Assignment 1.

2020249.

Section 1 theory.

Part a)

given $\bar{X}$ & $\bar{Y}$ are arithmetic mean of X & Y respectively

X -> independ variable.
Y -> Dependent variable.

$$\bar{Y} = w_0 + w_1 x.$$

with noise. (generisation)

$$Y_i = w_0 + w_i x_i + \varepsilon$$

n = number of samples in data

$$\sum_{i=1}^{n} Y_i = n(w_0 + w_i x_i + \varepsilon_i)$$

$$= n w_0 + w_1 \sum_{i=1}^{n} x_i + \underbrace{\sum_{i=1}^{n} \varepsilon_i}$$

noise considered zero.

$$\frac{1}{n} \sum_{i=1}^{n} \frac{Y_i}{\cancel{\varepsilon}} = w_0 + \frac{w_1}{n} \sum_{i=1}^{n} x_i$$

$$\bar{Y} = w_0 + w_1 \bar{X}$$

$\sum_{i=1}^{n} \frac{Y_i}{n}$ = arithmetic mean of Y.

$\frac{1}{n} \sum_{i=1}^{n} X_i$ = arithmetic mean of X.

b) No, (Not always)

example.

Let X, Y, Z are Random variables.

X & Y are positively correlated &

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~~$C(X,Y) = C(Y,Z) * C(Z,X) - \sqrt{1 - C(Y,Z)^2 \times (1-}$~~

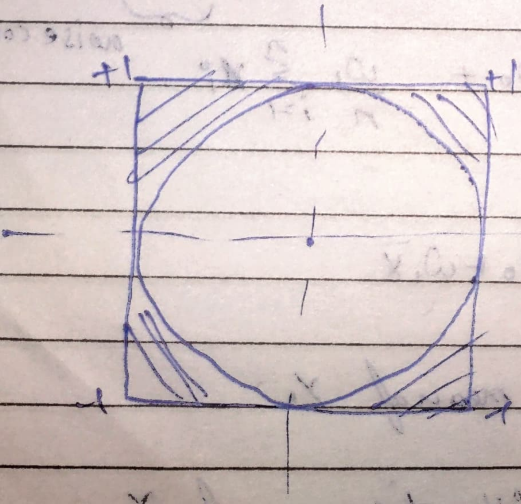
$$C(X,Y) = C(Y,Z) * C(Z,X) - \sqrt{(1 - C(Y,Z)^2) \times (1 - C(Z,X)^2)}$$

for $C(X,Y)$ to be more than zero.

$$C(Y,Z) \times C(Z,X) > \sqrt{(1 - C(Y,Z)^2)(1 - C(Z,X)^2)}$$

Squaring both sides.

$$C(Y,Z)^2 + C(Z,X)^2 > 1$$



if two correlations are in the circular Non shaded region we can't say precisely anything about the correlation of the 3rd pair.

c) Proof of LLN (WLLN)

let X_i --- X_n are ~~iati~~ i.i.d Random variables with mean give μ_x where n is very large.

$$S_n = \frac{X_1 + X_2 \cdots X_n}{n}$$

$$\lim_{n \to \infty} P\{|S_n - \mu_x| > \varepsilon\} = 0.$$

Also.

$E[S_n] = \mu_x$ and variance $= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$.

from cheby shev's inequality

$$P\{|S_n - \mu_x| > \varepsilon\} \leq \frac{Var[S_n]}{\varepsilon^2} = \frac{\sigma_x^2}{n\varepsilon^2}$$

$$\lim_{n \to \infty} P\{|S_n - \mu_x| \geq \varepsilon\} = 0$$

Example.

let's consider a fair die distribution.

$S = \{1, 2, 3, 4, 5, 6\}$.

$S_n = 3.5$.

When we roll the dies for a very large number(n) times.

the average value approaches 3.5.

Pseudo code

```
def FLLNdie(n)
  result = []
    for i in Range (1, n+1)
        result.append (random.choice(1, 2, 3, 4, 5, 6))
  return result

def LLN (n)
  result = LLNDie (n)
  average = []
  for i in Range (len (Results)):
      average.append (np.cumsum[i]/(i+1))

  return average.
```

Q.4 MAP Solution for linear regression

from Bayes th.

$$P(w|D) = \frac{P(D|w) \times P(w)}{P(D)}$$ ← unknow.

$$P(w|D) \propto P(D|w) \times P(w)$$

take log

$$\log(P(w|D)) = \log P(D|w) + \log(P(w)) \quad — ①$$

assume a gaussian distribution for w with mean 0 and variance σ, $N(0, \sigma^2)$

$$\log(P|w) = \log\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-\sigma}{2} w^T w\right)}\right) = \log(p|w)$$

$$= \left(\frac{1}{2\sigma^2} w^T w\right)$$

$$\log(P(D|w)) = \frac{1}{2\sigma^2}(y - Dw)^T (y - Dw)$$

put $P(D|w)$ & $P(w)$ in eq ① & differentiate with wrt w.

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$$\frac{d}{dw}\left[\frac{1}{2\sigma^2}(y-Dw)^T(y-Dw)\right]+\frac{d}{dw}\left(\frac{1}{2\sigma^2}w^Tw\right)$$

$$=\frac{1}{\sigma^2}\left(w^TD^TD-y^TD\right)+\frac{1}{\sigma^2}w^T$$

$$\frac{1}{\sigma^2}\left(w^TD^TD-y^TD\right)+\frac{1}{\sigma^2}w^T=0$$

$$\frac{w^T}{\sigma^2}\left(D^TD+I\right)=\frac{y^TD}{\sigma^2}$$

$$w^T=(y^TD)(D^TD+I)^{-1}$$

$$w^T_{map}=y^TD\,(D^TD+\lambda I)^{-1}$$

regulizer.