Homework 1
Mike Skalnik

2. For each digit position, there are 5 possible digits. Since there are 2n total digit positions, we can determine that there are a total of 5^{2n} possible positive integers.

3. (a) Bob's password rules only allow for $(26+26)^3 \times 10^2 \times 10 \times 26^2 \times 10 \times 10 = 9505100800000$ passwords

   (b) The total number of stings of length 10 made from the alphabet of all uppercase & lowercase English letters, decimal digits, and 10 symbols is $(26+26+10+10)^{10} = 3.7439 \times 10^{18}$

6. (a) Any one student from the whole group can take the first position. In the second position there are 9 possible students to choose from, and 8 for the third position. This continues until the final position, so there are simply $10! = 3628800$ possible ways for the students to line up.

   (b) For the 3 positions, it must be a student from group 1, 2, and then 3. The following 3 are the same, however there is now one less student to pick from in each group, so there are only 9 to choose from for those positions. The next 3 are the same way, but only with 8. This continues until the line is complete, so we can determine that there are a total of $10!^3 = 4.7785 \times 10^{19}$

11. (a) For every donut he purcahses, he can select any of the 12. This means that there are a total of $12^6 = 2985984$ possible ways of purchasing donuts.

    (b) Since he wants to choose 6 distinct donuts out of 12 total, we can determine all possible ways to purchase donuts with a permutation. $P(12,6) = 655280$

    (c) In this case, the order doesn't matter at all. So, using a combination, we can determine that the total number of ways to pick donuts is $\binom{12}{6} = 924$.

12. Assuming there are no positions in korfball, the selection order doesn't matter. So there are a total of $\binom{7}{4} \times \binom{11}{4} = 11550$

13. (a) There are $P(20,4) = 116280$ possible outcomes for the competition

    (b) With the honorable mention certificates, there are a total of $P(20,4) \times C(16,4) = 211629600$ possible outcomes.

15. For this problem, we simply turn it into a normal even distrbution problem by inflating and deflating the amount to fit the restrictions. There are 25 identical pencils, however both Ahmed and Dieter must get one, so the total goes up to 27. Also, Carlos cannot have more than 5 and Barbara needs at least 4, so the total is now 26. There are 25 gaps, between each of the pencils and we need to select 3 divides, so there are a total of $\binom{21}{3} = 1330$ possible ways to distribute the pencils.

17. (a) To deal with the constraints, we manipulate the total $132-1+3 = 134$. Using this new total we can determine that there are a total of $C(134,4) = 12840751$ possible solutions.

    (b) Factoring in the new restriction that $x_4 < 17$ we manipulate the total $134 - 17 = 117$. From here, we can determine that there are a total of $C(117,4) = 7413705$ possible solutions.

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