Homework 3
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Section 4.6

2. You will have to place, at most, $mn + 1$ pigeons into 1 hole.

3. No, this simply not possible. Any 2-element subset of X will also be included in the 3-element subsets of X. This is the same if X was a set of positive integers from 1 to 6.

Section 5.8

1. (e) 2
   (f) 3

3. Such a graph cannot exist, since $\sum_{v \in V} \deg(v) > 2|E|$.

6. For a tree with a single vertex, there are no edges. We then assume we have a tree with n-1 vertices that contains n-2 edges. We can construct a new tree with n vertices and n-1 edges by adding a leaf to that tree.

7. We can quickly determine that G_4 is not isomorphic with any other graph, since the degree of vertex x_5 is 5, which isn't seen in any other graph. G_2 is also not isomorphic with any other graph since both G_3 and G_1 have a triangle that cannot be replicated with G_2. This leaves G_1 and G_3 which are isomorphic, the isomorphism being:

$$\begin{aligned} f(v_1) &= w_1 \\ f(v_2) &= w_3 \\ f(v_3) &= w_4 \\ f(v_4) &= w_2 \\ f(v_5) &= w_6 \\ f(v_6) &= w_5 \end{aligned}$$

9. The given graph is eulerian since every vertex has an even degree. The eulerian circuit is:

$$\begin{aligned} C &= (a) \\ &\quad (a, b, l, a) \\ &\quad (a, b, l, d, j, l, a) \\ &\quad (a, b, l, d, h, m, i, d, j, l, a) \\ &\quad (a, b, l, d, h, m, g, n, m, i, d, j, l, a) \\ &\quad (a, b, l, d, h, m, g, n, m, i, c, f, i, d, j, c, f, e, a, k) \end{aligned}$$

13. The chromatic number is 3, and the graph coloring is:

a	:	2
b	:	2
c	:	1
d	:	1
e	:	3
f	:	2
g	:	1
h	:	1
i	:	2
j	:	2