

# 수치해석 HW#6

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## 수치해석 HW#6

### #12.1

$$\begin{cases} 3x_1 + 8x_2 = 11 \\ 7x_1 - x_2 = 5 \end{cases}, \quad \lambda = 1.25$$

$$\text{Sod} > \begin{pmatrix} 3 & 8 \\ 7 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$$

diagonal dominant 이기 Rearrange

$$\begin{pmatrix} 7 & -1 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

이때  $x_1^* = x_2^* = 0$  으로 시작 설정.

$$\textcircled{1} \quad x_1 = \frac{5 - (-1) \cdot 0}{7} = 0.7143$$

$$x_2 = \frac{11 - 3 \cdot 0}{8} = 1.375$$

$$x_1^{\text{new}} = 1.25(0.7143) + (1 - 1.25)(0) = 0.892875$$

$$x_2^{\text{new}} = 1.25(1.375) + (1 - 1.25)(0) = 1.71875$$

$$\textcircled{2} \quad x_1 = \frac{5 - (-1) \cdot (1.71875)}{7} = 0.956607$$

$$x_2 = \frac{11 - 3 \cdot (0.956607)}{8} = 1.04019$$

$$x_1^{\text{new}} = 1.25(0.956607) + (-0.25) \cdot 0.892875 = 0.97254$$

$$x_2^{\text{new}} = 1.25(1.04019) + (-0.25) \cdot 1.71875 = 0.89615$$

$$\textcircled{3} \quad x_1 = \frac{5 - (-1) \cdot (0.89615)}{7}$$

$$= 0.98231$$

$$x_2 = \frac{11 - 3 \cdot (0.98231)}{8}$$

$$= 1.06094$$

$$x_1^{\text{new}} = 1.25(0.98231) - 0.25 \cdot (0.97254)$$

$$x_2^{\text{new}} = 1.25(1.06094) - 0.25(0.89615)$$

third iteration start.

$$x_1 = 0.98231$$

$$x_2 = 1.0478815$$

true value

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{59} \begin{pmatrix} 8 & 1 \\ 7 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$$= \frac{1}{59} \begin{pmatrix} 51 \\ 62 \end{pmatrix}$$

$$= \begin{pmatrix} 0.86440 \\ 1.05085 \end{pmatrix}$$

$$\therefore \epsilon_{t,1} = \left| \frac{0.86440 - 0.98231}{0.86440} \right| \times 100$$

$$= 13.9\%$$

$$\epsilon_{t,2} = \left| \frac{1.05085 - 1.04788}{1.05085} \right| \times 100$$

$$= 0.66\%$$

# 12.2

$$\begin{bmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$d_1 = \frac{41 - (-0.4) \cdot 0 - 0 \cdot 0}{0.8} = 51.25$$

$$d_2 = \frac{25 - (-0.4) \cdot (51.25) - (-0.4) \cdot 0}{0.8} = 56.875$$

$$d_3 = \frac{105 - 0 \cdot 51.25 - (-0.4) \cdot 56.875}{0.8} = 159.6875$$

$$d_1 = \frac{41 - (-0.4) \cdot 56.875 - 0 \cdot 159.6875}{0.8} = 79.6875$$

$$d_2 = \frac{25 - (-0.4) \cdot 79.6875 - (-0.4) \cdot 159.6875}{0.8} = 150.9375$$

$$d_3 = \frac{105 - 0 - (-0.4) \cdot 150.9375}{0.8} = 206.71875$$

$$d_1 = \frac{41 - (-0.4) \cdot (150.9375)}{0.8} = 126.71875$$

$$d_2 = \frac{25 - (-0.4) \cdot (126.71875) - (-0.4) \cdot (206.71875)}{0.8} = 197.8125$$

$$d_3 = \frac{105 - (-0.4) \cdot 197.8125}{0.8} = 220.234$$

$$\begin{aligned} d_1 &= 180.234 \\ d_2 &= 221.484 \\ d_3 &= 241.991 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 4th.$$

$$\begin{aligned} d_1 &= 161.992 \\ d_2 &= 239.242 \\ d_3 &= 247.897 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 5th. \quad \left. \begin{array}{l} e_1 = 9.5\% \\ e_2 = 2.5\% \\ e_3 = 1.2\% \end{array} \right\}$$

$$\begin{aligned} d_1 &= 167.897 \\ d_2 &= 239.12 \\ d_3 &= 250.81 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 6th.$$

(b)  $\lambda = 1.2$ .

$$d_1^{new} = 1.2 \times 51.25 + (-0.2) \cdot 0 = 61.5$$

$$d_2^{new} = 1.2 \times 56.875 + (-0.2) \cdot 0 = 68.25$$

$$d_3^{new} = 1.2 \times 159.6875 = 202.14$$

$$d_1 = \frac{41 - (-0.4) \cdot 68.25}{0.8} = 85.375$$

$$d_1^{new} = 1.2 \times 85.375 + (-0.2) \cdot (61.5) = 90.15$$

$$d_2 = \frac{25 - (-0.4) \cdot (90.15) - (-0.4) \cdot (202.14)}{0.8} = 197.375$$

$$d_2^{new} = 1.2 \cdot 197.375 + (-0.2) \cdot 68.25 = 199.224$$

$$d_3 = \frac{105 + 0.4 \cdot (199.224)}{0.8} = 220.862$$

$$d_3^{new} = 1.2 \cdot (220.862) + (-0.2) \cdot 202.14 = 236.6064$$

#12.8

$$\left. \begin{aligned} y &= -x^2 + x + 0.75 \\ y + 5xy &= x^2 \end{aligned} \right) x=y=1.2$$

sol  $f_1(x,y) = -x^2 + x - y + 0.75$   
 $f_2(x,y) = x^2 - 5xy - y$

$$f_1 = -(1.2)^2 + 1.2 - 1.2 + 0.75 = -0.69$$

$$f_2 = (1.2)^2 - 5(1.2)(1.2) - (1.2) = -6.96$$

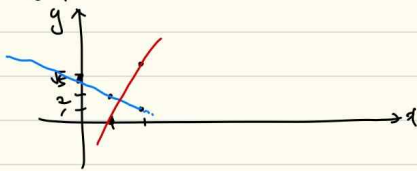
$$x_1 = 1.2 - \frac{(-0.69)(-1) - (-6.96)(-1)}{6.2} = 0.85645$$

$$y_1 = 1.2 - \frac{(-6.96)(-1.4) - (-0.69)(-3.6)}{6.2} = 0.029$$

#12.9

$$\left. \begin{aligned} x^2 &= 5 - y^2 \\ y + 1 &= x^2 \end{aligned} \right) \rightarrow \begin{aligned} y &= \sqrt{5 - x^2} \\ y &= x^2 - 1 \end{aligned}$$

(a) graphically



(b)  $x=y=1.5$   $\begin{cases} y = \sqrt{5 - x^2} \\ x = \sqrt{y + 1} \end{cases}$

$$\begin{cases} y = \sqrt{5 - (1.5)^2} = 1.6583 \\ x = \sqrt{1.6583 + 1} = 1.6304 \end{cases}$$

$$\begin{cases} y = \sqrt{5 - 1.6304^2} = 1.5302 \\ x = \sqrt{1.5302 + 1} = 1.5906 \end{cases}$$

(c)  $f_1 = 5 - x^2 - y^2$   
 $f_2 = x^2 - y - 1$

$$\left. \begin{aligned} \frac{df_1}{dx} &= -2x & \frac{df_1}{dy} &= -2y \\ \frac{df_2}{dx} &= 2x & \frac{df_2}{dy} &= -1 \end{aligned} \right)$$

$$x_1 = 1.5 - \frac{(0.5)(-1) - (0.25)(-2)}{12} = 1.60416$$

$$x_2 = 1.5 - \frac{(-0.25)(-1) - (0.5)(2)}{12} = 1.5625$$

12.14

```
function [J,f]=jacobian(f1,f2,x)
    delta=1E-6;
    df1dx1=(f1(x(1)+delta*x(1),x(2))-f1(x(1),x(2)))/(delta*x(1));
    df1dx2=(f1(x(1),x(2)+delta*x(2))-f1(x(1),x(2)))/(delta*x(2));
    df2dx1=(f2(x(1)+delta*x(1),x(2))-f2(x(1),x(2)))/(delta*x(1));
    df2dx2=(f2(x(1),x(2)+delta*x(2))-f2(x(1),x(2)))/(delta*x(2));
    J=[df1dx1,df1dx2;df2dx1,df2dx2];
    f11=f1(x(1),x(2));
    f22=f2(x(1),x(2));
    f=[f11;f22];
end
```

Jacobian function

```
function [x,f,ea,iter]=newton(x0,es,maxit,f1,f2)
    if isempty(es)
        es=0.0001;
    end
    if isempty(maxit)
        maxit=50;
    end
    iter=1;
    x=x0;
    while(1)
        [J,f]=jacobian(f1,f2,x);
        x=x-inv(J)*f;
        iter = iter+1;
        ea=100*max(abs((J*f)./x));
        if ea<=es, break, end
    end
```

x =  
0.1919  
0.9051  
f =  
0.0044  
0.0207  
ea =  
2.1963

newton funtion과 결과

#12.18

```
function [J,f]=jfreact(x,varargin)
    del = 0.000001;
    df1dx1 = (u(x(1)+del*x(1),x(2))-u(x(1),x(2)))/(del*x(1));
    df1dx2 = (u(x(1),x(2)+del*x(2))-u(x(1),x(2)))/(del*x(2));
    df2dx1 = (v(x(1)+del*x(1),x(2))-v(x(1),x(2)))/(del*x(1));
    df2dx2 = (v(x(1),x(2)+del*x(2))-v(x(1),x(2)))/(del*x(2));
    J=[df1dx1 df1dx2;df2dx1 df2dx2];
    f1=u(x(1),x(2));
    f2=v(x(1),x(2));
    f=[f1;f2];
end

function f = u(x,y)
    f = (5 + x + y) / (50 - 2 * x - y) ^ 2 / (20 - x) - 0.0004;
end

function f = v(x,y)
    f = (5 + x + y) / (50 - 2 * x - y) / (10 - y) - 0.037;
end
```

jfreact function

```
>> [x,f,ea,iter]=newtmult(@jfreact, x0)

x =

    3.3366e+00
    2.6772e+00

f =

   -7.1232e-17
    8.5959e-14

ea =

    5.2227e-10

iter =

     4
```

실행결과