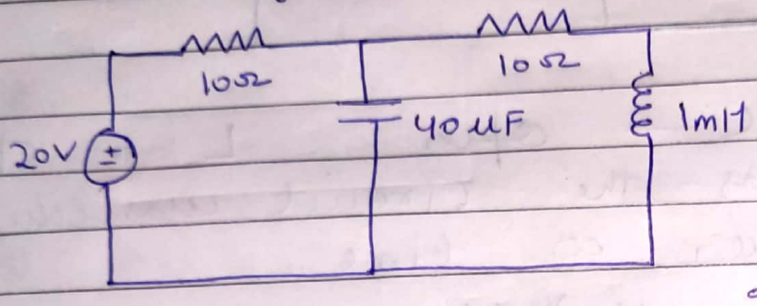
Q - 6

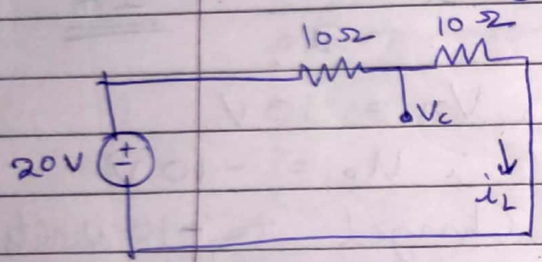
for $t < 0$



The circuit has been like this for ∞.

$\therefore$ Inductor → Short

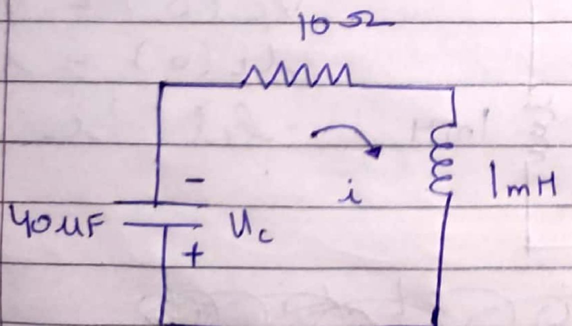
Capacitor → open



$i_L(\infty) = 1A$ $\quad V_L(\infty) = 0$

$V_c(\infty) = 10$ volts

The above should be our Initial Conditions for the natural response of RLC circuit.



We know the 2nd order D.E of Series RLC Circuit is given by :-

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

However for this to happen we need to assume opposite polarity for our Capacitor (as shown in figure)

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$\therefore$ Our Initial Condition will Change slightly

Although we → $V_c(0) = -10$ volts & not $+10$ volts
don't need this

$i_L(0) = 1A$

$$i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0$$

Characteristic Equation :-

$$m^2 + \left(\frac{R}{L}\right) m + \frac{1}{LC} = 0$$

$$m^2 + 10^4 m + 25 \times 10^6 = 0$$

$$m_1 = m_2 = -5000$$

Critically Damped Case

$$i(t) = A_1 e^{m_1 t} + A_2 t e^{m_2 t}$$

$$i(t) = A_1 e^{-5000t} + A_2 t e^{-5000t}$$

$i(0) = 1$ Amperes

$$\Rightarrow A_1 e^{-5000(0)} = 1$$

$$\boxed{A_1 = 1}$$

$$i(t) = e^{-5000t} + A_2 t e^{-5000t}$$

$$i'(t) = -5000 e^{-5000t} + A_2\left(e^{-5000t} - 5000 t e^{-5000t}\right)$$

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$$i'(t) = -5000e^{-5000t} + A_2 e^{-5000t}(1-t)$$

$V_L = L\,i'(t)$

at $t = 0$, $V_L = 0$

$$0 = 10^{-3}(-5000 + A_2)$$

$$\boxed{A_2 = 5000}$$

$$\therefore\ i_L(t) = e^{-5000t} + 5000t\,e^{-5000t}$$

$$i_L(t) = e^{-5000t}(1 + 5000t) \text{ Amperes}$$

at $t = 0^+$ $\quad v_c = -10\,V \quad i = 1A$

<u>KVL</u>

$$-v_c - 10i - V_L = 0$$

$$V_L = -(-10) - 10(1)$$

$$V_L = 0 \text{ volts}$$

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Method - 2 (If someone did using V_c)

till m_1, m_2 its same

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$$V_c(t) = A_1 e^{-5000t} + A_2 t e^{-5000t}$$

$$V_c(0) = -10 \text{ Volts}$$

$$\therefore \boxed{A_1 = -10}$$

$$V_c = -10 e^{-5000t} + A_2 t e^{-5000t}$$

$$i_c(0) = 1 A$$

$$40 \times 10^{-6} \left(50000 e^{-5000t} + A_2 \left(e^{-5000t} - 5000t\, e^{-5000t}\right)\right)$$

$$40 \times 10^{-6} \left(5 \times 10^4 + A_2\right) = 1$$

$$5 \times 10^4 + A_2 = \frac{10^6}{40}$$

$$A_2 = 25 \times 10^3 - 5 \times 10^4$$

$$A_2 = -25000$$

$$V_c = -\left(10 e^{-5000t} + 25000\, t\, e^{-5000t}\right)$$

$$i(t) = C\frac{dV_c}{dt} = e^{-5000t}(5000t + 1)$$ Amperes