CSE 102 - Data Structures and Algorithms
Quiz 1 - Solutions

19 May 2022

Name: **Total Marks: 20 Marks**

Roll No: **Duration: 20 mins**

Questions 1 is mandatory. Answer any 3 out of the other 4 questions. Each question carries 5 marks.

1. Write true or false (no justification required):
   (a) If $f(n) = O(s(n))$ and $g(n) = O(r(n))$, then $f(n)/g(n) = O(s(n)/r(n))$. **False**
   (b) An array is an Abstract Data Type but a record is a Data Structure. **False**
   (c) Let $\mathcal{P}$ be the problem of finding if a given input integer is prime or not. Let N be a fixed integer that is given as input to $\mathcal{P}$. Then, the size of the input is N. **False**
   (d) When the input is sorted, although the average case complexity of binary search is $O(log(n))$, the worst case complexity of binary search is $O(n)$. **False**
   (e) If $f(n) = \Theta(n)$, then it is always true that $f(n) = \Omega(n)$. **True**

2. Show that if $f(n) = O(s(n))$ and $g(n) = O(r(n))$, then $f(n) + g(n) = O(s(n) + r(n))$.

   Ans: We know that for any two function f_1 and f_2 if $f_1(n) = O(f_2(n))$, then there exists a constant $c > 0$ and a natural number n_0 such that $f_1(n) \leq c \cdot f_2(n)$ for all $n \geq n_0$. Using this, we have that,

$$\exists\ c_1 > 0 \ \text{ and } \ n_{01} \in \mathbb{N} \ \text{ such that } \ f(n) \leq c_1 \cdot s(n) \ \forall\ n \geq n_{01}: \tag{1}$$

   and

$$\exists\ c_2 > 0 \ \text{ and } \ n_{02} \in \mathbb{N} \ \text{ such that } \ g(n) \leq c_2 \cdot r(n) \ \forall\ n \geq n_{02} \tag{2}$$

   Let $c_3 = \max\{c_1, c_2\}$ and $n_0 = \max\{n_{01}, n_{02}\}$
   From the above equations, for all $n \geq \max\{n_{01}, n_{02}\}$ we have that

$$f(n) + g(n) \leq c_1 \cdot s(n) + c_2 \cdot r(n) \leq c_3 \cdot (s(n) + r(n)). \tag{3}$$

   This implies that

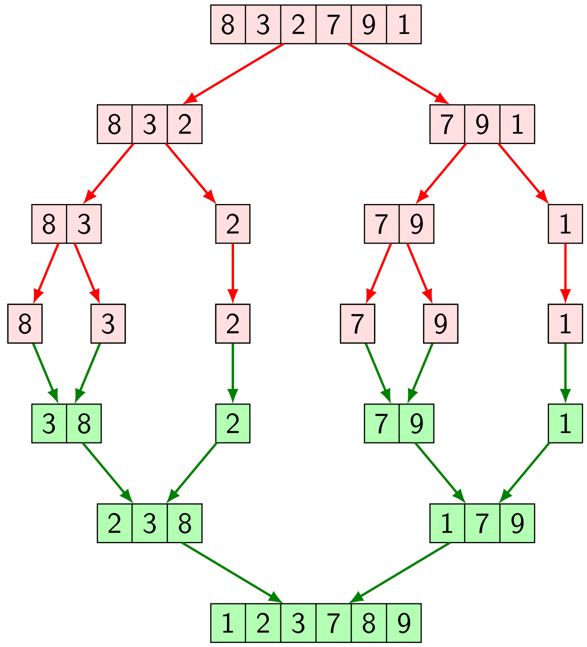
$$\exists\ c_3 > 0 \ \text{ and } \ n_0 \in \mathbb{N} \ \text{ such that } \ f(n) + s(n) \leq c_3 \cdot (s(n) + r(n)) \ \forall\ n \geq n_{01}. \tag{4}$$

   Hence, $f(n) + g(n) = O(s(n) + r(n))$.

3. Let $\mathcal{A}$ be an algorithm that takes an input x of size n. Then $\mathcal{A}$ makes $2 \cdot n$ iterations in total and the time taken for performing i^{th} iteration is i units of time. Is $\mathcal{A}$ an efficient algorithm? Explain in not more than 2 lines why you call it efficient or not efficient.

   Ans: The total number of iterations made by $\mathcal{A}$ is $2 \cdot n$. Since i^{th} iteration takes i units of time, the total time taken by the algorithm is $T = 1 + 2 + 3 + \cdots + (2n) = 2n \cdot (2n + 1)/2 = O(n^2)$. So, $\mathcal{A}$ is an efficient algorithm. We call $\mathcal{A}$ an efficient algorithm since it runs in polynomial units of time and not exponential units of time.

4. Say you are given a list $L = [8, 3, 2, 7, 9, 1]$. Show step by step how you would perform a merge sort on L.



5. Consider the following modified quick sort algorithm `ModQS` where in each iteration the pivot is chosen by finding the median of the list under consideration (assume that finding median of a list of size k takes $O(k)$ units of time.) Give the recurrence relation of `ModQS` algorithm and compute the time complexity of the algorithm (you can use the Master's theorem given below).

   Master's Theorem:

   The solution of the recurrence relation $T(n) = aT(n/b) + cnk$, where a and b are integer constants, $a \geq 1, b \geq 2$, and c and k are positive constants, is

$$T(n) = \begin{cases} O(n^{\log_b a}, \text{ if } a > b^k) \\ O(n^k \log(n)), \text{ if } a = b^k \\ O(n^k), \text{ if } a < b^k \end{cases}$$

   Ans: In `ModQS` each iteration, we find the median of the list and we break the list into two sub-lists. We also know that the time taken to find the median of a list of size k is $O(k)$. So, we can give the recurrence relation as

$$T(n) = 2 \cdot T(n/2) + O(n).$$

   Using Master's theorem, we get that the time complexity of `ModQS` is $O(nlog(n))$.