1. Solve the following recurrence relation using the expansion method. [10]

$$T(n) = T(n-1) + c_1n^2 + c_2$$

 $T(1) = c$

Answer:

$$\begin{split} T(n) &= T(n-1) + c_1 n^2 + c_2 \\ &= (T(n-2) + c_1 (n-1)^2 + c_2) + c_1 n^2 + c_2 \quad \text{// Give 0 marks if this step is not correct} \\ &= T(n-2) + c_1 (n^2 + (n-1)^2) + 2c_2 \\ &= (T(n-3) + c_1 (n-2)^2 + c_2) + c_1 (n^2 + (n-1)^2) + 2c_2 \\ &= T(n-3) + c_1 (n^2 + (n-1)^2 + (n-2)^2) + 3c_2 \quad \text{// total 3 marks if this step is} \\ \text{correct} \end{split}$$

The kth term is:

$$=T(n-k)+c_1(n^2+(n-1)^2+(n-2)^2+(n-k+1)^2)+kc_2$$
 // total 5 marks if this step is correct

Substituting, k = n-1

$$T(n) = T(1) + c_1(2^2 + 3^2 + \dots + n^2) + (n-1)c_2$$
 // total 10 marks if this step is correct
$$= c + c_1 \left(\frac{n(n+1)(2n+1)}{6} - 1 \right) + (n-1)c_2$$

2. Write an algorithm for the tower of Hanoi problem with **four** towers. In addition to the source and destination tower, there are two temporary towers. The goal is to move n discs from the source tower to the destination tower using temporary towers. For all $n \ge 4$, your algorithm should take less number of moves than the number of moves in the three towers solution. [15]

Give 0 marks if pseudo-code or a C code is not given

```
void move(int n, char *src, char *dst, char *tmp1, char *tmp2)
{
     If (n == 0) {
        return;
     }
     if (n == 1) {
                    // deduct 5-marks if the algorithm
                            // don't handle this base case
       printf("move from %s to %s\n", src, dst);
       return;
     }
     move(n-2, src, tmp1, dst, tmp2);  // give zero if n-1 discs
                                          // are moved at this step
     printf("move from %s to %s\n", src, tmp2);
     printf("move from %s to %s\n", src, dst)
     printf("move from %s to %s\n", tmp2, dst);
     // deduct 5-marks if all three print statements are not given
     move(n-2, tmp1, dst, src, tmp2); // give zero if this step is missing
}
```