

# Problem 1.

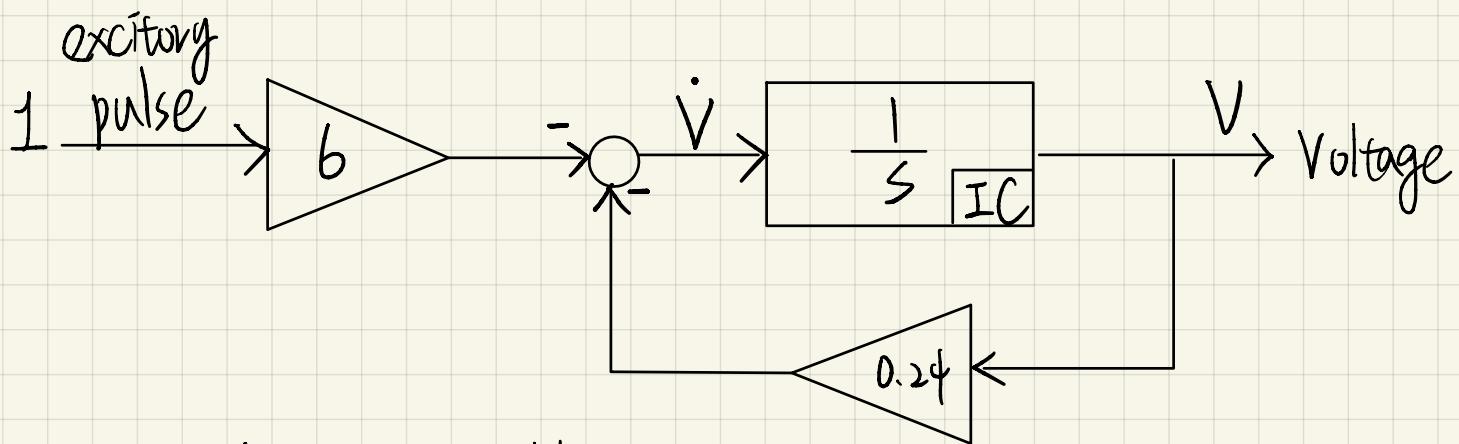
1-1. ODE ,  $t \leq 1 \text{ ms}$

$$12.5 \frac{dV}{dt} = -75 - V - \frac{g_e}{g_e} V + \frac{g_i}{g_e} (75 - V), \text{ where } g_i = 0, \frac{g_e}{g_e} = 2$$

$$\Rightarrow 12.5 \dot{V} = -75 - V - 2V + \frac{0}{g_e} (75 - V)$$

$$\Rightarrow \dot{V} = -0.24V - 6 \quad \#$$

1-2. Block Diagram .



1-3 Initial value problem

$$\dot{V} = -0.24V - 6, \quad V(0) = -75$$

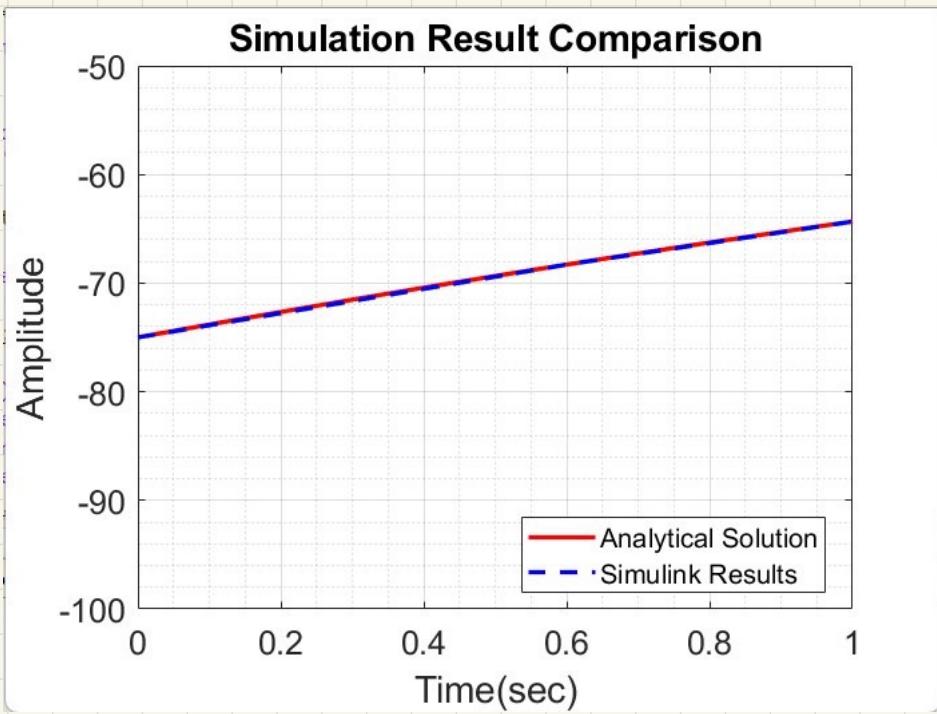
$$\dot{V} = -0.24V_p - 6$$

$$\Rightarrow e^{0.24t} V = -6 \int_0^t e^{0.24t} dt + C_1$$

$$\Rightarrow V = -6 e^{-0.24t} \int_0^t e^{0.24t} dt + C_1 e^{-0.24t}$$

$$\Rightarrow V = 25(e^{-0.24t} - 1) - 75e^{-0.24t} \quad \#$$

1-4 Simulation  $t \leq 1\text{ ms}$



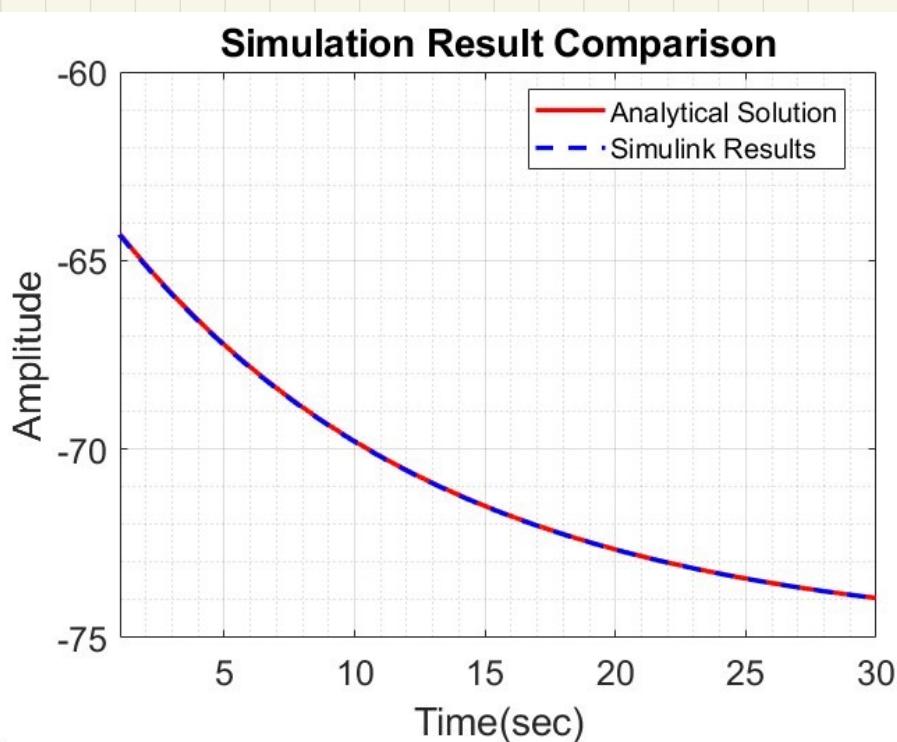
1-5 ODE,  $t > 1\text{ ms}$

$$12.5 \frac{dV}{dt} = -75 - V - \frac{g_e}{g_e} V + \frac{g_e}{g_e} (75 - V), \text{ where } g_c = 0, \frac{g_e}{g_e} = 0$$

$$\Rightarrow 12.5 \dot{V} = -75 - V - 0V + \frac{0}{g_e} (75 - V)$$

$$\Rightarrow \dot{V} = -0.0 \text{ V} - 6 \text{ ms}^{-1}$$

1-6 Simulation  $t > 1\text{ ms}$



Problem 2.

2-1 Find  $\bar{R}_f$

$$R_f = \frac{\sqrt{\Delta P}}{Q_{out}}$$

$$\Rightarrow \Delta P = R_f^2 Q_{out}^2$$

$$\begin{aligned}\Rightarrow \frac{d\Delta P}{dQ_{out}} &= 2R_f^2 Q_{out} \\ &= 2R_f(R_f Q_{out}) \\ &= 2R_f \sqrt{\rho g h_0}\end{aligned}$$

operation point is  $(Q_0, h_0)$

$$\Rightarrow \bar{R}_f = 2R_f \sqrt{\rho g h_0} *$$

2-2 Substitute  $\bar{R}_f$

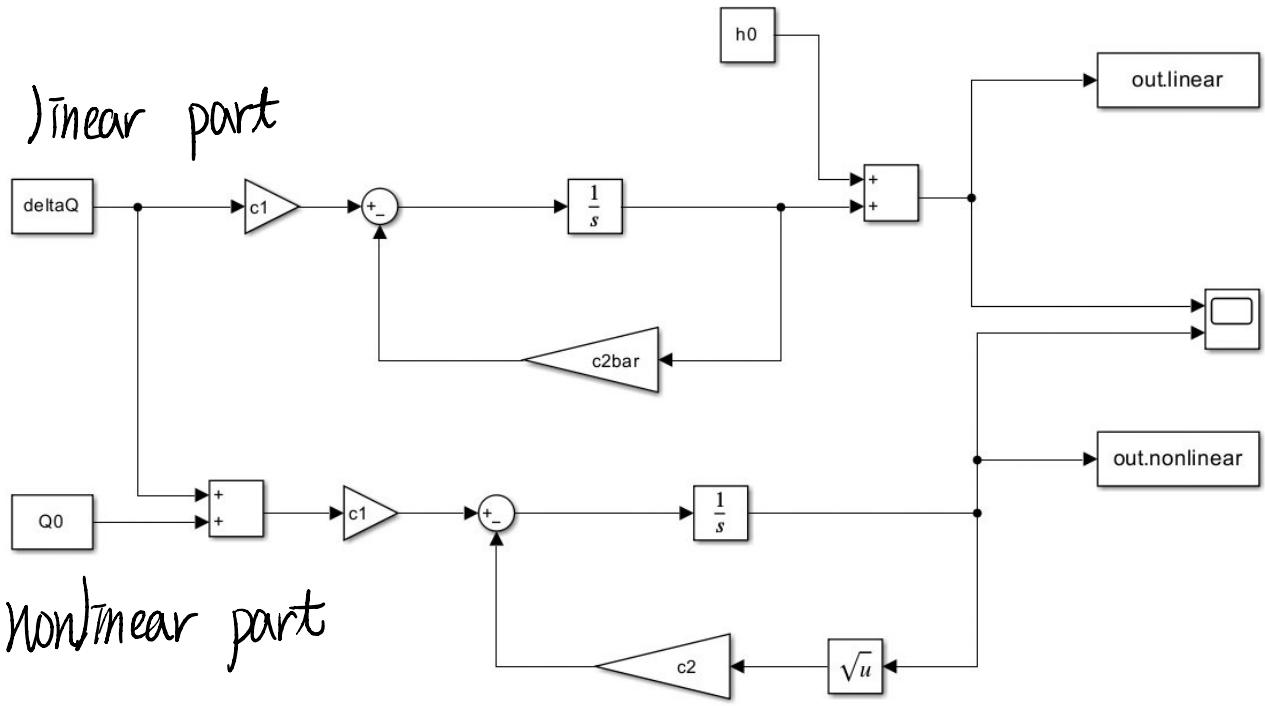
$$Eqn(7) \cdot \frac{\pi D^2}{4} \delta \dot{h}(t) = \delta Q(t) - \frac{1}{\bar{R}_f} \rho g \delta h(t)$$

$$\rightarrow \frac{\pi D^2}{4} \delta \dot{h}(t) = \delta Q(t) - \frac{1}{2R_f \sqrt{\rho g h_0}} \rho g \delta h(t)$$

$$\Rightarrow \delta \dot{h}(t) = \frac{4}{\pi D^2} \delta Q(t) - \frac{2\sqrt{\rho g}}{\pi D^2 R_f \sqrt{h_0}} \delta h(t)$$

the equation is identical to the results obtained in class. \*

## 2-3 Block Diagram



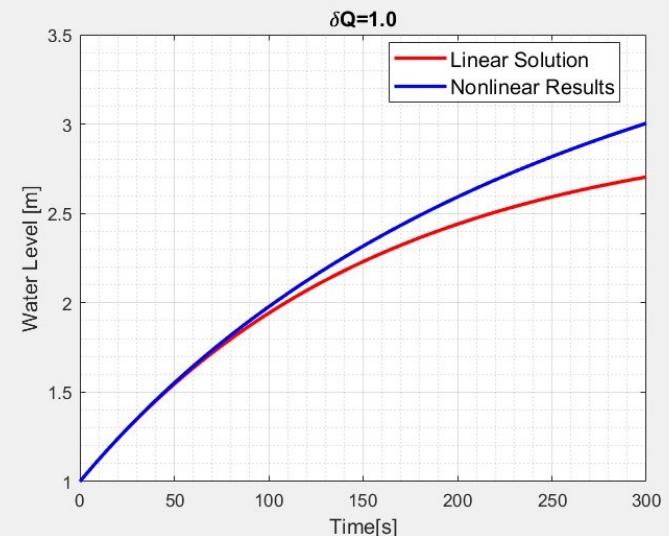
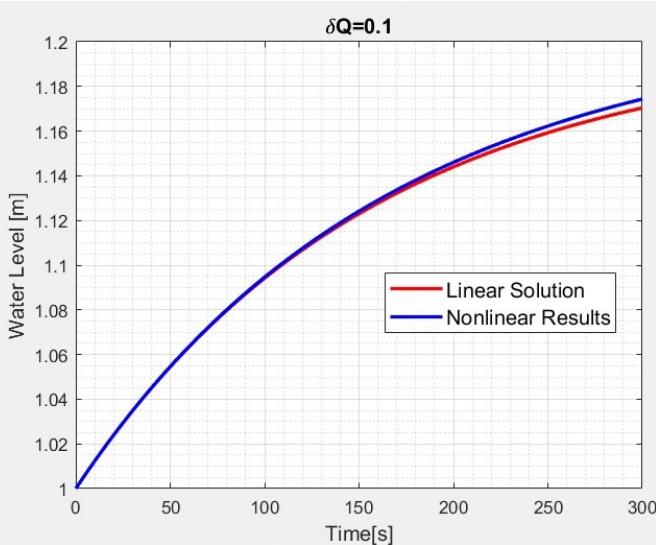
Follow the equations we can build up the linear and nonlinear models respectively, and the block diagrams are showed above.

$$\delta Q: \delta Q \quad C_2: \frac{4\sqrt{\rho g}}{R_f \pi D^2}$$

$$C_1: \frac{4}{\pi D^2} \quad C_{2\bar{}}: \frac{C_2}{2\sqrt{h_0}}$$

$\frac{1}{s}$ : Integrator  $\sqrt{u}$ : square root

## 2-4 Show the Result



# Problem 3.

## 3-1 IVP

$$\dot{X}(t) = -10X(t) + 10(1 + \sin 10t)$$

$$\Rightarrow SX(s) + 10X(s) = 0.75 + \frac{10}{s} + \frac{100}{s^2 + 100}$$

$$\Rightarrow X(s) = \frac{0.75}{s+10} + \frac{10}{s(s+10)} + \frac{100}{(s+10)(s^2+100)}$$

$$= \frac{0.75}{s+10} + \frac{1}{s} + \frac{-1}{s+10} + \frac{-\frac{1}{2}s + 5}{s^2 + 100} + \frac{1}{2} \cdot \frac{1}{s+10}$$

$$= \frac{1}{4} \frac{1}{s+10} + \frac{1}{s} + \left( -\frac{1}{2} \frac{s}{s^2 + 100} \right) + \frac{1}{2} \frac{10}{s^2 + 100}$$

$$\Rightarrow X(t) = \frac{1}{4} e^{-10t} + 1 - \frac{1}{2} (\cos 10t - \sin 10t)$$

## 3-2 plot

