SNR Wall for Generalized Energy Detection Under Noise Uncertainty in Cognitive Radio

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***Abstract*—Energy detection (ED) is a popular spectrum sensing technique in cognitive radio to detect the primary user. But the detection performance of ED deteriorates in the presence of noise uncertainty and exhibits associated SNR wall phenomenon. In this paper, the generalized energy detector (GED) is investigated, where the squaring operation of amplitude of received samples in conventional energy detector (CED) is replaced by an arbitrary positive operation p. Our aim is to study the effect of noise uncertainty on the detection performance of GED. We consider different distributions of noise uncertainty. Initially, uniform distribution of noise uncertainty is considered and an expression of the SNR wall for the same is derived. It is shown that the SNR wall for uniformly distributed noise uncertainty is independent of p. The study of the detection performance of GED is further extended for log-normally distributed noise uncertainty, where the SNR wall is calculated numerically.**

I. INTRODUCTION

In recent years, the problem of frequency scarcity is becoming worse with an enormous growth in wireless applications. In addition to frequency scarcity problem, it has been observed that the frequency spectrum allotted to licensed users or primary users (PU) is underutilized [1]. Thus to improve spectrum usage, cognitive radio [2] is proposed as a solution which allows unlicensed or secondary users (SU) to opportunistically access the frequency bands of PU. In cognitive radio, a SU senses PU bands, and if a PU band is found idle, SU may utilize the idle band for its own transmission. However, if a PU reappears, SU has to vacate the band immediately to avoid harmful interference that may cause to PU. Sensing an idle PU band and detection of PU reappearance require accurate spectrum sensing.

Spectrum sensing is an important and fundamental task in cognitive radio. Energy detection (ED) [3]–[5] is a popular spectrum sensing technique as it does not require any apriori knowledge about primary signals, and due to its low complexity, it is easy to implement. In conventional energy detector (CED) [4], amplitudes of received samples are squared, summed and compared with a predetermined threshold. In [6], authors have generalized CED by replacing squaring operation of the received signal samples by an arbitrary positive operation p. We call this modified CED as generalized energy detector (GED). It can be seen that CED is a special case of GED with p = 2. In [6]–[8], it is shown that the detection performance of a SU can be improved significantly by choosing an appropriate value of p, which may be different from 2, depending on the system parameters, that is, CED may not be the best ED. This motivates us to investigate GED.

In GED, the decision on the presence or absence of PU is taken by comparing the test statistic with a predetermined threshold, and the threshold depends on the noise power/variance. Thus to determine the threshold accurately, exact knowledge of noise power is required. Theoretically, if the noise power is known precisely, it is possible to detect PU even in low signal-to-noise ratio (SNR) if the sensing time is made sufficiently large [3]. But in practice, it is not possible to obtain exact knowledge about noise power since it changes with time and location. So there exists uncertainty in the noise power which is known as noise uncertainty. In [9], [10], the case of worst case of noise uncertainty is analysed. In [10], it is shown that if the noise power is not known precisely and confined in an interval, then there exists a phenomenon known as SNR wall, which states that, if the received SNR falls below a certain SNR known as SNR wall, then it is impossible to satisfy some detection requirements irrespective of sensing time, making ED an ineffective sensing method under the noise uncertainty.

In [11]–[13], the effect of uniform distribution of noise uncertainty is analysed on CED. The SNR wall expression is found for the same in [11]. In [12], the detection performance for cooperative spectrum sensing is presented. In [14], the detection performance of CED under log-normal approximated noise uncertainty is considered. Noise uncertainty model is considered in continuous and discrete forms in [15]. It is shown in [15] that different detection performances for CED can be obtained by choosing different statistical decision thresholds. In [16], an asymptotic analysis of noise power estimation is performed for CED. The detection performance of CED is studied with noise power estimation and the conditions in which the SNR wall exists are derived. In [17], the log-normal distribution of noise uncertainty is proposed based on the stochastic properties of noise uncertainty to study the detection performance of CED. In [18], the detection performance of GED is investigated for the worst case of noise uncertainty as well as uniformly distributed noise uncertainty. The effect of noise uncertainty is analysed on the receiver operating characteristics of GED. It is proved for the worst case of noise uncertainty that the SNR wall is independent of p. Further, it is shown that under uniform distribution of noise uncertainty, GED with p = 2, that is, CED is the best

ED, but in the absence of noise uncertainty CED may not be the best ED. Also, [18] shows numerically that for significant noise uncertainty (generally greater than 0.5 dB), the detection performance of GED becomes independent of p.

This paper is primarily built on [18]. We will investigate the detection performance of GED under different distributions of noise uncertainty with different values of p. First, an analytical expression of the SNR wall for GED under uniformly distributed noise uncertainty is derived. It is shown that the SNR wall is independent of value of p. The analytical results are supported by the numerical results. Then the log-normally distributed noise uncertainty [17] is assumed and the SNR wall for GED is calculated numerically.

The rest of the paper is organized as follows. In Section II, the system model is described, and conventional and generalized energy detectors are reviewed. In Section III, an analytical expression of the SNR wall for GED under uniformly distributed noise uncertainty is derived, and it shown that the SNR wall is independent of p. Log-normal distribution of noise uncertainty is studied in Section IV. Section V presents the numerical results that validate the analytical results for the SNR wall under the uniform distribution of noise uncertainty. Also, the SNR wall for GED under the log-normal distribution of noise uncertainty is calculated numerically. Conclusions are drawn in Section VI. Brief mathematical derivations are given in Appendix.

II. System Model

Primary user (PU) detection is a binary hypothesis problem [3] which can be presented as

$$r_n = \begin{cases} hs_n + w_n, & H_1 \\ w_n, & H_0, \end{cases} \tag{1}$$

where r_n is the nth sample of the received signal by a SU with $n = 1, \ldots, N$, h is the fading coefficient of the channel between PU and SU, s_n is the primary signal and w_n is additive white Gaussian noise (AWGN) with mean zero and variance σ^2. Hypotheses H_1 and H_0 respectively denote the presence and absence of PU. The primary signal samples and noise samples are assumed independent. Also, it is assumed that noise samples are independent. We also assume that primary signal samples are independent.

Effectiveness of a spectrum sensing technique is determined by the probability of detection (P_D) and the probability of false alarm (P_{FA}), which are given as

$$P_D \;=\; Pr(H_1|H_1) \;\text{ and }\; P_{FA} = Pr(H_1|H_0).$$

To avoid harmful interference to PU, P_D should be as high as possible, and P_{FA} should be as low as possible to increase the chances for opportunistic access of an idle PU band for a SU.

The test statistic for CED is given as [3]

$$T_{CED} = \frac{1}{N}\sum_{n=1}^{N} |r_n|^2, \tag{2}$$

where N is number of samples.

In GED, the squaring operation in CED is replaced by an arbitrary positive operation p. The test statistic for GED can be given as [6]

$$T_{GED} = \frac{1}{N}\sum_{n=1}^{N} |r_n|^p. \tag{3}$$

III. SNR Wall for GED Under Uniformly Distributed Noise Uncertainty

In this section, GED under uniform distribution of noise uncertainty is considered [11], [18] and an expression of the SNR wall for the same is derived. It is further shown that the SNR wall is independent of value of p.

Usually, the average noise power $\hat{\sigma}_n^2$ is known. However, the actual noise power may be different from the average noise power at a given time and location, giving rise to noise uncertainty. Let the actual noise power be σ_n^2. We define noise uncertainty factor η as $\frac{\hat{\sigma}_n^2}{\sigma_n^2}$. Let U be the upper bound on η in dB. Then U can be given as

$$U = \sup\{10\log_{10}\eta\}. \tag{4}$$

It is assumed that the noise uncertainty factor η in dB is uniformly distributed in $[-U, U]$. That is, η is confined in the range $[10^{-U/10}, 10^{U/10}]$. Since η in dB, that is, $10\log_{10}\eta$ is distributed uniformly in $[-U, U]$, we can write by simple transformation of random variable, the probability density function (pdf) of η as

$$f_\eta(x) = \begin{cases} 0, & x < 10^{-U/10} \\ \dfrac{5}{Ux\ln(10)}, & 10^{-U/10} < x < 10^{U/10} \\ 0, & x > 10^{U/10}, \end{cases} \tag{5}$$

where $\ln(z)$ is natural logarithm of z.

Let $\lambda\hat{\sigma}_n^2$ be the threshold for GED, where λ is a constant. Actual noise power is $\sigma_n^2 = \frac{\hat{\sigma}_n^2}{\eta}$. The average received SNR is γ. Then the average probability of false alarm $\bar{P}_{FA}$ and the average probability of detection $\bar{P}_D$ are given by [18]

$$\begin{aligned}\bar{P}_{FA} &= \int_{-\infty}^{\infty} Q\left(\left(\lambda x^{p/2} - G_p\right)\sqrt{\frac{N}{K_p}}\right) f_\eta(x)dx \\ &= \int_{10^{-U/10}}^{10^{U/10}} Q\left(\left(\lambda x^{p/2} - G_p\right)\sqrt{\frac{N}{K_p}}\right) \frac{5}{Ux\ln(10)}dx\end{aligned} \tag{6}$$

and

$$\begin{aligned}\bar{P}_D &= \int_{-\infty}^{\infty} Q\left(\left(\frac{\lambda x^{p/2} - G_p(1+x\gamma)^{p/2}}{(1+x\gamma)^{p/2}}\right)\sqrt{\frac{N}{K_p}}\right) \\ &\quad f_\eta(x)dx \\ &= \int_{10^{-U/10}}^{10^{U/10}} Q\left(\left(\frac{\lambda x^{p/2} - G_p(1+x\gamma)^{p/2}}{(1+x\gamma)^{p/2}}\right)\sqrt{\frac{N}{K_p}}\right) \\ &\quad \frac{5}{Ux\ln(10)}dx,\end{aligned} \tag{7}$$

$$\lim_{N\to\infty} \bar{P}_{FA} = \begin{cases} 1, & \lambda \le G_p \left(10^{-U/10}\right)^{p/2} \\ 0.5 - \dfrac{5\ln\left(\dfrac{\lambda}{G_p}\right)^{2/p}}{U\ln(10)}, & G_p \left(10^{-U/10}\right)^{p/2} < \lambda < G_p \left(10^{U/10}\right)^{p/2} \\ 0, & \lambda \ge G_p \left(10^{U/10}\right)^{p/2}. \end{cases} \tag{11}$$

$$\lim_{N\to\infty} \bar{P}_{D} = \begin{cases} 1, & \lambda \le G_p \left(10^{-U/10}+\gamma\right)^{p/2} \\ 0.5 - \dfrac{5\ln\left(\left(\dfrac{\lambda}{G_p}\right)^{2/p}-\gamma\right)}{U\ln(10)}, & G_p \left(10^{-U/10}+\gamma\right)^{p/2} < \lambda < G_p \left(10^{U/10}+\gamma\right)^{p/2} \\ 0, & \lambda \ge G_p \left(10^{U/10}+\gamma\right)^{p/2}. \end{cases} \tag{12}$$

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where

$$G_p = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right), \tag{8}$$

and

$$K_p = \frac{2^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right], \tag{9}$$

with $\Gamma(\cdot)$ denoting the ordinary gamma function.

As defined in [11], a sensing scheme is considered as *unlimitedly reliable* for a given SNR $\gamma > 0$ if there exists a threshold λ that satisfies the following conditions, given as

$$\lim_{N\to\infty} \bar{P}_{FA} = 0 \quad \text{and} \quad \lim_{N\to\infty} \bar{P}_{D} = 1. \tag{10}$$

This means, for a sufficiently long sensing time, an unlimitedly reliable sensing scheme can achieve any target probability of false alarm and probability of detection for any SNR level. This is true for the case when there is no noise uncertainty. However, in the presence of noise uncertainty, a sensing scheme may not be able to achieve the target probability of false alarm and probability of detection for some SNR level [10]. The SNR level below which the sensing scheme is not able to satisfy the target probability of false alarm and probability of detection is defined as *SNR wall* [10]. For SNR lower than the SNR wall, at least one condition in (10) is not satisfied, while for SNR greater than the SNR wall, there exists a threshold λ for which both conditions in (10) are satisfied. We state a theorem as follows.

Theorem 1. *For generalized energy detector, if noise uncertainty is uniformly distributed in an interval, then the SNR wall is independent of value of p.*

Proof: To find the SNR wall, given γ, λ and U, for GED, (6) and (7) for unlimited sample size ($N \to \infty$) can be written as (11) and (12) respectively. Derivations of (11) and (12) are given in Appendix. First, we will show that if there is no noise uncertainty ($U = 0$), GED is unlimitedly reliable. On the other hand, if there exists noise uncertainty ($U > 0$), GED is not unlimitedly reliable.

We try to find the threshold λ that satisfies the most stringent detection requirements, that is, the conditions given in (10). If GED needs to satisfy the condition $\lim_{N\to\infty} \bar{P}_{FA} = 0$, then from (11), it can be seen that the threshold λ must satisfy

$$\lambda \ge G_p \left(10^{U/10}\right)^{p/2}.$$

On the other hand, if the GED needs to satisfy the condition $\lim_{N\to\infty} \bar{P}_{D} = 1$, then from (12), it can be seen that the threshold λ must satisfy

$$\lambda \le G_p \left(10^{-U/10}+\gamma\right)^{p/2}.$$

Therefore, to satisfy both conditions at the same time, a threshold λ should exist such that

$$10^{U/10} \le \lambda \le 10^{-U/10} + \gamma,$$

that is,

$$\gamma \ge 10^{U/10} - 10^{-U/10}. \tag{13}$$

- When there is no noise uncertainty ($U = 0$), it can be seen from (13) that there exits a threshold that satisfies both conditions in (10) only if the SNR is non-negative ($\gamma \ge 0$), which is always the case. Hence, when there is no noise uncertainty, a threshold can always be found that satisfies both conditions given in (10), and thus GED is unlimitedly reliable.
- When there exists noise uncertainty ($U > 0$), it can be seen from (13) that there exits a threshold that satisfies both conditions in (10) only if the SNR satisfies $\gamma \ge 10^{U/10} - 10^{-U/10}$. In other words, if $0 < \gamma < 10^{U/10} - 10^{-U/10}$, it is impossible to find a threshold such that both conditions given in (10) are satisfied at the same time. Therefore, GED is not unlimitedly reliable.

As aforementioned, SNR wall is the lowest SNR for which there exists a threshold such that both conditions given in (10) are satisfied simultaneously. So it can be verified from the earlier discussion in this proof that the SNR wall for GED is

$$\text{SNR wall} = 10^{U/10} - 10^{-U/10}. \tag{14}$$

From (14), it can be seen that the SNR wall expression does not depend on p. Thus, the SNR wall for GED under uniformly distributed noise uncertainty is independent of p. ■

IV. GED Under Log-Normally Distributed Noise Uncertainty

In this section, the log-normal distribution of noise uncertainty for GED is considered [14], [17].

Let us define a new random variable $\delta = \hat{\sigma}_n^2 - \sigma_n^2$ (in dB) as the noise power random fluctuation and it is well-founded assumption on δ (in dB) to have Gaussian distribution with mean zero and variance σ_δ^2, that is, $\delta|_{dB} \sim \mathcal{N}(0, \sigma_\delta^2)$ [17].

The noise uncertainty factor β is $\frac{\hat{\sigma}_n^2}{\sigma_n^2}$. Then $\delta|_{dB} = 10\log_{10}\beta$, that is, $\beta = 10^{\delta/10}$. It can be shown that β has log-normal distribution [19] and is given by

$$\beta \sim Log\mathcal{N}\left(0, \left(\frac{\ln(10)}{10}\sigma_\delta\right)^2\right). \tag{15}$$

Thus the probability density function of β can be given as

$$f_\beta(x) = \begin{cases} \frac{10}{\sqrt{2\pi}x\sigma_\delta \ln 10} \exp\left(-\frac{50}{(\sigma_\delta \ln 10)^2}(\ln x)^2\right), & x > 0 \\ 0, & x < 0. \end{cases}$$

Then the average probability of false alarm $\bar{P}_{FA_{log}}$ and the average probability of detection $\bar{P}_{D_{log}}$ are given as

$$\bar{P}_{FA_{log}} = \int_{-\infty}^{\infty} Q\left(\left(\lambda x^{p/2} - G_p\right)\sqrt{\frac{N}{K_p}}\right) f_\beta(x)dx \tag{16}$$

and

$$\bar{P}_{D_{log}} = \int_{-\infty}^{\infty} Q\left(\left(\frac{\lambda x^{p/2} - G_p(1+x\gamma)^{p/2}}{(1+x\gamma)^{p/2}}\right)\sqrt{\frac{N}{K_p}}\right) f_\beta(x)dx, \tag{17}$$

where G_p and K_p are given by (8) and (9). We evaluate (16) and (17) numerically, as it is not possible to find the closed-form expressions for the same.

V. Numerical Results and Discussion

In this section, the numerical results are presented that show the effect of uniform and log-normal distributed noise uncertainty on the detection performance of generalized energy detector. The analytical expression of SNR wall is validated numerically, under the assumption that noise uncertainty is uniformly distributed. Also, the SNR wall is calculated numerically for log-normal distribution of noise uncertainty since it is not possible to find the closed-form expression.

A. Effect of uniform distribution of noise uncertainty

1) When SNR = SNR wall: In Fig. 1, the detection probabilities (at unlimited sample size) for p = 2, 4 are computed numerically and shown versus threshold λ for U = 0.5 dB and SNR = -6.3682 dB when noise uncertainty is uniformly distributed. The SNR is chosen to be -6.3682 dB as it is the lowest SNR for which there exists a threshold such that both conditions in (10) are satisfied. Thus this SNR corresponds to the SNR wall, and is computed by equating (6) to 0 and (7) to 1, and solving numerically for a very large number of samples (10^7 samples). The numerically computed SNR wall value matches exactly the SNR wall calculated analytically given by (14). From Fig. 1, it can be seen that, the SNR wall is the same for both p = 2 and p = 4. This numerical result also validates the theorem 1.

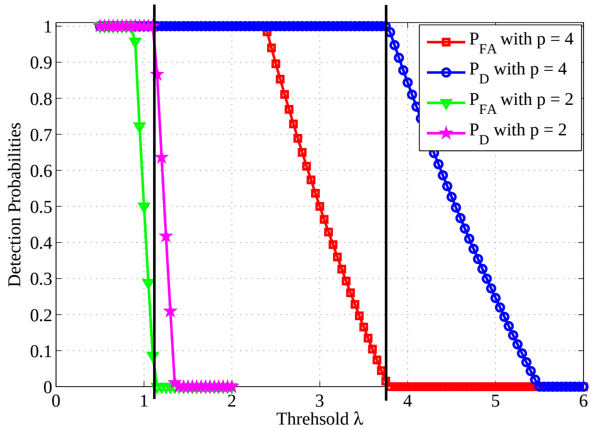

Fig. 1. Detection probabilities ($\bar{P}_{FA}$, $\bar{P}_D$) vs. Threshold λ for GED with p = 2, 4 when uniform distribution of noise uncertainty is assumed with $U = 0.5$ dB, SNR = -6.3682 dB (SNR = SNR wall) and $N = 10^7$ (approximated unlimited sample size for numerical simulation purpose). Vertical straight line denotes the condition $\lim_{N\to\infty} P_{FA} = 0$ and $\lim_{N\to\infty} P_D = 1$. SNR corresponds to this condition is SNR wall.

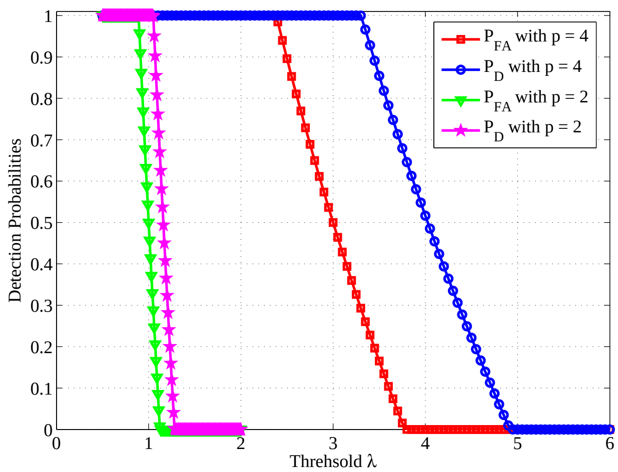

Fig. 2. Detection probabilities ($\bar{P}_{FA}$, $\bar{P}_D$) vs. Threshold λ for GED with p = 2, 4 when uniform distribution of noise uncertainty is assumed with $U = 0.5$ dB, SNR = -8 dB (SNR < SNR wall) and $N = 10^7$ (approximated unlimited sample size for numerical simulation purpose).

2) When SNR < SNR wall: In Fig. 2, the detection probabilities versus threshold λ are shown for U = 0.5 dB and SNR γ = -8 dB at unlimited sample size under the uniform distribution of noise uncertainty. It can be seen that some detection requirements are impossible to meet even for unlimited number of samples. For example, if the detection requirement for p = 4 is set as $\bar{P}_{FA} \leq 0.1$ and $\bar{P}_D \geq 0.9$, it is impossible to achieve this requirement for any number of samples. To achieve $\bar{P}_{FA} \leq 0.1$, the threshold should be greater than 3.6, but this threshold makes $\bar{P}_D < 0.9$. The impossibility to satisfy some detection requirements arises due to the SNR wall phenomenon. Here, SNR is chosen to be -8 dB which is less than the SNR wall (-6.3682 dB). Thus, it is impossible for GED to satisfy some detection requirements even with the unlimited sample size.

As aforementioned, the threshold is highly susceptible to noise uncertainty. For GED with $p = 2$, it can be seen from Fig. 2 that, the target detection performance is more sensitive to threshold change than that of GED with $p = 4$. For example, if the target $\bar{P}_D$ is 0.8 and the target $\bar{P}_{FA}$ is 0.2, the threshold should be chosen between 1.07 and 1.09 for GED with $p = 2$, while the threshold can be chosen between 3.45 and 3.55 for GED with $p = 4$, making latter more robust to noise uncertainty.

3) When SNR > SNR wall: Fig. 3 shows the detection probabilities versus threshold λ for $U = 0.5$ dB and SNR γ = -5 dB at unlimited sample size. Here, SNR is greater than the SNR wall, and it can be seen that, any detection requirement can be satisfied by choosing a suitable threshold with sufficient number of samples taken. For example, if the detection requirement for $p = 4$ is as $\bar{P}_{FA} \leq 0.1$ and $\bar{P}_D \geq 0.9$, the threshold $3.6 < \lambda < 4.5$ satisfies the detection requirement. Even the most stringent detection requirements for GED given by (18), can be satisfied by choosing the threshold between 3.8 and 4.35.

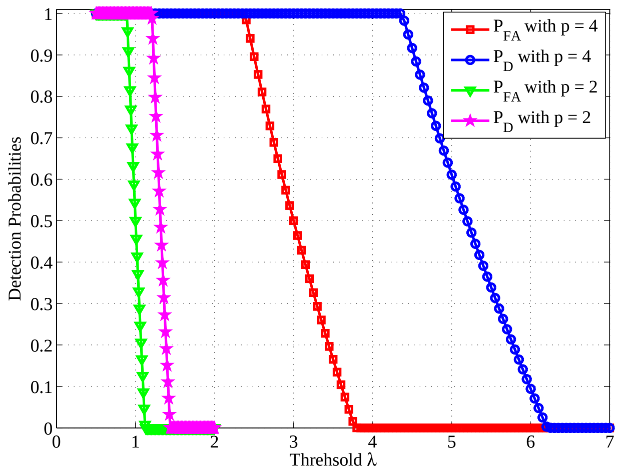

Fig. 3. Detection probabilities ($\bar{P}_{FA}$, $\bar{P}_D$) vs. Threshold λ for GED with p = 2, 4 when uniform distribution of noise uncertainty is assumed with $U = 0.5$ dB, SNR = -5 dB (SNR > SNR wall) and $N = 10^7$ (approximated unlimited sample size for numerical simulation purpose).

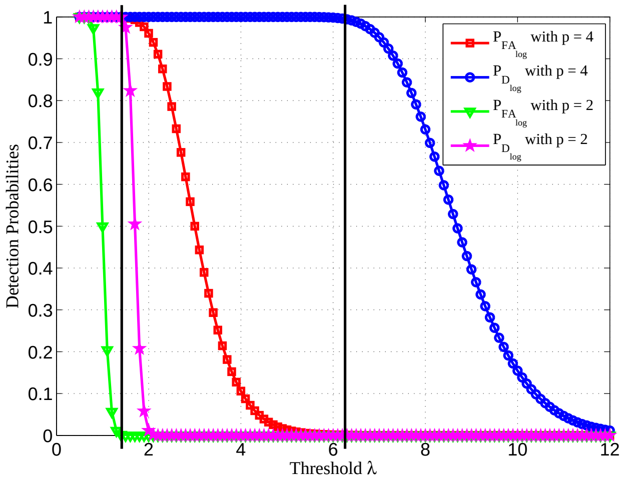

Fig. 4. Detection probabilities ($\bar{P}_{FA_{log}}$, $\bar{P}_{D_{log}}$) vs. Threshold λ for GED with p = 2, 4 when log-normal distribution of noise uncertainty is assumed with σ_δ = 0.5 dB, SNR = -1.54 dB (SNR = SNR wall) and $N = 10^7$ (approximated unlimited sample size for numerical simulation purpose). Vertical straight line denotes the condition $\lim_{N\to\infty} P_{FA} = 0$ and $\lim_{N\to\infty} P_D = 1$. SNR corresponds to this condition is SNR wall.

B. Effect of log-normal distribution of noise uncertainty

Fig. 4 shows the SNR wall of GED when noise uncertainty is log-normally distributed for p = 2, 4. Detection probabilities versus threshold are shown by computing (16) and (17) numerically for very high number of samples, when the standard deviation of noise uncertainty (σ_δ) is 0.5 dB and SNR = -1.54 dB. It can be seen that, -1.54 dB is the lowest SNR for which there exists a threshold such that both conditions in (10) are satisfied. Thus the SNR wall is -1.54 dB. For log-normal distribution of noise uncertainty also, the SNR wall is the same for both p = 2 and p = 4, that is, the SNR wall is independent of p. For SNR < SNR wall and SNR > SNR wall, the results follow the trend similar to that of uniform distributed noise uncertainty.

VI. Conclusions

In this paper, the detection performance of generalized energy detector is studied under different distributions of noise uncertainty. Uniform and log-normal distribution of noise uncertainty are considered and their effects on generalized energy detector are investigated. An expression for the SNR wall is derived under uniform distribution of noise uncertainty and it is shown that the SNR wall is independent of value of p. Analytical expression for the SNR wall is validated numerically. Then the SNR wall is calculated under the log-normal distribution of noise uncertainty numerically since the probability of detection and the probability of false alarm cannot be expressed in closed-form.

Appendix
Derivation of (11) and (12)

First, we will derive (11). Note that

$$\lim_{N\to\infty} Q(a\sqrt{N}) = \begin{cases} 0 & \text{if } a > 0 \\ 1 & \text{if } a < 0 \\ 0.5 & \text{if } a = 0. \end{cases} \quad (18)$$

Substituting $m = \ln(x)$ in (6), the average probability of false alarm can be written as

$$\bar{P}_{FA} = \frac{5}{U\ln(10)}\int_{-c}^{c} Q\left(\left(\lambda e^{mp/2} - G_p\right)\sqrt{\frac{N}{K_p}}\right) dm, \tag{19}$$

where $c = \left(\frac{U}{10}\right)\ln(10)$.

Based on the conditions given in (18), it is required to calculate (19) for three conditions given as

I. When $\lambda \le G_p\left(10^{-U/10}\right)^{p/2}$.

For this, $\lambda \le G_p\left(10^{-U/10}\right)^{p/2} = G_p\left(e^{-c}\right)^{p/2}$. So $\lambda \le \frac{G_p}{e^{cp/2}}$. Thus $\lambda < \frac{G_p}{e^{mp/2}}$ for all $m < c$. This gives $\lambda e^{mp/2} - G_p < 0$ for all $m < c$. Therefore, from (18) and (19), we can write

$$\lim_{N\to\infty} \bar{P}_{FA} = \frac{5}{U\ln(10)}\int_{-c}^{c} 1\, dm = 1. \tag{20}$$

II. When $G_p\left(10^{-U/10}\right)^{p/2} < \lambda < G_p\left(10^{U/10}\right)^{p/2}$.

Substituting $\frac{m\cdot p}{2} = t$ in (19), $\bar{P}_{FA}$ can be written as

$$\bar{P}_{FA} = \frac{10}{pU\ln(10)}\int_{-cp/2}^{cp/2} Q\left(\left(\lambda e^{t} - G_p\right)\sqrt{\frac{N}{K_p}}\right) dt,$$

that is,

$$\bar{P}_{FA} = \frac{10}{pU\ln(10)}\int_{-cp/2}^{cp/2} Q\left(G_p\left(\frac{\lambda}{G_p}e^{t} - 1\right)\sqrt{\frac{N}{K_p}}\right) dt. \tag{21}$$

Substituting $\frac{\lambda}{G_p} = \alpha$ in (21), $\bar{P}_{FA}$ becomes

$$\bar{P}_{FA} = \frac{10}{pU\ln(10)}\int_{-cp/2}^{cp/2} Q\left(G_p\left(\alpha e^{t} - 1\right)\sqrt{\frac{N}{K_p}}\right) dt. \tag{22}$$

The condition $G_p\left(10^{-U/10}\right)^{p/2} < \lambda < G_p\left(10^{U/10}\right)^{p/2}$ can be written as $-cp/2 < -\ln(\alpha) < cp/2$. The integral limits of (22) can be split into two parts, from $-cp/2$ to $-\ln(\alpha)$ and from $-\ln(\alpha)$ to $cp/2$. Then (22) can be written as

$$\begin{aligned}\bar{P}_{FA} &= \frac{10}{pU\ln(10)}\int_{-cp/2}^{-\ln(\alpha)} Q\left(G_p\left(\alpha e^{t} - 1\right)\sqrt{\frac{N}{K_p}}\right) dt \\ &\quad + \frac{10}{pU\ln(10)}\int_{-\ln(\alpha)}^{cp/2} Q\left(G_p\left(\alpha e^{t} - 1\right)\sqrt{\frac{N}{K_p}}\right) dt.\end{aligned} \tag{23}$$

It can be seen that for the first part when $-cp/2 < t < -\ln(\alpha)$, $\alpha e^t < 1$, and for second part when $-\ln(\alpha) < t < cp/2$, $\alpha e^t > 1$. Then from (18) and (23),

$$\begin{aligned}\lim_{N\to\infty} \bar{P}_{FA} &= \frac{10}{pU\ln(10)}\int_{-cp/2}^{-\ln(\alpha)} 1\, dt + \\ &\quad \frac{10}{pU\ln(10)}\int_{-\ln(\alpha)}^{cp/2} 0\, dt \\ &= 0.5 - \frac{5\ln\left(\frac{\lambda}{G_p}\right)^{2/p}}{U\ln(10)}.\end{aligned} \tag{24}$$

III. When $\lambda \ge G_p\left(10^{U/10}\right)^{p/2}$.

For this, $\lambda \ge G_p\left(10^{U/10}\right)^{p/2} = \lambda \ge G_p\left(e^{c}\right)^{p/2}$. So $\lambda \ge \frac{G_p}{e^{-cp/2}}$. That is, $\lambda > \frac{G_p}{e^{mp/2}}$ for all $m > -c$. Therefore, $\lambda e^{mp/2} - G_p > 0$ for all $m > -c$. From (18) and (19),

$$\lim_{N\to\infty} \bar{P}_{FA} = \frac{5}{U\ln(10)}\int_{-c}^{c} 0\, dm = 0. \tag{25}$$

The derivation of (12) can be done similar to (11).

References

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