Lets (u_1, u_2, u_3) are 3D co-ordinates where $\hat{u}_1, \hat{u}_2, \hat{u}_3$ are unit vector along u_1, u_2, u_3 respective

du_1, du_2, du_3 are infinitesimal increment along u_1, u_2, u_3 respectively.

$u_1, u_2, u_3 \rightarrow$ are orthogonal co-ordinate.

Change of length ~~along~~ $d\vec{l} = d\vec{r}$ where $\vec{r}$ = position vector.

$$\boxed{d\vec{r} = \hat{u}_1 h_1 du_1 + \hat{u}_2 h_2 du_2 + \hat{u}_3 h_3 du_3}$$

So, if x, y, z - co-ordinate,

$$d\vec{r} = \hat{x}\, dx + \hat{y}\, dy + \hat{z}\, dz$$

$$\Rightarrow h_1 = 1, \quad h_2 = 1, \quad h_3 = 1$$

$du_1 = dx$, $du_2 = dy$, $du_3 = dz$. $\hat{u}_1 = \hat{x}$, $\hat{u}_2 = \hat{y}$, $\hat{u}_3 = \hat{z}$

if (r, θ, ϕ) then, spherical co-ordinate.

$$d\vec{r} = \hat{r}\, dr + \hat{\theta}\, r\, d\theta + \hat{\phi}\, r\sin\theta\, d\phi$$

So,

u_1, u_2, u_3			$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$	h_1	h_2	h_3
r	θ	ϕ	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$	1	r	$r\sin$

if cylindrical (ρ, ϕ, z) co-ordinate.

u_1	u_2	u_3	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$	h_1	h_2	h_3
ρ	ϕ	z	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$	1	ρ	1

$$d\vec{r} = \hat{\rho}\, d\rho + \hat{\phi}\, \rho\, d\phi + \hat{z}\, dz$$

· f is a function, $f = f(x, y, z)$

$= f(r, \theta, \phi)$

$= f(\rho, \phi, z)$

$\bar{A}$ is a vector $\bar{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$

$= A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$

$= A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}$

In general, $f = f(u_1, u_2\ u_3)$

$\bar{A} = A_{u_1} \hat{u}_1 + A_{u_2} \hat{u}_2 + A_{u_3} \hat{u}_3$

$$\bar{\nabla} f = \frac{1}{h_1} \hat{u}_1 \frac{\partial f}{\partial u_1} + \frac{1}{h_2} \hat{u}_2 \frac{\partial f}{\partial u_2} + \frac{1}{h_3} \hat{u}_3 \frac{\partial f}{\partial u_3}$$

(x, y, z) $\bar{\nabla} f = \hat{x} \frac{\partial f}{\partial x} + \hat{y} \frac{\partial f}{\partial y} + \hat{z} \frac{\partial f}{\partial z}$

(r, θ, ϕ) $\bar{\nabla} f = \hat{r} \frac{\partial f}{\partial r} + \frac{1}{r} \hat{\theta} \frac{\partial f}{\partial \theta} + \frac{1}{r \sin\theta} \hat{\phi} \frac{\partial f}{\partial \phi}$

(ρ, θ, z) $\bar{\nabla} f = \hat{\rho} \frac{\partial f}{\partial \rho} + \frac{1}{\rho} \hat{\phi} \frac{\partial f}{\partial \phi} + \hat{z} \frac{\partial f}{\partial z}$

$f \rightarrow$ scaler, $\bar{\nabla} f \rightarrow$ vector.

$\Rightarrow$ normal to surface f.

$$\nabla\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3A_1) + \frac{\partial}{\partial u_2}(h_1h_3A_2) + \frac{\partial}{\partial u_3}(h_1h_2A_3)\right.$$

$(x,y,z) \rightarrow$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad [\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z$$

$(r,\theta,\phi) \Rightarrow$ spherical $\Rightarrow \vec{A} = \hat{r}A_r + \hat{\theta}A_\theta + \hat{\phi}A_\phi$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta A_r) + \frac{\partial}{\partial\theta}(r\sin\theta A_\theta) + \frac{\partial}{\partial\phi}(rA_\phi)\right]$$

[as, $h_1=1$, $h_2=r$, $h_3=r\sin\theta$]

$(\rho,\phi,z) \rightarrow$ cylindrical $\rightarrow \vec{A} = \hat{\rho}A_\rho + \hat{\phi}A_\phi + \hat{z}A_z$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{\partial}{\partial\phi}(A_\phi) + \frac{\partial}{\partial z}(\rho A_z)\right]$$

Curl $\Rightarrow \vec{\nabla}\times\vec{A}$

u_1, u_2, u_3

$$(x,y,z) \Rightarrow \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{u}_1 & h_2\hat{u}_2 & h \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \\ h_1A_1 & h_2A_2 & \end{vmatrix}$$

$(r,\theta,\phi) \Rightarrow \vec{\nabla}\times\vec{A}$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$