin{\theta} \\ -\sin{\theta} & \cos{\theta} \end{bmatrix} \\ &= \begin{bmatrix} \cos{\theta} & -e^{i\delta} \sin{\theta} \\ \sin{\theta} & e^{i\delta} \cos{\theta} \end{bmatrix} \begin{bmatrix} \cos{\theta} & \sin{\theta} \\ -\sin{\theta} & \cos{\theta} \end{bmatrix} \\ \therefore J &= \begin{bmatrix} e^{i\delta} \sin^2{\theta} + \cos^2{\theta} & \sin{\theta} \cos{\theta} (1 - e^{i\delta}) \\ \sin{\theta} \cos{\theta} (1 - e^{i\delta}) & \sin^2{\theta} + e^{i\delta} \cos^2{\theta} \end{bmatrix} \\ \end{aligned} $$

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# 1/2□□□

□□□□$\delta=\pi$□□□□□□**1/2□□□(HWP:Half Wave Plate)** □□□□

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$$ \begin{aligned} J_{HWP(\theta)} &= \begin{bmatrix} -\sin^2{\theta} + \cos^2{\theta} & 2\sin{\theta} \cos{\theta} \\ 2\sin{\theta} \cos{\theta} & \sin^2{\theta} - \cos^2{\theta} \end{bmatrix} \\ &= \begin{bmatrix} \cos{2\theta} & \sin{2\theta} \\ \sin{2\theta} & -\cos{2\theta} \end{bmatrix} \end{aligned} $$

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□□$\theta = 0$□□□□

$$ J_{HPW(\theta=0)} = \begin{bmatrix} 1 & 0 \ 0 & -1 \end{bmatrix} $$

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□□□$\frac{\pi}{4}$ □□□□□□□□□

$$ \boldsymbol{E}_{in} = \begin{bmatrix} 1 \ 1 \end{bmatrix} $$

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$$ \boldsymbol{E}_{out} = \begin{bmatrix} 1 \ -1 \end{bmatrix} $$

□□□□$-\frac{\pi}{4}$□□□□□□□□□□□□□□□□□□

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□□□ $\theta = \frac{\pi}{4}$ □□□□□□□□

$$ J_{HWP(\theta=\frac{\pi}{4})} = \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} $$

□□□ **x,y**□□□□□□□□□□□□□□□□

# 1/4□□□

□□□ $\delta=\frac{\pi}{2}$ □□□□□

$$ \begin{aligned} J_{QWP(\theta)} &= \begin{bmatrix} e^{i \frac{\pi}{2}} \sin^2{\theta} + \cos^2{\theta} & \sin{\theta} \cos{\theta} (1 - e^{i \frac{\pi}{2}}) \ \sin{\theta} \cos{\theta} (1 - e^{i \frac{\pi}{2}}) & \sin^2{\theta} + e^{i \frac{\pi}{2}} \cos^2{\theta} \end{bmatrix} \ &= \begin{bmatrix} i \sin^2{\theta} + \cos^2{\theta} & \sin{\theta} \cos{\theta} (1 - i) \ \sin{\theta} \cos{\theta} (1 - i) & \sin^2{\theta} + i \cos^2{\theta} \end{bmatrix} \ &= \begin{bmatrix} 1 - i\cos{2\theta} & -i\sin{\theta}\ -i\sin{\theta} & 1 + i\cos{2\theta} \end{bmatrix} \end{aligned} $$

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$\theta = 0$ □□□□

$$ J_{QWP(\theta=0)} = \begin{bmatrix} 1 & 0 \ 0 & i \end{bmatrix} $$

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$\frac{\pi}{4}$ □□□□□□□□□

$$ \boldsymbol{E}_{in} = \begin{bmatrix} 1 \ 1 \end{bmatrix} $$

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$$ \boldsymbol{E}_{out} = \begin{bmatrix} 1 \ i \end{bmatrix} $$

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$$ \begin{bmatrix} \alpha \ \beta \end{bmatrix} (\alpha, \beta \in \mathbb{C}) $$

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$$ \begin{bmatrix} S_1 \ S_2 \ S_3 \end{bmatrix} = \begin{bmatrix} | \alpha | ^2 - | \beta | ^2 \ 2 Re( \alpha \beta ^) \ -2 Im( \alpha \beta ^) \end{bmatrix} $$

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$$ | \alpha | ^2 + | \beta | ^2 = 1 $$

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$$ P = \begin{bmatrix} \alpha \ \beta \end{bmatrix} \begin{bmatrix} \alpha \beta \end{bmatrix} ^* $$

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$$ \begin{aligned} □ P^2 &= 1 \ □ P^ \dagger &= P \end{aligned} $$

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$$ \begin{aligned} P^2 &= ( \begin{bmatrix} \alpha \ \beta \end{bmatrix} \begin{bmatrix} \alpha \beta \end{bmatrix} ^* ) ( \begin{bmatrix} \alpha \ \beta \end{bmatrix} \begin{bmatrix} \alpha \beta \end{bmatrix} ^* ) \ &= \begin{bmatrix} \alpha \ \beta \end{bmatrix} ( \begin{bmatrix} \alpha \beta \end{bmatrix} ^* \begin{bmatrix} \alpha \ \beta \end{bmatrix} ) \begin{bmatrix} \alpha \beta \end{bmatrix} ^* \ &= \begin{bmatrix} \alpha \ \beta \end{bmatrix} ( | \alpha | ^2 + | \beta | ^2 ) \begin{bmatrix} \alpha \beta \end{bmatrix} ^* \ \therefore P^2 &= P \end{aligned} $$

□□

$$ \begin{aligned} P ^{\dagger} &= ( \begin{bmatrix} \alpha \ \beta \end{bmatrix} \begin{bmatrix} \alpha \beta \end{bmatrix} ^* ) ^{\dagger} \ &= ( \begin{bmatrix} \alpha \ \beta \end{bmatrix} \begin{bmatrix} \alpha \beta \end{bmatrix} ^* ) ^{\dagger} \ &= ( \begin{bmatrix} \alpha \ \beta \end{bmatrix} ^* )^* \begin{bmatrix} \alpha \beta \end{bmatrix} ^* \ \therefore P^{\dagger} &= P \end{aligned} $$

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https://electrodynamics.hatenablog.com/entry/2018/12/01/233744
https://electrodynamics.hatenablog.com/entry/2018/12/16/000833

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$$ \sigma_0 = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}, \quad \sigma_1 = \begin{bmatrix} 1 & 0 \ 0 & -1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} $$

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$$ \begin{aligned} \sigma_2 ^2 &= \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} \ &= \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} \end{aligned} $$

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$$ \begin{aligned} Tr( \sigma_3 \sigma_2 ) &= Tr \Bigl ( \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} \Bigr )\ &= Tr \Bigl ( \begin{bmatrix} -i & 0 \ 0 & i \end{bmatrix} \Bigr ) \ &= -i + i \ \therefore Tr( \sigma_3 \sigma_2 ) &= 0 \end{aligned} $$

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$$ \begin{aligned} P &= \frac{1}{2} ( h_0 \sigma_0 + h_1 \sigma_1 + h_2 \sigma_2 + h_3 \sigma_3 ) \ &= \frac {1}{2} \Bigl ( h_0 \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} +h_1 \begin{bmatrix} 1 & 0 \ 0 & -1 \end{bmatrix} +h_2 \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} +h_3 \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} \Bigr ) \end{aligned} $$

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$$ \begin{aligned} Tr(P \sigma_3) &= \frac{1}{2} \Bigl ( h_0 Tr(\sigma_0 \sigma_3) +h_1 Tr(\sigma_1 \sigma_3) +h_2 Tr(\sigma_2 \sigma_3) +h_3 Tr(\sigma_3 \sigma_3) \Bigr) \ &= h_3 \ h_3 &= 2 Tr \Bigl ( \begin{bmatrix} | \alpha | ^2 & \alpha \beta ^* \ \alpha ^* \beta & | \beta | ^2 \end{bmatrix} \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} \Bigr ) \ &= Tr \Bigl ( \begin{bmatrix} i \alpha \beta ^* & -i | \alpha | ^2 \ i | \beta | ^2 & -i \alpha ^* \beta \end{bmatrix} \Bigr ) \ &= i (\alpha \beta ^* - \alpha ^* \beta) \ &= -2 Im( \alpha \beta ^*) \ \therefore h_3 &= -2 Im( \alpha \beta ^* ) \end{aligned} $$

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$$ \begin{aligned} P &= \frac{1}{2} ( h_0 \sigma_0 +h_1 \sigma_1 +h_2 \sigma_2 +h_3 \sigma_3 ) \ &= \frac{1}{2} \Bigl ( h_0 \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} +h_1 \begin{bmatrix} 1 & 0 \ 0 & -1 \end{bmatrix} +h_2 \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} +h_3 \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} \Bigr ) \ &= \frac{1}{2} \Bigl ( \bigl (| \alpha | ^2 + | \beta | ^2 \bigr ) \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} +\bigl (| \alpha | ^2 - | \beta | ^2 \bigr ) \begin{bmatrix} 1 & 0 \ 0 & -1 \end{bmatrix} +2 Re( \alpha \beta ^*) \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} -2 Im( \alpha \beta ^*) \begin{bmatrix} 0 & -i \ i & 0 \end{bmatrix} \Bigr ) \end{aligned} $$

□□$| \alpha | ^2 + | \beta | ^2 = 1$□□□□□□□□□□□□□□□□1□□□□□□□□□□$h_1, h_2, h_3$□□□□□□□

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$$ \begin{aligned} \boldsymbol{ S } &= \begin{bmatrix} S_1 \ S_2 \ S_3 \end{bmatrix} \ &= \begin{bmatrix} h_1 \ h_2 \ h_3 \end{bmatrix} \ \therefore \boldsymbol{ S } &= \begin{bmatrix} | \alpha | ^2 - | \beta | ^2 \ 2 Re( \alpha \beta ^*) \ -2 Im( \alpha \beta ^*) \end{bmatrix} \end{aligned} $$

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$$ S^2_1 + S^2_2 + S^2_3 = 1 $$

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**$S_1, S_2, S_3$□□□□□□□□□□□1□□□□□□□□**

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$(S_1, S_2, S_3) = (1, 0, 0)$

$(S_1, S_2, S_3) = (0, 0, 1)$

$$ \begin{aligned} S_1 &= \begin{bmatrix} \alpha ^* & \beta ^* \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ &= \begin{bmatrix} \alpha ^* & \beta \end{bmatrix} \begin{bmatrix} \alpha \\ -\beta \end{bmatrix} \\ \therefore S_1 &= | \alpha | ^2 - | \beta | ^2 \end{aligned} $$

$$ \begin{aligned} S_2 &= \begin{bmatrix} \alpha ^* & \beta ^* \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ &= \begin{bmatrix} \alpha ^* & \beta \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \\ &= \alpha ^* \beta + \alpha \beta ^* \\ \therefore S_2 &= 2Re(\alpha \beta ^*) \end{aligned} $$

$$ \begin{aligned} S_3 &= \begin{bmatrix} \alpha ^* & \beta ^* \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ &= \begin{bmatrix} \alpha ^* & \beta \end{bmatrix} \begin{bmatrix} -i \beta \\ i \alpha \end{bmatrix} \\ &= -i \alpha ^* \beta + i \alpha \beta ^* \\ &= i (\alpha \beta ^* - \alpha ^* \beta) \\ \therefore S_3 &= -2Im(\alpha \beta ^*) \end{aligned} $$

$$ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} $$

$\alpha = \frac{1}{\sqrt{2}}, \beta = \frac{i}{\sqrt{2}}$

$$ \begin{aligned} S_1 &= \Bigl|\frac {1} {\sqrt{2}} \Bigr| ^2 - \Bigl|\frac {i} {\sqrt{2}} \Bigr| ^2 \\ &= \frac {1} {2} - \frac {1} {2} \\ &= 0 \\ S_2 &= 2Re\Bigl(\frac {1} {\sqrt{2}} \times \frac {i} {\sqrt{2}}^* \Bigr) \\ &= 2Re(-i) \\ &= 0 \\ S_3 &= -2Im\Bigl(\frac {1} {\sqrt{2}} \times \frac {i^*} {\sqrt{2}}\Bigr) \\ &= -2Im \Bigl(- \frac {i} {2} \Bigr) \\ &= 1 \end{aligned} $$

$$ S = (S_1, S_2, S_3) = (0, 0, 1) $$

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# SLM

$$ J_{SLM} = A_0 \begin{bmatrix} -e^{i\delta} & 0 \\ 0 & 1 \end{bmatrix} $$

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- $\delta = 2[n_e(V)-n_o]kd$
- $A_0 = e^{i2n_okd}$
- $k = \frac{2\pi}{\lambda}$
- $n_o$:□□□□□□□□□□□
- $n_e$:□□□□□□□□□□□□□□□□□□□□□□□□□□□

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$$ \boldsymbol{E}_{out} = J_{QWP(\gamma)} [J_{SLM1} + J_{HWP(-\beta)} J_{SLM2} J_{HWP(\beta)}] J_{PBS} J_{HWP(\alpha)} \boldsymbol{E}_{in} $$

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$$ \nabla \times \boldsymbol{E}(\boldsymbol{r}, t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} $$

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$$ \nabla \times (\nabla \times \boldsymbol{E}(\boldsymbol{r}, t)) = - \mu_0 \nabla \times \frac{\partial \boldsymbol{H}(\boldsymbol{r}, t)}{\partial t} $$

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$$ \nabla \times \boldsymbol {H} (\boldsymbol {r} , t) = \epsilon_0 \frac{\partial \boldsymbol {E} (\boldsymbol {r} , t) }{\partial t} $$

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$$ \nabla \times (\nabla \times \boldsymbol {E} (\boldsymbol {r} , t) ) = - \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} , t) }{\partial t^2} $$

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$$ \nabla \times (\nabla \times \boldsymbol {E} (\boldsymbol {r} , t) ) = \nabla(\nabla \cdot \boldsymbol {E} (\boldsymbol {r} , t) ) - \nabla^2 \boldsymbol {E} (\boldsymbol {r} , t) $$

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$$ \nabla(\nabla \cdot \boldsymbol {E} (\boldsymbol {r} , t) ) - \nabla^2 \boldsymbol {E} (\boldsymbol {r} , t) = - \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} , t) }{\partial t^2} $$

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$$ \nabla \cdot \boldsymbol {E} (\boldsymbol {r} , t) = 0 $$

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$$ \begin{aligned} -\nabla^2 \boldsymbol {E} (\boldsymbol {r} , t) &= -\epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} , t) }{\partial t^2} \\ \nabla^2 \boldsymbol {E} (\boldsymbol {r} , t) &= \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} , t) }{\partial t^2} \end{aligned} $$

$$ \therefore \nabla^2 \boldsymbol{E} (\boldsymbol {r} , t) - \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} , t)}{\partial t^2} = 0 $$

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$$ \boldsymbol {E} (\boldsymbol {r} , t) = \boldsymbol {E} (\boldsymbol {r}) f(t) $$

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$$ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) f(t) - \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} ) f(t)}{\partial t^2} = 0 $$

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$$ F(\omega) = \frac{1}{\sqrt{2 \pi}}\int_{- \infty}^{\infty}f(t) e^{i \omega t}dt $$

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$$ f(t) = \frac{1}{\sqrt{2 \pi}}\int_{- \infty}^{\infty}F(\omega) e^{-i \omega t}d \omega $$

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$$ \begin{aligned} \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) \frac{1}{\sqrt{2 \pi}}\int_{- \infty}^{\infty}F(\omega) e^{-i \omega t}d \omega - \epsilon_0 \mu_0 \frac{\partial^2 \boldsymbol {E} (\boldsymbol {r} )}{\partial t^2} \frac{1}{\sqrt{2 \pi}}\int_{- \infty}^{\infty}F(\omega) e^{-i \omega t}d \omega &= 0 \\ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) \frac{1}{\sqrt{2 \pi}}\int_{- \infty}^{\infty}F(\omega) e^{-i \omega t}d \omega + \frac{\epsilon_0 \mu_0 \omega ^2}{\sqrt{2 \pi}} \boldsymbol {E} (\boldsymbol {r} ) \int_{- \infty}^{\infty}F(\omega) e^{-i \omega t}d \omega &= 0 \\ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) + \epsilon_0 \mu_0 \omega ^2 \boldsymbol{E} (\boldsymbol {r} ) &= 0 \\ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) + \frac{\omega ^2}{c^2} \boldsymbol{E} (\boldsymbol {r} ) &= 0 \\ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) + \biggl (\frac{2 \pi f}{f \lambda} \biggr ) ^2 \boldsymbol{E} (\boldsymbol {r} ) &= 0 \\ \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) + \biggl (\frac{2 \pi}{\lambda} \biggr ) ^2

\boldsymbol{E} (\boldsymbol {r} ) &= 0 \ \therefore \nabla^2 \boldsymbol{E} (\boldsymbol {r} ) + k ^2 \boldsymbol{E} (\boldsymbol {r} ) &= 0 \end{aligned} $$

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$$ \boldsymbol{E}(\boldsymbol{r}) = u(x, y, z) e^{ikz} $$

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$$ \begin{aligned} \Bigl (\frac{\partial ^2}{\partial ^2 x} + \frac{\partial ^2}{\partial ^2 y} + \frac{\partial ^2}{\partial ^2 z} \Bigr ) \bigl (u(x, y, z) e^{ikz} \bigr ) + k^2 u(x, y, z) e^{ikz} &= 0 \\ \Bigl (\frac{\partial ^2 u}{\partial ^2 x} + \frac{\partial ^2 u}{\partial ^2 y} + \frac{\partial ^2 u}{\partial ^2 z} + 2ik \frac{\partial u}{\partial z} - k^2 u + k^2 u \Bigr ) e^{ikz} &= 0 \\ \frac{\partial ^2 u}{\partial ^2 x} + \frac{\partial ^2 u}{\partial ^2 y} + \frac{\partial ^2 u}{\partial ^2 z} + 2ik \frac{\partial u}{\partial z} &= 0 \end{aligned} $$

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$$ \frac{\partial ^2 u}{\partial ^2 x} + \frac{\partial ^2 u}{\partial ^2 y} + 2ik \frac{\partial u}{\partial z} = 0 $$

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$$ \nabla ^2 f = \nabla \cdot (\nabla f) $$

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$$ \nabla ^2 \boldsymbol{f} = \nabla (\nabla \cdot \boldsymbol{f}) -\nabla \times (\nabla \times \boldsymbol{f}) $$

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$$ \nabla \times \nabla \times \boldsymbol{E} -k^2 \boldsymbol{E} = 0 $$

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$$ \boldsymbol{E}(r, z) = U(r, z) e^{ikz} \boldsymbol{e}_\phi $$

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$$ \nabla = \frac{\partial}{\partial r} \boldsymbol{e}_r + \frac{1}{r} \frac{\partial}{\partial \phi} + \frac{\partial}{\partial z} \boldsymbol{e}_z $$

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$$ \nabla = \frac{\partial}{\partial r} \boldsymbol{e}_r + \frac{\partial}{\partial z} \boldsymbol{e}_z $$

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$$ \nabla \times \boldsymbol{E} = \Bigl (\frac{1}{r} \frac{\partial}{\partial \phi} E_z - \frac{\partial}{\partial z} E_\phi \Bigr ) \boldsymbol{e}_r + \Bigl (\frac{\partial}{\partial z} E_r - \frac{\partial}{\partial r} E_z \Bigr ) \boldsymbol{e}_\phi + \frac{1}{r} \Bigl (\frac{\partial}{\partial r} (r E_\phi) - \frac{\partial}{\partial \phi} E_r \Bigr )\boldsymbol{e}_z $$

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$$ E_r = E_z = 0 $$

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$$ \nabla \times \boldsymbol{E} = - \frac{\partial}{\partial z} E_\phi \boldsymbol{e}_r + \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \boldsymbol{e}_z $$

$$ \begin{aligned} \nabla \times (\nabla \times \boldsymbol{E}) &= \frac{1}{r} \frac{\partial}{\partial \phi} \Bigl ( \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \Bigr ) \boldsymbol{e}_r - \Bigl ( \frac{\partial ^2}{\partial z ^2} E_\phi + \frac{\partial}{\partial r} \Bigl ( \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \Bigr ) \Bigr ) \boldsymbol{e}_\phi \ &= - \Bigl ( \frac{\partial ^2}{\partial z ^2} E_\phi + \frac{\partial}{\partial r} \Bigl ( \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \Bigr ) \Bigr ) \boldsymbol{e}_\phi \ &※ E_\phi □ \phi □□□□□□□□1□□□□ \end{aligned} $$

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$$ \begin{aligned} \nabla \times \nabla \times \boldsymbol{E} -k^2 \boldsymbol{E} &= 0 \ -\frac{\partial ^2}{\partial z ^2} E_\phi -\frac{\partial}{\partial r} \Bigl ( \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \Bigr ) -k^2 E_\phi &= 0 \ \end{aligned} $$

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$$ \begin{aligned} \Bigl ( -\frac{\partial ^2 U}{\partial z ^2} -2ik \frac{\partial U}{\partial z} +k^2 U +\frac{1}{r^2} U -\frac{1}{r} \frac{\partial U}{\partial r} -\frac{\partial ^2 U}{\partial r ^2} -k^2 U \Bigr ) e^{ikz} &= 0 \ -\frac{\partial ^2 U}{\partial z ^2} -2ik \frac{\partial U}{\partial z} +\frac{1}{r^2} U -\frac{1}{r} \frac{\partial}{\partial r} \Bigl ( r \frac{\partial U}{\partial r} \Bigl ) &= 0 \ \therefore \quad \frac{1}{r} \frac{\partial}{\partial r} \Bigl ( r \frac{\partial U}{\partial r} \Bigl ) -\frac{1}{r^2} U +2ik \frac{\partial U}{\partial z} +\frac{\partial ^2 U}{\partial z ^2} &= 0 \end{aligned} $$

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$$ \frac{1}{r} \frac{\partial}{\partial r} \Bigl ( r \frac{\partial U}{\partial r} \Bigl ) - \frac{1}{r^2} U + 2ik \frac{\partial U}{\partial z} = 0 $$

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# Gouy□□□□

# Vector Beam□□□□□

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# Vector Beam□□□□□

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$$ \boldsymbol{E} = \begin{bmatrix} E_{x0} e^{i\varphi_x} \ E_{y0} e^{i\varphi_y} \end{bmatrix} $$

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$\alpha$ □□□□□HWP□□□□□□□□□

$$ J_{HWP(\alpha)} = \begin{bmatrix} \cos{2\alpha} & \sin{2\alpha} \ \sin{2\alpha} & -\cos{2\alpha} \end{bmatrix} $$

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$$ J_{PBS} = \begin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} $$

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$$ \vec{a} _{in} \vec{b} _{out} $$

$$ \vec{a} \stackrel{\text{in}}{\longrightarrow} \vec{b} \stackrel{\text{out}}{\longrightarrow} $$

$$ \vec{a} _{\text{in}} \vec{b} _{\text{out}} $$

$$ \vec{a} _{\mathrm{in}} \vec{b} _{\mathrm{out}} $$

$$ \vec{a} _{\text{in}} \quad \vec{b} _{\text{out}} $$

$\vec{a} _{\text{in}}$ $\vec{b} _{\text{out}}$

$$\begin{aligned} \boldsymbol{E}{out1} &= J{PBS} J_{HWP(\alpha)} \boldsymbol{E}_{in} \ &= \begin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos{2\alpha} & \sin{2\alpha} \ \sin{2\alpha} & -\cos{2\alpha} \end{bmatrix} \begin{bmatrix} E_{x0} e^{i\varphi_x} \ E_{y0} e^{i\varphi_y} \end{bmatrix} \ &= \begin{bmatrix} \cos{2\alpha} & \sin{2\alpha} \ 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x0} e^{i\varphi_x} \ E_{y0} e^{i\varphi_y} \end{bmatrix} \ \therefore \boldsymbol{E}_{out1}&= \begin{bmatrix} E_{x0} e^{i\varphi_x} \cos{2\alpha} + E_{y0} e^{i\varphi_y} \sin{2\alpha} \ 0 \end{bmatrix} \end{aligned}$$

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$$ \boldsymbol{E}_{out1} = \begin{bmatrix} 1 \ 0 \end{bmatrix} $$

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□,SLM1□□□□□BS□□□□□□□□□

$$ J_{SLM} = A_0 \begin{bmatrix} -e^{i\delta} & 0 \ 0 & 1 \end{bmatrix} $$

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$$ J_{SLM1} = \begin{bmatrix} -e^{i\delta_1} & 0 \ 0 & 1 \end{bmatrix} $$

$\delta_1 = p\varphi + \delta_{10} \quad (p \in \mathbb{Z})$

$$ J_{SLM2} = \begin{bmatrix} -e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} $$

$\delta_2 = q\varphi + \delta_{20} \quad (q \in \mathbb{Z})$

- $\varphi:$ □□□(azimuthal angle)
- $\delta_{10}, \delta_{20}:$ constant phase

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$$ \begin{aligned} \boldsymbol{E} _{out2} &= \begin{bmatrix} -e^{i\delta_1} & 0 \ 0 & 1 \end{bmatrix} \boldsymbol{E} _{out1} \ \boldsymbol{E} _{out2} &= \begin{bmatrix} -e^{i\delta_1} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \ \therefore \boldsymbol{E} _{out2} &= \begin{bmatrix} -e^{i\delta_1} \ 0 \end{bmatrix} \end{aligned} $$

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$$ \boldsymbol{E}_{out2} = \begin{bmatrix} 1 \ 0 \end{bmatrix} $$

SLM1□□□□□□□□□□□

□, $\beta$ □□□HWP□□□□□□SLM2□□□□□□□□□□□□□□□HWP□□□□□□BS□□□□□□□□□□□□□

$$ \begin{aligned} \boldsymbol{E} *{out3} &= J*{HWP(-\beta)} J_{SLM2} J_{HWP(\beta)} \boldsymbol{E} _{out1}\ &= \begin{bmatrix} \cos{2\beta} & -\sin{2\beta} \ -\sin{2\beta} & -\cos{2\beta} \end{bmatrix} \begin{bmatrix} -e^{i \delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{2\beta} & \sin{2\beta} \ \sin{2\beta} & -\cos{2\beta} \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \ &= \begin{bmatrix} \cos{2\beta} & -\sin{2\beta} \ -\sin{2\beta} & -\cos{2\beta} \end{bmatrix} \begin{bmatrix} -e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{2\beta} \ \sin{2\beta} \end{bmatrix} \ &= \begin{bmatrix} \cos{2\beta} & \sin{2\beta} \ \sin{2\beta} & -\cos{2\beta} \end{bmatrix} \begin{bmatrix} -e^{i\delta_2}\cos{2\beta} \ \sin{2\beta} \end{bmatrix} \ \therefore \boldsymbol{E} _{out3} &= \begin{bmatrix} -e^{i\delta_2} \cos^2{2\beta} + \sin^2{2\beta} \ -\frac{e^{i\delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2} \end{bmatrix} \end{aligned} $$

□□SLM□□□□□□□□□□□□□□□□□□□BS□□□□□□□□□□□□□

$$ \begin{aligned} \boldsymbol{E} _{out4} &= \boldsymbol{E} _{out2} + \boldsymbol{E} _{out3} \ &= \begin{bmatrix} 1 \ 0 \end{bmatrix} + \begin{bmatrix} -e^{-i\delta_2} \cos^2{2\beta} - \sin^2{2\beta} \ -\frac{e^{-i \delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2} \end{bmatrix} \ \therefore \boldsymbol{E} _{out4} &= \begin{bmatrix} 1 -e^{-i\delta_2} \cos^2{2\beta} + \sin^2{2\beta}\ -\frac{e^{-i\delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2} \end{bmatrix} \end{aligned} $$

□,□□□□□□□$\gamma$ □□□QWP□□□□□□□□□□□□

$$ \begin{aligned} \boldsymbol{E} *{out} &= J*{QWP(\gamma)} \boldsymbol{E}_{out4}\ &= \begin{bmatrix} i \sin^2{\gamma} + \cos^2{\gamma} & \sin{\gamma} \cos{\gamma} (1 - i) \ \sin{\gamma} \cos{\gamma} (1 - i) & \sin^2{\gamma} + i \cos^2{\gamma} \end{bmatrix} \begin{bmatrix} 1 -e^{-i\delta_2} \cos^2{2\beta} + \sin^2{2\beta} \ -\frac{e^{-i\delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2} \end{bmatrix} \ &= \begin{bmatrix} (1 -e^{-i\delta_2} \cos^2{2\beta} + \sin^2{2\beta}) (i \sin^2{\gamma} + \cos^2{\gamma}) + (-\frac{e^{-i\delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2}) (\sin{\gamma} \cos{\gamma} (1 - i)) \ (1 -e^{-i\delta_2} \cos^2{2\beta} + \sin^2{2\beta}) (\sin{\gamma} \cos{\gamma} (1 - i)) + (-\frac{e^{-i\delta_2}}{2} \sin{4\beta} - \frac{\sin{4\beta}}{2}) (\sin^2{\gamma} + i \cos^2{\gamma}) \end{bmatrix} \end{aligned} $$

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$$ \boldsymbol{a} = e^{i\theta} \begin{bmatrix} \cos{\varphi} \ \sin{\varphi} \end{bmatrix} $$

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□□

$$ \boldsymbol{b} = e^{i\theta} \begin{bmatrix} -\sin{\varphi} \ \cos{\varphi} \end{bmatrix} $$

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$$ \begin{aligned} Re[(1 -e^{i\delta_2} \cos^2{2\beta} - \sin^2{2\beta}) (i \sin^2{\gamma} + \cos^2{\gamma}) + \frac{\sin4\beta}{2}(-e^{i\delta_2} + 1) (\sin{\gamma} \cos{\gamma} (1 - i))] &= \cos{\varphi} \ Re[(1 -e^{i\delta_2} \cos^2{2\beta} - \sin^2{2\beta}) (\sin{\gamma} \cos{\gamma} (1 - i)) + \frac{\sin{4\beta}}{2}(-e^{i\delta_2} + 1) (\sin^2{\gamma} + i \cos^2{\gamma})] &= \sin{\varphi} \end{aligned} $$

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$$ \delta_2 = 2\varphi - \frac{\pi}{2} $$

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$$ Re[(1 - \sin{2\varphi} \cos^2{2\beta} - \sin^2{2\beta} -i\cos{2\varphi} \cos^2{2\beta}) (i\sin^2{\gamma} + \cos^2{\gamma}) + \frac{\sin{4\beta}}{2} (1 - \sin{2\varphi} - i\cos{2\varphi})] $$

[□□] □□SLM2□□□□□□□□□□□□□□HWP□1□□□□□□□□□□□□□□

$$ \boldsymbol{E}{out} = J{QWP(\gamma)} [J_{SLM1} + J_{SLM2} J_{HWP(\beta)}] J_{PBS} J_{HWP(\alpha)} \boldsymbol{E}_{in} $$

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$$ \begin{aligned} \boldsymbol{E}{out3} &= J{SLM2} J_{HWP(\beta)} \boldsymbol{E}_{out1}\ &= \begin{bmatrix} -e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{2\beta} & \sin{2\beta} \ \sin{2\beta} & -\cos{2\beta} \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \ &= \begin{bmatrix} -e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{2\beta} \ \sin{2\beta} \end{bmatrix} \ \therefore \boldsymbol{E}_{out3} &= \begin{bmatrix} -e^{i\delta_2} \cos{2\beta} \ \sin{2\beta} \end{bmatrix} \end{aligned} $$

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$$ \begin{aligned} \boldsymbol{E}{out4} &= \boldsymbol{E}{out2} + \boldsymbol{E}{out3} \ &= \begin{bmatrix} 1 \ 0 \end{bmatrix} + \begin{bmatrix} -e^{-i\delta_2} \cos{2\beta} \ \sin{2\beta} \end{bmatrix} \ \therefore \boldsymbol{E}{out4} &= \begin{bmatrix} 1 -e^{-i\delta_2} \cos{2\beta}\ \sin{2\beta} \end{bmatrix} \end{aligned} $$

$$ \begin{aligned} \boldsymbol{E}{out} &= J{QWP(\gamma)} \boldsymbol{E}{out4}\ &= \begin{bmatrix} i \sin^2{\gamma} + \cos^2{\gamma} & \sin{\gamma} \cos{\gamma} (1 - i) \ \sin{\gamma} \cos{\gamma} (1 - i) & \sin^2{\gamma} + i \cos^2{\gamma} \end{bmatrix} \begin{bmatrix} 1 -e^{-i\delta_2} \cos{2\beta}\ \sin{2\beta} \end{bmatrix} \ \therefore \boldsymbol{E}{out} &= \begin{bmatrix} (1 -e^{-i\delta_2} \cos{2\beta}) (i \sin^2{\gamma} + \cos^2{\gamma}) + \sin{2\beta} (\sin{\gamma} \cos{\gamma} (1 - i)) \ (1 -e^{-i\delta_2} \cos{2\beta}) (\sin{\gamma} \cos{\gamma} (1 - i)) +\sin{2\beta} (\sin^2{\gamma} + i \cos^2{\gamma}) \end{bmatrix} \end{aligned} $$

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$$ \delta_2 = 2\varphi + \theta $$

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$$ \begin{aligned} \therefore \boldsymbol{E}_{out} &= \begin{bmatrix} (1 -\cos{(2\varphi + \theta)} -i\sin{(2\varphi + \theta)} \cos{2\beta}) (i \sin^2{\gamma} + \cos^2{\gamma}) + \sin{2\beta} (\sin{\gamma} \cos{\gamma} (1 - i)) \ (1 - \cos{(2\varphi + \theta)} -i\sin{(2\varphi + \theta)} \cos{2\beta}) (\sin{\gamma} \cos{\gamma} (1 - i)) +\sin{2\beta} (\sin^2{\gamma} + i \cos^2{\gamma}) \end{bmatrix} \ &= \begin{bmatrix} (1 - \cos{2\varphi} \cos{\theta} + \sin{2\varphi} \sin{\theta} -i \cos{2\beta} (\sin{2\varphi} \cos{\theta} + \cos{2\varphi} \sin{\theta})) (i \sin^2{\gamma}

+ \cos^2{\gamma}) + \sin{2\beta} (\sin{\gamma} \cos{\gamma} (1 - i)) \ (1 - \cos{2\varphi} \cos{\theta} + \sin{2\varphi} \sin{\theta} -i \cos{2\beta} (\sin{2\varphi} \cos{\theta} + \cos{2\varphi} \sin{\theta})) (\sin{\gamma} \cos{\gamma} (1 - i)) +\sin{2\beta} (\sin^2{\gamma} + i \cos^2{\gamma}) \end{bmatrix} \ \end{aligned} $$

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$$ \boldsymbol{E}{out} = J{QWP(\gamma)} J_{HWP(-\beta)} J_{SLM2(\delta_2)} J_{HWP(\beta)} J_{SLM1(\delta_1)}J_{PBS} J_{HWP(\alpha)} \boldsymbol{E}_{in} $$

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$$ \boldsymbol{E}{out} = J{QWP(\frac{\pi}{4})} (J_{SLM1} \boldsymbol{E}_{1} + J_{HWP(-\frac{\pi}{4})} J_{SLM2} J_{HWP(\frac{\pi}{4})} \boldsymbol{E}_{2}) \ (\boldsymbol{E}_{in} = \boldsymbol{E}_{1} + \boldsymbol{E}_{2}) $$

□□ $PBS$ □□□□

$$ \boldsymbol { E } _ { in } = \boldsymbol {E} _ {1} + \boldsymbol {E} _ {2} = \begin{bmatrix} 1 \ 0 \end{bmatrix} + \begin{bmatrix} 0 \ 1 \end{bmatrix} $$

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$$ \begin{aligned} \boldsymbol {E} _ {out} &= J_{QWP(\frac{\pi}{4})} (J_{SLM1} \begin{bmatrix} 1 \ 0 \end{bmatrix} +J_{HWP(-\frac{\pi}{4})} J_{SLM2} J_{HWP(\frac{\pi}{4})} \begin{bmatrix} 0 \ 1 \end{bmatrix} ) \ &= \begin{bmatrix} 1 & -i \ -i & 1 \end{bmatrix} \biggl( \begin{bmatrix} e^{i\delta_1} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} +\begin{bmatrix} 0 & -1 \ -1 & 0 \end{bmatrix} \begin{bmatrix} e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \ 1 \end{bmatrix} \biggr) \ &= \begin{bmatrix} 1 & -i \ -i & 1 \end{bmatrix} \biggl( \begin{bmatrix} e^{i\delta_1} \ 0 \end{bmatrix} +\begin{bmatrix} 0 & -1 \ -1 & 0 \end{bmatrix} \begin{bmatrix} e^{i\delta_2} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \biggr) \ &= \begin{bmatrix} 1 & -i \ -i & 1 \end{bmatrix} \biggl( \begin{bmatrix} e^{i\delta_1} \ 0 \end{bmatrix} +\begin{bmatrix} 0 & -1 \ -1 & -0 \end{bmatrix} \begin{bmatrix} e^{i\delta_2} \ 0 \end{bmatrix} \biggr) \ &= \begin{bmatrix} 1 & -i \ -i & 1 \end{bmatrix} \biggl( e^{i\delta_1} \begin{bmatrix} 1 \ 0 \end{bmatrix} +e^{i \delta_2} \begin{bmatrix} 0 \ 1 \end{bmatrix} \biggr) \ &= e^{i\delta_1} \begin{bmatrix} 1 \ -i \end{bmatrix} +e^{i \delta_2} \begin{bmatrix} -i \ 1 \end{bmatrix} \ \therefore \boldsymbol{E}_{out} &= e^{i\delta_1} \begin{bmatrix} 1 \ -i \end{bmatrix} +e^{i\delta_2} \begin{bmatrix} 1 \ i \end{bmatrix} \end{aligned} $$

$$ J_{SLM1} \begin{bmatrix} 1 \ 0 \end{bmatrix} = \begin{bmatrix} \cos{\phi} + i \sin{\phi} \ 0 \end{bmatrix} \ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 + i \ 0 \end{bmatrix} \ \begin{bmatrix} i \ 0 \end{bmatrix} $$

$$ J_{SLM2} \begin{bmatrix} 1 \ 0 \end{bmatrix} = \begin{bmatrix} \cos{\phi} - i \sin{\phi} \ 0 \end{bmatrix} $$

□ $x, y$ □□□□□□□□□□□

$$ \begin{bmatrix} 0 \ \cos{\phi} - i \sin{\phi} \end{bmatrix} $$

$$ 0 \ \begin{bmatrix} 0 \ 1 \end{bmatrix} \ 45 \ \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \ 1 + i \end{bmatrix} \ 90 \ \begin{bmatrix} 0 \ i \end{bmatrix} $$

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$$ \begin{bmatrix} \cos{\phi} + i \sin{\phi} \ \cos{\phi} - i \sin{\phi} \end{bmatrix} $$

□□□QWP□□□□□□□

$$ \begin{bmatrix} 1 & -i \ -i & 1 \end{bmatrix} \begin{bmatrix} \cos{\phi} + i \sin{\phi} \ \cos{\phi} - i \sin{\phi} \end{bmatrix} = \begin{bmatrix} \cos{\phi} - \sin{\phi} + i(\sin{\phi} - \cos{\phi}) \ \cos{\phi} + \sin{\phi} - i(\cos{\phi} + \sin{\phi}) \end{bmatrix} \begin{bmatrix} \cos{\phi} - \sin{\phi} \ \cos{\phi} + \sin{\phi} \end{bmatrix} \ \begin{bmatrix} \cos{\bigl (\phi + \frac{\pi}{4} \bigr )} \ \cos{\bigl ( \phi - \frac{\pi}{4} \bigr )} \end{bmatrix} \ \begin{bmatrix} \cos{\phi} \ \sin{\phi} \end{bmatrix} $$

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$$ \begin{aligned} \delta_{1} &= \phi + \phi_{0} \\ \delta_{2} &= -(\phi + \phi_{0}) \end{aligned} $$

□□□□□SLM□□□□□□□□□□□□□

$$ \begin{aligned} \delta_1 &= \phi \\ \delta_2 &= -\phi \end{aligned} $$

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$$ \begin{bmatrix} e^{i \phi} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} $$

$$ \begin{aligned} \boldsymbol{E}_{out} &= e^{i\phi} \begin{bmatrix} 1 \\ -i \end{bmatrix} +e^{-i\phi} \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= (\cos{\phi} + i\sin{\phi}) \begin{bmatrix} 1 \\ -i \end{bmatrix} +(\cos{\phi} - i\sin{\phi}) \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= \begin{bmatrix} \cos{\phi} + i\sin{\phi} \\ \sin{\phi} - i\cos{\phi} \end{bmatrix}+ \begin{bmatrix} \cos{\phi} - i\sin{\phi} \\ \sin{\phi} + i\cos{\phi} \end{bmatrix} \\ &= \begin{bmatrix} 2\cos{\phi} \\ 2\sin{\phi} \end{bmatrix} \\ \therefore \boldsymbol{E}_{out} &\propto \begin{bmatrix} \cos{\phi} \\ \sin{\phi} \end{bmatrix} \end{aligned} $$

□□□□ $\phi$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

SLM1

$$ \begin{bmatrix} \cos{\phi} \cos{t} - \sin{\phi} \sin{t} \\ \sin{\phi} \cos{t} + \cos{\phi} \sin{t} \end{bmatrix} $$

$(\cos{\phi}, \sin{\phi})$ □□□□□□□ $\boldsymbol{E}_{out}$ □□□□□□□

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$$ \begin{aligned} \delta_1 &= \phi + \frac{\pi}{2}\\ \delta_2 &= -(\phi + \frac{\pi}{2}) \end{aligned} $$

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$$ \begin{aligned} \boldsymbol{E}_{out} &= e^{i(\phi + \frac{\pi}{2})} \begin{bmatrix} 1 \\ -i \end{bmatrix} +e^{-i(\phi + \frac{\pi}{2})} \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= \Bigl( \cos{(\phi + \frac{\pi}{2})} + i\sin{(\phi + \frac{\pi}{2})} \Bigr) \begin{bmatrix} 1 \\ -i \end{bmatrix} +\Bigl(\cos{(\phi + \frac{\pi}{2})} - i\sin{(\phi + \frac{\pi}{2})} \Bigr) \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= \Bigl( -\sin{\phi} + i\cos{\phi} \Bigr) \begin{bmatrix} 1 \\ -i \end{bmatrix} + \Bigl(-\sin{\phi} - i\cos{\phi} \Bigr) \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= \begin{bmatrix} -\sin{\phi} + i\cos{\phi} \\ \cos{\phi} + i\sin{\phi} \end{bmatrix}+ \begin{bmatrix} -\sin{\phi} - i\cos{\phi} \\ \cos{\phi} - i\sin{\phi} \end{bmatrix} \\ &= \begin{bmatrix} -2\sin{\phi} \\ 2\cos{\phi} \end{bmatrix} \\ \therefore \boldsymbol{E}_{out} &\propto \begin{bmatrix} \sin{\phi} \\ -\cos{\phi} \end{bmatrix} \end{aligned} $$

□□□□ $\phi$ □□□□□□□□□□□□□□□□□□□□

$$ \begin{bmatrix} \cos{\phi} \\ \sin{\phi} \end{bmatrix} \cdot \begin{bmatrix} \sin{\phi} \\ -\cos{\phi} \end{bmatrix} = 0 $$

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$(\cos{\phi}, \sin{\phi})$ □□□□□□□ $\boldsymbol{E}_{out}$ □□□□□□□

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$$ \begin{aligned} \delta_1 &= 2\phi \\ \delta_2 &= -2\phi \end{aligned} $$

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$$ \begin{aligned} \boldsymbol{E}_{out} &= e^{i2\phi} \begin{bmatrix} 1 \\ -i \end{bmatrix} +e^{-i2\phi} \begin{bmatrix} 1 \\ i \end{bmatrix} \\ &= (\cos{2\phi} + i\sin{2\phi}) \begin{bmatrix} 1 \\ -i \end{bmatrix} +(\cos{2\phi} -

i\sin{2\phi}) \begin{bmatrix} 1 \ i \end{bmatrix} \ &= \begin{bmatrix} \cos{2\phi} + i\sin{2\phi} \ \sin{2\phi} - i \cos{2\phi} \end{bmatrix}+ \begin{bmatrix} \cos{2\phi} - i\sin{2\phi} \ \sin{2\phi} + i\cos{2\phi} \end{bmatrix} \ &= \begin{bmatrix} 2\cos{2\phi} \ 2\sin{2\phi} \end{bmatrix} \ \therefore \boldsymbol{E}{out} &\propto \begin{bmatrix} \cos{2\phi} \ \sin{2\phi} \end{bmatrix} \end{aligned} $$

$(\cos{\phi}, \sin{\phi})$ □□□□□□□$\boldsymbol{E}_{out}$ □□□□□□□□□

# □□4

$$ \begin{aligned} \boldsymbol{E}{out} &= \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{i \beta} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{i \alpha} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \ &= \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{i \beta} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{i \alpha} \ 0 \end{bmatrix} \ &= \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{i \beta} & 0 \ 0 & 1 \end{bmatrix} e^{i \alpha} \begin{bmatrix} 1 \ 1 \end{bmatrix} \ &= e^{i \alpha} \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{i \beta} \ 1 \end{bmatrix} \ &= e^{i \alpha} \begin{bmatrix} e^{i \frac{\pi}{4}} e^{i \beta} + e^{-i \frac{\pi}{4}} \ e^{-i \frac{\pi}{4}} e^{i \beta} + e^{i \frac{\pi}{4}} \end{bmatrix} \ &= e^{i \alpha} \begin{bmatrix} e^{i \frac{\pi}{2}} e^{i \beta} + 1 \ e^{i \beta} + e^{i \frac{\pi}{2}} \end{bmatrix} \ \therefore \boldsymbol{E}{out} &= e^{i \alpha} \begin{bmatrix} i \cos{\beta} - \sin{\beta} + 1 \ \cos{\beta} + i \sin{\beta} + i \end{bmatrix} \end{aligned} $$

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# SLM2□□□□□□□□□□□□□□

## "Polarization distribution control of parallel femtosecond pulses with spatial light modulators"

$$ \begin{aligned} \boldsymbol{E}{out} &= J{QWP(\frac{\pi}{4})} J_{SLM2(\beta)} J_{HWP(\frac{\pi}{8})} J_{SLM1(\alpha)} \boldsymbol{E}_{in} \ &\propto \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{-i\beta} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \ 0 \end{bmatrix} \ &= \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{-i\beta} & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \ 1 \end{bmatrix} \ &= \begin{bmatrix} 1+i & 1-i \ 1-i & 1+i \end{bmatrix} \begin{bmatrix} e^{-i\beta} \ 1 \end{bmatrix} \ &= \begin{bmatrix} e^{-i\beta}(1+i) + 1-i \ e^{-i\beta}(1-i) + 1+i \end{bmatrix} \ &= \begin{bmatrix} \cos{\beta} + \sin{\beta} + 1 \ \cos{\beta} - \sin{\beta} + 1 \end{bmatrix} +i \begin{bmatrix} \cos{\beta} - \sin{\beta} - 1 \ -(\cos{\beta} + \sin{\beta} - 1) \end{bmatrix} \ \therefore \boldsymbol{E}_{out} &= \begin{bmatrix} \cos{\frac{\beta}{2}} + \sin{\frac{\beta}{2}} \ \cos{\frac{\beta}{2}} - \sin{\frac{\beta}{2}} \end{bmatrix} +i \begin{bmatrix} \cos{\frac{\beta}{2}} - \sin{\frac{\beta}{2}} - 2 \ -(\cos{\frac{\beta}{2}} + \sin{\frac{\beta}{2}} - 2) \end{bmatrix} \ \therefore Re(\boldsymbol{E}_{out}) &= \begin{bmatrix} \cos{\frac{\beta}{2}} + \sin{\frac{\beta}{2}} \ \cos{\frac{\beta}{2}} - \sin{\frac{\beta}{2}} \end{bmatrix} \end{aligned} $$

$$ \beta = 2\varphi + \theta $$

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$$ \begin{aligned} \boldsymbol{E}{out} &\propto \begin{bmatrix} \cos{(\frac{2\varphi + \theta}{2})} + \sin{(\frac{2\varphi + \theta}{2})} \ \cos{(\frac{2\varphi + \theta}{2})} - \sin{(\frac{2\varphi + \theta}{2})} \end{bmatrix} \ &= \begin{bmatrix} \cos{(\varphi + \frac{\theta}{2})} + \sin{(\varphi + \frac{\theta}{2})} \ \cos{(\varphi + \frac{\theta}{2})} - \sin{(\varphi + \frac{\theta}{2})} \end{bmatrix} \ \therefore \boldsymbol{E}{out} &\propto \begin{bmatrix} \cos{\varphi}(\cos{\frac{\theta}{2}} + \sin{\frac{\theta}{2}}) + \sin{\varphi}(\cos{\frac{\theta}{2}} - \sin{\frac{\theta}{2}}) \ \cos{\varphi}(\cos{\frac{\theta}{2}} - \sin{\frac{\theta}{2}}) - \sin{\varphi}(\cos{\frac{\theta}{2}} + \sin{\frac{\theta}{2}}) \end{bmatrix} \end{aligned} $$

□□$\theta = -\frac{\pi}{2}$□□

$$ \begin{aligned} \boldsymbol{E}_{out} &\propto \begin{bmatrix} \cos{\varphi}(\cos{\frac{\pi}{4}} - \sin{\frac{\pi}{4}}) + \sin{\varphi}(\cos{\frac{\pi}{4}} + \sin{\frac{\pi}{4}}) \ \cos{\varphi}(\cos{\frac{\pi}{4}} + \sin{\frac{\pi}{4}}) - \sin{\varphi}(\cos{\frac{\pi}{4}} - \sin{\frac{\pi}{4}}) \end{bmatrix} \ &\propto \begin{bmatrix} \sin{\varphi} \ \cos{\varphi} \end{bmatrix} \end{aligned} $$

## "Holographic femtosecond laser manipulation for advanced material processing"

$$ \boldsymbol{E}_{out} \propto \begin{bmatrix} -i\cos{\beta} + \sin{\beta} + 1 \ -\cos{\beta} -i\sin{\beta} + i \end{bmatrix} $$

$\beta = \varphi + \theta$□□□□

$$ \begin{aligned} \boldsymbol{E}_{out} &\propto \begin{bmatrix} -i\cos{(\varphi + \theta)} + \sin{(\varphi + \theta)} + 1 \ -\cos{(\varphi + \theta)} -i\sin{(\varphi + \theta)} + i \end{bmatrix} \ &\propto \begin{bmatrix} -i(\cos{\varphi} \cos{\theta} -\sin{\varphi} \sin{\theta}) + \sin{\varphi} \cos{\theta} + \cos{\varphi} \sin{\theta} + 1 \ -(\cos{\varphi} \cos{\theta} -\sin{\varphi} \sin{\theta}) -i(\sin{\varphi} \cos{\theta} +\cos{\varphi} \sin{\theta})+ i \end{bmatrix} \end{aligned} $$

$\beta = 2\varphi + \theta$□□□□

$$ \begin{aligned} \boldsymbol{E}_{out} &\propto \begin{bmatrix} -i\cos{(2\varphi + \theta)} + \sin{(2\varphi + \theta)} + 1 \ -\cos{(2\varphi + \theta)} -i\sin{(2\varphi + \theta)} + i \end{bmatrix} \ &\propto \begin{bmatrix} -i(\cos{2\varphi} \cos{\theta} -\sin{2\varphi} \sin{\theta}) + \sin{2\varphi} \cos{\theta} + \cos{2\varphi} \sin{\theta} + 1 \ -(\cos{2\varphi} \cos{\theta} -\sin{2\varphi} \sin{\theta}) -i(\sin{2\varphi} \cos{\theta} +\cos{2\varphi} \sin{\theta})+ i \end{bmatrix} \end{aligned} $$

□□□$\theta = \frac{\pi}{2}$□□□□

$$ \boldsymbol{E}_{out} \propto \begin{bmatrix} i\sin{2\varphi} + \cos{2\varphi} +1 \ \sin{2\varphi} - i\cos{2\varphi} + i \end{bmatrix} $$

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## "Flexible generation of the generalized vector vortex beams"(2021)

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$$ \boldsymbol{E} = e^{iA\varphi}[ \cos{2\alpha} e^{-i(B\varphi+C)} \boldsymbol{S_1}(2\gamma, 2(2\beta-\gamma)) +\sin{2\alpha} e^{i(B\varphi+C)} \boldsymbol{S_2}(2\gamma+\pi, -2(2\beta-\gamma))] $$

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$$\boldsymbol{S}(2\psi, 2\chi) = \begin{bmatrix} \frac{1}{\sqrt{2}}[\sin{(\chi + \frac{\pi}{4})} e^{-i\psi} + \cos{(\chi + \frac{\pi}{4}) e^{i\psi}}] \ \frac{i}{\sqrt{2}}[\sin{(\chi + \frac{\pi}{4})} e^{-i\psi} - \cos{(\chi + \frac{\pi}{4}) e^{i \psi}}] \end{bmatrix} $$

SLM□□□□□□□□□□□□

- SLM1: $\delta_1 = p\varphi + \delta_{10}$
- SLM2: $\delta_2 = q\varphi + \delta_{20}$

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- $\alpha$:1□□□□□□□□□□□□□
- $\beta$:2□□□□□□□□□□□□□
- $\gamma$:4□□1□□□□□□□□□
- $\boldsymbol{S_i}(i \in 1, 2)$□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
- $A = p + \frac{q}{2}$
- $B = -\frac{q}{2}$
- $C = -\frac{2\delta_{20} + \pi}{4}$

□□□ $\alpha = \frac{\pi}{8}, \beta = 0, \gamma = -\frac{\pi}{4}$ □□□□

$$ \boldsymbol{S_1}(-\frac{\pi}{2}, \frac{\pi}{2}) \propto \begin{bmatrix} 1 \ -i \end{bmatrix}(□□□□□□) $$

$$ \boldsymbol{S_2}(\frac{\pi}{2}, -\frac{\pi}{2}) \propto \begin{bmatrix} 1 \\ i \end{bmatrix}(□□□□□□) $$

□□□□ □□□ $p = 1, q = 2, \delta_{20} = -\frac{\pi}{2}$ □□□□ $A = 2, B = -1, C = 0$ □□□□□□□□□□

$$ \boldsymbol{E} = \sqrt{2} e^{i2\varphi} \begin{bmatrix} \cos{\varphi} \\ \sin{\varphi} \end{bmatrix} $$

$$ Re(\boldsymbol{E}) = \sqrt{2} \cos{2\varphi} \begin{bmatrix} \cos{\varphi}\\ \sin{\varphi} \end{bmatrix} \propto \begin{bmatrix} \cos{\varphi}\\ \sin{\varphi} \end{bmatrix} $$

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□□□□□□□ $\varphi$ □□□□□□□□□□

$$ \begin{aligned} Re(\boldsymbol{E_{\varphi=0}}) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} (□□□□)\\ Re(\boldsymbol{E_{\varphi=\frac{\pi}{4}}}) &\propto \begin{bmatrix} 1 \\ 1 \end{bmatrix} (\frac{\pi}{4}□□□□□□□)\\ Re(\boldsymbol{E_{\varphi=\frac{\pi}{2}}}) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} (□□□□) \\ \end{aligned} $$

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□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ $\alpha = \frac{\pi}{8}, \beta = -\frac{\pi}{4}, \gamma = -\frac{3\pi}{16}$

$$ \begin{aligned} \boldsymbol{S_1}(-\frac{\pi}{2}, -\frac{\pi}{4}) &\propto \begin{bmatrix} \sin{\frac{\pi}{8} e^{i\frac{\pi}{4}}} + \cos{\frac{\pi}{8}} e^{-i\frac{\pi}{4}} \\ i[\sin{\frac{\pi}{8} e^{i\frac{\pi}{4}}} - \cos{\frac{\pi}{8}} e^{-i\frac{\pi}{4}}] \end{bmatrix} \\ \boldsymbol{S_2}(\frac{\pi}{2}, \frac{\pi}{4}) &\propto \begin{bmatrix} \sin{\frac{3\pi}{8} e^{-i\frac{\pi}{4}}} + \cos{\frac{3\pi}{8}} e^{i\frac{\pi}{4}} \\ i[\sin{\frac{3\pi}{8} e^{-i\frac{\pi}{4}}} - \cos{\frac{3\pi}{8}} e^{i\frac{\pi}{4}}] \end{bmatrix} \end{aligned} $$

□□□□□□□□□□□ $p = 1, q = 2, \delta_{20} = -\frac{\pi}{2}$ □□ $A = 2, B = -1, C = 0$ □□□

$$ \boldsymbol{E} \propto \begin{bmatrix} e^{i\varphi} [\sin{\frac{\pi}{8} e^{i\frac{\pi}{4}}} + \cos{\frac{\pi}{8}} e^{-i\frac{\pi}{4}}] + e^{-i\varphi} [\sin{\frac{3\pi}{8} e^{-i\frac{\pi}{4}}} + \cos{\frac{3\pi}{8}} e^{i\frac{\pi}{4}}] \\ i (e^{i\varphi} [\sin{\frac{\pi}{8} e^{i\frac{\pi}{4}}} - \cos{\frac{\pi}{8}} e^{-i\frac{\pi}{4}}] + e^{-i\varphi} [\sin{\frac{3\pi}{8} e^{-i\frac{\pi}{4}}} - \cos{\frac{3\pi}{8}} e^{i\frac{\pi}{4}}]) \end{bmatrix} $$

$$ \begin{aligned} \boldsymbol{E}_{\varphi = \frac{\pi}{4}} &\propto \begin{bmatrix} e^{i\frac{\pi}{2}} \sin{\frac{\pi}{8}} + e^{-\frac{i\pi}{2}} \sin{\frac{3\pi}{8}} + \cos{\frac{\pi}{8}} + \cos{\frac{3\pi}{8}}\\ i[e^{i\frac{\pi}{2}} \sin{\frac{\pi}{8}} + e^{-\frac{i\pi}{2}} \sin{\frac{3\pi}{8}} - \cos{\frac{\pi}{8}} - \cos{\frac{3\pi}{8}}] \end{bmatrix} \\ &\propto \begin{bmatrix} \cos{\frac{\pi}{8}} + \cos{\frac{3\pi}{8}} + i[\sin{\frac{\pi}{8}} - \sin{\frac{3\pi}{8}}]\\ \sin{\frac{3\pi}{8}} - \sin{\frac{\pi}{8}} - i[\cos{\frac{\pi}{8}} + \cos{\frac{3\pi}{8}}] \end{bmatrix} \\ &\propto \begin{bmatrix} 1.3 - i0.54 \\ 0.54 - i1.3 \end{bmatrix} \end{aligned} $$

□□□□□□ $e^{i\theta}$ □□□□□□□□□□ $\theta$□ $0\rightarrow 2\pi$ □□□□□□□□□□□□□□□□□□□□

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□□□□□□ $\varphi = \frac{\pi}{4}$ □□□□□□□□□□□□□□□□□□□

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## "Machine learning-based classification of vector vortex beams"

[https://blog.tsurubee.tech/entry/2022/enns](https://blog.tsurubee.tech/entry/2022/enns)-overview#%E5%8C%BB%E7%99%82%E7%94%BB%E5%83%8F

[https://github.com/Chen](https://github.com/Chen)-Cai-OSU/awesome-equivariant-network

[https://ds](https://ds)-notes.com/%E5%90%8C%E5%A4%89%E3%83%8B%E3%83%A5%E3%83%BC%E3%83%A9%E3%83%AB%E3%83%8D%E3%83%83%E3%83%88%E3%83%AF%E3%83%BC%E3%82%AF

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**"Observation of optical polarization Möbius strips"**

**"Full Poincare´ beams"**

**"Generation of A Space-Variant Vector Beam with Catenary-Shaped Polarization States"**

## Gouy□□

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**"□□□□□□□□□□□□□□□□□□□□□□□□"**

**"Manifestation of the Gouy phase in strongly focused, radially polarized beams"**

### □□□□□□□□□□□□□□

**"Higher-Order Poincaré Sphere, Stokes Parameters, and the Angular Momentum of Light"**

**"Generalized Poincare sphere"**

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**"Measuring the nonseparability of vector vortex beams"**

**"Beam quality measure for vector beams"**

**"ENTANGLEMENT OF FORMATION AND CONCURRENCE"**

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**"From Maxwell to paraxial wave optics"**

**"Gaussian Beam □□□□□□□□"**

**[http://solidstatephysics.blog.fc2.com/blog](http://solidstatephysics.blog.fc2.com/blog)-entry-47.html**

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□□:**"Separability and Applications"**

**"Vector-beam solutions of Maxwell's wave equation"**

**"Vector Helmholtz–Gauss and vector Laplace–Gauss beams"**

**http://solidstatephysics.blog.fc2.com/blog-entry-31.html**

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**"Nonparaxial Propagation Properties of Specially Correlated Radially Polarized Beams in Free Space"**

**"Closed-form bases for the description of monochromatic, strongly focused, electromagnetic fields"**

**"Measuring the nonseparability of vector vortex beams"**

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**"A practical algorithm for the determination of phase from image and diffraction plane pictures"**

**"Kinoform design with an optimal-rotation-angle method"**

**"New iterative algorithm for the design of phaseonly gratings"**

**"Continuous-relief diffractive optical elements for two-dimensional array generation"**

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**"Cylindrical vector beams: from mathematical concepts to applications"**

**"Vector Beams for Fundamental Physics and Applications"**

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**"Orbital angular momentum: origins, behavior and applications"**

## SLM

**"Creation and detection of optical modes with spatial light modulators"**

$( i^2 = -1)$

$$ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ 1 \end{bmatrix} $$

Alt text