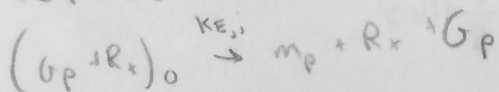
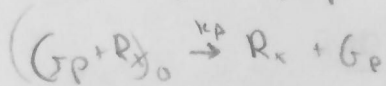
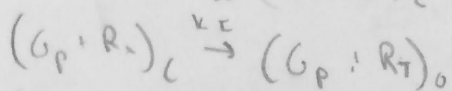
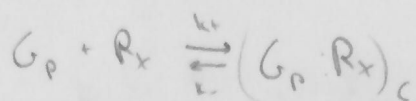


(a)

Sneha Kabaria

$$r_{x,j} = k_{E,j} (G_j \cdot R_x)_o \quad ; \quad \hat{r}_{x,j} = r_{x,j} \cdot U(I)$$

Material Balances

$$\text{closed complex} \rangle \frac{d}{dt} (G_j \cdot R_x)_c = k_1 (G_j)(R_x) - k_{-1} (G_p \cdot R_x)_c - k_{-2} (G_j \cdot R_x)_c = 0 \quad (1)$$

$$\text{open complex} \rangle \frac{d}{dt} (G_j \cdot R_x)_o = k_2 (G_j \cdot R_x)_c - k_A (G_j \cdot R_x)_o - k_{E,j} (G_j \cdot R_x)_o = 0 \quad (2)$$

$$\text{RNAP} \rangle R_{x,T} = R_x + (G_j \cdot R_x)_c + (G_j \cdot R_x)_o \quad (3)$$

↑
Assume
Steady-state

Solve (1) and (2)

$$(G_j \cdot R_x)_c \approx \left(\frac{k_1}{k_{-1} + k_{-2}} \right) (G_j)(R_x) \quad (4)$$

$$(G_j \cdot R_x)_o \approx \left(\frac{k_2}{k_A + k_{E,j}} \right) (G_j \cdot R_x)_c \quad (5)$$

$$K_{x,j} = \left(\frac{k_1}{k_{-1} + k_{-2}} \right)$$

$$\tau_{x,j} = \left(\frac{k_2}{k_A + k_{E,j}} \right)$$

$$(G_j \cdot R_x)_o = K_{x,j} (G_j)(R_x) (\tau_{x,j})$$

$$R_x = \frac{R_{x,T} (\tau_{x,j} K_{x,j})}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$(G_j \cdot R_x)_o = \frac{R_{x,T} G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$\rightarrow r_{x,j} = k_{E,j} \left(\frac{G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j} \right)$$

a. Constants

(see constants pdf)
for citations

$$k_{E,j} = e_x L^{-1} = \frac{412 \text{ nt/l}}{307 \text{ lnt}} = .0135 \text{ s}^{-1} \quad (\text{Julia})$$

$$R_{\lambda,T} = 30 \times 10^{-6} \text{ mM} \quad (\text{Simulator})$$

$$G_j = \frac{2500 \text{ copies}}{\text{cell}} \times \frac{\text{mol}}{6.02 \times 10^{23} \text{ copies}} \times \frac{\text{cell}}{6.7 \times 10^{-8} \text{ L}} = 3093.3 \text{ mM} \quad (\text{Julia})$$

$$k_I = .024 \text{ s}^{-1}$$

$$k_{F,j} = \left(\frac{k_- + k_I}{k_+} \right) = k_I \left(\frac{k_- + k_I}{k_+ k_+} \right) = 2.49 \times 10^{-5} \text{ mM} \quad (\text{Julia})$$

McClure Paperslope

$$\tau_{x,j} = \frac{k_{E,j} + k_A}{k_I} = \frac{k_{E,j}}{k_I} = .569 \text{ s} \quad (\text{Julia})$$

Assume $k_A \ll k_{E,j}$
 $k_A \ll k_I$

(b) $\tau = .56915$

since $\tau \approx \frac{k_{E,j}}{k_I} < 1$

$\tau \approx$ elongation limited

$k_I < k_{E,j}$

(c) $\frac{dm_j}{dt} = \underbrace{\sum_{k=1}^R \sigma_{jk} r_k}_{\text{rates of production and degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution volume change}}$

mRNA balance $\frac{dm_j}{dt} = \underbrace{r_{2,j} U_j}_{\text{transcription}} - \underbrace{k_{2,j}^d m_j}_{\text{degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution}} = 0$
 @ steady state

[mRNA]

$m_j = \frac{r_{2,j} U_j}{k_{2,j}^d + B^{-1} \dot{B}}$

$u(I)$

$U_j(I) = \frac{w_1 + w_2 f_I}{1 + w_1 + w_2 f_I}$

fraction

$f_I = \frac{I^n}{K + I^n}$

dilution

$B^{-1} \dot{B} = \frac{1}{30 \text{ min}} \cdot \left(\frac{\text{min}}{60 \text{ s}} \right) = .005 \text{ s}^{-1}$
 specific growth rate

$k_{2,j}^d$

$= \frac{.693}{\text{half-life}} = \frac{.693}{5(60 \text{ s})}$

(assuming 1st order)

$= .0023 \text{ s}^{-1}$

all in Julia
 (constants given in problem)
 or
 constants document

plot is in Julia Code