

~~Section~~

$$L = T - V$$

$T \rightarrow$ Total energy $\rightarrow T_1 + T_2 + T_3 + \dots$

$V \rightarrow$ Full potential

$$\vec{F} = -\vec{\nabla} V$$

Cartesian (x, y, z) co-ordinates, $\vec{F}(x, y, z) = -\vec{\nabla} V(x, y, z)$

$$F_x \hat{x} + F_y \hat{y} + F_z \hat{z} = - \left[\hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z} \right]$$

$$F_x = -\frac{\partial V}{\partial x}, \quad F_y = -\frac{\partial V}{\partial y}, \quad F_z = -\frac{\partial V}{\partial z}$$

$(\rho, \phi, z) \rightarrow$ cylindrical co-ordinates $\vec{F}(\rho, \phi, z) = -\vec{\nabla} V(\rho, \phi, z)$

$$F_\rho \hat{\rho} + F_\phi \hat{\phi} + F_z \hat{z} = - \left[\hat{\rho} \frac{\partial V}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z} \right]$$

$(r, \theta, \phi) \rightarrow$ Spherical co-ordinates $\vec{F} = -\vec{\nabla} V(r, \theta, \phi)$

$$\vec{F} = F_r \hat{r} + F_\theta \hat{\theta} + F_\phi \hat{\phi} = - \left[\hat{r} \frac{\partial V}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \right]$$

$$F_r = -\frac{\partial V}{\partial r}, \quad F_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta}, \quad F_\phi = -\frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2} m (\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2) \quad (x, y, z)$$

$$= \frac{1}{2} m (\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2), \quad (\rho, \phi, z) \text{ cylindrical}$$

$$= \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2), \quad (r, \theta, \phi) \rightarrow \text{spherical co-ordinates}$$

$$2D \text{ planes, } T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) = \frac{1}{2} m (r^2 + r^2 \dot{\theta}^2) \rightarrow (r, \theta)$$

$$\vec{F} = -\vec{\nabla} V$$

Express F in (x, y, z) / (r, θ, ϕ) / (ρ, ϕ, z)
and V as well.

$$\text{Ex: } F_n = -\frac{\partial V}{\partial n} \quad V = -\int F_n \, dn$$

$$F_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} \quad V = -\int r F_\theta \, d\theta$$

$$F_\rho = -\frac{\partial V}{\partial \rho} \quad V = -\int F_\rho \, d\rho$$

$$L = T - V$$

use formula of Lagrange's

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \quad \text{for each } q$$

$$\text{i.e., } q = r, \theta, z$$

$$q = r, \theta, \phi$$

$$q = \rho, \phi, z$$

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