Department of Mathematical Sciences

Rajiv Gandhi Institute Of Petroleum Technology, Jais

MA111 / REAL ANALYSIS & CALCULUS

Week 3 / August 2023	Problem Set 1	GR

Real Analysis

Notations : Set of Real Numbers ($\mathbb{R}$), Set of Natural Numbers ($\mathbb{N}$), sup is *supremum / least upper bound*, inf is *infimum / greatest lower bound*.

Properties of $\mathbb{R}$: Archimedean Property, lub property etc.

■ Tutorial Problems

1. If a be a real number such that $0 \leq a < \frac{1}{n}$ for every natural number n, then show that $a = 0$. [**Hint:** Use Archimedean Property]

2. If y be a positive real number, show that there exists a natural number m such that $0 < \frac{1}{2^m} < y$. [**Hint:** Use Archimedean Property]

3. Find the solution set of the inequality $\left|\dfrac{x+3}{2x-6}\right| \leq 1$.

4. Find sup A and inf A, where

   (i) $A = \{x \in \mathbb{R} \,:\, x^2 < 1\}$,

   (ii) $A = \{x \in \mathbb{R} \,:\, 3x^2 + 8x - 3 \,<\, 0\}$,

   (iii) $A = \{\frac{1}{m} + \frac{1}{n} \,:\, m, n \in \mathbb{N}\}$,

   (iv) $A = \left\{\frac{n+(-1)^n}{n} \,:\, n \in \mathbb{N}\right\}$.

5. Let S be a non-empty subset of $\mathbb{R}$ bounded below. A lower bound ℓ of S is such that for each natural number n there exists an elements s_n in S satisfying $s_n < \ell + \frac{1}{n}$. Prove that $\ell = \inf S$.

   [**Hint:** Use the definition of infimum and the Archimedean Property]

■ Assignment Problems

1. A and B are non-empty bounded subsets of $\mathbb{R}$. Prove that

   (i) $\sup A \cup B = \max\{\sup A, \sup B\}$,

   (ii) $\inf A \cup B = \min\{\inf A, \inf B\}$.

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