Problem 1.

1-1 Governing Equation

$$m_b \ddot{h} = m_b g - \alpha \frac{i(t)^2}{h(t)^2}$$

$$\ddot{h} = g - \frac{\alpha}{m_b} \frac{i(t)^2}{h(t)^2}$$

Let $\begin{cases} h(t) = \bar{h} + \delta h(t) \\ i(t) = \bar{i} + \delta i(t) \end{cases}$

$$\Rightarrow (\bar{h} + \delta h(t))'' = g - \frac{\alpha}{m_b} \cdot \frac{[\bar{i} + \delta i(t)]^2}{[\bar{h} + \delta h(t)]^2}$$

$$[\bar{i} + \delta i(t)]^2 \xrightarrow{\text{Taylors}} (\bar{i}^2 + 2\delta i(t)\, \bar{i})$$

$$[\bar{h} + \delta h(t)]^{-2} \xrightarrow{\text{Taylors}} \left(\frac{1}{\bar{h}^2} - \frac{2\delta h(t)}{\bar{h}^3}\right)$$

$$\Rightarrow \delta \ddot{h}(t) = g - \frac{\alpha}{m_b} \cdot \frac{\bar{i}}{\bar{h}^3} (\bar{i} + 2\delta i(t))(\bar{h} - 2\delta h(t))$$

$$= g - \frac{\alpha \bar{i}^2}{m_b \bar{h}^2} + \frac{2\alpha \bar{i}^2}{m_b \bar{h}^3} \delta h(t) - \frac{2\alpha \bar{i}}{m_b \bar{h}^2} \delta i(t)$$

$$= g - \frac{\alpha \bar{i}^2}{m_b \bar{h}^2} + \frac{2\alpha \bar{i}^2}{m_b \bar{h}^3} (h(t) - \bar{h}) - \frac{2\alpha \bar{i}}{m_b \bar{h}^2} (i(t) - \bar{i})$$

To find $\bar{i}$, plug in $h(t) = \bar{h}$ and $i(t) = \bar{i}$, $\delta \ddot{h}(t) \to 0 \Rightarrow \bar{i} = \sqrt{\frac{g m_b \bar{h}^2}{\alpha}}$

$$a_0 = \frac{2\alpha \bar{i}^2}{m_b \bar{h}^3}, \quad b_0 = -\frac{2\alpha \bar{i}}{m_b \bar{h}^2}$$

$$\Rightarrow \begin{bmatrix} \dot{h} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2\alpha \bar{i}^2}{m_b \bar{h}^3} & 0 \end{bmatrix} \begin{bmatrix} h - \bar{h} \\ \dot{h} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{2\alpha \bar{i}}{m_b \bar{h}^2} \end{bmatrix} \left(i(t) - \sqrt{\frac{g m_b \bar{h}^2}{\alpha}} \right)$$

$$h = \bar{h} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} h - \bar{h} \\ \dot{h} \end{bmatrix}$$

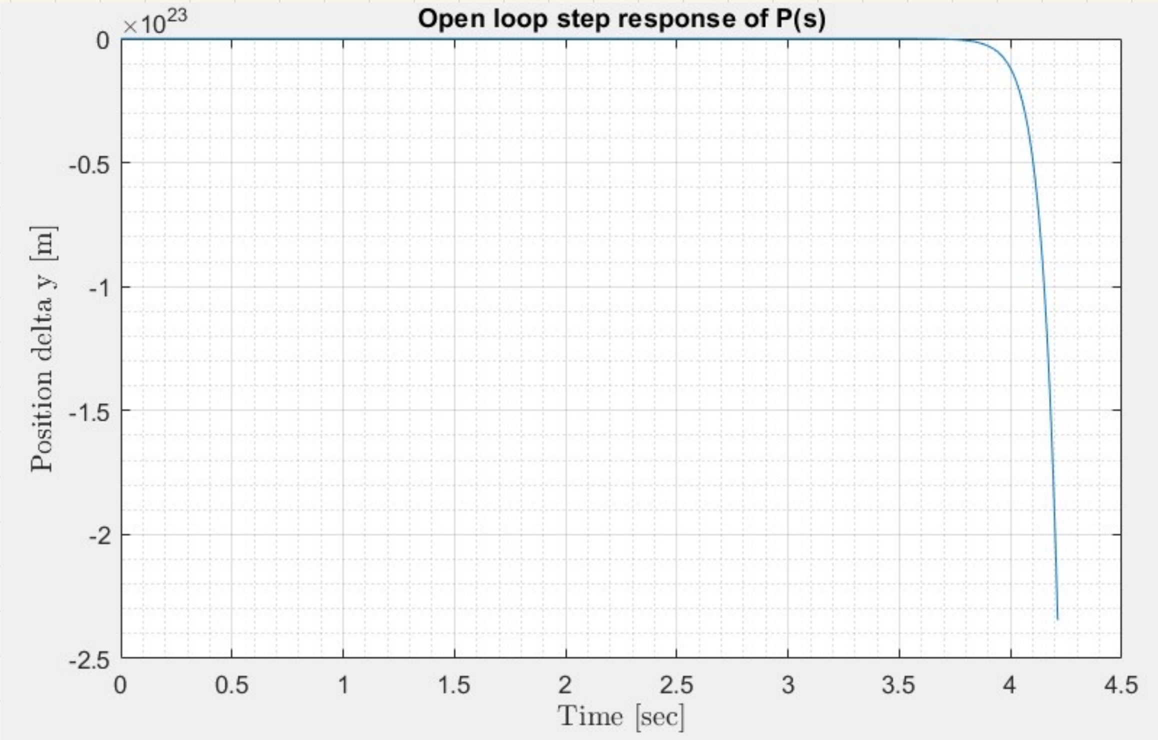
1-2 Modeling

$$\delta \ddot{h} = a_0 \delta h(t) + b_0 \delta i(t)$$

$$s^2 \delta H(s) = a_0 \delta H(s) + b_0 \delta I(s)$$

$$\Rightarrow \frac{\delta H(s)}{\delta I(s)} = \frac{b_0}{s^2 - a_0} = \frac{\delta Y(s)}{\delta U(s)} \quad \#$$

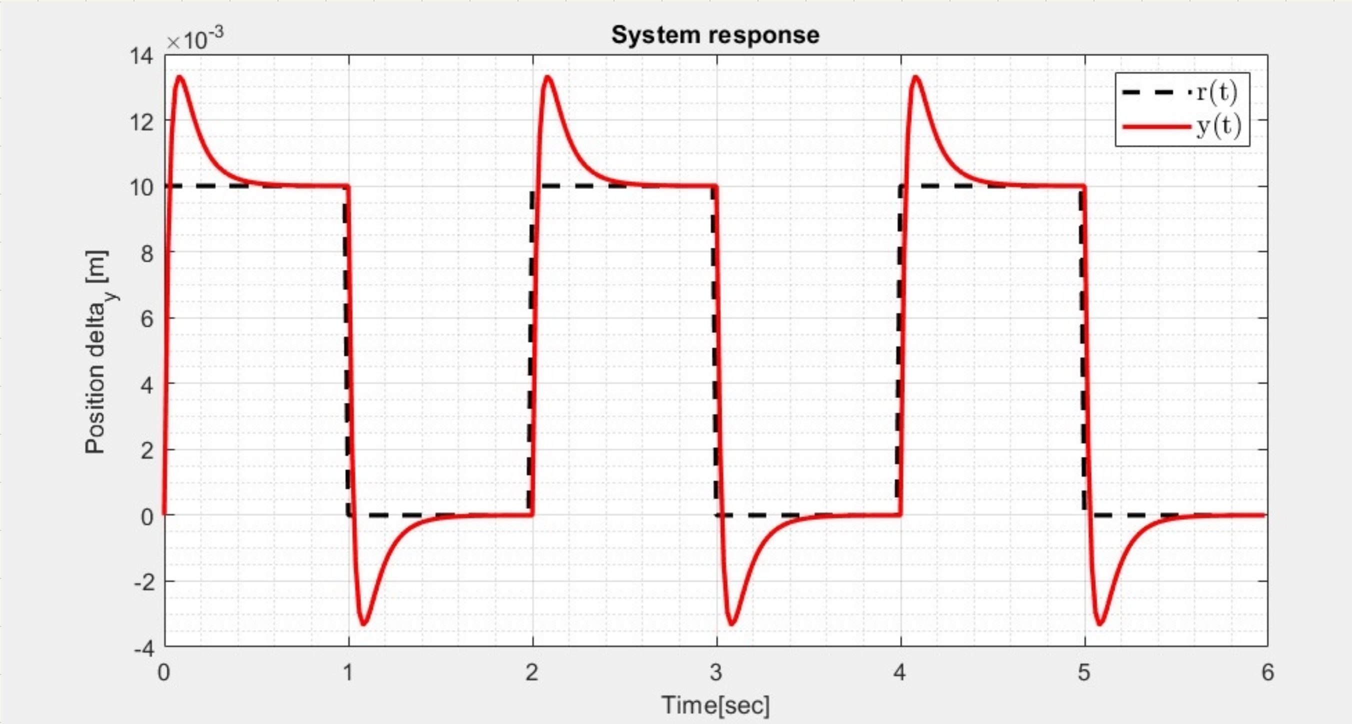
pole locations: $\pm\sqrt{\frac{2a\bar{i}^2}{m_b \frac{1}{h}^3}}$ one pole is greater than zero, so the system is unstable



1-3 Feedback Control

By matlab computation, the pole locations are

$-29.4799, -16.7750, -8.9268$



Problem 2.

2-1 Equation of motion

ODE: $m\ddot{x} = -c(\dot{x}-\dot{z}) - k(x-z)$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = c\dot{z} + kz$$

$$\overset{\mathcal{L}}{\Rightarrow} ms^2X(s) + csX(s) + kX(s) = csZ(s) + kZ(s)$$

$$\Rightarrow H(s) = \frac{X(s)}{Z(s)} = \frac{cs+k}{ms^2+cs+k} \quad \#$$

2-2 Critically Damped Design

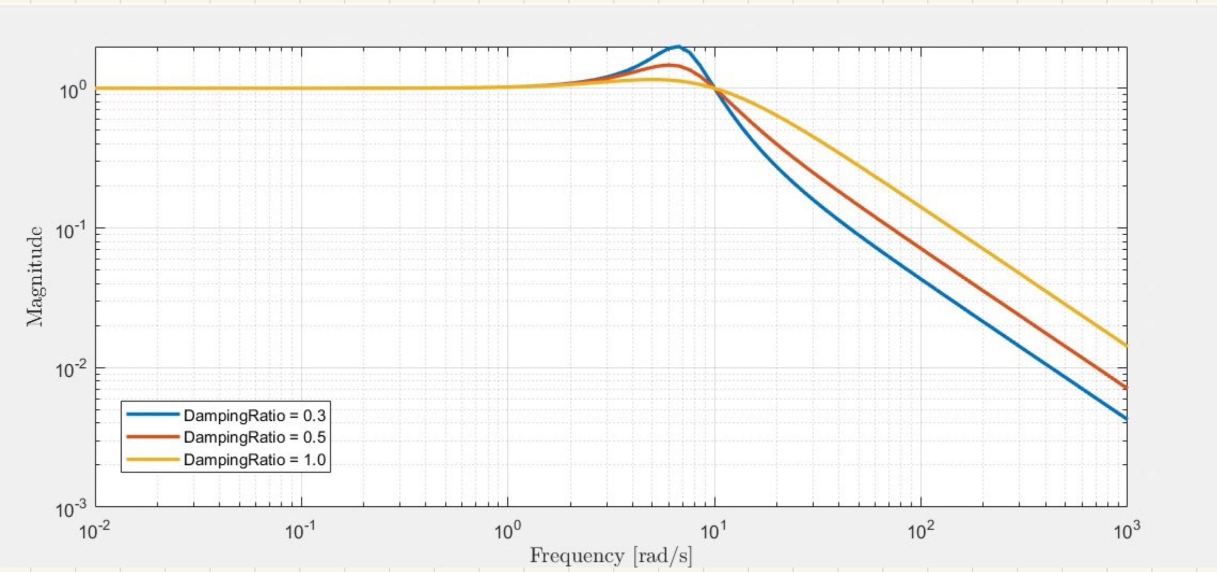
plug in: $m = 1.0\,kg$, $k = 50\,N/m$

$$H(s) = \frac{cs+50}{s^2+cs+50}$$

$$s^2 + cs + 50 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

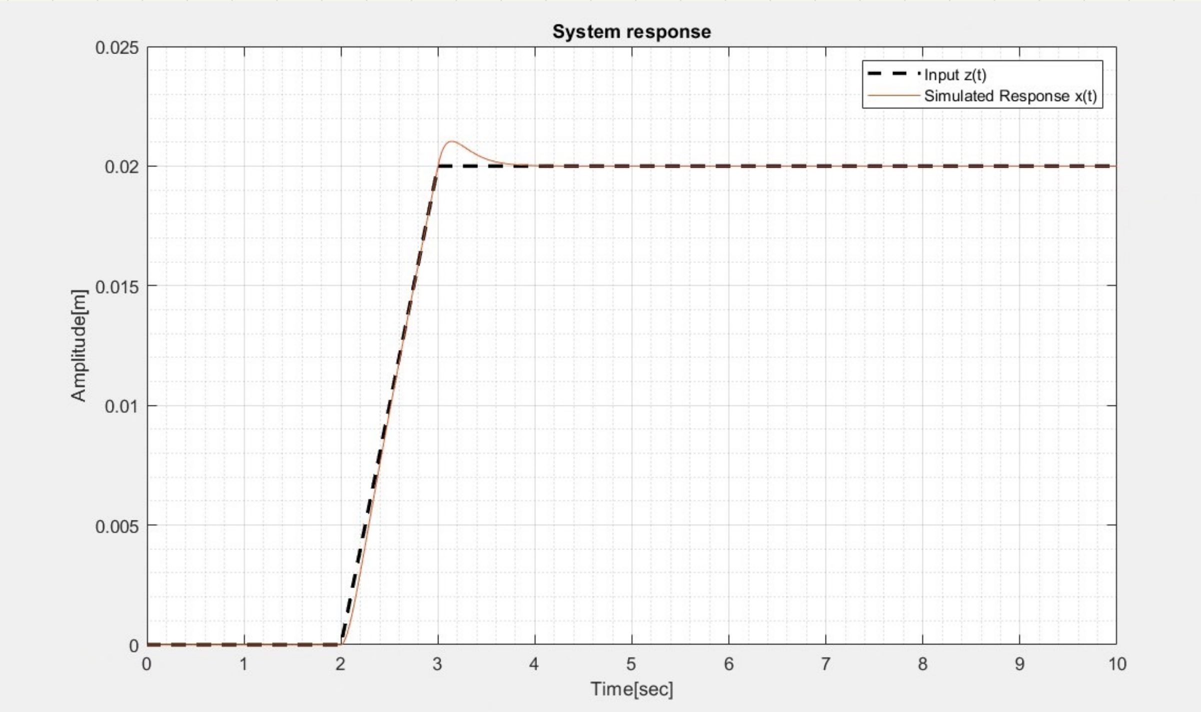
$$\Rightarrow \omega_n = 5\sqrt{2}\ rad/s,\ c = 10\sqrt{2}\,\zeta$$

$\therefore$ when critically damped $\zeta = 1$, $c = 10\sqrt{2}$ #



2-3 Response for Device Transferring

Assume $\zeta = 1$



2-4 Anti-vibration Table

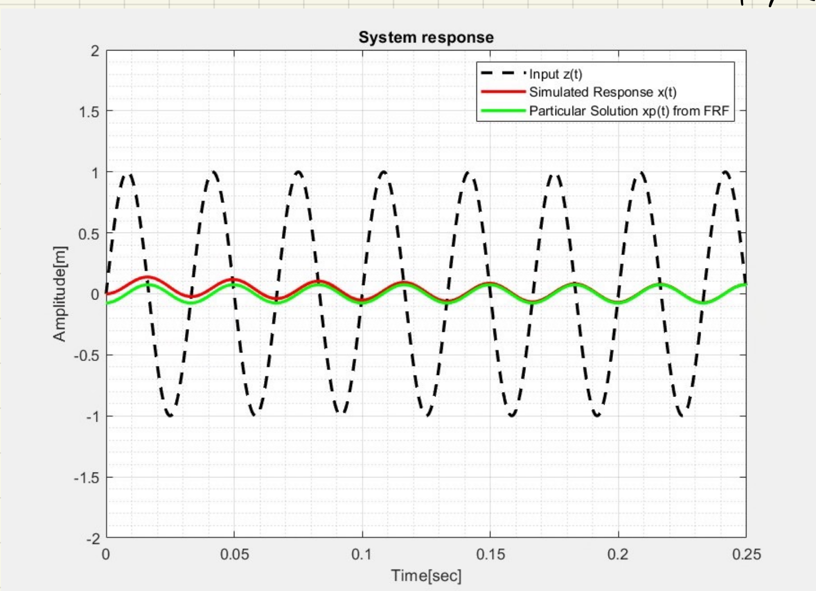
$$H(s) = \frac{cs + 50}{s^2 + cs + 50}$$

$$\Rightarrow H(j\omega_0) = \frac{cj\omega_0 + 50}{-\omega_0^2 + cj\omega_0 + 50}$$

$, \ \omega_0 = 2\pi f_0$ with $f_0 = 30\,Hz$

$C = 10\sqrt{2}$ when $\zeta = 1$

$$\Rightarrow H(j\omega_0) = \frac{50 + j\,600\sqrt{2}\pi}{(-3600\pi^2 + 50) + j\,600\sqrt{2}\pi} \Rightarrow \begin{cases} A(\omega_0) = 0.0749 \\ \phi(\omega_0) = 93.22^\circ \end{cases}$$



3-1

$$\zeta = \frac{\frac{1}{11-1}\ln\frac{0.4016}{0.3116}}{\sqrt{4\pi^2+\left(\frac{1}{11-1}\ln\frac{0.4016}{0.3116}\right)^2}} = 4.0384\times10^{-3}$$

$$W_d \approx \frac{2\pi}{t_{i+1}-t_i} = \sqrt{1-\zeta^2}\,W_n \qquad W_d \approx \frac{2\pi}{0.515} = \sqrt{1-(4.0384\times10^{-3})^2}\,W_n$$

$$W_n = 12.2005\left(\tfrac{1}{s}\right)$$

$$2\zeta W_n = \frac{c}{I_0}$$

$$W_n^2 = \frac{mg\ell}{I_0} \Rightarrow I_0 = \frac{15.06\times10^{-3}(kg)\times9.81(m/s^2)\times57.03\times10^{-3}(m)}{(12.2005)^2(1/s^2)} = 5.66\times10^{-5}\,(kg\cdot m^2) \quad \#$$

$$C = 2\zeta W_n I_0 = 2\times4.0384\times10^{-3}\times12.2005\left(\tfrac{1}{s}\right)\times5.66\times10^{-5}(kg\cdot m^2)$$

$$= 5.5774\times10^{-6}\left(kg\cdot m^2/s\right) \quad \#$$

3-2