Q 1) For the position vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

show that

$$\boxed{\nabla r^n = n r^{n-2} \vec{r}}$$

Q 2) Find a unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 11$, at the point $(4, 2, 3)$

Q 3) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at point $(2, -1, 2)$

Q 4 For the position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Show that

(i) $\operatorname{div} \vec{r} = 3$

(ii) $\operatorname{div}\left(\frac{\vec{r}}{r^3}\right) = 0$

(iii) $\operatorname{div}(r^n \vec{r}) = (3+n) r^n$

$\vec{r} \rightarrow$ stands for vector

$r \rightarrow$ magnitude of $\vec{r}$

Q 5] Show that the vector field

$$\vec{V} = \frac{-x\hat{i} - y\hat{j}}{\sqrt{x^2+y^2}}$$

is a sink.

Q6 For the position vector

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

Show that

i) $\text{curl}\ \vec{r} = 0$

ii) $\text{curl}\ \dfrac{\vec{r}}{r} = \dfrac{-\hat{i}y + \hat{j}x}{r^3}$

iii) $\text{curl}\ r^n \vec{r} = 0$

$\vec{r} \rightarrow$ stands for vector

r is its magnitude

Q7 Find the values of a, b, c so that the function

$$\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$$ is irrotational

Q8 Show that

$$\vec{F} = (2xy+z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$ is a conservative force field

Q9) Find a unit normal to the surface

$$2x^2 + 4yz - 5z^2 = -10$$

at the point $P(3, -1, 2)$

Q10) (a) If $\vec{A} = (2xy + z^3)\hat{i} + (x^2 + 2y)\hat{j} + (3xz^2 - 2)\hat{k}$

Show that $\nabla \times \vec{A} = 0$

(b) Find a scalar function ϕ such that $\vec{A} = \nabla \phi$

Q11) If $\phi = x^2 y z^3$ and $A = xz\,\hat{i} - y^2\,\hat{j} + 2x^2 y\,\hat{k}$

find (a) $\nabla \phi$

(b) $\nabla \cdot \vec{A}$

(c) $\vec{\nabla} \times \vec{A}$

(d) $\operatorname{div}(\phi \vec{A})$

(e) $\operatorname{curl}(\phi \vec{A})$

Q12 Let $\vec{\mathscr{r}}$ be the seperation vector from a fixed point (x', y', z') to the point (x, y, z), and let $\mathscr{r}$ be its length. Show that

(a) $\nabla(\mathscr{r}^2) = 2\vec{\mathscr{r}}$

(b) $\nabla\left(\frac{1}{\mathscr{r}}\right) = -\hat{\mathscr{r}}/\mathscr{r}^2$

(c) What is the general formula for $\nabla(\mathscr{r}^n)$?