

Secondary Outage Analysis of Amplify-and-Forward Cognitive Relays with Direct Link and Primary Interference

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Abstract—The use of cognitive relays is an emerging and promising solution to overcome the problem of spectrum underutilization while achieving the spatial diversity. In this paper, we perform an outage analysis of the secondary system with amplify-and-forward relays in a spectrum sharing scenario, where a secondary transmitter communicates with a secondary destination over a direct link as well as the best relay. Specifically, under the peak power constraint, we derive a closed-form expression of the secondary outage probability provided that the primary outage probability remains below a predefined value. We also take into account the effect of primary interference on the secondary outage performance. Finally, we validate the analysis by simulation results.

Index Terms—Amplify-and-forward relays, cognitive radio, outage probability, spectrum sharing.

I. INTRODUCTION

A. Relays in Cognitive Radio

In future wireless networks, cognitive radio [1] is an exciting solution to overcome the inefficient use of spectrum as it allows spectrum sharing between the licensed user (primary user) and the unlicensed user (secondary user). In a spectrum sharing scenario [2], [3], a secondary user (SU) may share the spectrum with the primary user (PU), provided that SU does not violate the interference constraint at the PU receiver—which prompts SU to limit its transmit power to satisfy the interference constraint.

The use of relays for secondary communication in cognitive radio, at the same time, offers better reliability and improved coverage for SU's transmission [4]–[8]. In addition, the cognitive relays provide increased spatial diversity compared to only direct link transmission. However, the secondary system with relays, in spectrum sharing, faces particularly following two challenges that hinder its performance:

- 1) Limitations on its transmit power to satisfy the interference constraint at PU receiver.
- 2) Harmful interference from primary transmissions.

Among various relaying protocols, amplify-and-forward (AF) and decode-and-forward (DF) are the most popular due to their low complexity. In AF relaying, a relay amplifies the signal received from the secondary transmitter and forwards it to the secondary destination [9], [10], whereas in DF relaying, the relay decodes the received signal and forwards it to the secondary destination [6], [11].

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B. Contributions and Related Work

1) *Contributions*: We perform an analysis for the outage probability of a secondary system with AF relaying, provided that the outage probability of PU remains below a predefined threshold—we characterize the interference to PU as its outage probability. We couple the primary outage constraint with the peak power constraint. We then choose the best relay that maximizes the end-to-end signal-to-interference noise ratio (SINR), and derive a closed-form expression for the secondary outage probability considering the interference from the primary transmission. We assume the presence of the direct link between the secondary transmitter and the secondary destination, and use the maximum ratio combining (MRC) to combine two copies of signal—one via direct link and second via the best relay—at the secondary destination.

2) *Related Work*: In [6], [12], authors derive a closed-form expression of the secondary outage probability with the direct link and primary interference under PU's outage probability constraint. In [13], authors consider a spectrum sharing scenario, where a single AF relay assists the secondary direct link communication, and the signals at the secondary destination are combined by selection combining; but the PU interference is ignored. In [14], authors study the effect of PU's interference on secondary outage probability for AF relays in absence of the direct link, while [15] uses similar setup like [14] for DF relays. Authors in [16], [17] study a secondary system with DF relays under direct link and primary interference with the interference power constraint at PU. The references [4], [18] consider the direct secondary link along with DF relays and calculate the secondary outage probability. However, they ignore the effect of PU's interference on the secondary transmission.

II. SYSTEM MODEL

Consider a cognitive radio network consisting of a primary transmitter (PT), a primary destination (PD), a secondary transmitter (ST), a secondary destination (SD), and N AF secondary relays (SR), as shown in Fig. 1. The ST communicates with SD via the direct link as well as i th AF relay ($i = 1, 2, \dots, N$). The relays operate in a half-duplex mode. The communication between ST and SD happens over two time slots, each of T -second duration. In the first time slot, ST transmits the signal with power P_{ST} to SD over the direct link, and to secondary relays; while in the second time slot, the best relay amplifies the received signal and forwards it to SD with power P_{SR_i} . At SD, two received signal copies—first via direct link and second via the best relay—are combined by the maximum ratio combining. Relay selection can be employed by a centralized

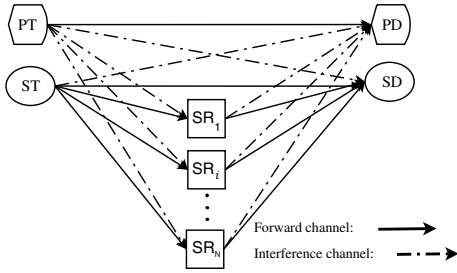


Fig. 1. Secondary transmissions via AF relays in spectrum sharing.

entity, such as the secondary source or a secondary network-manager or in a distributed manner using timers [19]. We consider the peak power constraint P_{pk} on transmit powers of ST and i th secondary relay. In addition, the constraint that the primary outage probability should be below a predefined value regulates the transmit powers of ST and i th secondary relay. Denote the powers of ST and i th secondary relay, when they are regulated by the primary outage constraint alone, by $P_{u,ST}$ and P_{u,SR_i} , respectively. Then, combining both above constraints, the maximum allowable powers for ST and i th secondary relay become

$$P_{ST} = \min(P_{pk}, P_{u,ST}) \quad (1)$$

and

$$P_{SR_i} = \min(P_{pk}, P_{u,SR_i}), \quad (2)$$

respectively. The channel between a transmitter $a \in \{PT, ST, SR_i\}$ and a receiver $b \in \{PD, SD, SR_i\}$ is a Rayleigh fading channel with its channel gain denoted by h_{a-b} . Therefore, the channel power gain $|h_{a-b}|^2$ is exponentially distributed with the mean channel power gain Ω_{a-b} . Thus, we can write the probability density function (PDF) and cumulative distribution function (CDF) of $X = |h_{a-b}|^2$ as

$$f_X(x) = \frac{1}{\Omega_{a-b}} \exp\left(-\frac{x}{\Omega_{a-b}}\right), x \geq 0, \quad (3)$$

$$F_X(x) = 1 - \exp\left(-\frac{x}{\Omega_{a-b}}\right), x \geq 0, \quad (4)$$

respectively, where $\exp(\cdot)$ represents the exponential function. We consider that the channels are independent of each other, experience block-fading, and remain constant for two slots of the secondary communication, i.e., for $2T$ -second, as in [4], [6], [7].

III. MAXIMUM AVERAGE ALLOWABLE TRANSMIT POWER FOR SECONDARY TRANSMITTER AND RELAYS

We use the primary outage probability to characterize the quality of service (QoS) of primary transmissions. The outage probability of the primary user should be below a certain value λ_p , given the interference from the secondary transmitter and relay. For a constant primary transmit power P_{PT} , we can calculate the primary outage probability as

$$P_{out_p} = \Pr\left(\log_2\left(1 + \frac{P_{PT}|h_{PT-PD}|^2}{P_{t,ST}|h_{ST-PD}|^2 + N_0}\right) < R_p\right) \leq \lambda_p, \quad (5)$$

where R_p is the primary user's desired data rate, N_0 is additive white Gaussian noise (AWGN) power at all receivers, and $P_{t,ST}$ is the transmit power of ST. The term $\frac{P_{PT}|h_{PT-PD}|^2}{P_{t,ST}|h_{ST-PD}|^2 + N_0}$

represents the received SINR at PD. In (5), at the maximum allowed average power $P_{u,ST}$ for ST, i.e., when $P_{t,ST} = P_{u,ST}$, the weak inequality becomes equality. Thus, from (5), conditioned on $|h_{ST-PD}|^2 = x$, we can write

$$P_{out_p} \Big|_{|h_{ST-PD}|^2=x} = \Pr\left(|h_{PT-PD}|^2 < \frac{\theta_p(P_{u,ST}x + N_0)}{P_{PT}}\right) = \lambda_p, \quad (6)$$

where $\theta_p = 2^{R_p} - 1$. Thus, we can write (6) as

$$P_{out_p} \Big|_{|h_{ST-PD}|^2=x} = 1 - \exp\left(-\frac{\theta_p(P_{u,ST}x + N_0)}{\Omega_{PT-PD}P_{PT}}\right). \quad (7)$$

Taking expectation with respect to $|h_{ST-PD}|^2$, we obtain

$$P_{out_p} = 1 - \frac{\exp\left(-\frac{\theta_p N_0}{\Omega_{PT-PD}P_{PT}}\right)}{1 + \frac{\theta_p P_{u,ST}\Omega_{ST-PD}}{\Omega_{PT-PD}P_{PT}}}. \quad (8)$$

Rearranging the terms and using (6), we find the maximum secondary transmit power $P_{u,ST}$ under alone primary outage constraint as

$$P_{u,ST} = \frac{P_{PT}\Omega_{PT-PD}}{\theta_p\Omega_{ST-PD}} \left(\frac{\exp\left(\frac{\theta_p N_0}{\Omega_{PT-PD}P_{PT}}\right)}{1 - \lambda_p} - 1 \right)^+, \quad (9)$$

where $(x)^+ = \max(x, 0)$. After combining with the peak power constraint, the maximum average allowable transmit power P_{ST} for the secondary transmitter can be given by (1). Similar to (9), the transmit power of i th secondary relay regulated alone by the primary outage constraint can be readily found as

$$P_{u,SR_i} = \frac{P_{PT}\Omega_{PT-PD}}{\theta_p\Omega_{SR_i-PD}} \left(\frac{\exp\left(\frac{\theta_p N_0}{\Omega_{PT-PD}P_{PT}}\right)}{1 - \lambda_p} - 1 \right)^+. \quad (10)$$

After combining with the peak power constraint, the maximum average allowable transmit power P_{SR_i} for relay i can be given by (2).

IV. DERIVATION OF SECONDARY OUTAGE PROBABILITY

The AF relays cooperate opportunistically, where the relay with the largest end-to-end SINR at the secondary destination is selected to forward the received signal in the second time slot. Thus, after receiving the signal from both time slots, SD combines them using MRC technique. The end-to-end SINR is given by [20], [21]

$$\begin{aligned} \gamma_{eq} &= \gamma_{SD} + \max_{SR_i \in \mathbb{R}} \left(\frac{\gamma_{SR_i}\gamma_{R_iD}}{1 + \gamma_{SR_i} + \gamma_{R_iD}} \right) \\ &\leq \gamma_{SD} + \max_{SR_i \in \mathbb{R}} (\min(\gamma_{SR_i}, \gamma_{R_iD})) = \gamma_{tot}, \end{aligned} \quad (11)$$

where \mathbb{R} is the set of relays given as $\mathbb{R} = \{SR_1, \dots, SR_i, \dots, SR_N\}$, γ_{SR_i} , γ_{SD} , and γ_{R_iD} denote SINR at the i th relay, and SINR at SD due to direct transmission and relaying respectively, which are given by

$$\gamma_{SR_i} = \frac{P_{ST}|h_{ST-SR_i}|^2}{P_{PT}|h_{PT-SR_i}|^2 + N_0}, \quad (12)$$

$$\gamma_{SD} = \frac{P_{ST}|h_{ST-SD}|^2}{P_{PT}|h_{PT-SD}|^2 + N_0}, \quad (13)$$

$$\gamma_{R_iD} = \frac{P_{SR_i}|h_{SR_i-SD}|^2}{P_{PT}|h_{PT-SD}|^2 + N_0}. \quad (14)$$

For analytical tractability, we use the upper bound given in (11), which is tight in medium to high SINR range [20], [21]. We can obtain P_{ST} and P_{SR_i} from (1) and (2). The secondary outage occurs when the instantaneous SINR of the secondary transmission falls below the designated threshold, θ_s . Thus, we can write the secondary outage probability as

$$\mathcal{P}_o = \Pr(\gamma_{SD} + \max_{SR_i \in \mathbb{R}} (\min(\gamma_{SR_i}, \gamma_{R_iD})) < \theta_s), \quad (15)$$

where $\theta_s = 2^{2R_s} - 1$ with R_s is the desired secondary data rate. From (11), we can see that, γ_{SD} , γ_{R_iD} , and γ_{R_jD} ($i \neq j$) contain a common term $|h_{PT-SD}|^2$, that makes them dependent. Thus, conditioning on $|h_{PT-SD}|^2 = y$ and denoting $Z = \max_{SR_i \in \mathbb{R}} (\min(\gamma_{SR_i}, \gamma_{R_iD}))$, we can write

$$\begin{aligned} \Pr(\gamma_{tot} < \theta_s) \big|_{|h_{PT-SD}|^2=y} &= \Pr(\gamma_{SD} < \theta_s - Z) \\ &= \int_0^{\theta_s} F_{\gamma_{SD}}(\theta_s - z) f_Z(z) dz. \end{aligned} \quad (16)$$

Now, we have

$$\begin{aligned} F_{\gamma_{SD}}(z) \big|_{|h_{PT-SD}|^2=y} &= \Pr\left(|h_{ST-SD}|^2 < \frac{z(P_{PT}y + N_0)}{P_{ST}}\right) \\ &= 1 - \exp\left(-\frac{z(P_{PT}y + N_0)}{\Omega_{ST-SD}P_{ST}}\right). \end{aligned} \quad (17)$$

We also have

$$\begin{aligned} F_Z(z) \big|_{|h_{PT-SD}|^2=y} &= \Pr\left(\max_{SR_i \in \mathbb{R}} (\min(\gamma_{SR_i}, \gamma_{R_iD})) < z\right) \\ &= \prod_{i=1}^N \Pr(\min(\gamma_{SR_i}, \gamma_{R_iD}) < z) \\ &= \prod_{i=1}^N [1 - \Pr(\gamma_{SR_i} > z) \Pr(\gamma_{R_iD} > z)], \end{aligned} \quad (18)$$

where (18) results from the independence of γ_{SR_i} and γ_{R_iD} , given y . For ease of presentation and without compromising the insight into analysis, we assume that mean channel gains of ST-SR_i are the same for all relays and so is for SR_i-SD, PT-SR_i, and SR_i-PD channels. Thus, we have $P_{SR_i} = P_{SR}$. Next, given $|h_{PT-SD}|^2 = y$, we compute $\Pr(\gamma_{SR_i} > z)$ as

$$\begin{aligned} \Pr(\gamma_{SR_i} > z) &= \Pr\left(|h_{ST-SR_i}|^2 > \frac{z(P_{PT}|h_{PT-SR_i}|^2 + N_0)}{P_{ST}}\right) \\ &= \int_0^\infty \Pr\left(|h_{ST-SR_i}|^2 > \frac{z(P_{PT}w + N_0)}{P_{ST}}\right) f_{|h_{PT-SR_i}|^2}(w) dw \\ &= \int_0^\infty \exp\left(-\frac{z(P_{PT}w + N_0)}{\Omega_{ST-SR}P_{ST}}\right) \frac{\exp\left(-\frac{w}{\Omega_{PT-SR}}\right)}{\Omega_{PT-SR}} dw \\ &= \frac{\exp\left(-\frac{zN_0}{\Omega_{ST-SR}P_{ST}}\right)}{1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}}. \end{aligned} \quad (20)$$

We also compute $\Pr(\gamma_{R_iD} > z)$ as

$$\begin{aligned} \Pr(\gamma_{R_iD} > z) &= \Pr\left(|h_{SR_i-SD}|^2 > \frac{z(P_{PT}|h_{PT-SD}|^2 + N_0)}{P_{ST}}\right) \\ &= \exp\left(-\frac{z(P_{PT}y + N_0)}{\Omega_{SR-SD}P_{SR}}\right). \end{aligned} \quad (21)$$

Thus, by substituting (20) and (21) in (19), we have

$$\begin{aligned} F_Z(z) \big|_{|h_{PT-SD}|^2=y} &= \left(1 - \frac{\exp\left(-zN_0\left(\frac{1}{\Omega_{ST-SR}P_{ST}} + \frac{1}{\Omega_{SR-SD}P_{SR}}\right)\right)}{1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}}\right)^N \\ &\times \exp\left(-\frac{zP_{PT}y}{\Omega_{SR-SD}P_{SR}}\right) \\ &= \left[\sum_{n=0}^N \binom{N}{n} (-1)^n \frac{\exp\left(-nzN_0\left(\frac{1}{\Omega_{ST-SR}P_{ST}} + \frac{1}{\Omega_{SR-SD}P_{SR}}\right)\right)}{\left(1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}\right)^n}\right] \\ &\times \exp\left(-\frac{nzP_{PT}y}{\Omega_{SR-SD}P_{SR}}\right). \end{aligned} \quad (22)$$

Hence, PDF of Z is given by

$$\begin{aligned} f_Z(z) \big|_{|h_{PT-SD}|^2=y} &= \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \\ &\times \exp\left(-nzN_0\left(\frac{1}{\Omega_{ST-SR}P_{ST}} + \frac{1}{\Omega_{SR-SD}P_{SR}}\right)\right) \\ &\times \exp\left(-\frac{nzP_{PT}y}{\Omega_{SR-SD}P_{SR}}\right) \\ &\times \left[\frac{\left(\frac{N_0}{\Omega_{ST-SR}P_{ST}} + \frac{N_0}{\Omega_{SR-SD}P_{SR}} + \frac{P_{PT}y}{\Omega_{SR-SD}P_{SR}}\right)}{\left(1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}\right)^n}\right. \\ &\left. + \frac{\frac{\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}}{\left(1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}\right)^{n+1}}\right]. \end{aligned} \quad (23)$$

From (16), we have

$$\begin{aligned} \Pr(\gamma_{tot} < \theta_s) \big|_{|h_{PT-SD}|^2=y} &= \int_0^{\theta_s} \left(1 - \exp\left(-\frac{\theta_s(P_{PT}y + N_0)}{\Omega_{ST-SD}P_{ST}}\right)\right) \\ &\times \exp\left(\frac{z(P_{PT}y + N_0)}{\Omega_{ST-SD}P_{ST}}\right) f_Z(z) \big|_{|h_{PT-SD}|^2=y} dz \\ &= F_Z(\theta_s) \big|_{|h_{PT-SD}|^2=y} \\ &- \exp\left(-\frac{\theta_s(P_{PT}y + N_0)}{\Omega_{ST-SD}P_{ST}}\right) \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \\ &\times \int_0^{\theta_s} \exp\left(-zN_0\left(\frac{n}{\Omega_{ST-SR}P_{ST}} + \frac{n}{\Omega_{SR-SD}P_{SR}}\right) - \frac{1}{\Omega_{ST-SD}P_{ST}}\right) \\ &\times \frac{\exp\left(-P_{PT}yz\left(\frac{n}{\Omega_{SR-SD}P_{SR}} - \frac{1}{\Omega_{ST-SD}P_{ST}}\right)\right)}{\left(1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}\right)^n} \\ &\times \left(\frac{N_0}{\Omega_{ST-SR}P_{ST}} + \frac{N_0}{\Omega_{SR-SD}P_{SR}} + \frac{\frac{\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}}{\left(1 + \frac{z\Omega_{PT-SR}P_{PT}}{\Omega_{ST-SR}P_{ST}}\right)}\right. \\ &\left. + \frac{P_{PT}y}{\Omega_{SR-SD}P_{SR}}\right) dz. \end{aligned} \quad (24)$$

Hence, the outage probability can be expressed as

$$\mathcal{P}_o = E_Y \left[\Pr(\gamma_{tot} < \theta_s) \big|_{Y=y} \right] = \mathcal{I}_1 - \mathcal{I}_2 - \mathcal{I}_3, \quad (25)$$

where $E_Y[\cdot]$ is the expectation operator on Y and

$$\begin{aligned} \mathcal{I}_1 &= \sum_{n=0}^N \binom{N}{n} (-1)^n \\ &\times \frac{\exp\left(-n\theta_S N_0 \left(\frac{1}{\Omega_{ST-SR} P_{ST}} + \frac{1}{\Omega_{SR-SD} P_{SR}}\right)\right)}{\left(1 + \frac{\theta_S \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n} \\ &\times \int_0^\infty \exp\left(-\frac{n\theta_S P_{PT} y}{\Omega_{SR-SD} P_{SR}}\right) \frac{\exp\left(-\frac{y}{\Omega_{PT-SD}}\right)}{\Omega_{PT-SD}} dy, \quad (26) \end{aligned}$$

$$\begin{aligned} \mathcal{I}_2 &= \exp\left(-\frac{\theta_S N_0}{\Omega_{ST-SD} P_{ST}}\right) \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \\ &\times \int_0^{\theta_S} \exp\left(-z N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right)\right) \\ &\times \frac{\left(\frac{N_0}{\Omega_{ST-SR} P_{ST}} + \frac{N_0}{\Omega_{SR-SD} P_{SR}} + \frac{\frac{\Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)}\right)}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n} \\ &\times \int_{y=0}^\infty \exp\left(-P_{PT} y \left(\frac{nz}{\Omega_{SR-SD} P_{SR}} + \frac{\theta_S}{\Omega_{ST-SD} P_{ST}} - \frac{z}{\Omega_{ST-SD} P_{ST}}\right)\right) \frac{\exp\left(-\frac{y}{\Omega_{PT-SD}}\right)}{\Omega_{PT-SD}} dy dz, \quad (27) \end{aligned}$$

$$\begin{aligned} \mathcal{I}_3 &= \exp\left(-\frac{\theta_S N_0}{\Omega_{ST-SD} P_{ST}}\right) \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \\ &\times \int_0^{\theta_S} \exp\left(-z N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right)\right) \frac{\frac{P_{PT}}{\Omega_{SR-SD} P_{SR}}}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n} \\ &\times \int_0^\infty y \exp\left(-P_{PT} y \left(\frac{nz}{\Omega_{SR-SD} P_{SR}} + \frac{\theta_S}{\Omega_{ST-SD} P_{ST}} - \frac{z}{\Omega_{ST-SD} P_{ST}}\right)\right) \frac{\exp\left(-\frac{y}{\Omega_{PT-SD}}\right)}{\Omega_{PT-SD}} dy dz. \quad (28) \end{aligned}$$

We use the following results in (29) and (30) to derive the integrations $\mathcal{I}_i, i = 1, 2, 3$: When Y is an exponential random variable with mean Ω_Y , we have

$$\begin{aligned} E_Y[\exp(-RY)] &= \frac{1}{\Omega_Y} \int_0^\infty \exp\left(-\left(R + \frac{1}{\Omega_Y}\right)y\right) dy \\ &= \frac{1}{1 + \Omega_Y R}, \quad (29) \end{aligned}$$

$$\begin{aligned} E_Y[Y \exp(-RY)] &= \frac{1}{\Omega_Y} \int_0^\infty y \exp\left(-\left(R + \frac{1}{\Omega_Y}\right)y\right) dy \\ &= \frac{\Omega_Y}{(1 + \Omega_Y R)^2}, \quad (30) \end{aligned}$$

with $R \geq 0$. Using (29), we compute \mathcal{I}_1 as

$$\begin{aligned} \mathcal{I}_1 &= \sum_{n=0}^N \binom{N}{n} (-1)^n \\ &\times \frac{\exp\left(-n\theta_S N_0 \left(\frac{1}{\Omega_{ST-SR} P_{ST}} + \frac{1}{\Omega_{SR-SD} P_{SR}}\right)\right)}{\left(1 + \frac{\theta_S \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n \left(1 + \frac{n\theta_S \Omega_{PT-SD} P_{PT}}{\Omega_{SR-SD} P_{SR}}\right)}. \quad (31) \end{aligned}$$

To compute \mathcal{I}_2 , we write it as

$$\begin{aligned} \mathcal{I}_2 &= \exp\left(-\frac{\theta_S N_0}{\Omega_{ST-SD} P_{ST}}\right) \sum_{n=1}^N n \binom{N}{n} (-1)^{n+1} \\ &\times (\mathcal{I}_{2,1,n} + \mathcal{I}_{2,2,n}), \quad (32) \end{aligned}$$

where

$$\begin{aligned} \mathcal{I}_{2,1,n} &= \int_0^{\theta_S} \frac{\left(\frac{N_0}{\Omega_{ST-SR} P_{ST}} + \frac{N_0}{\Omega_{SR-SD} P_{SR}}\right)}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n} \\ &\times \frac{\exp\left(-z N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right)\right) dz}{\left(1 + \Omega_{PT-SD} P_{PT} \left(\frac{nz}{\Omega_{SR-SD} P_{SR}} + \frac{\theta_S}{\Omega_{ST-SD} P_{ST}} - \frac{z}{\Omega_{ST-SD} P_{ST}}\right)\right)} \quad (33) \end{aligned}$$

and

$$\begin{aligned} \mathcal{I}_{2,2,n} &= \int_0^{\theta_S} \frac{\frac{\Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^{n+1}} \\ &\times \frac{\exp\left(-z N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right)\right) dz}{\left(1 + \Omega_{PT-SD} P_{PT} \left(\frac{nz}{\Omega_{SR-SD} P_{SR}} + \frac{\theta_S}{\Omega_{ST-SD} P_{ST}} - \frac{z}{\Omega_{ST-SD} P_{ST}}\right)\right)}. \quad (34) \end{aligned}$$

To compute $\mathcal{I}_{2,1,n}$ and $\mathcal{I}_{2,2,n}$, we use the following notations for convenience of presentation:

$$\begin{aligned} S &= N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right), \\ \mu &= \Omega_{PT-SD} P_{PT} \left(\frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}}\right), \\ \tau &= \frac{\frac{\Omega_{PT-SD} P_{PT} \theta_S}{\Omega_{ST-SD} P_{ST}} + 1}{\mu}, \\ \pi_1 &= \left(\frac{\Omega_{ST-SR} P_{ST}}{\Omega_{PT-SR} P_{PT}}\right). \quad (35) \end{aligned}$$

Thus, we can write

$$\begin{aligned} \mathcal{I}_{2,1,n} &= \frac{\pi_1^n}{\mu} \left(\frac{N_0}{\Omega_{ST-SR} P_{ST}} + \frac{N_0}{\Omega_{SR-SD} P_{SR}}\right) \\ &\times \underbrace{\int_0^{\theta_S} \frac{\exp(-Sz)}{(z + \pi_1)^n (z + \tau)} dz}_{\mathcal{J}_{2,1,n}}. \quad (36) \end{aligned}$$

For $\Omega_{ST-SD} > \Omega_{ST-SR}$ and $\Omega_{ST-SD} > \Omega_{SR-SD}$, we have $S > 0, \mu > 0, \tau > 0$ and we can write (36) in terms of the exponential integral as shown later in this section. Using the substitution, $r = z + \pi_1$ and denoting $\chi = \tau - \pi_1$, we write

$$\mathcal{J}_{2,1,n} = \exp(S\pi_1) \int_{\pi_1}^{\pi_1 + \theta_S} \frac{\exp(-Sr)}{r^n (r + \chi)} dr. \quad (37)$$

Using the partial fraction expansion, we have

$$\frac{1}{r^n(r+\chi)} = \sum_{m=0}^{n-1} \frac{(-1)^m}{\chi^{m+1}r^{n-m}} + \frac{1}{(-\chi)^n(r+\chi)}, \quad (38)$$

Thus, we can write

$$\begin{aligned} \mathcal{J}_{2,1,n} &= \exp(S\pi_1) \int_{\pi_1}^{\pi_1+\theta_S} \exp(-Sr) \\ &\times \left(\sum_{m=0}^{n-1} \frac{(-1)^m}{\chi^{m+1}r^{n-m}} + \frac{1}{(-\chi)^n(r+\chi)} \right) dr \\ &= \exp(S\pi_1) \sum_{m=0}^{n-1} \frac{(-1)^m}{\chi^{m+1}} \int_{\pi_1}^{\pi_1+\theta_S} \frac{\exp(-Sr)}{r^{n-m}} dr \\ &+ \frac{\exp(S\pi_1)}{(-\chi)^n} \int_{\pi_1+\chi}^{\pi_1+\chi+\theta_S} \frac{\exp(-S(p-\chi))}{p} dp \end{aligned} \quad (39)$$

$$\begin{aligned} &= \exp(S\pi_1) \sum_{m=0}^{n-1} \frac{(-1)^m}{\chi^{m+1}} S^{n-m-1} \int_{S\pi_1}^{S(\pi_1+\theta_S)} \frac{\exp(-z)}{z^{n-m}} dz \\ &+ \frac{\exp(S(\pi_1+\chi))}{(-\chi)^n} \int_{S\tau}^{S(\tau+\theta_S)} \frac{\exp(-y)}{y} dy \quad (40) \\ &= \exp(S\pi_1) \left(\sum_{m=0}^{n-2} \frac{(-1)^m}{\chi^{m+1}} S^{n-m-1} \right. \\ &\times [\Gamma(m-n+1, S\pi_1) - \Gamma(m-n+1, S(\pi_1+\theta_S))] \\ &+ \exp(S\pi_1) \frac{(-1)^{n-1}}{\chi^n} [E_1(S\pi_1) - E_1(S(\pi_1+\theta_S))] \\ &+ \exp(S\tau) \frac{(-1)^n}{\chi^n} [E_1(S\tau) - E_1(S(\tau+\theta_S))] \Big], \end{aligned} \quad (41)$$

where in (39), we use the substitution $p = r + \chi$, and in (40), we use $z = Sr$ and $y = Sp$; $\Gamma(\cdot, \cdot)$ and $E_1(\cdot)$ are upper incomplete gamma function and exponential integral [22], respectively with $E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt$. Similarly, we compute $\mathcal{I}_{2,2,n}$ by representing it as

$$\mathcal{I}_{2,2,n} = \frac{\pi_1^n}{\mu} \underbrace{\int_0^{\theta_S} \frac{\exp(-Sz)}{(z+\pi_1)^{n+1}(z+\tau)} dz}_{\mathcal{J}_{2,2,n}}, \quad (42)$$

where $\mathcal{J}_{2,2,n}$ is computed as

$$\begin{aligned} \mathcal{J}_{2,2,n} &= \exp(S\pi_1) \left(\sum_{m=0}^{n-1} \frac{(-1)^m}{\chi^{m+1}} S^{n-m-1} \right. \\ &\times [\Gamma(m-n, S\pi_1) - \Gamma(m-n, S(\pi_1+\theta_S))] \\ &+ \exp(S\pi_1) \frac{(-1)^n}{\chi^{n+1}} [E_1(S\pi_1) - E_1(S(\pi_1+\theta_S))] \\ &+ \exp(S\tau) \frac{(-1)^{n+1}}{\chi^{n+1}} [E_1(S\tau) - E_1(S(\tau+\theta_S))] \Big]. \end{aligned} \quad (43)$$

We note that $\mathcal{J}_{2,2,n} = \mathcal{J}_{2,1,n}$ with n replaced by $n+1$. Thus, we can use the same procedure to compute both these

expressions. Using (30), we write \mathcal{I}_3 as

$$\begin{aligned} \mathcal{I}_3 &= \exp\left(-\frac{\theta_S N_0}{\Omega_{ST-SD} P_{ST}}\right) \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \\ &\times \int_0^{\theta_S} \exp\left(-z N_0 \left(\frac{n}{\Omega_{ST-SR} P_{ST}} + \frac{n}{\Omega_{SR-SD} P_{SR}} - \frac{1}{\Omega_{ST-SD} P_{ST}} \right)\right) \\ &\times \frac{\frac{P_{PT}}{\Omega_{SR-SD} P_{SR}}}{\left(1 + \frac{z \Omega_{PT-SR} P_{PT}}{\Omega_{ST-SR} P_{ST}}\right)^n} \\ &\times \frac{\Omega_{PT-SD} dz}{\left(1 + \Omega_{PT-SD} P_{PT} \left(\frac{nz}{\Omega_{SR-SD} P_{SR}} + \frac{\theta_S}{\Omega_{ST-SD} P_{ST}} - \frac{z}{\Omega_{ST-SD} P_{ST}} \right)\right)^2} \\ &= \frac{\Omega_{PT-SD} P_{PT}}{\Omega_{SR-SD} P_{SR}} \exp\left(-\frac{\theta_S N_0}{\Omega_{ST-SD} P_{ST}}\right) \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} n \frac{\pi_1^n}{\mu^2} \\ &\times \int_0^{\theta_S} \underbrace{\frac{\exp(-Sz)}{(z+\pi_1)^n (z+\tau)^2}}_{\mathcal{J}_{3,n}} dz, \end{aligned} \quad (44)$$

where we use the same notations as for the case of \mathcal{I}_2 . With the substitution of $t = z + \pi_1$, $\mathcal{J}_{3,n}$ can be written as

$$\mathcal{J}_{3,n} = \exp(S\pi_1) \int_{\pi_1}^{\pi_1+\theta_S} \frac{\exp(-St)}{t^n (t+\chi)^2} dt \quad (45)$$

For computation of $\mathcal{J}_{3,n}$, we use the following partial fraction expansion:

$$\frac{1}{r^n(r+\chi)^2} = \left(\sum_{m=0}^{n-1} \frac{(-1)^m(m+1)}{\chi^{m+2}r^{n-m}} \right) + \frac{1}{(-\chi)^n(r+\chi)^2} - \frac{n}{(-\chi)^{n+1}(r+\chi)}. \quad (46)$$

Using the steps similar to that of the derivation of $\mathcal{I}_{2,1,n}$ given in (36), we, hereby, can write the expression of $\mathcal{J}_{3,n}$ as

$$\begin{aligned} \mathcal{J}_{3,n} &= \exp(S\pi_1) \left(\sum_{m=0}^{n-2} \frac{(-1)^m(m+1)}{\chi^{m+2}} S^{n-m-1} \right. \\ &\times [\Gamma(m-n+1, S\pi_1) - \Gamma(m-n+1, S(\pi_1+\theta_S))] \\ &+ \exp(S\pi_1) \frac{(-1)^{n-1}}{\chi^{n+1}} [E_1(S\pi_1) - E_1(S(\pi_1+\theta_S))] \\ &+ \exp(S\tau) \frac{(-1)^n}{\chi^n} [\Gamma(-1, S\tau) - \Gamma(-1, S(\tau+\theta_S))] \\ &+ \exp(S\tau) \frac{(-1)^{n+1}}{\chi^{n+1}} [E_1(S\tau) - E_1(S(\tau+\theta_S))] \Big]. \end{aligned} \quad (47)$$

V. RESULTS AND DISCUSSIONS

Using the analysis performed in previous sections, we investigate the effects of direct link, primary interference, primary outage constraint, and the peak power constraint on the outage performance of the secondary system. We also validate the analysis by simulation results. The simulation parameters are as follow: $\Omega_{ST-SD} = 1.5$, $\Omega_{PT-PD} = \Omega_{ST-SR} = \Omega_{SR-SD} = 1$; $\Omega_{PT-SR} = \Omega_{PT-SD} = \Omega_{ST-PD} = \Omega_{SR-PD} = 0.5$, $N_0 = 1$, $R_p = 0.4$ bits/s/Hz, $R_s = 0.1$ bits/s/Hz.

Fig.2 shows the effect of primary power P_{PT} on the secondary outage probability \mathcal{P}_o . The increase in P_{PT} has

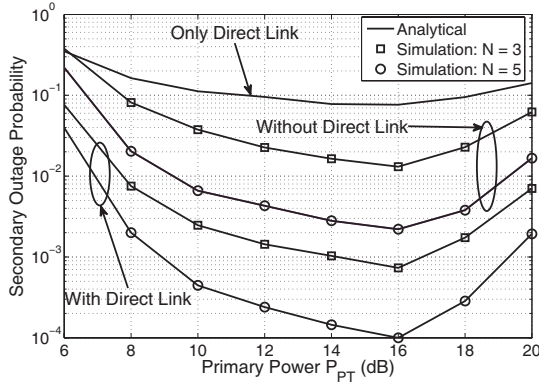


Fig. 2. Secondary outage probability vs. Primary power (P_{PT}) for different number of relays N , with and without direct link, $P_{pk} = 15\text{dB}$, $\lambda_p = 10^{-1}$.

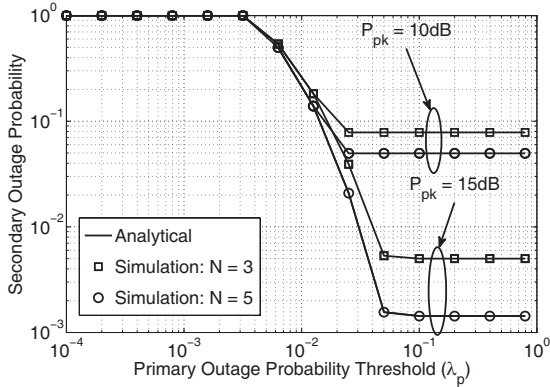


Fig. 3. Secondary outage probability vs. Primary outage probability threshold (λ_p) for different values of peak power constraint P_{pk} and number of relays N , $P_{PT} = 20\text{dB}$.

two opposite effects on P_o : 1) It improves the quality of the primary link, in turn, increases SINR at the primary destination. This leads to decrease in the primary outage probability, providing an extra margin for transmit powers of secondary transmitter ST (P_{ST}) and the selected relay SR (P_{SR}), which further helps in reducing the secondary outage probability; 2) it increases the interference to the secondary system, thereby increasing the secondary outage probability. From Fig. 2, we can observe that, initially, the secondary outage probability reduces as P_{PT} increases. However, if P_{PT} is increased beyond a level, the peak power constraint is reached for SU, which does not allow further increase in P_{ST} and P_{SR} . Thus, with an additional increase in the primary power, SINR at the secondary destination reduces as P_{ST} and P_{SR} cannot be increased further, degrading SU's outage performance. We can also see from Fig. 2 that the presence of direct link effectively helps in improving SU's performance. Also, the increase in the number of relays improves secondary's outage performance due to the increase in the diversity gain.

Fig. 3 shows the effect of the primary outage probability threshold λ_p on the secondary outage probability. We can see that increase in λ_p relaxes the constraint on P_{ST} and P_{SR} . But, if we increase λ_p beyond a level, the peak power constraint is reached, and ST and SR may transmit with the maximum power P_{pk} even though they are allowed, by the primary, to transmit with higher power than P_{pk} . In this case, unlike in

Fig. 2, the primary power, in turn, the primary interference to SD remains constant. Thus, irrespective of the increase in λ_p , the secondary outage probability remains constant—we call it as floor—once the peak power constraint is reached. We can also notice from Fig. 3 that relaxing the peak power constraint delays the arrival of the floor as expected.

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