Fast Node Cardinality Estimation for 5 types of Nodes

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In this table we specify 5 types and their corresponding symbols.

Types	Symbols
1	α 0 0
2	α α 0
3	α α α
4	0 0 β
5	0 β β

3 slots are required in 1st phase. In slot 2 and 3 there are 4 possibilities (0 - Empty slot, α, β, and C – Collision, where α and β are two different symbols) and in slot 1 there are 3 possibilities (0, C, α). Total of possible cases are ($4^2 * 3 = 48$).

At least one collision cases are considered, since zero collision cases do not need additional phase. Exactly One collision cases are $\binom{3}{1} * 3 * 3 = 27$ and exactly two collision cases are $\binom{3}{2} * 3 = 9$. In Tables 1, 2 and 3 consider exactly one collision cases. Here 0 implies empty slot and 'Not Sure' outcome means additional slots are required to identify the presence of these types of nodes. Tables 4 denote exactly two collisions cases

In case of all Three slots result in 'Collisions', first we check for types 3 and 6. By eliminating and reassigning the nodes, we can form a new set of remaining 4 symbols, which can solved using 4 Type method. It is an iterative procedure. For (C, C, β). case, to resolve the ambiguity between [Type 1, One of Type 4, 5, 6], one slot is used for resolving the ambiguity of Type 1 device. To resolve the ambiguity of one of 4, 5, 6, we can reassign symbols (0, α, β) and ambiguity can be resolved in one slot only. So Total slots required to resolve the ambiguity is 2. Similar method is used for (α, C, C) case.

Outcome in Block i			Types	
Slot 1	Slot 2	Slot 3	Sure	Not Sure
C	0	0	1	-
C	0	α	#	#
C	0	β	1,4	-
C	α	0	1,2	-
C	α	α	1,3	-
C	α	β	1,2,4	-
C	β	0	#	#
C	β	α	#	#
C	β	β	1,5	-

Table 1: Exactly one collision case (Part 1). #, C and – denote "Invalid Case", "Collision" and "Nil" respectively.

Outcome in Block i			Types	
Slot 1	Slot 2	Slot 3	Sure	Not Sure
0	C	0	#	#
0	C	α	#	#
0	C	β	#	#
α	C	0	#	#
α	C	α	#	#
α	C	β	2,5	-

Table 2: Exactly one collision case (Part 2). #, C and $-$ denote "Invalid Case", "Collision" and "Nil" respectively.

Outcame in Block i			Types	
Slot 1	Slot 2	Slot 3	Sure	Not Sure
0	0	C	4	-
0	α	C	#	#
0	β	C	5,4	-
α	0	C	1,4	-
α	α	C	4	One of {2,3}
α	β	C	1,5,4	-

Table 3: Exactly one collision case (Part 3). #, C and $-$ denote "Invalid Case", "Collision" and "Nil" respectively.

Outcame in Block i			Types	
Slot 1	Slot 2	Slot 3	Sure	Not Sure
C	C	0	2	1
C	C	α	2,3	1
C	C	β	2	1, One of {4,5}
0	C	C	5	4
α	C	C	5	4, One of {1,2,3}
C	0	C	1,4	-
C	α	C	1,4	One of {2,3}
C	β	C	1,4,5	-

Table 4: Exactly two collisions case. #, C and $-$ denote "Invalid Case", "Collision" and "Nil" respectively.