Tut-6

(1)

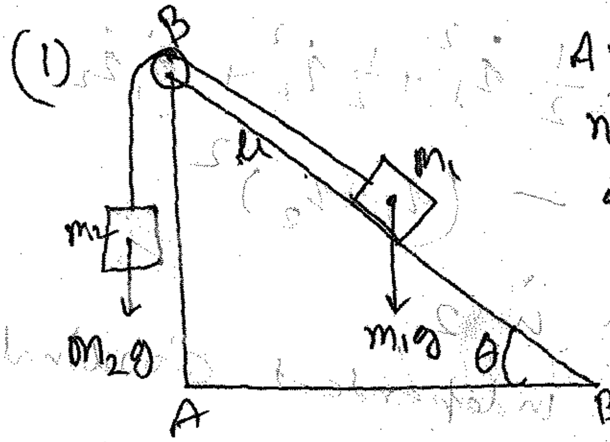


Fig-1

An inextensible string of negligible mass hanging over 'B' as shown in figure-1 connects one mass m_1 on a inclined plane of angle θ to another mass m_2.

Inclined plane have a friction.

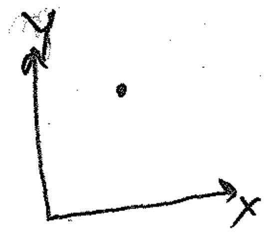
(a) Coefficient of friction of inclined plane is 'μ'

(b) Using D'Alembert's principle, prove that

$$\sin\theta - \mu\cos\theta = \frac{m_2}{m_1}$$

in the equilibrium condition.

(2) A particle of mass 'm' is constrained to move on the plane $xy = c$, where $c =$ constant, under gravity ('y' axis is verticle here). Set up the Langrangian of particle & find out the equation of motion.



(3) Consider a Langrangian of a particle is expressed as $L = \frac{1}{2}\dot{q}_1^2 + \frac{1}{2}\dot{q}_2^2 + \dot{q}_1\dot{q}_2 - (q_1+q_2)^2$

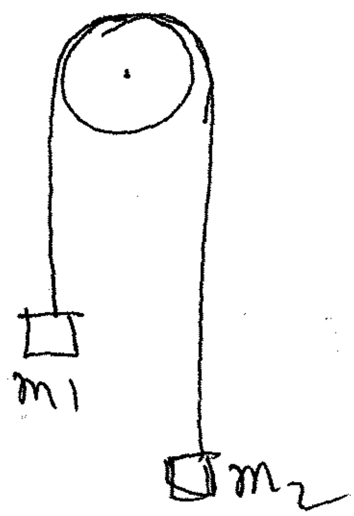
~~Read~~ Degrees of freedom is 2,

q_1 & q_2 are the independent coordinates.

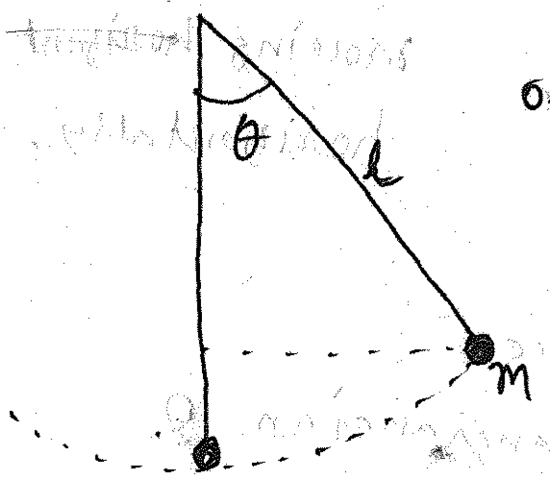
Find out the equation of motion of the particle.

[Formula: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$ for all q.]

(4) Obtain the equation of motion of a system of two masses, connected by an inextensible string passing over a smooth pulley.



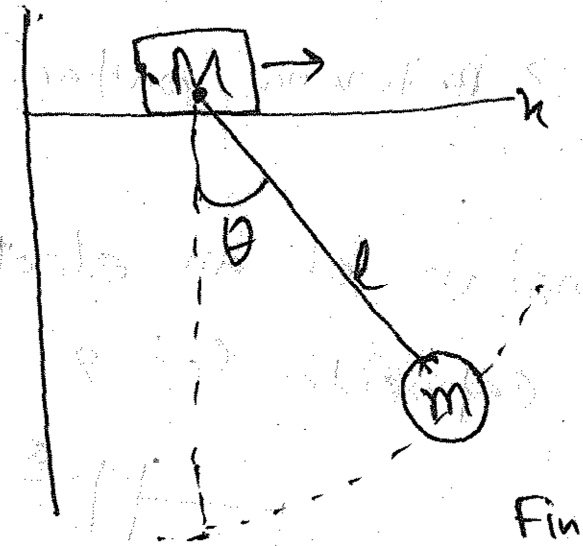
(5) Simple pendulum



Obtain the equation of motion of a simple pendulum by using Langrangian method.

[Hints: use polar co-ordinate (r, θ), $r = l \to$ constant.
potential energy $V = mgh$
calculate ~~to~~ height $\to h$

(6)

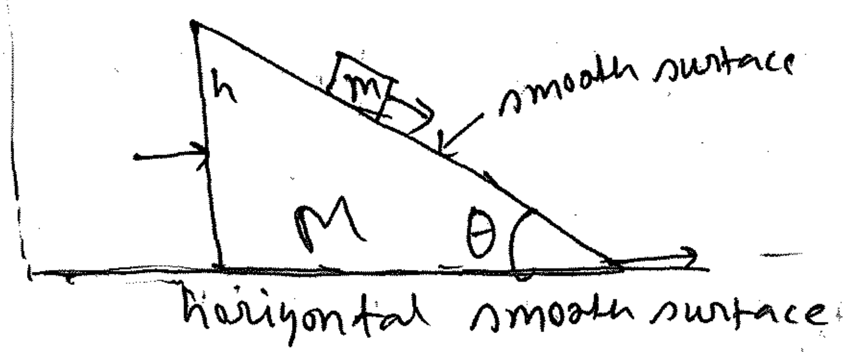


A pendulum of mass 'm' is attached to a block 'M'. The block is also moving horiyondtally as shown in Figure.

Find out the equation of motion of the system.

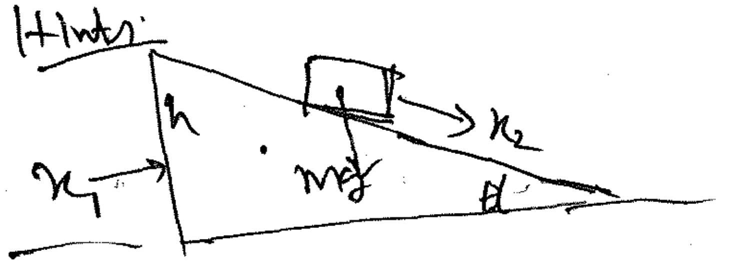
[Hints: $\boxed{M} \to$ one co-ordinate 'x'
2nd particle (m) $\to$ co-ordinate 'x' and 'θ'

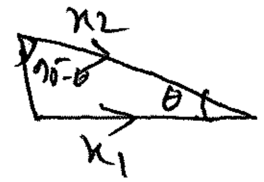
(7) i. The small block 'm' is sliding on a ~~hoom~~ inclined plane surface of a triangular block 'M'. The Block 'M' is also moving ~~horizont~~ horizontally..



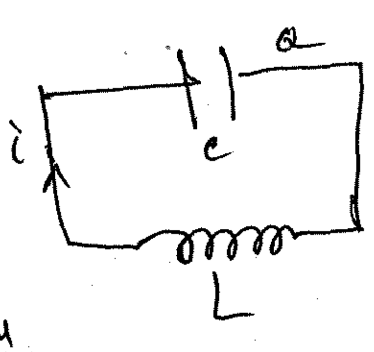
Ei. Find out the ~~or~~ Langrangian & equation of motion;

Hints:





(8) Find the Langrangian of an electrical circuit containing capacitor 'c' & Inductor 'L'.



Hints:-

energy of capacitor, $= E_c = \frac{1}{2}cv^2$

$= \frac{q^2}{2c}$ → potential energy (T) $i = \frac{dq}{dt}$, $q = cv$

energy of inductor, $E_L = \frac{1}{2}L\dot{i}^2$ $[i = \frac{dq}{dt} = \dot{q}]$

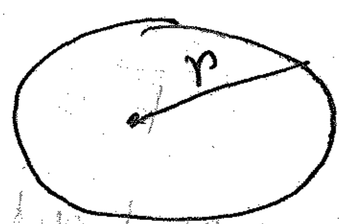
$= \frac{1}{2}L\dot{q}^2$ → kinetic energy (V)

So, $L = T - V = \frac{1}{2}L\dot{q}^2 - \frac{1}{2}\frac{q^2}{c}$

(9) A particle is moving under the influence of a central force $\vec{F} = -\frac{k\vec{r}}{r^3}$ (planetary motion)

Set up the Langrangian.

[Hints: express K.E ~~in~~ in $(r, \theta, \dot{r}, \dot{\theta})$



$\vec{F} = -\vec{\nabla} V$ (express $\vec{\nabla}$ in (r, θ) & calculate V in]

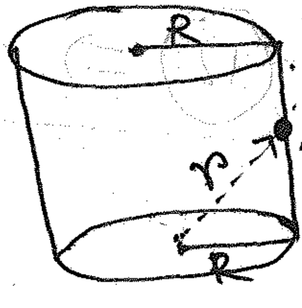
(10) Consider same problem (9) where the particle is moving under different potential where force can be expressed as,

(i) $\vec{F} = -kr\sin\theta\,\hat{r}$

(ii) $\vec{F} = -kr\cos\theta\,\hat{\theta}$

Set up the Langrangian & equation of motion.

(11) ~~Consi~~ A particle is moving on the surface of a cylinder as shown in Figure ~~of~~ under ~~a potential~~ a force $\vec{F} = -k\vec{r}$. [illegible]



Find out Langrangian. solve in r, θ, z

Hints: $\vec{r} = R\hat{\rho} + z\hat{z}$, $\rho = R$

(12) A particle is moving on a surface of sphere (Radius of sphere = 'R') ~~to~~ under a central force ~~F(θ)~~

$$\vec{F} = -kr\sin\theta\,\hat{r} + kr\cos\theta\,\hat{\theta}.$$

find out the equation of motion using Langrangian.

[Hints: Express K.E in ($\dot{r}, \dot{\theta}, \dot{\phi}, r, \theta, \phi$).

Find out potential V using $\bar{\nabla}$ operator

$\bar{F} = -\bar{\nabla} V$ where $V = V(r, \theta)$.

~~(13) Solve the same problem~~ if

$\cancel{\vec{F} = -kr\sin\theta\,\hat{\phi}}$