15. If we number the chemicals 0-9, then we get a graph with vertex and edge sets
$$V = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$
$$E = 01, 03, 04, 06, 19, 13, 14, 25, 27, 28, 34, 48, 59, 56, 67$$

The chromatic number is 4, which means we would need 4 rooms to safely store all 10 chemicals.

21. The cardinality of the independent set is $t(n_t - 1) + 1$. Then $\binom{t(n_t-1)+1}{n_t} n_t$ which leads us to $n_t = t(n_t - 1) + 1 + \binom{t(n_t-1)+1}{n_t} n_t$.

23. The girth is 5 since there is a cycle of 5 coming from $t = 3$. The rest are independent sets.

28. $\#\text{ofvertices} - \#\text{ofedges} + \#\text{offaces} = 2$. $14 - 22 + 10 = 2$, so the graph is planar.

29. $12 - 18 + 14 = 8$, so the graph isn't planar.

32. Since every graph larger than K_5 is either homeomorphic to K_5 or non-planar, the most edges a vertex of a planar graph has is 5.

33. (a) c
    (b) a, e
    (c) a
    (d) b
    (e) d

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40.

Prüfer Code	Label Set	Edge Added
96113473	$1, 2, 3, 4, 5, 6, 7, 8, 9$	2-9
6113473	$1, 3, 4, 5, 6, 7, 8, 9$	5-6
113473	$1, 3, 4, 6, 7, 8, 9$	6-1
13473	$1, 3, 4, 7, 8, 9$	8-1
3473	$1, 3, 4, 7, 9$	1-3
473	$3, 4, 7, 9$	9-4
73	$3, 4, 7$	4-7
3	$3, 7$	7-3
-	3	-