Tut-5

(1) $f(x,y,z,t)$ is an explicit function of position co-ordinates x, y, z & time t.

$$f(x,y,z,t) = x^3 y + y^2 + z^2 xyt$$

Find out (i) $\frac{\partial f}{\partial t}$

Find out (ii) $\frac{df}{dt}$ when $\frac{dx}{dt} \neq 0$, $\frac{dy}{dt} \neq 0$, $\frac{dz}{dt} \neq$

(iii) does $\frac{df}{dt} = \frac{\partial f}{\partial t}$??

(iv) Show that $\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x}$

(2) $f(x,y,z,t)$ is an implicit function of t. $\frac{dx}{dt} \neq 0$, $\frac{dy}{dz} \neq 0$, $\frac{dz}{dt} \neq 0$

$$f(x,y,z,t) = x^4 y^2 + y^2 z + z^3$$

Find out (i) $\frac{\partial f}{\partial t}$ and (ii) $\frac{df}{dt}$

(3) $f = \sin(\omega t + k)$ $\omega, k \rightarrow$ constant

Find out. (i) $\frac{\partial f}{\partial t}$ & (ii) $\frac{df}{dt}$

(4) ~~$\frac{d\vec{r}}{}$~~ is

(4) $(x, y, z) \longrightarrow (r, \theta, \phi)$

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

(i) Calculate kinetic energy of a free particle

$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$ in $(r, \theta, \phi, \dot{r}, \dot{\theta}, \dot{\phi}$ form)

where $\dot{x} = \frac{dx}{dt}$

Hints: use the partial derivative of $\frac{d\vec{r}}{dt}$ where

$\Rightarrow \vec{r} = \vec{r}(r, \theta, \phi, t)$

(ii) if $V = 0$, calculate

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} \Rightarrow ?$$

& $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \Rightarrow ?$$

[$$d\vec{r} = \frac{\partial r}{\partial q_1}dq_1 + \frac{\partial r}{\partial q_2}dq_2 + \cdots + \frac{\partial r}{\partial t}dt$$

$$\dot{r} = \frac{d\vec{r}}{dt} = \frac{\partial r}{\partial q_1}\dot{q}_1 + \frac{\partial r}{\partial q_2}\cdot\dot{q}_2 + \cdots + \frac{\partial r}{\partial t}$$

where $\dot{q} = \frac{dq}{dt}$]