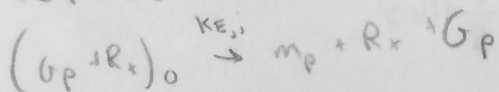
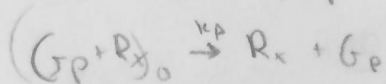
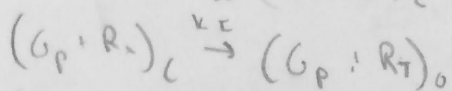
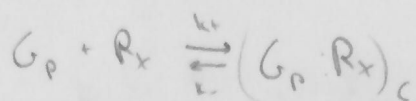


(a)

Sneha Kabaria

$$r_{x,j} = k_{E,j} (G_j \cdot R_x)_o \quad ; \quad \hat{r}_{x,j} = r_{x,j} \cdot U(I)$$

Material Balances

$$\text{closed complex} \rangle \frac{d}{dt} (G_j \cdot R_x)_c = k_1 (G_j)(R_x) - k_{-1} (G_p \cdot R_x)_c - k_{-2} (G_j \cdot R_x)_c = 0 \quad (1)$$

$$\text{open complex} \rangle \frac{d}{dt} (G_j \cdot R_x)_o = k_2 (G_j \cdot R_x)_c - k_A (G_j \cdot R_x)_o - k_{E,j} (G_j \cdot R_x)_o = 0 \quad (2)$$

$$\text{RNAP} \rangle R_{x,T} = R_x + (G_j \cdot R_x)_c + (G_j \cdot R_x)_o \quad (3)$$

Assume  
Steady-state

Solve (1) and (2)

$$(G_j \cdot R_x)_c \approx \left( \frac{k_1}{k_{-1} + k_2} \right) (G_j)(R_x) \quad (4)$$

$$(G_j \cdot R_x)_o \approx \left( \frac{k_2}{k_A + k_{E,j}} \right) (G_j \cdot R_x)_c \quad (5)$$

$$K_{x,j} = \left( \frac{k_1}{k_{-1} + k_2} \right)$$

$$\tau_{x,j} = \left( \frac{k_2}{k_A + k_{E,j}} \right)$$

$$(G_j \cdot R_x)_o = K_{x,j} (G_j)(R_x)(\tau_{x,j})$$

$$R_x = \frac{R_{x,T} (\tau_{x,j} K_{x,j})}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$(G_j \cdot R_x)_o = \frac{R_{x,T} G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j}$$

$$\rightarrow r_{x,j} = k_{E,j} \left( \frac{G_j}{\tau_{x,j} K_{x,j} + (\tau_{x,j} + 1) G_j} \right)$$

close gives:

a. Constants

(see constants pdf)  
for citations

$$k_{E,j} = e_x L^{-1} = \frac{42 \text{ nt/l}}{307 \text{ lnt}} = .0135 \text{ s}^{-1}$$

$$R_{\lambda,T} = 30 \times 10^{-6} \text{ mM}$$

$$G_j = \frac{2500 \text{ copies}}{\text{cell}} \times \frac{\text{mol}}{6.02 \times 10^{23} \text{ copies}} \times \frac{\text{cell}}{6.7 \times 10^{-8} \text{ L}} = 3093.3 \text{ mM}$$

$$k_I = .024 \text{ s}^{-1}$$

$$K_{E,j} = \left( \frac{k_- + k_I}{k_r} \right) = k_I \left( \frac{k_- + k_I}{k_r k_r} \right) = 2.49 \times 10^{-5} \text{ mM} \quad (\text{Julia})$$

McClure

$$\tau_{\lambda,j} = \frac{k_{E,j}}{k_I + k_A} = \frac{k_{E,j}}{k_I} \approx .5695 \quad (\text{Julia})$$

slope

Assume  $k_A \ll k_{E,j}$   
 $k_A \ll k_I$

Now that we have  $R_x$  we can get the

(b)  $\tau = .569125$

since  $\tau = \frac{1/E_{1.5}}{K_I} < 1$

$\tau \Rightarrow$  elongation limited

(c)  $\frac{dm_j}{dt} = \underbrace{\sum_{k=1}^R \sigma_{jk} r_k}_{\text{rates of production and degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution volume change}}$

mRNA balance  $\frac{dm_j}{dt} = \underbrace{r_{x,j} U_j}_{\text{transcription}} - \underbrace{k_{x,j}^d m_j}_{\text{degradation}} - \underbrace{m_j B^{-1} \dot{B}}_{\text{dilution}} = 0$   
 @ steady state

[mRNA]  $m_j = \frac{r_{x,j} U_j}{k_{x,j}^d + B^{-1} \dot{B}}$

$u(I) \quad U_j(I) = \frac{w_1 + w_2 f_I}{1 + w_1 + w_2 f_I}$   
 fraction  $f_I = \frac{I^n}{K + I^n}$

dilution  $B^{-1} \dot{B} = \frac{1}{30 \text{ min}} = \left( \frac{\text{min}}{60 \text{ s}} \right) = .0055 \text{ s}^{-1}$   
 specific growth rate

$k_{x,j}^d = \frac{.693}{\text{half-life}} = \frac{.693}{5(60 \text{ s})} = .00235 \text{ s}^{-1}$   
 (assuming 1st order)

all in Julia  
 (constants given in problem)  
 or  
 constants document