

# Cosc341 - Assignment 3

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## 1 Longest Path problem

The **Longest path problem** is the problem for finding the maximum length Simple-path in a graph,  $G(V, E)$  with  $|V| = n : n \in \mathbb{N}$ . The graph is undirected and all edges weights are assumed to be 1.

### 1.1 Simple-path

Suppose in our graph,  $G(V, E)$ , we want the Simple-path between two vertices  $s$  and  $t$ . The Simple-path is a path between any two nodes in the graph that does not go over the same node twice, and for two consecutive nodes there is a path between them.

As such a simple path between two nodes  $s, t \in V$  is the sequence of vertices  $(v_1, v_2, v_3, \dots, v_k)$  that satisfy these conditions:

- $s = v_1$  and  $t = v_k$
- Each consecutive nodes  $(v_i, v_{i+1})$  there is an edge  $e = (v_i, v_{i+1}) \in E$
- No node appears more than once in the sequence.

### 1.2 Longest-simple path and Decision version

The figure below shows the longest simple path, in blue, of a graph.

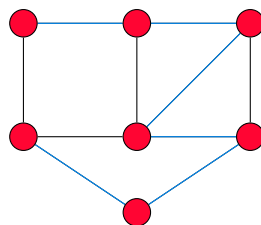


Figure 1: Longest simple path of a graph

In the decision version of the Longest path problem, yes or no version; we input a graph,  $G$ , and an integer  $k$  and outputs a yes or no if a simplest path of at least length,  $k$ , exists in the graph.

### 1.3 NP

A problem is in NP if we can verify the solution in polynomial time. A corresponding certificate can be produced from the decision version of the Longest path problem. If we can verify this certificate in polynomial time then Longest path problem is in  $NP$ . The certificate is a path at least of length  $k$  with no duplicate nodes.

To check that the path has no duplicate nodes we can iterate over each node in the path for every node,  $O(k^2)$  complexity.

To check the length of the path we just need to count each node and check it is greater than or equal to  $k$ , this operation has a  $O(k)$  complexity.

In some implementations like most programming languages getting the length of a list is constant complexity  $O(1)$ , in most cases and the worst case we can check a certificate with  $O(n^2)$  complexity, therefore the Longest path problem is in  $NP$ .

## 2 NP-complete

If a problem,  $X$ , is in  $NP$ -complete and  $X$  polynomial reduces to another problem  $Y$  in NP, then  $Y$  is also in  $NP$ -complete.

We have shown that the Longest path problem is  $NP$ , all that's left is to show that another problem in  $NP$ -complete polynomial reduces to the Longest path problem.

### 2.1 Hamiltonian path

A Hamiltonian path problem - Determining if a Hamiltonian path (a path which visits every node in a graph once) exists in a graph, is a known  $NP$ -complete problem. It stands to show that the Hamiltonian path problem polynomial reduces to the Longest path problem.