(a) Sneha Kabaria

$$G_p + R_x \underset{k_-}{\overset{k_+}{\rightleftharpoons}} (G_p : R_x)_c$$
$$(G_p : R_x)_c \xrightarrow{k_I} (G_p : R_T)_o$$
$$(G_p : R_x)_o \xrightarrow{k_A} R_x + G_p$$
$$(G_p : R_x)_o \xrightarrow{k_{E,j}} m_p + R_x + G_p$$

$r_{x,j} = k_{E,j} (G_j : R_x)_o$, $\hat{r}_{x,j} = r_{x,j} \cdot u(I)$

Material Balances

closed complex > $\frac{d}{dt}(G_j : R_x)_c = k_+ (G_j)(R_x) - k_- (G_p : R_x)_c - k_I (G_j : R_x)_c = 0$ (1)

open complex > $\frac{d}{dt}(G_j : R_x)_o = k_I (G_j : R_x)_c - k_A (G_j : R_x)_o - k_{E,j} (G_j : R_x)_o = 0$ (2)

RNAP > $R_{x,T} = R_x + (G_j : R_x)_c + (G_j : R_x)_o$ (3)

Assume steady-state

solve (1) and (2)

$$(G_j : R_x)_c \approx \left(\frac{k_+}{k_- + k_I}\right)(G_j)(R_x) \quad (4)$$

$$(G_j : R_x)_o \approx \left(\frac{k_I}{k_A + k_E}\right)(G_j : R_x)_c \quad (5)$$

$$K_{x,j}^{-1} = \left(\frac{k_+}{k_- + k_I}\right)$$

$$\tau_{x,j}^{-1} = \left(\frac{k_I}{k_A + k_E}\right)$$

$$(G_j : R_x)_o = (K_{x,j}^{-1})(G_j)(R_x)(\tau_{x,j}^{-1})$$

$$R_x = \frac{R_{x,T}(\tau_{x,j} K_{x,j})}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$(G_j : R_x)_o = \frac{R_{x,T} G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$\rightarrow \boxed{r_{x,j} = k_{E,j}\left(\frac{G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}\right)}$$

a. Constants

(see constants pdF)
For citations

- $k_{E,j} = e_x L^{-1} = \frac{42 \text{ nt/s}}{3075 \text{ nt}} = .0135 \ s^{-1}$
(Julia)

- $R_{X,T} = 30 \times 10^{-6}$ mM

- $G_j = \frac{2500 \text{ copies}}{\text{cell}} \times \frac{\text{mol}}{6.02 \times 10^{23} \text{ copies}} \times \frac{\text{cell}}{6.7 \times 10^{-13} L} = 3093.3$ mM
(Julia)

- $k_I = .024 \ s^{-1}$

- $K_{F,j} = \left(\frac{k_- + k_I}{k_+}\right) = k_I \left(\frac{k_- + k_I}{k_I \ k_+}\right) = 2.49 \times 10^{-5}$ mM
(Julia)
McClure Paper slope

- $\tau_{X,j} = \frac{k_{E,j} + k_A}{k_I} = \frac{k_{E,j}}{k_I} = .569$ (Julia)

- Assume $k_A << k_{E,j}$
$k_A << k_I$

Now that we have R_X we can get the ...

(b) $\tau = .5691$

since $\tau = \frac{k_{E,J}}{k_I} < 1$

$\tau \Rightarrow \boxed{\text{elongation limited}}$

(c) $\frac{dm_j}{dt} = \underbrace{\sum_{k=1}^{R} \sigma_{jk} r_k}_{\text{rates of production and degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution volume change}}$

mRNA balance $\frac{dm_j}{dt} = \underbrace{r_{x,j} u_j}_{\text{transcription}} - \underbrace{k^d_{x,j} m_j}_{\text{degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution}} = \underbrace{0}_{\text{@ steady state}}$

[mRNA] $\boxed{m_j = \frac{r_{x,j} u_j}{k^d_{x,j} + B^{-1} \dot{B}}}$

u(I) $u_j(I) = \frac{W_1 + W_2 f_I}{1 + W_1 + W_2 f_I}$

fraction $f_I = \frac{I^n}{K + I^n}$

dilution $\underbrace{B^{-1} \dot{B}}_{\text{specific growth rate}} = \frac{1}{30 \text{min}} \cdot \left(\frac{\text{min}}{60 \text{s}} \right) = .0005 s^{-1}$

$k^d_{x,j} = \frac{.693}{\text{half-life}}$ (assuming 1st order)

$= \frac{.693}{5(60 s)} = .0023 s^{-1}$

all in Julia (constants given in problem) or constants document