Complex convolution filters

A pulsar can be accelerated due to orbital motion. This acceleration can be approximated in a constant value of the spin frequency derivative for a short integration time. A constant spin frequency derivative can curve the time vs phase plot.

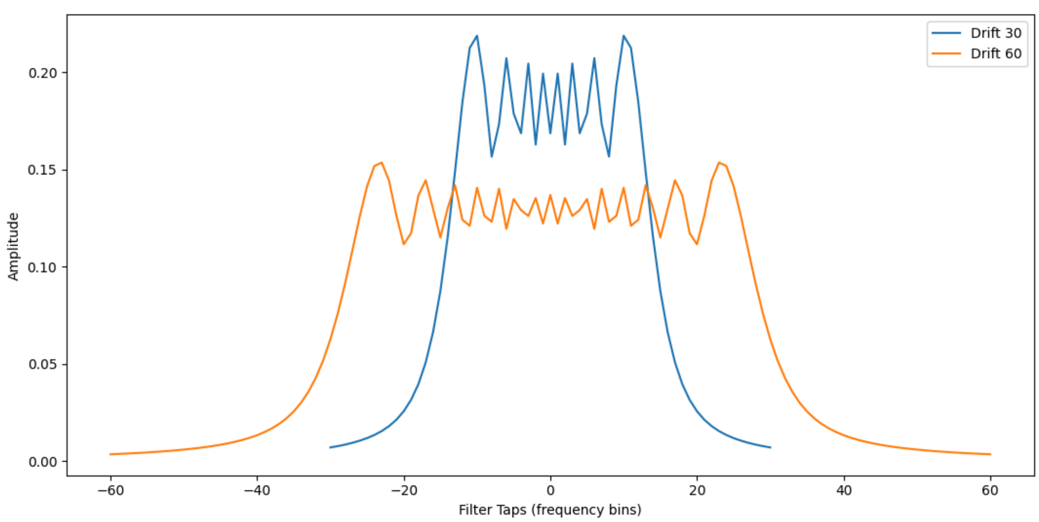
The frequency domain response of a constant spin frequency derivative was derived by Ransom et al 2002 (Scott M. Ransom *et al* 2002 *AJ* 124 1788). The harmonic signals suffer redistribution of signal in neighboring frequency bins due to acceleration, known as drifts. There are ways to recover the harmonic power of an accelerated signal in the frequency domain. One direct method is by matched filtering process with a set of complex filters matching the frequency response of the acceleration.

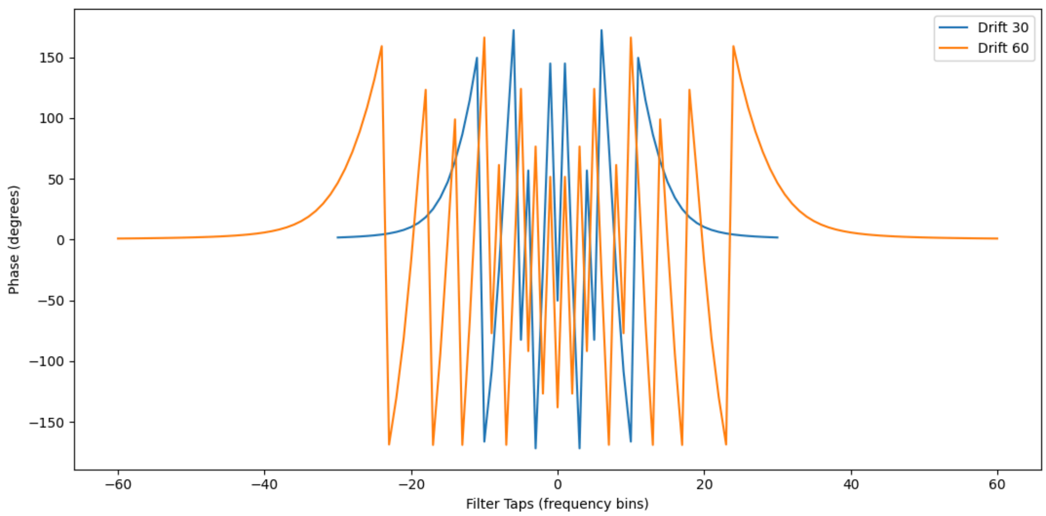
All the results discussed below are for a particular harmonic signal. In a real pulsar search, recovered power from a number of harmonics are added together but these results are just considering a single harmonic of a periodic signal.

Filter Shape and Characteristics:

The filters are a combination of Fresnel integrals along with an extra phase term. The shape of amplitudes and phase terms are determined by the drift being corrected.

The following figures show the amplitude and phase of two filters, one for a drift of 30 bins (blue curve) and the second for a drift of 60 bins (orange curve).





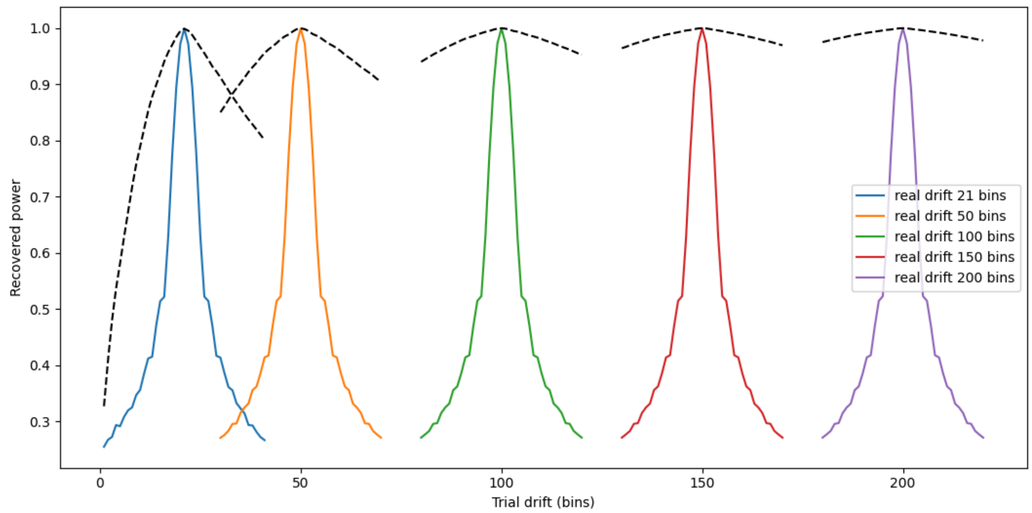
I note two features of these filters,

1. The amplitude as well as the phase plot show modulations.
2. The number of nodes in the phase plot is directly proportional to the drift. We have 8 peaks/dips in the 30 bin drift, and 16 peaks/dips in 60 drift bins, roughly 4 bins per peak/dip.

This shows that the number of nodes will increase linearly as the drift bins increase. While selecting filter widths, we should keep in mind that the mismatch should not exceed 4-5 bins as that will result in a 180-degree phase difference. This would result in degradation in recovered power.

Degradation in recovered power

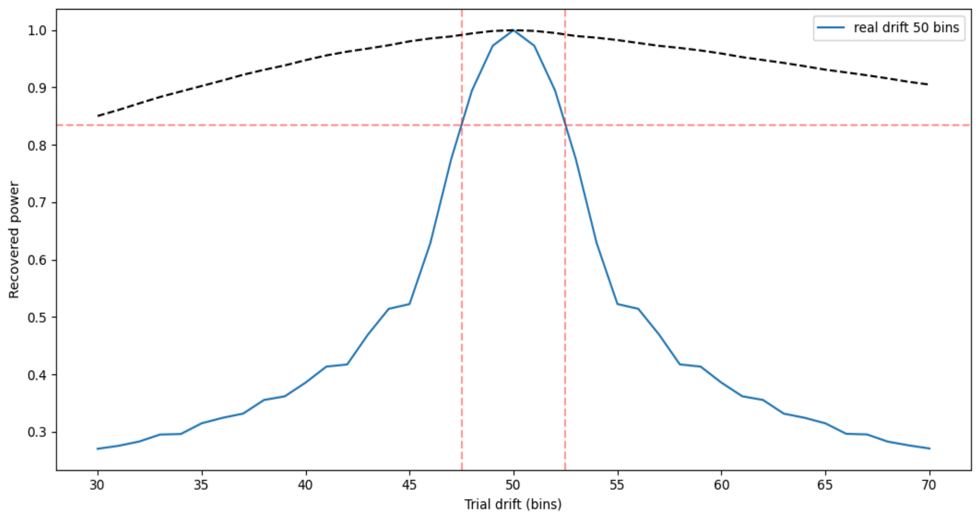
Here I try to visualize the power recovered by the convolution using different sizes of filters. I try five different values for the actual drift of the signal and then convolve it with several trial filters and note the highest recovered power for each trial filter. We see that the power degrades by a factor of more than two if the mismatch in filter widths is larger than 5 bins. This degradation is independent of the actual drift in the harmonic signal and is the same for both large and small drifts. The black dashed curves show the trend in the recovered power if we use only the amplitude of the filters and ignore the phase.



This suggests against using a variable width separation between trial filters.

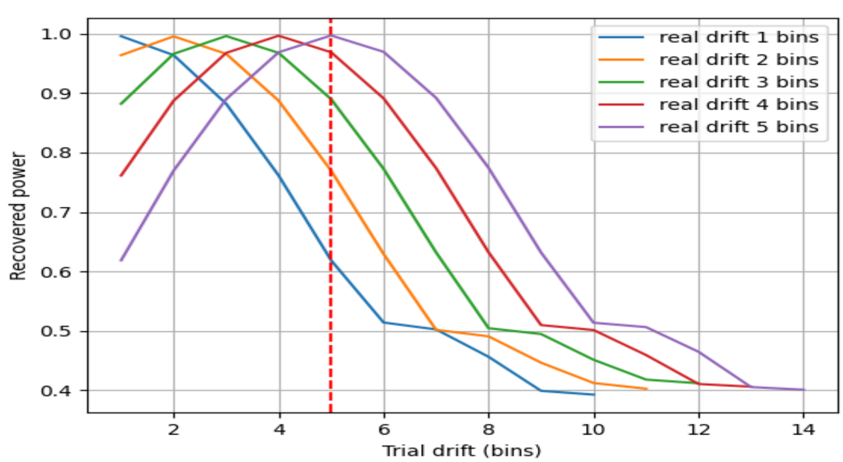
Acceptable separation for filter widths

The following figure shows recovered power vs trial width for a harmonic signal with 50 bins drift. The red dashed lines represent the possible mismatch in filter widths if we use a width separation of 5 bins in the trial widths (the maximum possible mismatch is 2.5 bins). The maximum loss in this case is less than 20%.



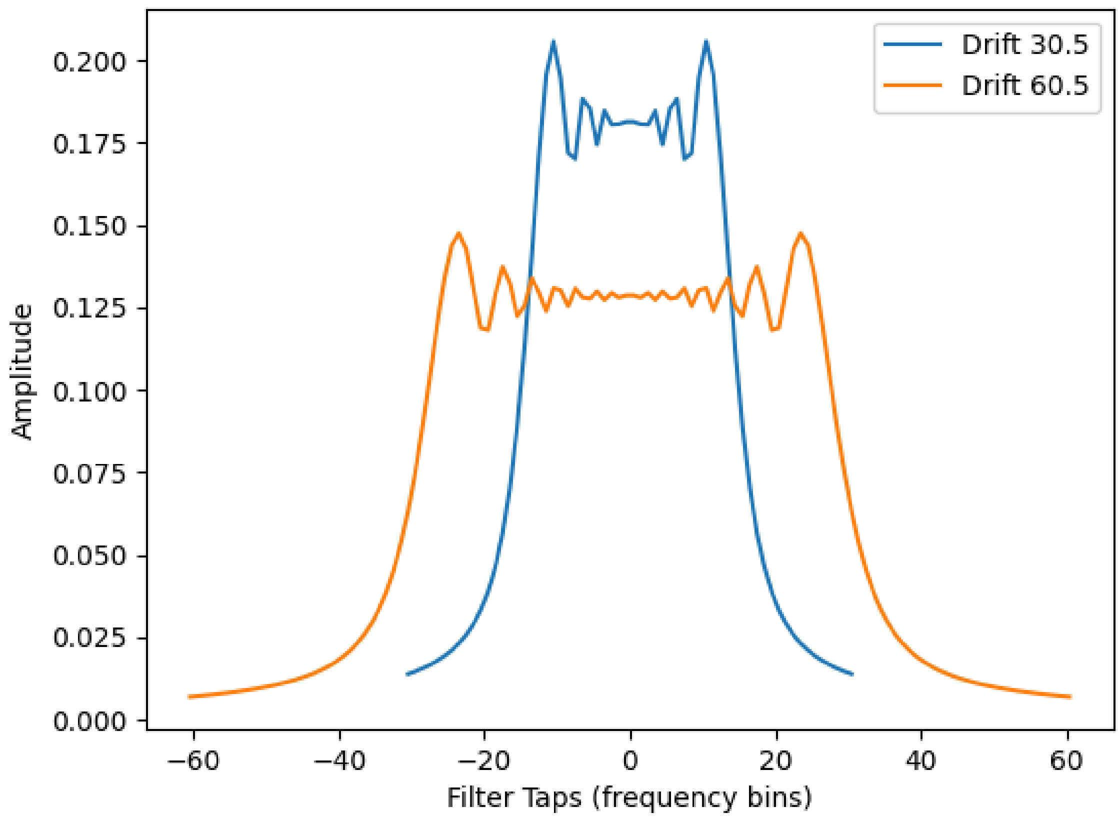
Performance at very small drifts:

One of the main concerns with the 5-bin separation in filter trial widths is that sampling is too coarse for small drifts. The following figure shows the recovered power by different trial filters on a harmonic with drifts of 1,2,3,4, and 5 bins. The red dashed lines represent the trial width of 5 bins. The trial width of 5 bins is recovering more than 70% of the signal power for the actual signal drift of 2 bins.

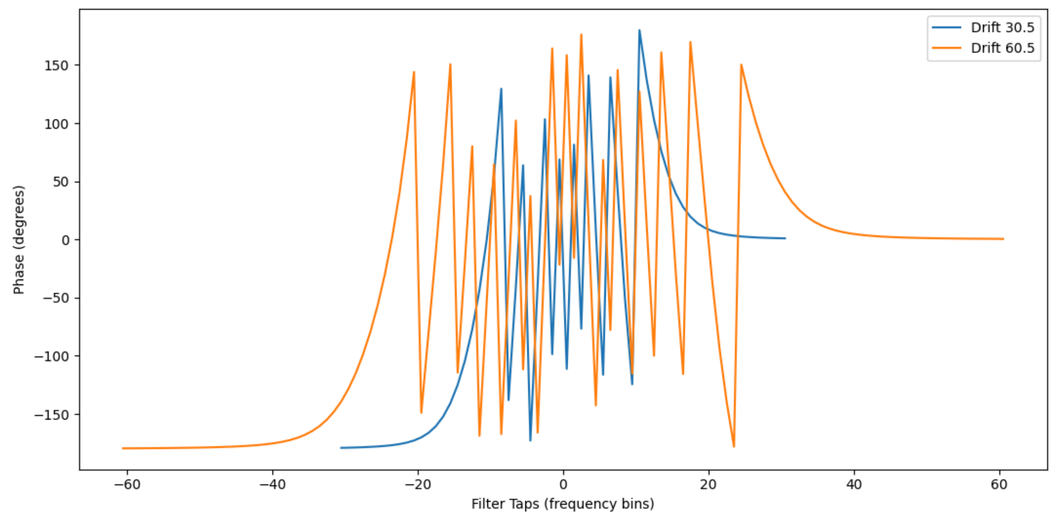


Filters with fractional length:

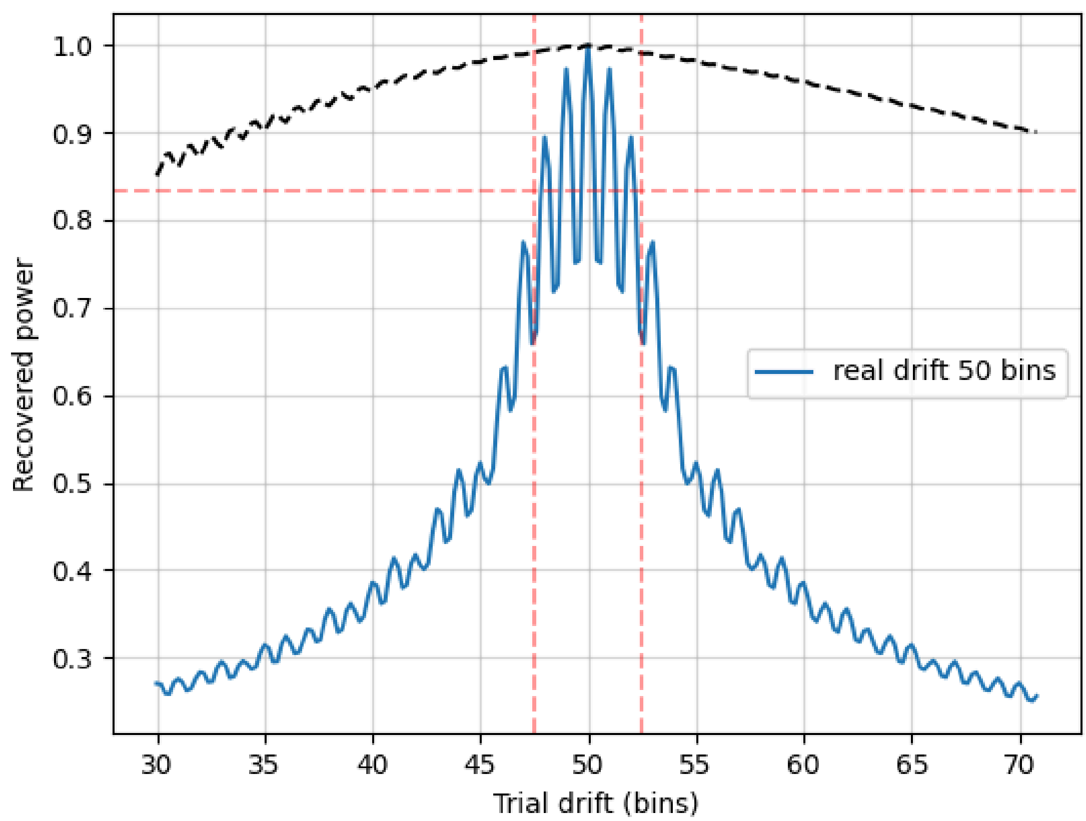
The actual drift caused by acceleration can be fractional in terms of the number of bins. But with fractional drifts, the convolution results show peculiar behavior. Even the filters produced for these fractional widths have very different characteristics.



The filter amplitudes show weaker modulations on them.

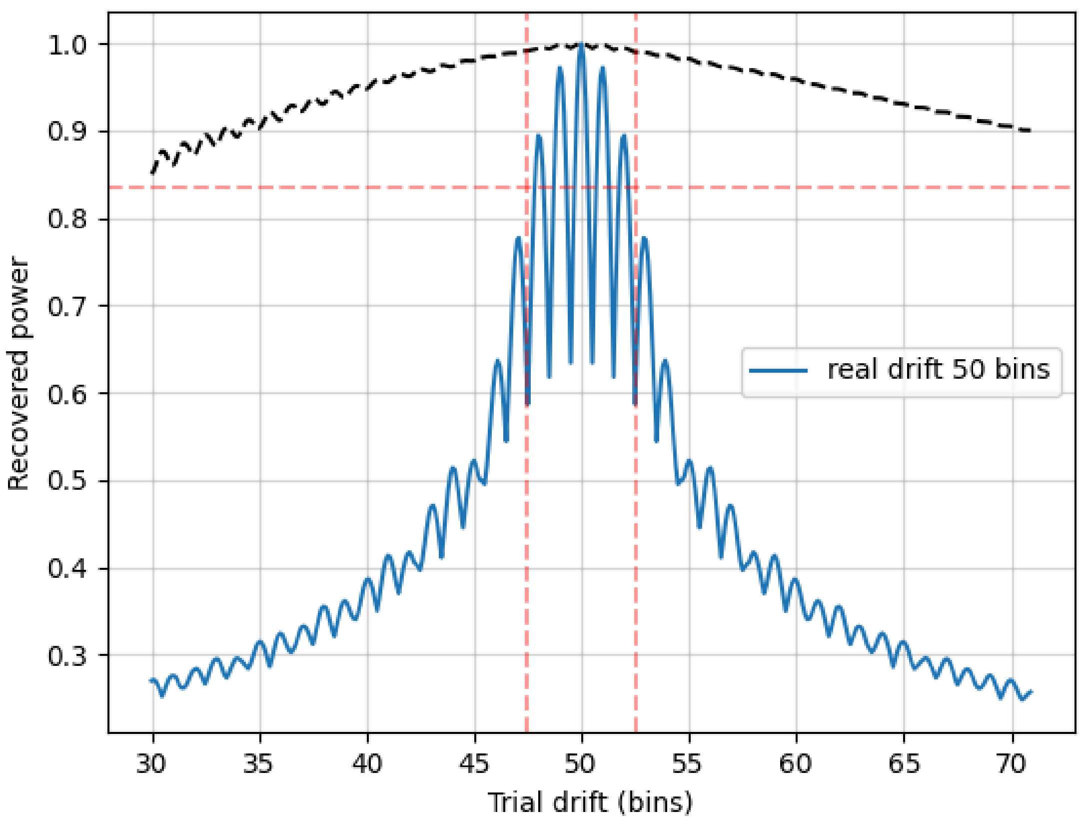


Filters have non-zero phases outside the drift range and they are no more symmetric.

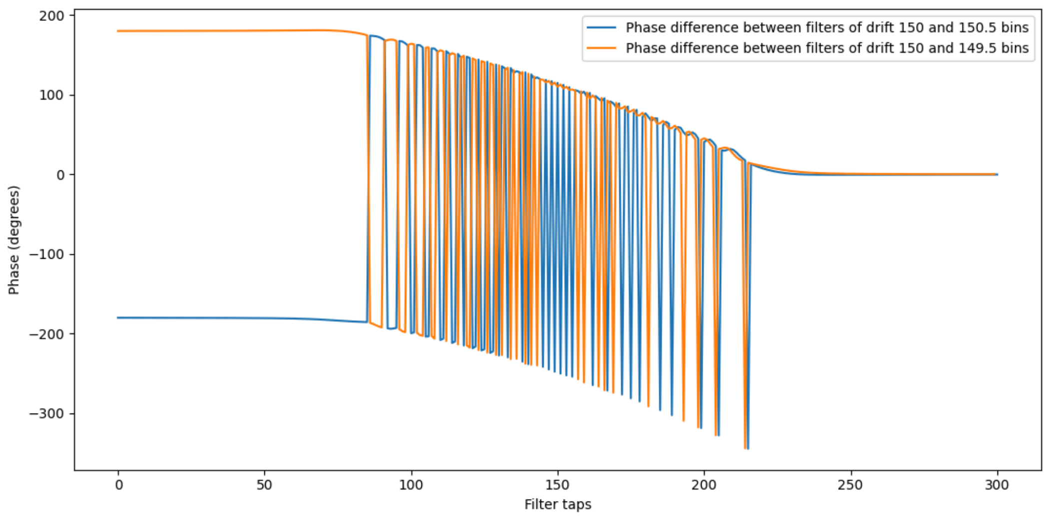


Even if we try fractional trial widths on an integer drift, we get oscillations in the overall response curve. The black curve is the degradation trend for incoherent recovery. We use a spacing of 0.2 bins in the trial widths, while the actual drift of the signal is 50 bins.

The depth of modulation increases when we have smaller spacing in the trial widths. In the following figure, we use 0.1 bin spacing. It turns out that the deepest point in dip appears at a separation of 0.5 from each integer drift (i.e. the recovered power degrades the most when the mismatch is an odd multiple of 0.5).



Phase difference due to fractional drift



There is a major change in the phase of the filter if the drift is a fractional number. The above plot shows phase differences for drifts 150, 150.5, and 149.5 bins. As one may notice, the phase difference can have values near 180 degrees at some filter taps. This is potentially causing the dips in the recovered power when filter mismatch is an odd multiple of 0.5 drift.