In [1]:
```
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
```

In [2]:
```
df=pd.read_csv("Real estate.csv",index_col=False)
```

In [3]:
```
df=df.drop("No",axis=1)
df
```

Out[3]:

	X1 transaction date	X2 house age	X3 distance to the nearest MRT station	X4 number of convenience stores	X5 latitude	X6 longitude	Y house price of unit area
0	2012.917	32.0	84.87882	10	24.98298	121.54024	37.9
1	2012.917	19.5	306.59470	9	24.98034	121.53951	42.2
2	2013.583	13.3	561.98450	5	24.98746	121.54391	47.3
3	2013.500	13.3	561.98450	5	24.98746	121.54391	54.8
4	2012.833	5.0	390.56840	5	24.97937	121.54245	43.1
...	...	...	...	...	...	...	...
409	2013.000	13.7	4082.01500	0	24.94155	121.50381	15.4
410	2012.667	5.6	90.45606	9	24.97433	121.54310	50.0
411	2013.250	18.8	390.96960	7	24.97923	121.53986	40.6
412	2013.000	8.1	104.81010	5	24.96674	121.54067	52.5
413	2013.500	6.5	90.45606	9	24.97433	121.54310	63.9

414 rows × 7 columns

In [4]:
```
for i in df.columns[:-1]:
    print(i)
```

```
X1 transaction date
X2 house age
X3 distance to the nearest MRT station
X4 number of convenience stores
X5 latitude
X6 longitude
```

In [5]:
```
df.describe()
```

Out[5]:

	X1 transaction date	X2 house age	X3 distance to the nearest MRT station	X4 number of convenience stores	X5 latitude	X6 longitude	Y house price of unit area
count	414.000000	414.000000	414.000000	414.000000	414.000000	414.000000	414.000000

	X1 transaction date	X2 house age	X3 distance to the nearest MRT station	X4 number of convenience stores	X5 latitude	X6 longitude	Y house price of unit area
mean	2013.148971	17.712560	1083.885689	4.094203	24.969030	121.533361	37.980193
std	0.281967	11.392485	1262.109595	2.945562	0.012410	0.015347	13.606488
min	2012.667000	0.000000	23.382840	0.000000	24.932070	121.473530	7.600000
25%	2012.917000	9.025000	289.324800	1.000000	24.963000	121.528085	27.700000
50%	2013.167000	16.100000	492.231300	4.000000	24.971100	121.538630	38.450000
75%	2013.417000	28.150000	1454.279000	6.000000	24.977455	121.543305	46.600000
max	2013.583000	43.800000	6488.021000	10.000000	25.014590	121.566270	117.500000

In [6]:

```
df1=pd.read_csv("Real estate.csv")
```

In [7]:

```
class Linear_Regression():
    def __init__(self,l_rate,Epochs):

        self.l_rate=l_rate
        self.Epochs=Epochs

    def fit(self, X, Y ):

        self.m,self.n = X.shape
        on=np.ones((self.m,1))
        self.X=np.concatenate((on,X),axis=1)
        self.n=self.n+1
        self.Y=Y
        self.W=np.zeros(self.n)
        self.H=np.dot(self.X,self.W)
        self.Cost=np.zeros(self.Epochs)

        for i in range(self.Epochs):
            self.upd_weight()
            self.Cost[i]=np.sum(np.square(self.H-self.Y))/(2*self.m)
        return self

    def upd_weight(self):

        self.W[0]=self.W[0]-(self.l_rate/self.m)*sum(self.H-self.Y)
        for i in range(1,self.n):
            self.W[i]=self.W[i]-(self.l_rate/self.m)*sum((self.H-self.Y)*self.X[:,i])
        self.H=np.dot(self.X,self.W)
        return self

    def W_C(self):

        print(self.W[0])
        print(self.W[1:])

    def predict(self,X):

        return X.dot(self.W[1:])+self.W[0]
```

```
def norm(df1):
    for i in df1.columns[:]:
        df1[i]=df1[i]-df1[i].mean()
        df1[i]=df1[i]/np.sqrt(df1[i].var())
    #return np.array(df1)
    return df1
```

Part a

You will need to perform K-Fold cross-validation (K=2-5) in this exercise (implement from scratch). What is the optimal value of K? Justify it in your report along with the table for the mean accuracy of K-values and K-value.

In [8]:

```
def K_fold_validation(k,df):
    n=len(df.index)
    fold_size=n//k
    if(n%k>0):
        fold_size+=1
    df = df.reindex(np.random.permutation(df.index))
    df = df.reset_index(drop=True)
    df=np.array(df)
    #print(fold_size)
    l=[]
    k=0
    while(k<n):
        l.append(df[k:k+fold_size,:])
        k+=fold_size


#         print("==================================================================

    X_Trains=[]
    Y_Trains=[]
    X_Tests=[]
    Y_Tests=[]
    for i in range(len(l)):
        temp=[]
        for j in range(len(l)):
            if(j!=i):
                temp.append(pd.DataFrame(l[j]))
        t_df=np.array(pd.concat(temp))
        X_Trains.append(t_df[:,:-1])
        Y_Trains.append(t_df[:,-1])
        X_Tests.append(l[i][:,:-1])
        Y_Tests.append(l[i][:,-1])

    return X_Trains,Y_Trains,X_Tests,Y_Tests
```

In [9]:

```
results_mse=[]
# results_mle=[]
for i in range(2,6):
    l=[]
#     l1=[]
```

```
    X_Trains, Y_Trains, X_Tests, Y_Tests = K_fold_validation(i,norm(pd.read_csv("Real e
    for j in range(i):
        model=Linear_Regression(0.01,20000)
        model.fit(X_Trains[j],Y_Trains[j])
        y_pred=model.predict(X_Tests[j])
        difference_array = np.subtract(y_pred, Y_Tests[j])
        squared_array = np.square(difference_array)
        mse = squared_array.mean()
        l.append(mse)
#         l1.append(difference_array.mean())
    results_mse.append(np.sum(l)/len(l))
#     results_mle.append(np.sum(l1)/len(l1))
```

In [10]:

```
print("K| MSE ")
for i in range(2,6):
    print("{}| {}".format(i,results_mse[i-2]))
```

```
K| MSE
2| 0.48138327722866797
3| 0.43327257038518413
4| 0.42797314694324884
5| 0.4320416914330435
```

If we look at Mse k=5 is the best

Part B

Plot the RMSE V/s iteration graph for all models trained with optimal value of K for K-Fold cross-validation. RMSE should be reported on the train and val set.

Choosen K = 5

In [11]:

```
#added the RMSE list for validation as well tranning set to the model

class Linear_Regression1():
    def __init__(self,l_rate,Epochs):

        self.l_rate=l_rate
        self.Epochs=Epochs

    def fit(self, X, Y ,X_test,Y_test):

        self.m,self.n = X.shape
        on=np.ones((self.m,1))
        self.X=np.concatenate((on,X),axis=1)
        self.n=self.n+1
        self.Y=Y
        self.W=np.zeros(self.n)
        self.H=np.dot(self.X,self.W)
        self.Cost=np.zeros(self.Epochs)
        self.val_err=[]
        self.tr_err=[]
```

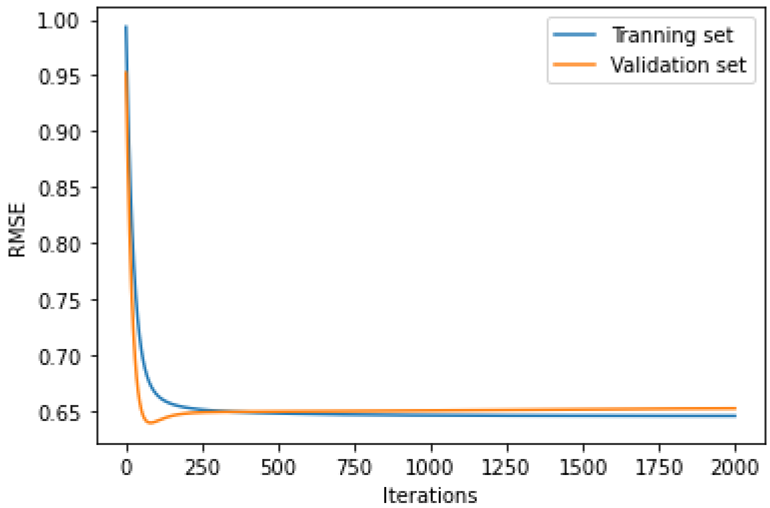
```
        for i in range(self.Epochs):
            self.upd_weight()
            self.val_err.append(self.rmse(Y_test,self.predict(X_test)))
            self.tr_err.append(self.rmse(Y,self.predict(X)))
            self.Cost[i]=np.sum(np.square(self.H-self.Y))/(2*self.m)
        #self.plot1()
        print("RMSE for tranning set => {}".format(self.tr_err[-1]))
        print("RMSE for validation set => {}".format(self.val_err[-1]))
        return self

    def upd_weight(self):

        self.W[0]=self.W[0]-(self.l_rate/self.m)*sum(self.H-self.Y)
        for i in range(1,self.n):
            self.W[i]=self.W[i]-(self.l_rate/self.m)*sum((self.H-self.Y)*self.X[:,i])
        self.H=np.dot(self.X,self.W)
        return self

    def W_C(self):
        print(self.W[0])
        print(self.W[1:])

    def rmse(self,y1,y2):
        mse = (np.square(y1 - y2)).mean()
        return np.sqrt(mse)
    def plot1(self):
        list1 = list(range(0,self.Epochs))

        plt.plot(list1,self.tr_err,label='Tranning set')
        plt.plot(list1,self.val_err,label='Validation set')
        plt.xlabel('Iterations')
        plt.ylabel('RMSE')
#         plt.title("k= "+str(i)+" and model number="+str(j+1)+"th")
        plt.legend()
        plt.show()

        plt.plot(list1,self.Cost,color='g',label='cost')
#         plt.title("Cost function for K = "+str(i)+" and model number="+str(j+1))
        plt.xlabel('Iterations')
        plt.ylabel('Cost')
        plt.legend()
        plt.show()
    def predict(self,X):

        return X.dot(self.W[1:])+self.W[0]
```

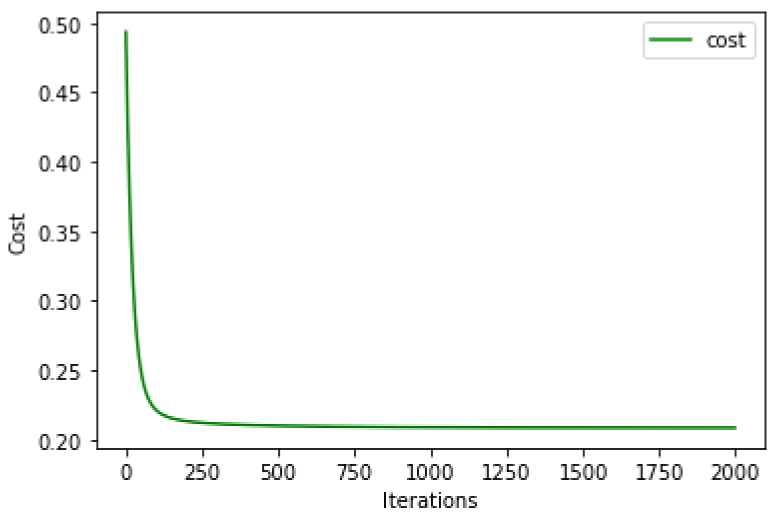
In [12]:

```
# the range for which one needs to run K-fold (K=5 ) so range (5,6)
for i in range(5,6):
    l=[]
    l1=[]
    X_Trains, Y_Trains, X_Tests, Y_Tests = K_fold_validation(i,norm(pd.read_csv("Real e
    for j in range(i):
        model=Linear_Regression1(0.01,2000)
        print("k= "+str(i)+" and model number="+str(j+1))
        model.fit(X_Trains[j],Y_Trains[j],X_Tests[j],Y_Tests[j])
        model.plot1()
```

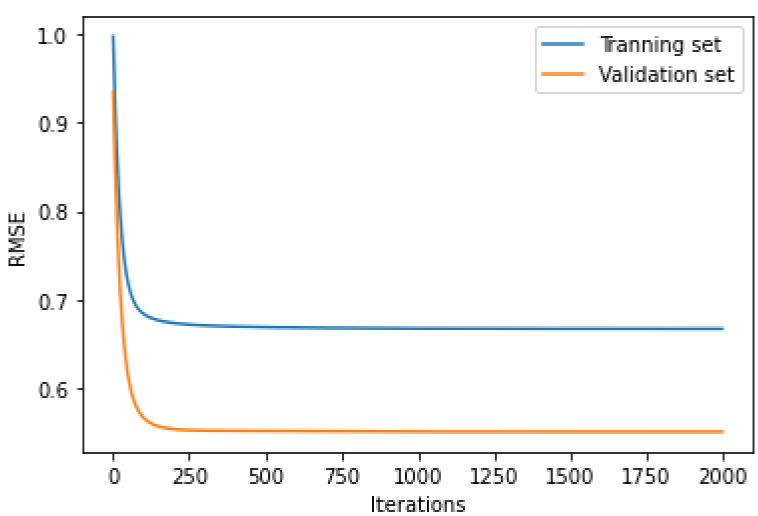
```
k= 5 and model number=1
```

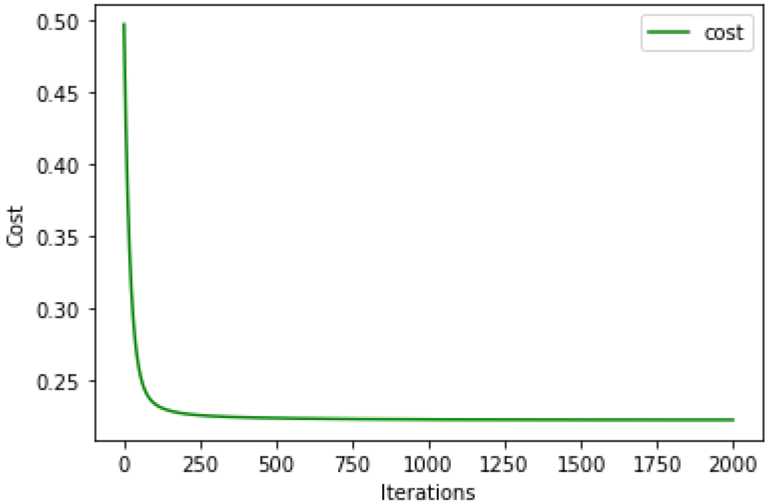
```
RMSE for tranning set => 0.6455807116763345
RMSE for validation set => 0.652294284531606
```



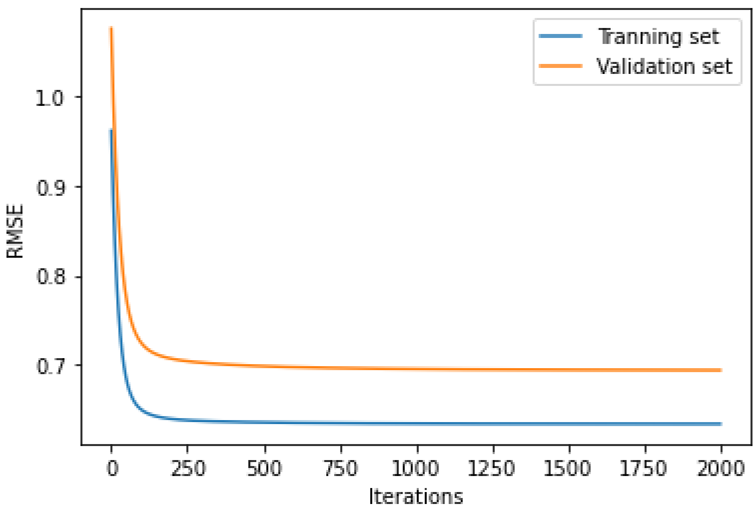


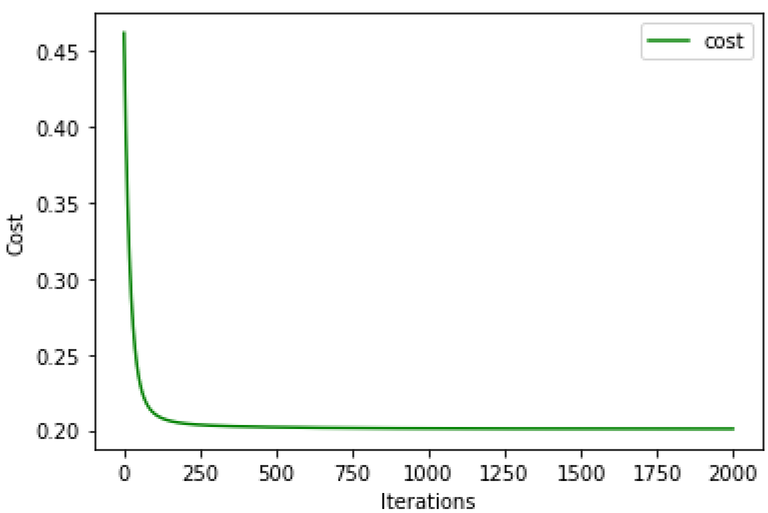

```
k= 5 and model number=2
RMSE for tranning set => 0.6673322954749914
RMSE for validation set => 0.5511216204356201
```



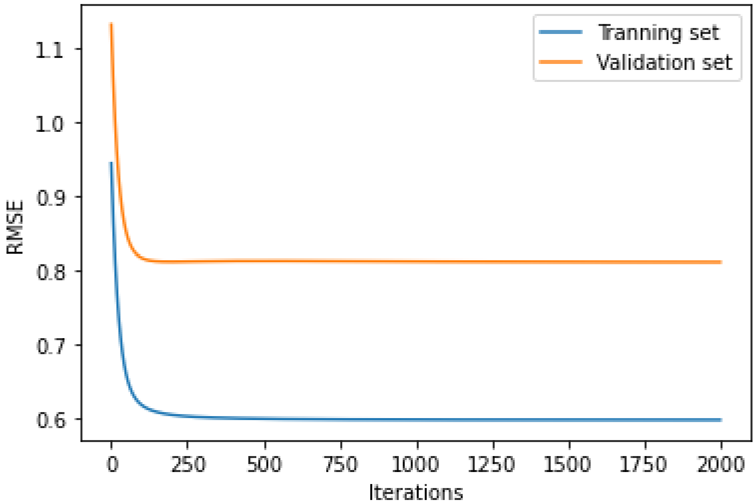


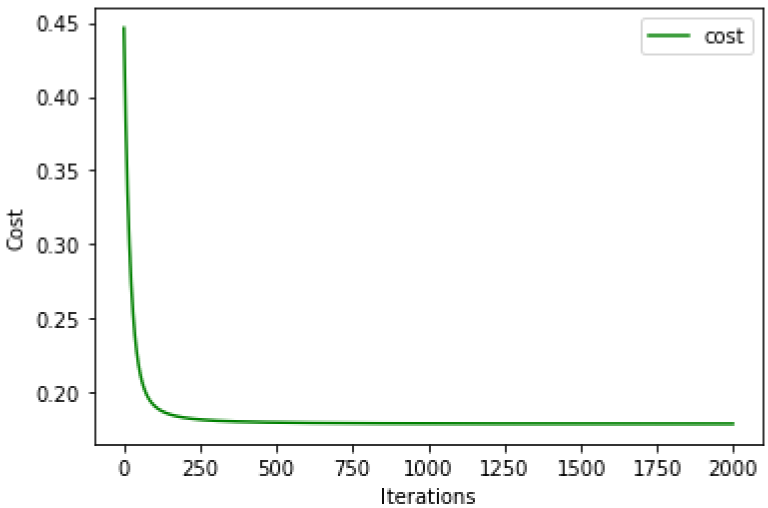
k= 5 and model number=3
RMSE for tranning set => 0.6337213018795999
RMSE for validation set => 0.6938312819257478



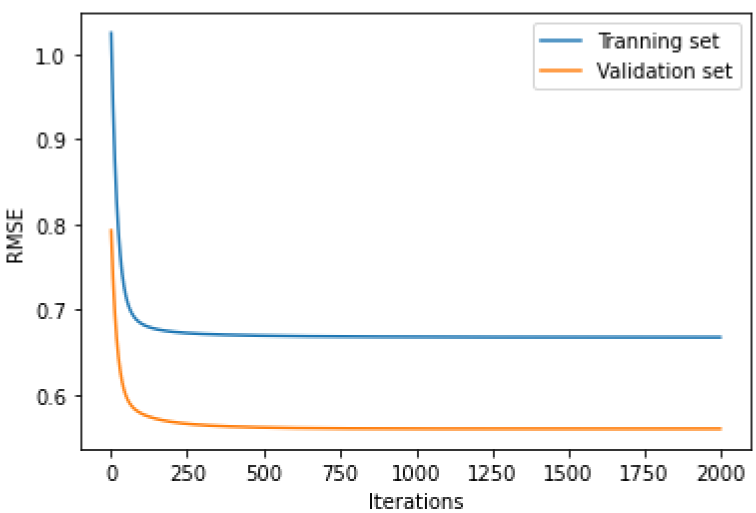


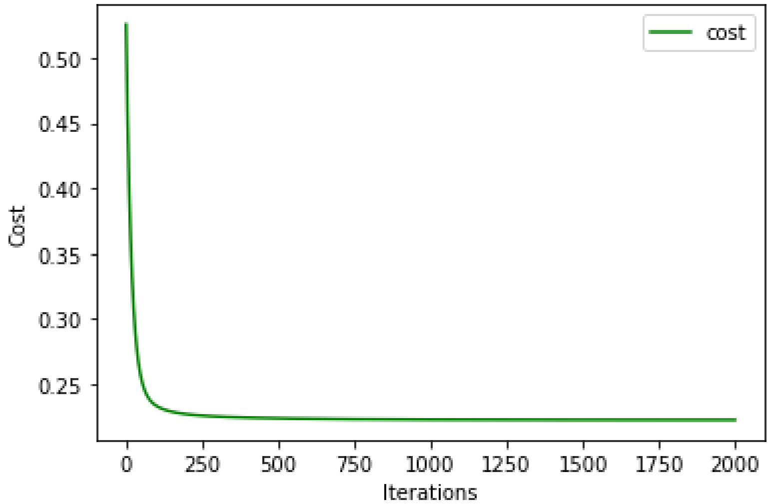
k= 5 and model number=4
RMSE for tranning set => 0.5978329170873334
RMSE for validation set => 0.8115815436018022






```
k= 5 and model number=5
RMSE for tranning set => 0.6673537073840736
RMSE for validation set => 0.5595258522593299
```





Part C

Modify your Regression implementation by including L1 (LASSO) and L2 (Ridge Regression) regularization. Implement both regularization function from scratch and train the model again. Try different values of the regularization parameter and report the best one. Plot similar RMSE V/s iteration graph as before

In [13]:

```
class Linear_Regression_lasso():
    def __init__(self,l_rate,Epochs,lmd):

        self.l_rate=l_rate
        self.Epochs=Epochs
        self.lmd=lmd

    def fit(self, X, Y ,X_test,Y_test):

        self.m,self.n = X.shape
        on=np.ones((self.m,1))
        self.X=np.concatenate((on,X),axis=1)
        self.n=self.n+1
        self.Y=Y
        self.W=np.zeros(self.n)
        self.H=np.dot(self.X,self.W)
        self.Cost=np.zeros(self.Epochs)
        self.val_err=[]
        self.tr_err=[]



        for i in range(self.Epochs):
            self.upd_weight()
            self.val_err.append(self.rmse(Y_test,self.predict(X_test)))
            self.tr_err.append(self.rmse(Y,self.predict(X)))
            self.Cost[i]=np.sum(np.square(self.H-self.Y))/(2*self.m)
        #self.plot1()
        return self

    def upd_weight(self):
        p=0
```

```
        if(self.W[0]<0):
            p=1
        elif(self.W[0]>0):
            p=-1

        self.W[0]=self.W[0]-(self.l_rate/self.m)*sum(self.H-self.Y)+(p*self.lmd)
        for i in range(1,self.n):
            p=0
            if(self.W[0]<0):
                p=1
            elif(self.W[0]>0):
                p=-1
            self.W[i]=self.W[i]-(self.l_rate/self.m)*sum((self.H-self.Y)*self.X[:,i])+(

        self.H=np.dot(self.X,self.W)
        return self

    def W_C(self):
        print(self.W[0])
        print(self.W[1:])

    def rmse(self,y1,y2):
        mse = (np.square(y1 - y2)).mean()
        return np.sqrt(mse)
    def plot1(self):
        list1 = list(range(0,self.Epochs))

        plt.plot(list1,self.tr_err,label='Tranning set')
        plt.plot(list1,self.val_err,label='Validation set')
#         plt.title("k= "+str(i)+" and model number="+str(j+1)+"th")
        plt.legend()
        plt.xlabel('Iterations')
        plt.ylabel('RMSE')
        plt.show()

        plt.plot(list1,self.Cost,color='g',label='cost')
#         plt.title("Cost function for K = "+str(i)+" and model number="+str(j+1))
        plt.legend()
        plt.xlabel('Iterations')
        plt.ylabel('Cost')
        plt.show()

    def predict(self,X):

        return X.dot(self.W[1:])+self.W[0]
```

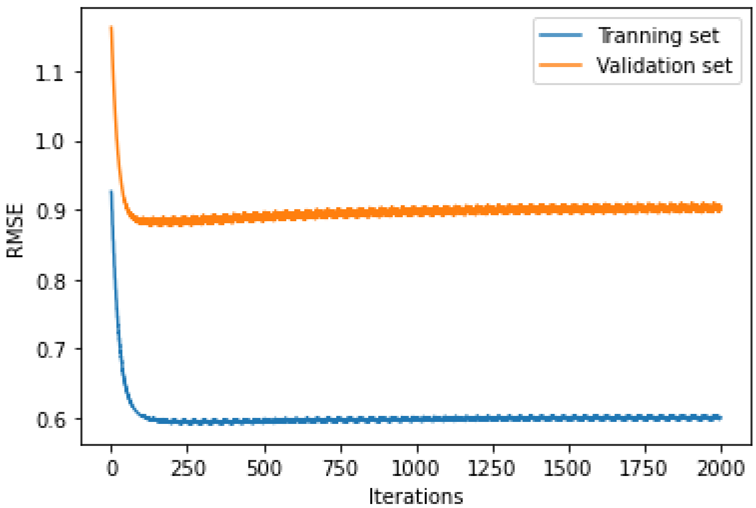
In [14]:

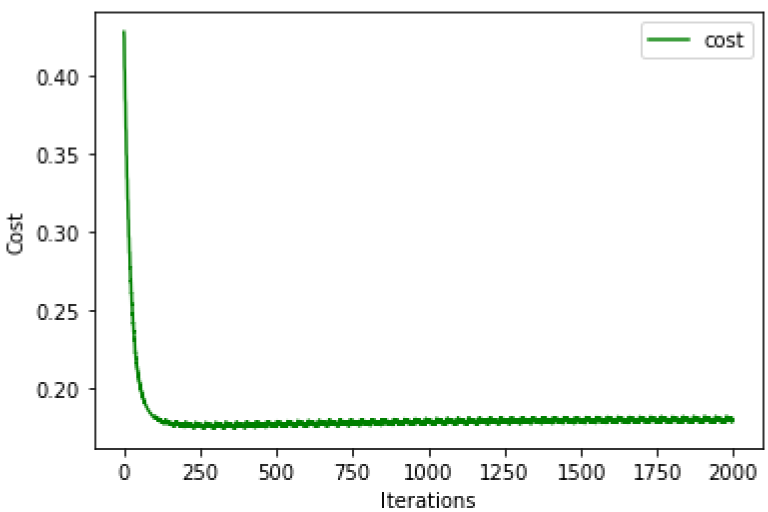
```
class Linear_Regression_ridge():
    def __init__(self,l_rate,Epochs,lmd):

        self.l_rate=l_rate
        self.Epochs=Epochs
        self.lmd=lmd

    def fit(self, X, Y ,X_test,Y_test):

        self.m,self.n = X.shape
        on=np.ones((self.m,1))
        self.X=np.concatenate((on,X),axis=1)
```

```
        self.n=self.n+1
        self.Y=Y
        self.W=np.zeros(self.n)
        self.H=np.dot(self.X,self.W)
        self.Cost=np.zeros(self.Epochs)
        self.val_err=[]
        self.tr_err=[]



        for i in range(self.Epochs):
            self.upd_weight()
            self.val_err.append(self.rmse(Y_test,self.predict(X_test)))
            self.tr_err.append(self.rmse(Y,self.predict(X)))
            self.Cost[i]=np.sum(np.square(self.H-self.Y))/(2*self.m)
        #self.plot1()
        return self

    def upd_weight(self):

        self.W[0]=((1-2*self.l_rate*self.lmd)*self.W[0])-(self.l_rate/self.m)*sum(self.

        for i in range(1,self.n):
            self.W[i]=((1-2*self.l_rate*self.lmd)*self.W[i])-(self.l_rate/self.m)*sum((

        self.H=np.dot(self.X,self.W)
        return self

    def W_C(self):
        print(self.W[0])
        print(self.W[1:])

    def rmse(self,y1,y2):
        mse = (np.square(y1 - y2)).mean()
        return np.sqrt(mse)

    def plot1(self):
        list1 = list(range(0,self.Epochs))

        plt.plot(list1,self.tr_err,label='Tranning set')
        plt.plot(list1,self.val_err,label='Validation set')
#         plt.title("k= "+str(i)+" and model number="+str(j+1)+"th")
        plt.legend()
        plt.xlabel('Iterations')
        plt.ylabel('RMSE')
        plt.show()

        plt.plot(list1,self.Cost,color='g',label='cost')
#         plt.title("Cost function for K = "+str(i)+" and model number="+str(j+1))
        plt.legend()
        plt.xlabel('Iterations')
        plt.ylabel('Cost')
        plt.show()

    def predict(self,X):

        return X.dot(self.W[1:])+self.W[0]
```

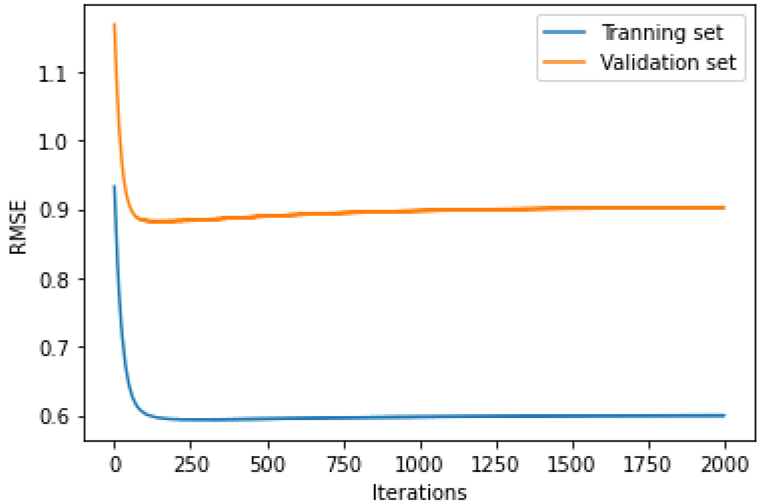
In [15]:

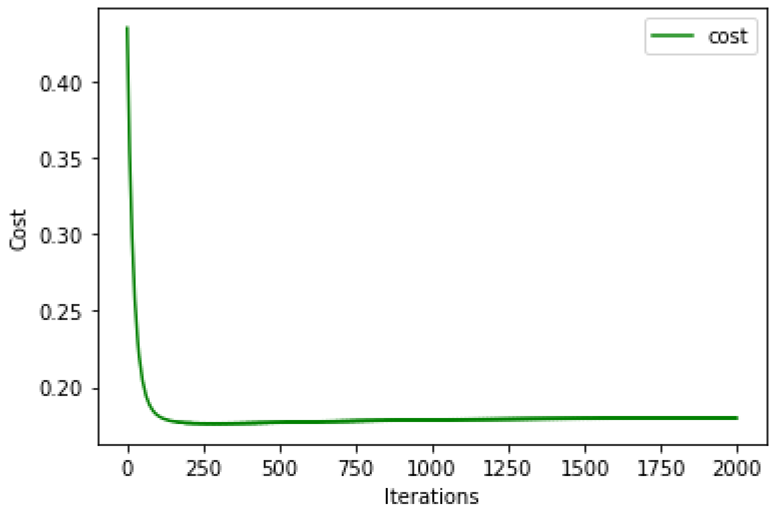
```
for i in range(5,6):
    l=[]
    l1=[]
    X_Trains, Y_Trains, X_Tests, Y_Tests = K_fold_validation(i,norm(pd.read_csv("Real e
    for j in range(i):
        mp=[0.01,0.001,0.000005,0.0006,0.00007]
        for m in range(5):
            model1=Linear_Regression_lasso(0.01,2000,mp[m])
            model1.fit(X_Trains[j],Y_Trains[j],X_Tests[j],Y_Tests[j])

            #we got the best split at k=5 and the 1th split so only showing that one
            if(j==1):
                model1.plot1()
                print("Regularisation parameter {}".format(mp[m]))
                print("Validation Set RMSE=> {}".format(model1.val_err[-1]))
                print("Tranning Set RMSE => {}".format(model1.tr_err[-1]))
```



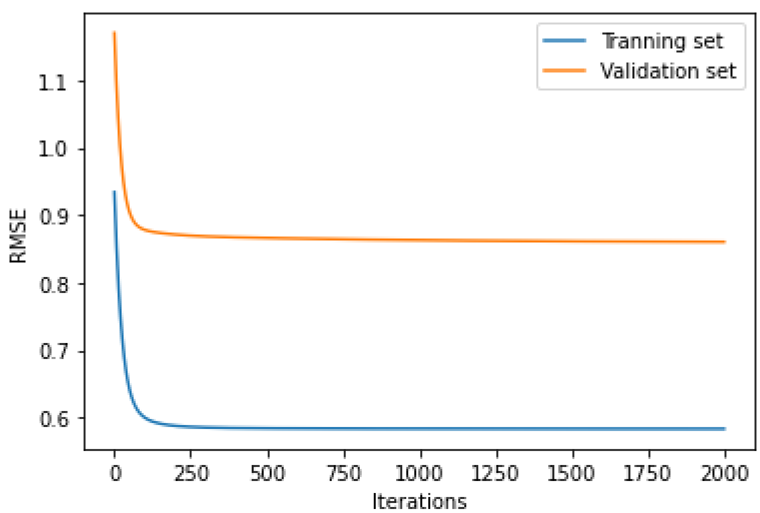


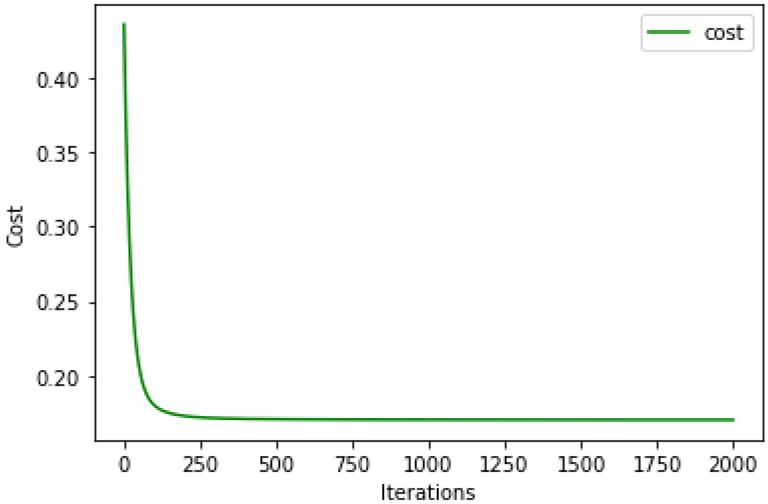

```
Regularisation parameter 0.01
Validation Set RMSE=> 0.9052595364454838
Tranning Set RMSE => 0.6004640526698816
```



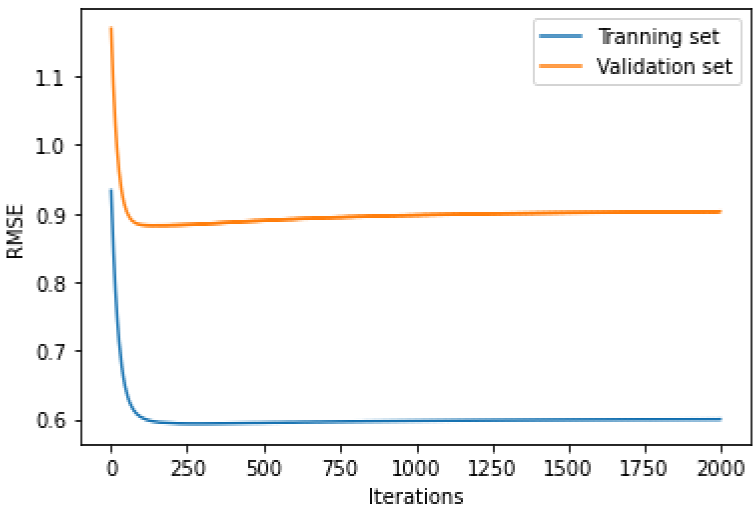


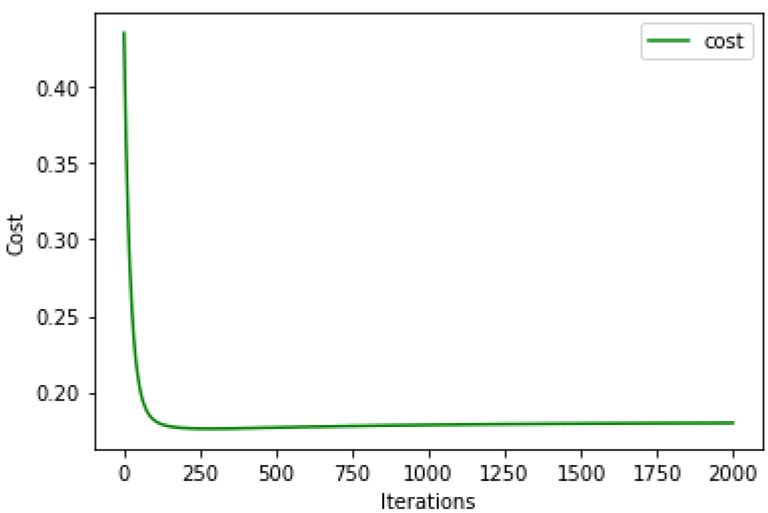

```
Regularisation parameter 0.001
Validation Set RMSE=> 0.9021533241674045
Tranning Set RMSE => 0.5996499679753795
```



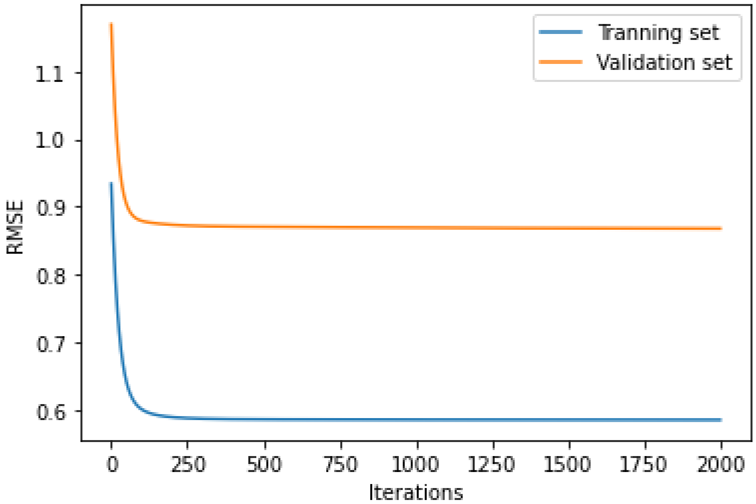


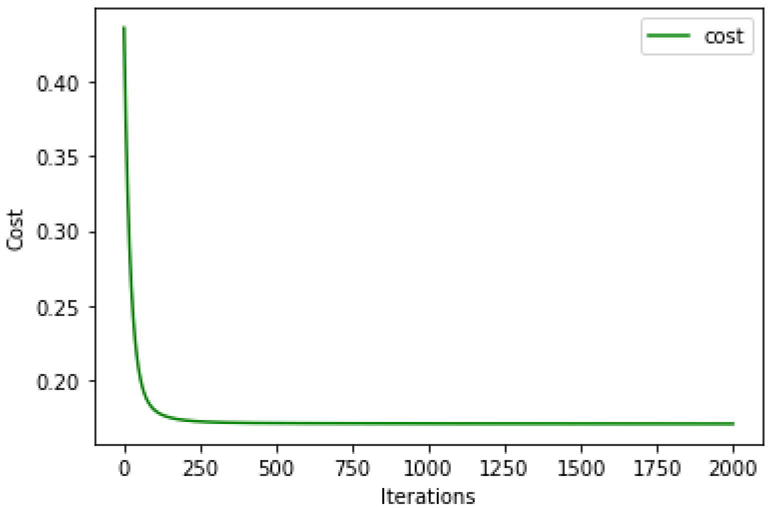

```
Regularisation parameter 5e-06
Validation Set RMSE=> 0.8599830377739207
Tranning Set RMSE => 0.5833127613815478
```






```
Regularisation parameter 0.0006
Validation Set RMSE=> 0.9027627515683939
Tranning Set RMSE => 0.5998633095266118
```





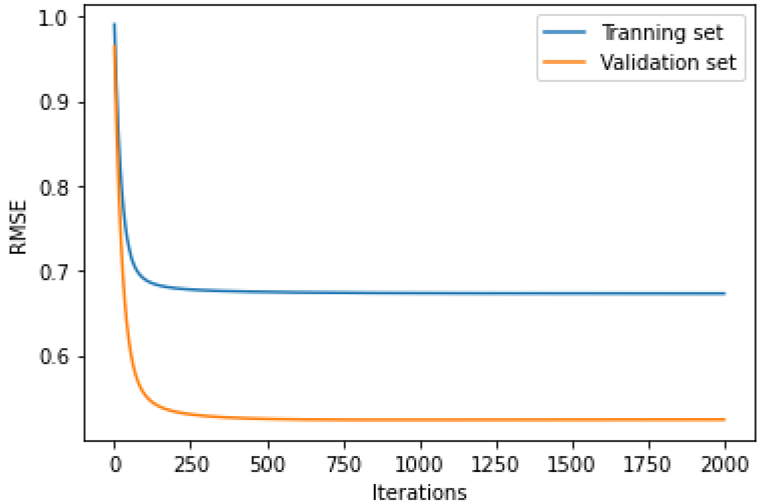

```
Regularisation parameter 7e-05
Validation Set RMSE=> 0.8673471706837934
Tranning Set RMSE => 0.5844403951382474
```

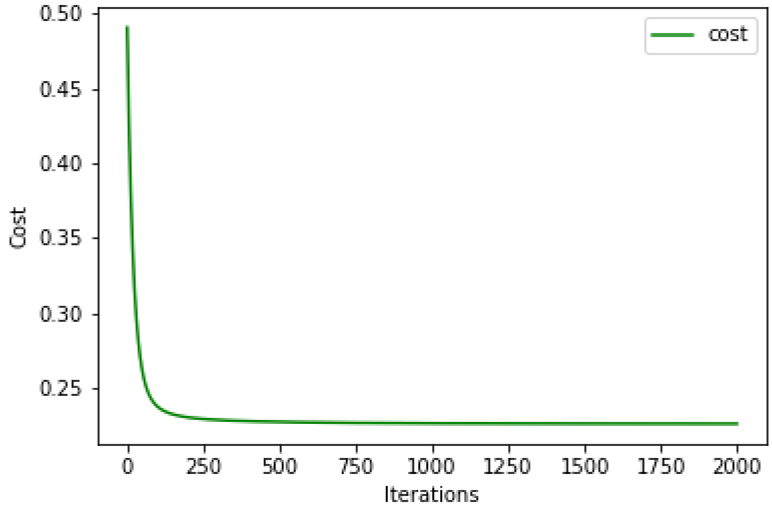
The best parameter is 0.01

In [16]:

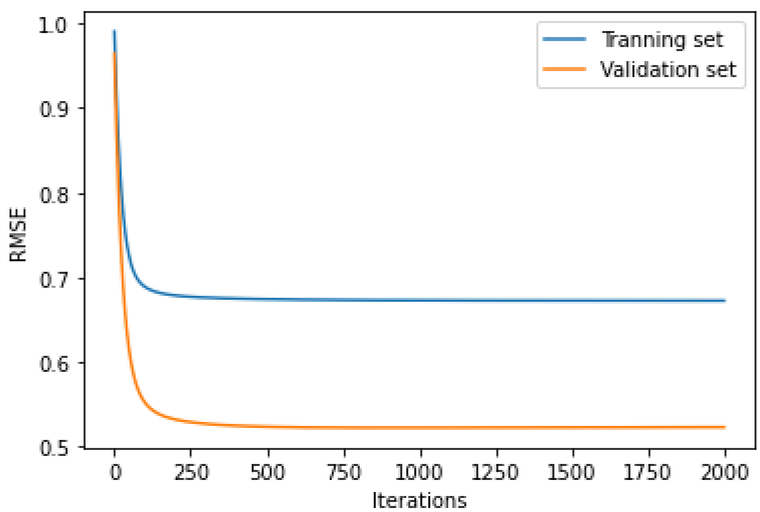
```
for i in range(5,6):
    l=[]
    l1=[]
    X_Trains, Y_Trains, X_Tests, Y_Tests = K_fold_validation(i,norm(pd.read_csv("Real e
    for j in range(i):
        mp=[0.01,0.001,0.000005,0.0006,0.00007]
        for m in range(5):
            model1=Linear_Regression_ridge(0.01,2000,mp[m])
            model1.fit(X_Trains[j],Y_Trains[j],X_Tests[j],Y_Tests[j])

            #we got the best split at k=5 and the 1th split so only showing that one
            if(j==1):

                print("Regularisation parameter {}".format(mp[m]))
                print("Validation Set RMSE=> {}".format(model1.val_err[-1]))
                print("Tranning Set RMSE => {}".format(model1.tr_err[-1]))
                model1.plot1()
```

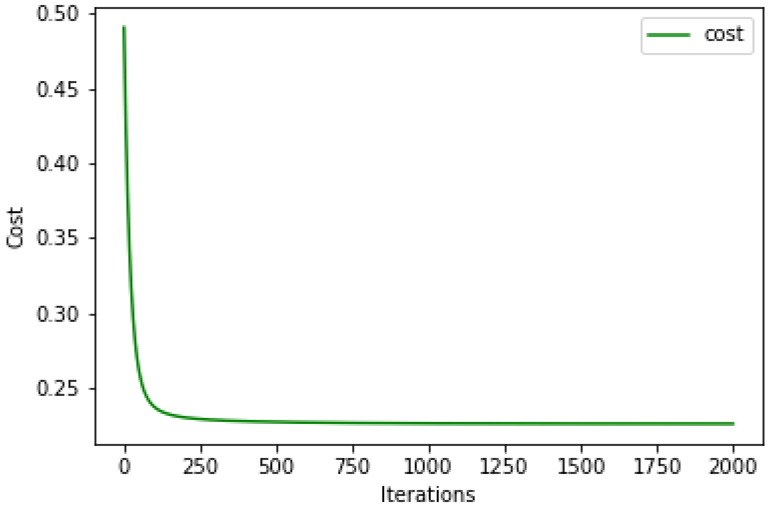
```
Regularisation parameter 0.01
Validation Set RMSE=> 0.5239991291862338
Tranning Set RMSE => 0.6730139364760966
```



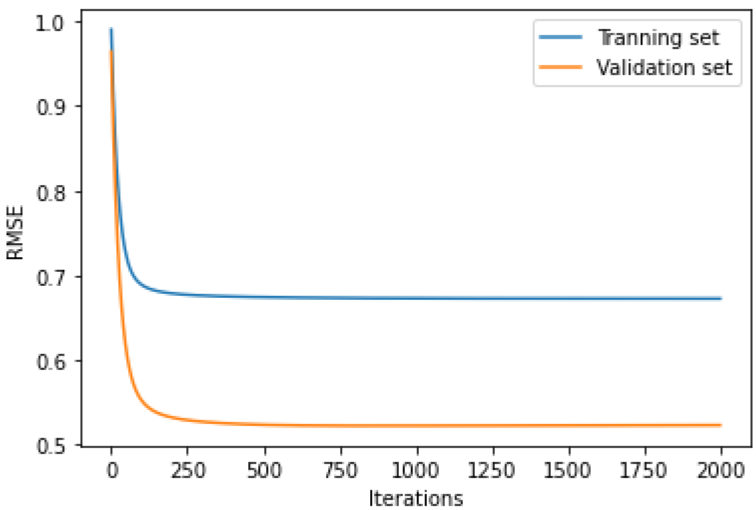


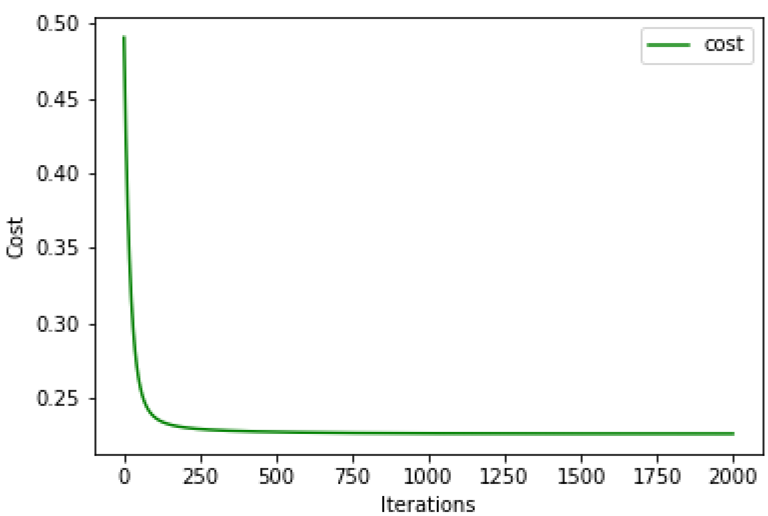

```
Regularisation parameter 0.001
Validation Set RMSE=> 0.5232712602950086
Tranning Set RMSE => 0.6728304370646259
```



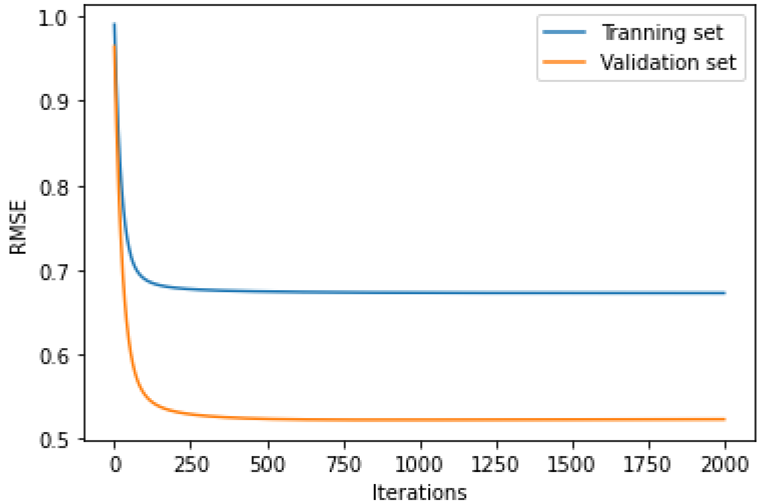


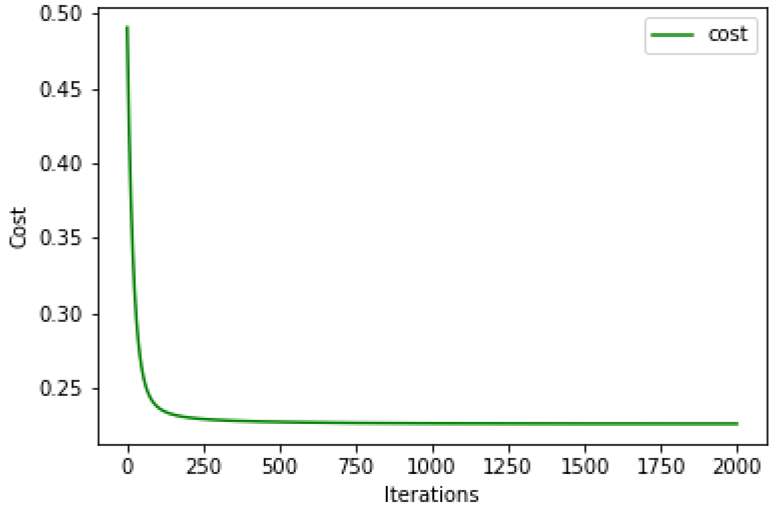

```
Regularisation parameter 5e-06
Validation Set RMSE=> 0.523205503196435
Tranning Set RMSE => 0.6728216809390176
```



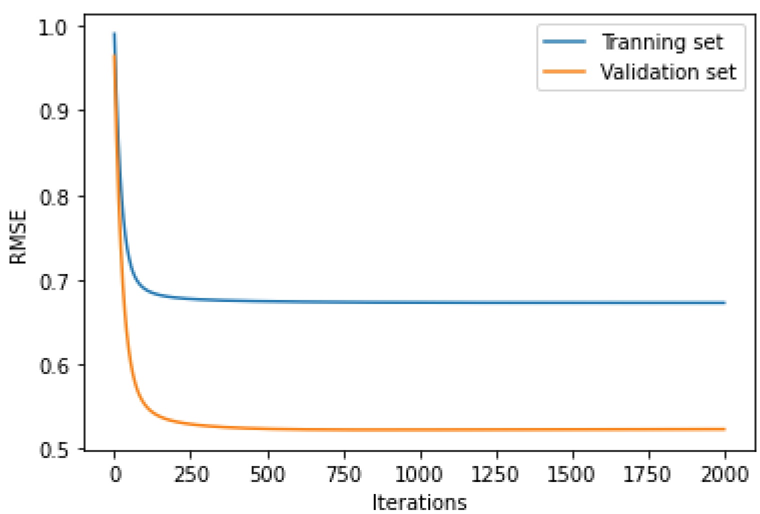


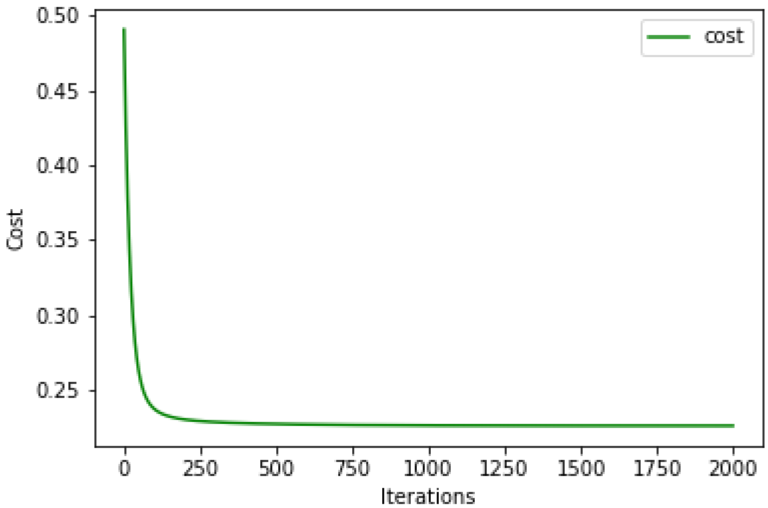

```
Regularisation parameter 0.0006
Validation Set RMSE=> 0.5232444313708448
Tranning Set RMSE => 0.672826604880376
```






```
Regularisation parameter 7e-05
Validation Set RMSE=> 0.5232096981710419
Tranning Set RMSE => 0.6728221730869017
```





The best parameter is 0.005

In [25]:

```
class LR_NORM():
    def __init__(self):
        self.name=0
    def fit(self, X, Y):

        self.m,self.n = X.shape
        on=np.ones((self.m,1))
        self.X=np.concatenate((on,X),axis=1)
        self.n=self.n+1
        self.Y=Y
        self.normal_eq()

    def normal_eq(self):
        self.W = np.linalg.inv(self.X.T.dot(self.X)).dot(self.X.T).dot(self.Y)

    def predict(self,X):
        X=np.array(X)
        return X.dot(self.W[1:])+self.W[0]
```

In [26]:

```
for i in range(2,6):
    X_Trains, Y_Trains, X_Tests, Y_Tests = K_fold_validation(i,norm(pd.read_csv("Real e
    l_c=[]
    for j in range(i):
        model=LR_NORM()
        model.fit(X_Trains[j],Y_Trains[j])
        y_pr=model.predict(X_Tests[j])
        l_c.append(np.sqrt(np.mean(np.square(model.predict(X_Tests[j])-Y_Tests[j]))))
        print("Rmse for k={} and split ={} =>".format(i,j)+str(sum(l_c)/len(l_c)))
    print()
```

```
Rmse for k=2 and split =0 =>0.6756446887548269
Rmse for k=2 and split =1 =>0.6604265204712334
```

```
Rmse for k=3 and split =0 =>0.6541133917854346
Rmse for k=3 and split =1 =>0.6976576051546526
Rmse for k=3 and split =2 =>0.6537020357875981

Rmse for k=4 and split =0 =>0.6770590790553567
Rmse for k=4 and split =1 =>0.6281281547675499
Rmse for k=4 and split =2 =>0.6191999641902847
Rmse for k=4 and split =3 =>0.6602538105727593

Rmse for k=5 and split =0 =>0.8688665619458582
Rmse for k=5 and split =1 =>0.7588676235103131
Rmse for k=5 and split =2 =>0.7204739580173932
Rmse for k=5 and split =3 =>0.663132149920377
Rmse for k=5 and split =4 =>0.6483658795405521
```

In [ ]: