$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\,\sin(nt)\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -1\cdot\sin(nt)\,dt + \int_{0}^{\pi} 1\cdot\sin(nt)\,dt$$

$$= \frac{1}{\pi}\left[\frac{\cos(nt)}{n}\right]_{-\pi}^{0} - \frac{1}{\pi}\left[\frac{\cos(nt)}{n}\right]_{0}^{\pi}$$

$\cos(-\theta)=\cos\theta$

$$= \frac{1}{\pi}\left[\frac{1}{n} - \frac{\cos(-n\pi)}{n}\right] - \frac{1}{\pi}\left[\frac{\cos(n\pi)}{n} - \frac{1}{n}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n} - \frac{\cos(n\pi)}{n} - \frac{\cos(n\pi)}{n} + \frac{1}{n}\right]$$

$$= \frac{1}{\pi}\left[\frac{2}{n} - \frac{2}{n}\cos(n\pi)\right]$$

$$= \frac{2}{n\pi}\left(1-\cos(n\pi)\right)$$

$$= 2/n\pi\left(1-(-1)^n\right) \qquad \text{con}(n\pi) = (-1)^n$$

$b_1 = \frac{4}{\pi}$; $b_2 = 0$; $b_3 = \frac{4}{3\pi}$; $b_4 = 0$, $b_5 = \frac{4}{5\pi}$,

$$x(t) = b_1 \sin(t) + b_3 \sin(3t) + b_5 \sin(5t) + \cdots$$

$$= \frac{4}{\pi}\left(\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\right)$$

$$= \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\sin((2n-1)t)}{2n-1}$$

20. Starting from the principle of orthogonality for continuous-time sinusoids, show that a piece-wise regular real-valued signal $x(t)$ periodic in $[-\pi, \pi]$ can be expressed as a sum of sinusoids (under certain conditions) as

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\,t) + b_n \sin(n\,t).$$

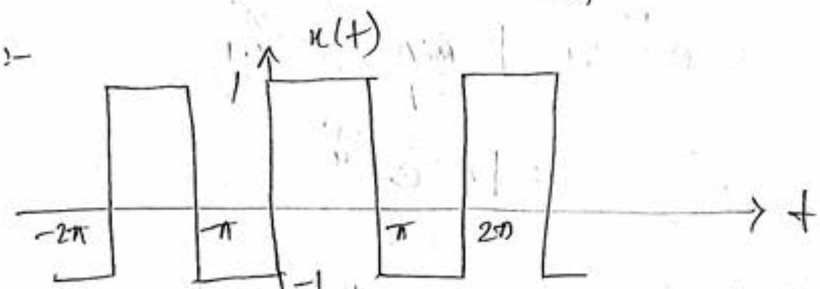
20. $$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(nt) + b_n \sin(nt)\right)$$

DC = avg. value of the signal

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\,dt \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\cos(nt)\,dt$$

$$= \frac{1}{2} a_0 \qquad a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\,dt$$

Example :-



$$x(t) = \begin{cases} -1, & -\pi < t < 0 \\ 0, & t = 0 \\ 1, & 0 < t < \pi \end{cases}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x(t)\cos(nt)\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -1\cdot\cos(nt)\,dt + \frac{1}{\pi}\int_{0}^{\pi} 1\cdot\cos(nt)\,dt$$

$$= -\frac{1}{\pi}\left[\frac{\sin(nt)}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{\sin(nt)}{n}\right]_{0}^{\pi}$$

$= 0$

$a_0 = 0 \Rightarrow DC = 0$; $a_n = 0 \Rightarrow$ there are no cosine components.

23. Classify the following systems in terms of memory and causality:

(a) $y(t) + y(t-1) = x(t)$

(b) $y(t) = x(t+2)$

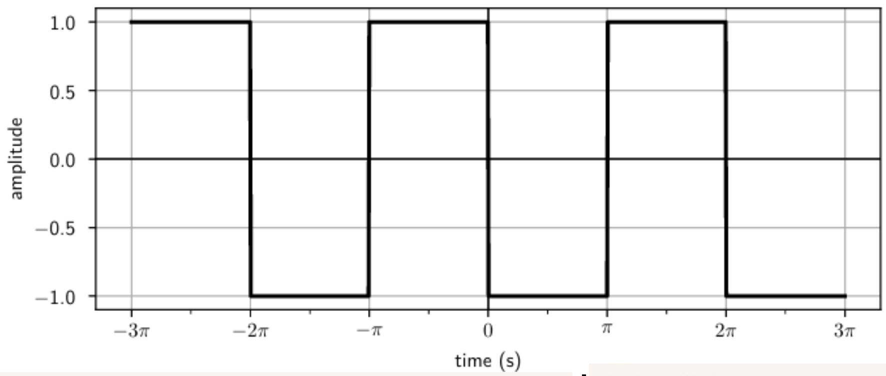
(c) $y(t) = x(-t+1)$

(d) $\dfrac{dy(t)}{dt} = 5y(t) + x(t)$

(e) $y(t) = 5t$

23. i) Memory, causal
ii) Memory, non-causal
iii) Memory, non-causal
iv) Memory, causal
v) Memoryless, Acausal.

21. Consider the following signal $x(t)$:



Is $x(t)$ bounded?

Yes, it is. The function only takes values **+1 and -1**, so it's bounded.

Bounds:

$$-1 \leq x(t) \leq 1$$

Is $x(t)$ periodic?

Yes. From the waveform, we observe that it repeats every 2π units.

Period:

$$T = 2\pi$$

Is $x(t)$ even or odd?

The waveform is **odd**. This is because it satisfies:

$$x(-t) = -x(t)$$

This odd symmetry tells us a lot about its Fourier series structure.

Which Fourier components are absent?

For an odd function:

- **All cosine terms** a_n are **zero**
- **The DC term** a_0 is **zero**

Only **sine components** b_n will remain in the Fourier series.

Express $x(t)$ as a piecewise function:

Over one period, say from $-\pi$ to π, you can write:

$$x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & -\pi < t < 0 \end{cases}$$

And extend this definition periodically.

Fourier Series for $x(t)$

Since it's odd and periodic with $T = 2\pi$, the Fourier sine coefficients :

$$b_n = \frac{2}{T}\int_0^{\pi} x(t)\sin(nt)\,dt = \frac{2}{\pi}\int_0^{\pi} \sin(nt)\,dt$$

Computing this integral:

$$b_n = \frac{2}{\pi}\left[\frac{-\cos(nt)}{n}\right]_0^{\pi} = \frac{2}{\pi}\cdot\frac{1-(-1)^n}{n}$$

So:

$$b_n = \begin{cases} \frac{4}{n\pi}, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

19. With suitable examples, define signals and systems. With suitable examples, explain the following properties of a system: memory, causality, invertiblity, stability, linearity and time-invariance.

19. Signal :- It is a function that carries information.

→ It can represent physical quantities like voltage, temp or sound etc.

→ Eg. Audio signal, Image etc.

Systems :- A system is a process that operates on a signal to produce a new signal.

→ It can be a phisical device or a set of mathematical rules.

→ Eg: Amplifier, filter, communication, system etc.

Memory :- A system has memory if its output at any given time depends on past or future.
eg: moving average filter.

Causality :- The present output depends on the past and present input.

eg: A simple amplifier.

Invertibitity: A system is invertible if its input can be uniquely recovered from its output.

$$y(t) = 2x(t) \longleftrightarrow x(t) = \frac{1}{2} y(t)$$

Stability :- A stable system produces a bounded output for any bounded input.

eg. a system that outputs the squre root of the input, as long as the input is positive.

linearity :- A linear system satisfies two properties additiry & homogenetive.

eg) A simplg resistor.

18. Find the Fourier transform of the signal $x(t) = e^{-t^2}$.

18) Fourier transform of the Gaussian signal $x(t) = e^{-t^2}$

The signal is $x(t) = e^{-t^2}$

~~That~~ Fourier transform of a Gaussian function,

$$e^{-at^2} \text{ is } \sqrt{\frac{\pi}{a}}\, e^{\frac{\omega^2}{4a}}$$

Comparing it by general form e^{-at^2}, we find $a=1$,

$\therefore$ The fourier transform of $x(t)$,

$$= e^{-t^2} \text{ is } \sqrt{\frac{\pi}{1}}\, e^{-\frac{\omega^2}{4\cdot 1}}$$

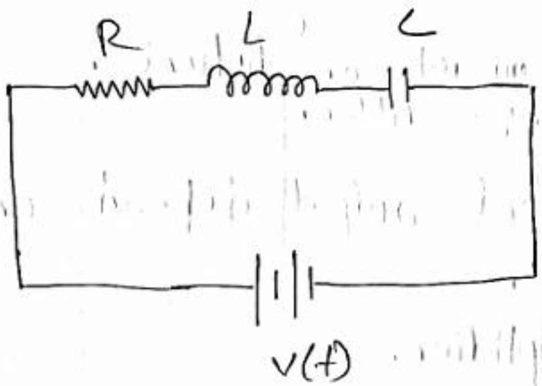
$$= \sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$$

Time-invariance :- A system is called time invariant if any advacement or delay in the input produces the same amount of delay or advancement on output.

eg: Electrical Circuit with ~~fri~~fixed component.

17. For a series R-L-C circuit, if the input voltage, and the voltage across the capacitor are considered to be $x(t)$ and $y(t)$, respectively, find the relationship between them.

17.



By kirchoff's Law,

$$x(t) = V_R(t) + V_L(t) + V_C(t)$$

Now, expressing V in terms of i(t),

$$V_R(t) = Ri(t) = RC\, dy(t)/dt$$

$$V_L(t) = L\frac{di(t)}{dt} = LC\frac{d^2y(t)}{dt^2}$$

$$V_C(t) = \frac{1}{C}\int i(t)\,dt = y(t)$$

So, current in terms of capacitor voltage:

$$i(t) = C\frac{dy(t)}{dt}$$

$$\therefore x(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t)$$

$$x(t) = LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t)$$

16. What are the differences between discrete-time signals and digital signals? What is sam-

16.

Discrete time Signal	Digital Signals
→ This signal exsist only at specific, uniformely spaced time instance → Can be processed theoretically using mathematical.	→ These Signals are discrete in both time and amplitude → Can be processed practically using digital systems.

Sampling :- It is the process of converting a continuous -time signal into a discrete-time signal by talking mesurement at regular intervals.

→ The frequency at which it takes snapshot of a antiyous signal at specific point in time is called sample rate.

→ Aliasing occurs when the sampling rate is insufficient to capture the changes in the original continuous time signal.

→ If signal ~~continu~~ contains frequency higher than the ~~Nyqu~~ Nyquist frequency (half the sansling rate), these frequencies with be misrepresented, as lower frequencies in the sampled.

Nyquist Frequency :- The nyquist frequency is half the sampling rate. of a discrete -time- signal.

- It represents the highest frequency component that can be accurately represented in the sampled signal without aliasing.

- If the signal a sampled at 1000 Hz. the Nyquist frequency is 500 Hz. But any frequency above 580 Hz will be aliased.

14. If a signal $x(t)$ is given, describe the steps for obtaining $-2x\left(-\frac{t}{2}+1\right)-1$.

14. $-2x\left(-\frac{t}{2}+1\right)-1 = -2x\left(-\frac{1}{2}(t-2)\right)-1$

i) Time reflection → ✓

ii) Time Scaling → $\frac{1}{2}$ (expansion)

iii) Time shift → Delay or Right shift (-2 second)

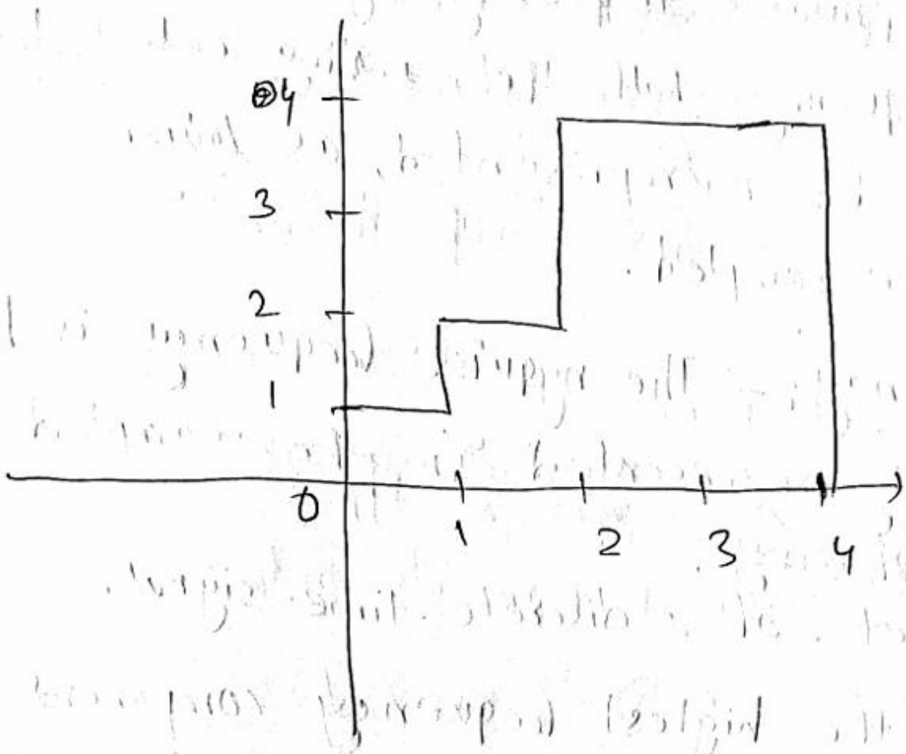
iv) Amplitude reflection → ✓

v) Amplitude scaling → 2 (~~attenuation~~ amplification)

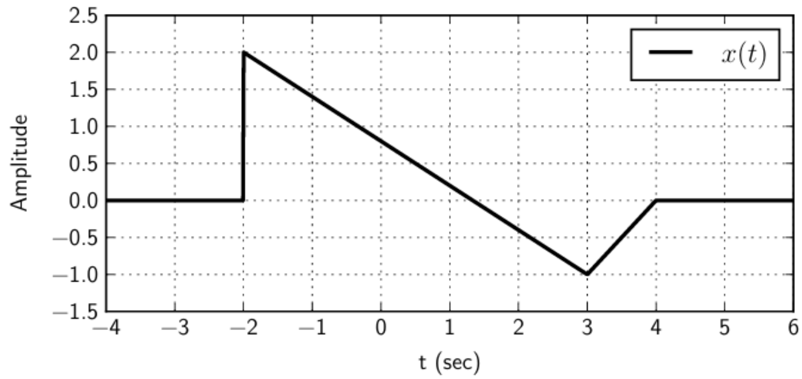
vi) Amplitude shift → -1 down shift.

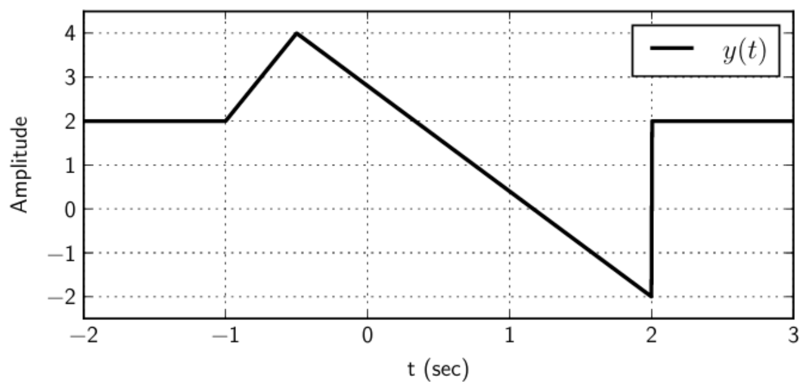
15. Draw the signal $x(t) = u(t) + u(t-1) + 2u(t-2) - 4u(t-4)$. (A linear graph paper will be provided if needed).

15.



13. Given two signals $x(t)$ and $y(t)$. Express $y(t)$ in terms of $x(t)$ and $x(t)$ in terms of $y(t)$.





13. i) Time reflection →

ii) Time scaling → $\frac{AD}{A'D'} = \frac{4-(-2)}{2-(-1)} = \frac{6}{3} = 2$ 6

(Contraction)

iii) Time shift → Delay or Right shift (−1 seconds)

iv) Amplitude reflection →

v) Amplitude scaling → $\frac{B'C'}{BC} = \frac{6}{3} = 2$ (Amplification)

vi) Amplitude shift → +2 upshift.

$$y(t) = -2x(-2(t-1)) + 2$$

11. The impulse response of a system is given by $h(t)$. What would be the output of the system to an arbitrary input signal $x(t)$?

11. The output of a system with impulse response $h(t)$ to an arbitary input signal $x(t)$ is given by the convolution of $x(t)$ and $h(t)$. This means the output, $y(t)$ can be calculated by integral of convoleved input signal.

$$y(t) = \int_{-\infty}^{\infty} x(t) h(t-T) dT$$

9. Calculate the Fourier transform of $\delta(t + a)$.

9) The fourier transform of a shifted delta function

$\delta(t+a)$

Formula: $-F\{\delta(t+a)\} = e^{jua}$

$$\int_{-\infty}^{\infty} \delta(t+a)\, e^{-j\omega t}\, dt$$

using the shift property

$e^{-j\omega(-a)} = e^{j\omega a}$

$F\{\delta(t+a)\} = e^{j\omega a}$ (Ans)

10. Is the system given by the input-output relation $y(t) = \dfrac{1}{x(t)+1}$, BIBO stable?

10) $y(t) = \dfrac{1}{x(t)+1}$

BIBO stability criteria:

A system is BIBO Stable if every bounded input $x(t)$ produces a bounded output $y(t)$.

This is:

if $|x(t)| \leq M$ for all t, them

$|y(t)| \leq N$ for some finite N

$y(t) = \dfrac{1}{x(t)+1}$

let suppose the input is bounded i-e,

$|x(t)| \leq M$ for all t,

Then, $x(t) \in [-M, M]$ so,

$x(t)+1 \in [-M+1, M+1]$

if $x(t) = -1$ then,

$y(t) = \dfrac{1}{t+1} = \dfrac{1}{0} \rightarrow \infty$

So, if the bounded input $x(t)$ can reach -1 the output becomes unbounded.

Therefor, the system is not BIBO stable.

7. What is the force-displacement relationship of a viscous damper? What is the unit of damping coefficient?

7. In a varions damper the force is proportional to velocity the relationship is given by

$$F = \frac{c \cdot dx}{dt}$$

where,

F = damping force (in Newton N)

e = damping efficient

$\frac{dx}{dt}$ = velocity (rate of chang of displacement with respect of time)

$$\boxed{F = c_i\, v(t)}$$

Unit of Damping efficient (c) is kg/s or Ns/m.

8. Evaluate: $\int_{-\infty}^{\infty} \frac{\sin(t-\frac{\pi}{2})}{t}\delta(t+\pi)dt.$

8. Evaluate $\int_{-\infty}^{\infty} \frac{\sin(t-\pi/2)}{t} \cdot \delta(t+\pi)dt$

Using the delta function shifting Properly.

$\delta(t+\pi)dt$

$\delta = (t-(-\pi))\, dt$

there the value of function at $t = -\pi$

$$f(t) = \frac{\sin(t-\pi/2)}{t}$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t+\pi)\,dt = f(-\pi)$$

Now, compute $f(-\pi)$

$$f(-\pi) = \frac{\sin(-\pi-\pi/2)}{-\pi}$$

$$= \frac{\sin(-3\pi)}{2}$$

$$= -\sin(3\pi/2)$$

$$= -(-1) = 1$$

So, $f(-\pi) = \frac{1}{-\pi} = \frac{1}{-\pi}$ (Ans)

6. Determine the even component of the signal $x(t) = u(t) - u(t-3)$.

6. $x(t) = u(t) - u(t-3)$

$$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)]$$

The unit step function $u(t)$ is defined as,

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$$

So,

$$x(t) = u(t) - u(t-3)$$

$$= \begin{cases} 0, & t<0 \\ 1, & 0 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Now, computing $x(-t)$

we flip the signal about vertical axis

$$x(-t) = u(-t) - u(-t-3)$$

$$\begin{cases} 1, & -3 < t \leq 0 \\ 0, & \text{otherwise} \end{cases}$$

Now, computing the even component,

$$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)]$$

Combine the value of $x(t)$ & $x(-t)$

t value	$x(t)$	$x(-t)$	$x_{even}(t)$
$t<-3$	0	0	0
$-3<t<3$	0	1	$\frac{1}{2}(0+1) = \frac{1}{2}$
$t=0$	1	1	$\frac{1}{2}(1+1) = 1$
$0<t<3$	1	0	$\frac{1}{2}(1+0) = \frac{1}{2}$
$t>3$	0	0	0

$$x_{even}(t) = \begin{cases} \frac{1}{2}, & -3<t<0 \\ 1, & t=0 \\ \frac{1}{2}, & 0<t<3 \\ 0, & \text{otherwise} \end{cases}$$ (Ans)

3. Write an example of an anti-causal system.

3. All static system are causal because they depends Only on the current input but all causal system are not static because causal system can depend on past input which is making the dynamic.

4. "Every static system is causal but not every causal system is static," – explain.

4. Example of anti-causal system,

$$y(t) = ax(t+2)$$

5. Find the period of the signal $\sin(t) + \cos(\pi t)$.

5. $x(t) = \sin(t) + \cos(\pi t)$

Period of $\sin(t)$

$T_1 = 2\pi$

Period of $\cos(\pi t)$

$\cos(\pi t) = \cos(\omega t)$, where $\omega = \pi$

$$T_2 = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$$

The period of the signal $\sin(t) + \cos(\pi t)$ in 2π.

1. State two differences between static and dynamic systems.

1.

Static	Dynamic
→ These are time invariant	→ These are time-varing.
→ The systems are memory-less.	→ They have memory.
→ They are hystresis free.	→ They have hystresis.

2. Is the system given by the input-output relation $y(t) = 3x(t) + 4$, linear?

2. $y(t) = 3x(t) + 4$

Additivity :-

$y_1(t) = 3x_1(t) + 4$

$y_2(t) = 3x_2(t) + 4$

Input :- $x_1(t) + x_2(t)$

output :- $y(t) = 3(x_1(t) + x_2(t)) + 4$

$= 3x_1(t) + 3x_2(t) + 4$

$(3x_1(t) + 4) + (3x_2(t) + 4)$

$\Rightarrow 3x_1(t) + 3x_2(t) + 8$

So, it ~~dose~~ doesn't hold additivity.

Homogenity :-

ut the input scaled.

If input $= x(t)$, output $= y(t) = 3x(t) + 4$

Now scale the input $ax(t)$

Output beomes,

$y_a(t) = 3ax(t) + 4$

$\Rightarrow a.y(t) = a(3x(t) + 4) = 3ax(t) + 4a$