Homework 5
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Section 6.9

3. The binary relation $P$ is not a partial order on the set $X$ since it isn't reflexive. If the relation $5, 5$ to $P$, it would then be a partial order on the set $X$.

6. (a) $\{2\} \leq \{3\} \leq \{2,5\} \leq \{3,7\} \leq \{2,3,11\} \leq \{2,5,25\} \leq \{2,3,5,7\}$
   (b) $2 \leq 3 \leq 10 \leq 21 \leq 66 \leq 50 \leq 210$
   (c) $(1,2,1) \leq (3,3,2) \leq (4,1,5) \leq (2,5,4) \leq (5,7,3) \leq (6,4,5) \leq (7,6,7)$

10. (a) Right Poset:
    width $w = 7$
    antichain $= \{(7,6,7), (5,7,3), (6,4,5), (2,5,4), (3,3,2), (4,1,5), (1,2,1)\}$.
    If each element is placed in a chain by itself, the poset is partitioned into 7 chains.
    (b) Center Poset:
    width $w = 5$
    antichain $= \{10, 21, 66, 210, 50\}$
    partition $\{50, 10, 210\}, \{210, 21, 3\}, \{3, 66, 2\}, \{2, 10, 50\}, \{2, 10, 210\}$

18. width $w = 3$ $C_1 = \{f, d, j, k\}$
    $C_2 = \{b, h, c, l, e, m, o\}$
    $C_3 = \{g, n, a, i\}$
    antichain $= \{f, b, g\}$

Section 7.6

5. 1000− #s divisible by 3 − #s divisible by 8 − #s divisible by 25 + #s divisible by 3 & 8 + #s divisible by 8 & 25 + #s divisible by 3 & 25 − #s divisible by 3, 8, & 25
   $1000 - 333 - 125 - 40 + 41 + 5 + 13 - 1 = 560$

6. If we use $x_1, x_2, \cdots + x_5$ to represent Fulton, Gwinett, DeKalb, Cobb, and Clayton counties, then we see that $x_1 + x_2 + x_3 + x_4 + x_5 = 173$, $x_1, x_2, x_3, x_4, x_5, \geq 1$, $x_5 \leq 10$, & $x_4 \leq 30$. We then calculate the total number of solutions with $\binom{173}{4} - \binom{162}{4} - \binom{142}{4} + \binom{131}{4} = 3877170$

8.

12. (a) $\sigma$ does not satisfy $P_2$ since $\sigma(2) = 1 \neq 2$. It does satisfy $P_2$ since $\sigma(6) = 6$. $P_4, P_6$ are the only properties satisfied by $\sigma$.

(b)

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\tau(i)$ | 1 | 8 | 8 | 4 | 8 | 8 | 8 | 8 |

(c)

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\pi(i)$ | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

14.

$$\begin{aligned} S(n,m) &= \sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n \\ S(8,6) &= \sum_{k=0}^{6} (-1)^k \binom{6}{k} (6-k)^8 \\ &= 191520 \end{aligned}$$

17.

$$\begin{aligned} S(n,m) &= \sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n \\ S(12,6) &= \sum_{k=0}^{6} (-1)^k \binom{6}{k} (m-k)^{12} \\ &= 953029440 \end{aligned}$$

21.

$$\begin{aligned} \binom{7}{4} d_4 &= \binom{7}{4} \sum_{k=0}^{4} (-1)^k \binom{4}{k} (4-k)! \\ &= 315 \end{aligned}$$

25.

$$\begin{aligned}\phi\left(1625190883965792\right) &= 1625190883965792\frac{1}{2}\frac{2}{3}\frac{10}{11}\frac{12}{13}\frac{22}{23}\frac{180}{181} \\ &= 432431285299200\end{aligned}$$