Problem 1.

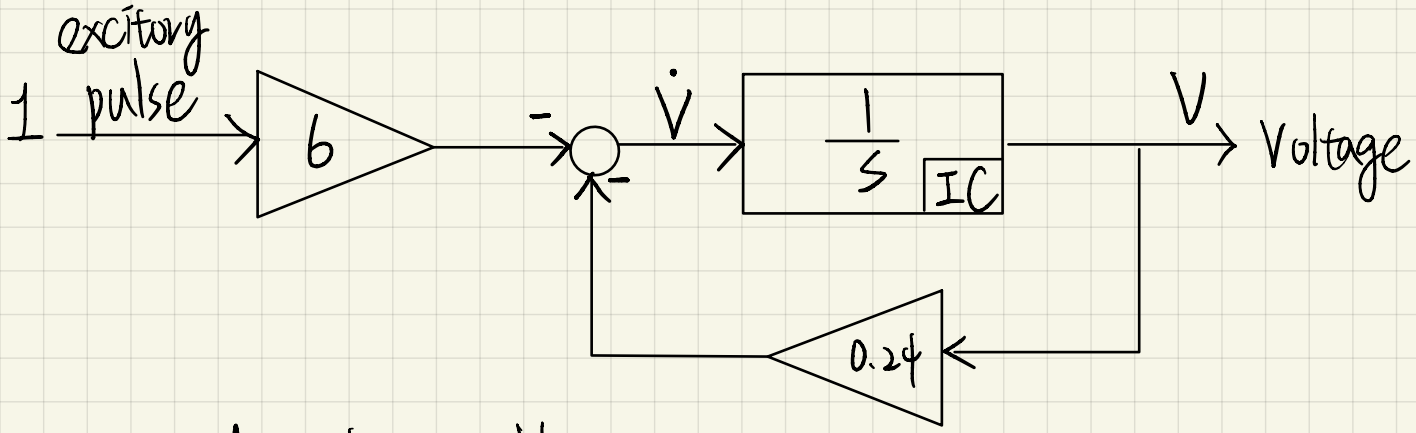
1-1. ODE, $t \le 1ms$

$12.5\frac{dV}{dt} = -75 - V - \frac{g_i}{g_l}V + \frac{g_i}{g_l}(75-V)$, where $g_i = 0$, $g_e/g_l = 2$

$\Rightarrow 12.5\dot{V} = -75 - V - 2V + \frac{0}{g_l}(75-V)$

$\Rightarrow \dot{V} = -0.24V - 6$ #

1-2. Block Diagram.



1-3 Initial value problem

$\dot{V} = -0.24V - 6$, $V(0) = -75$

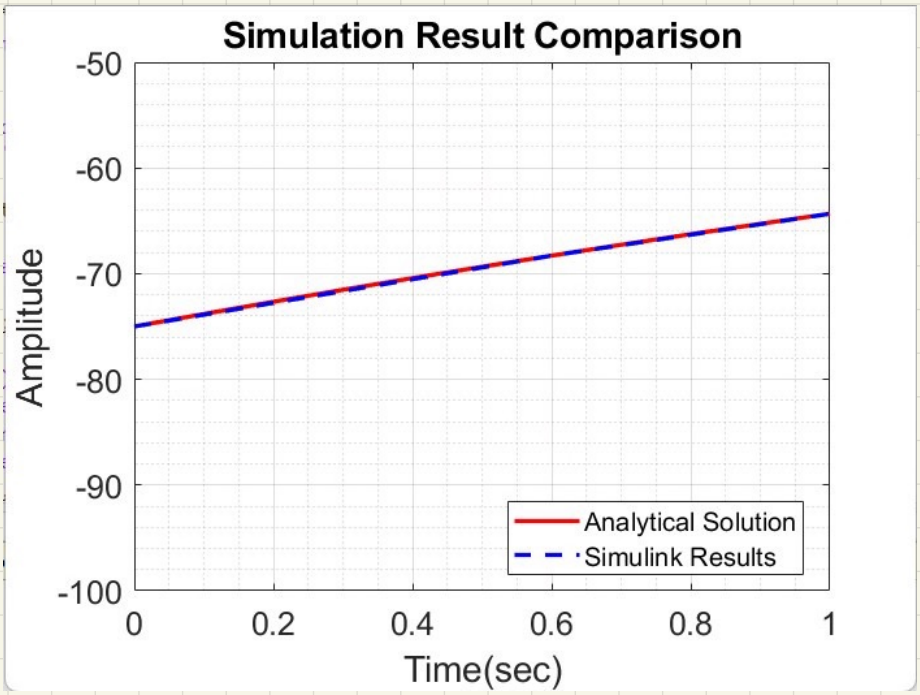
$\dot{V} = -0.24V_p - 6$

$\Rightarrow e^{0.24t}V = -6\int_0^t e^{0.24t}dt + C_1$

$\Rightarrow V = -6e^{-0.24t}\int_0^t e^{0.24t}dt + C_1e^{-0.24t}$

$\Rightarrow V = 25(e^{-0.24t} - 1) - 75e^{-0.24t}$ #

1-4 Simulation $t \leq 1$ ms



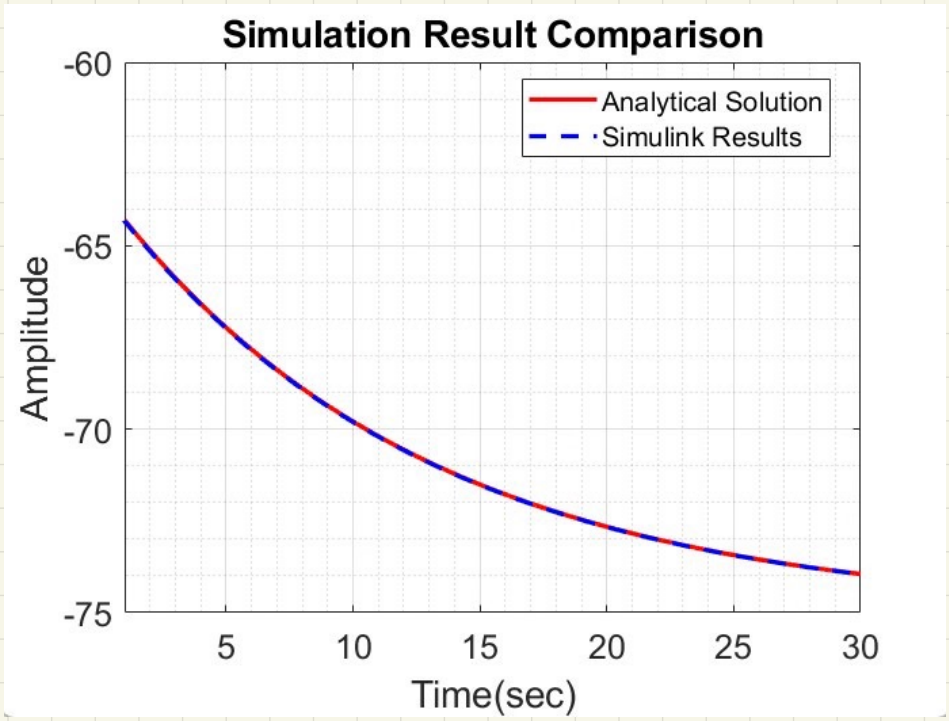
1-5 ODE, $t > 1$ ms

$$12.5\frac{dV}{dt} = -75 - V - \frac{g_e}{g_l}V + \frac{g_i}{g_l}(75 - V)$$, where $g_i = 0$, $g_e/g_e = 0$

$$\Rightarrow 12.5\dot{V} = -75 - V - 0V + \frac{0}{g_l}(75 - V)$$

$$\Rightarrow \dot{V} = -0.0\ V - 6$$ #

1-6 Simulation $t > 1$ ms



Problem 2.

2-1 Find $\bar{R}_f$

$$R_f = \frac{\sqrt{\Delta P}}{Q_{out}}$$

$$\Rightarrow \Delta P = R_f^2 Q_{out}^2$$

$$\Rightarrow \frac{d\Delta P}{dQ_{out}} = 2R_f^2 Q_{out}$$

$$= 2R_f (R_f Q_{out})$$

$$= 2R_f \sqrt{\rho g h(t)}$$

operation point is (Q_0, h_0)

$$\Rightarrow \bar{R}_f = 2R_f \sqrt{\rho g h_0} \quad \#$$

2-2 Substitude $\bar{R}_f$

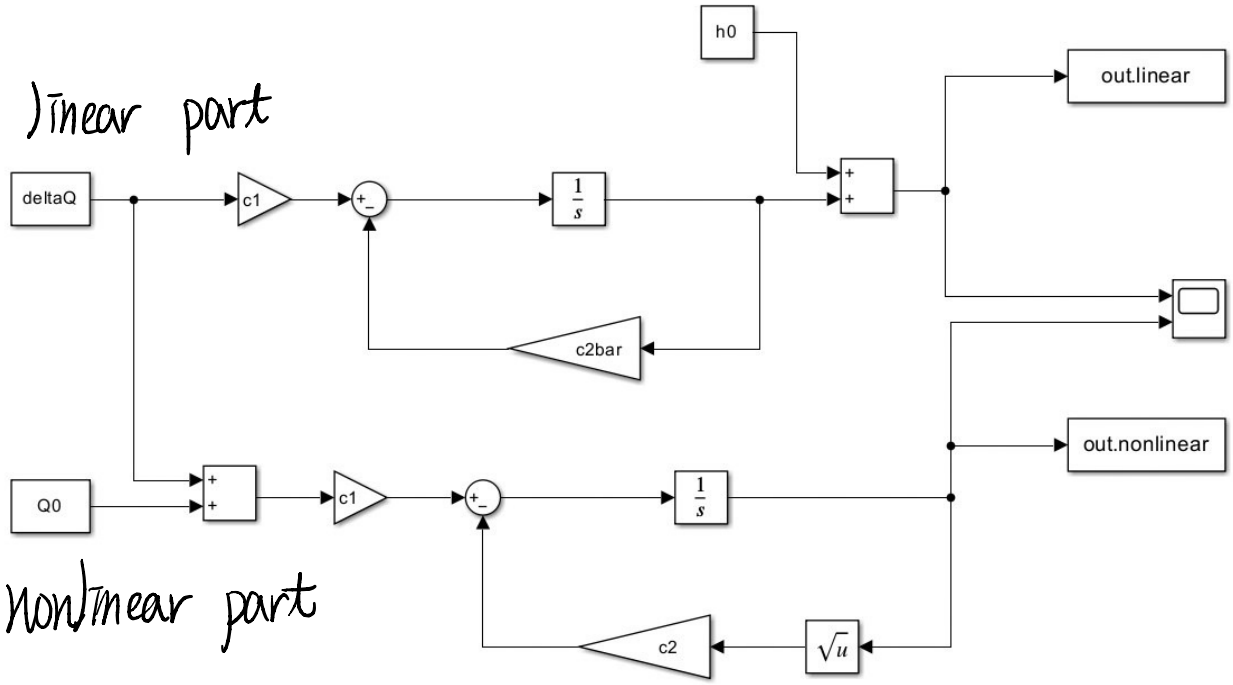
Eqn(7), $\frac{\pi D^2}{4} \delta\dot{h}(t) = \delta Q(t) - \frac{1}{\bar{R}_f} \rho g \delta h(t)$

$$\Rightarrow \frac{\pi D^2}{4} \delta\dot{h}(t) = \delta Q(t) - \frac{1}{2R_f\sqrt{\rho g h_0}} \rho g \delta h(t)$$

$$\Rightarrow \delta\dot{h}(t) = \frac{4}{\pi D^2} \delta Q(t) - \frac{2\sqrt{\rho g}}{\pi D^2 R_f \sqrt{h_0}} \delta h(t) \quad \#$$

the equation is identical to the results obtained in class.

2-3 Block Diagram



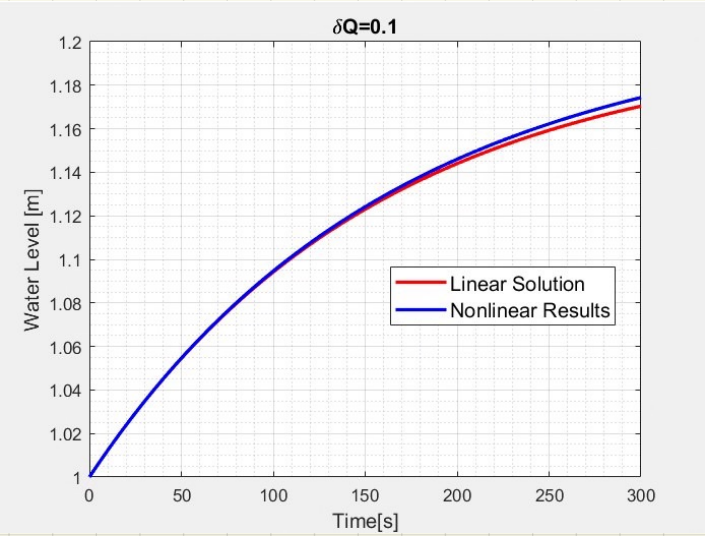
Follow the equations we can build up the linear and nonlinear models respectively, and the block diagrams are showed above.

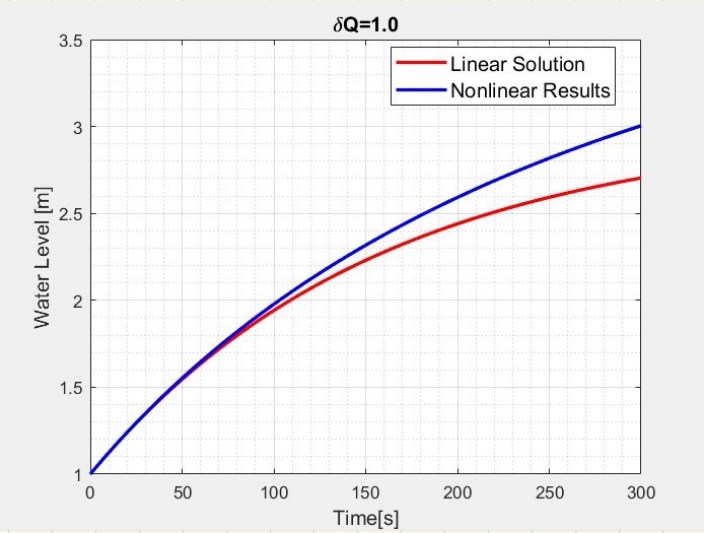
delta Q: δQ C_2: $\frac{4\sqrt{\rho g}}{R_f \pi D^2}$

C_1: $4/\pi D^2$ C_2bar: $\frac{C_2}{2\sqrt{h_0}}$

$\frac{1}{s}$: Integrator $\sqrt{u}$: square root

2-4 Show the Result





Problem 3.

3-1 IVP

$$\dot{x}(t) = -10x(t) + 10(1 + \sin 10t)$$

$\mathcal{L}$

$$\Rightarrow sX(s) + 10X(s) = 0.75 + \frac{10}{s} + \frac{100}{s^2+100}$$

$$\Rightarrow X(s) = \frac{0.75}{s+10} + \frac{10}{s(s+10)} + \frac{100}{(s+10)(s^2+100)}$$

$$= \frac{0.75}{s+10} + \frac{1}{s} + \frac{-1}{s+10} + \frac{-\frac{1}{2}s + 5}{s^2+100} + \frac{1}{2} \cdot \frac{1}{s+10}$$

$$= \frac{1}{4}\frac{1}{s+10} + \frac{1}{s} + \left(-\frac{1}{2}\frac{s}{s^2+100}\right) + \frac{1}{2}\frac{10}{s^2+100}$$

$\mathcal{L}^{-1}$

$$\Rightarrow \underline{x(t) = \frac{1}{4}e^{-10t} + 1 - \frac{1}{2}(\cos 10t - \sin 10t)} \#$$