

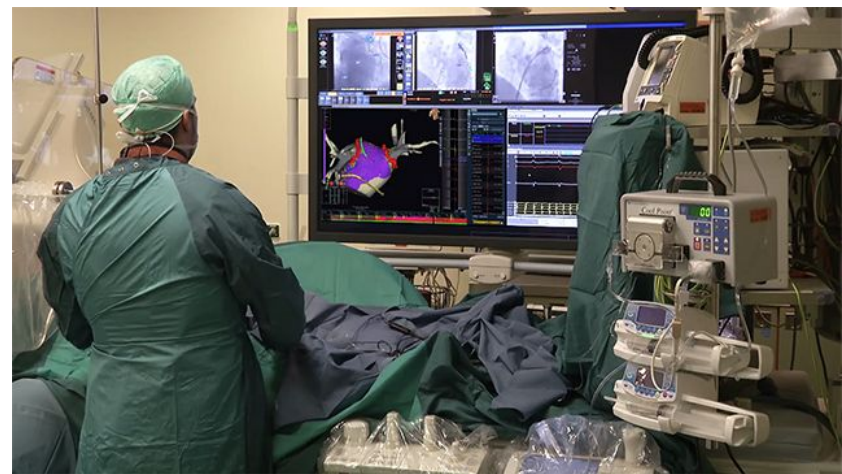
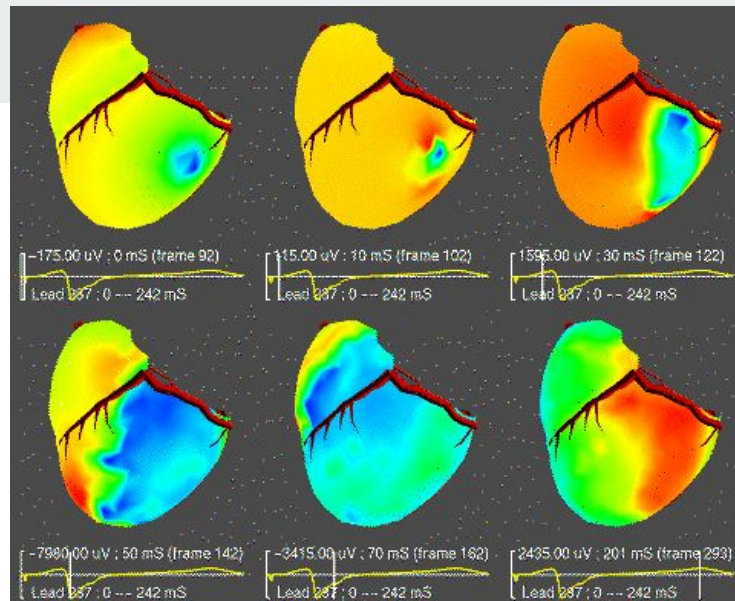


# Physics Informed Neural Operators (PINO) for Atrial Fibrillation

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# Motivation

- **Atrial Fibrillation (Afib):** arrhythmia (irregular heart beat) that begins in your heart's atria (upper chambers)
  - Risk factors: high blood pressure, coronary artery disease and obesity
  - Symptoms: fatigue, heart palpitations, trouble breathing, and dizziness.
  - Affects 0.5% of the world's population
- **Cardiac Electrophysiology (EP):** diagnosing and treating heart arrhythmias and other electrical activity issues
- **Cardiac mapping:** electrocardiogram (ECG) sensors placed on the surface of the atria, record electrical activity, construct an arrival time (AT) map to show whether certain regions of the heart have abnormal EP properties (low conduction velocity, heightened excitability or shortened action potential duration (APD))
- **Catheter ablation:** a catheter is inserted into a blood vessel (usually in the groin), guided to the heart, and various medications are administered to increase or decrease the electrical activity on the surface of the atria
- Identifying the problematic regions is a **difficult task**, so a **model** that can accurately construct AT maps from these sparse measurements would be useful in **aiding diagnosis**



# Background: PINN vs PINO

PINN - Neural Network with Physics-Informed Loss term, learns mapping between finite-dimensional vector space

PINO - Neural Operator with Physics-Informed Loss term, learns mapping between infinite dimensional function space

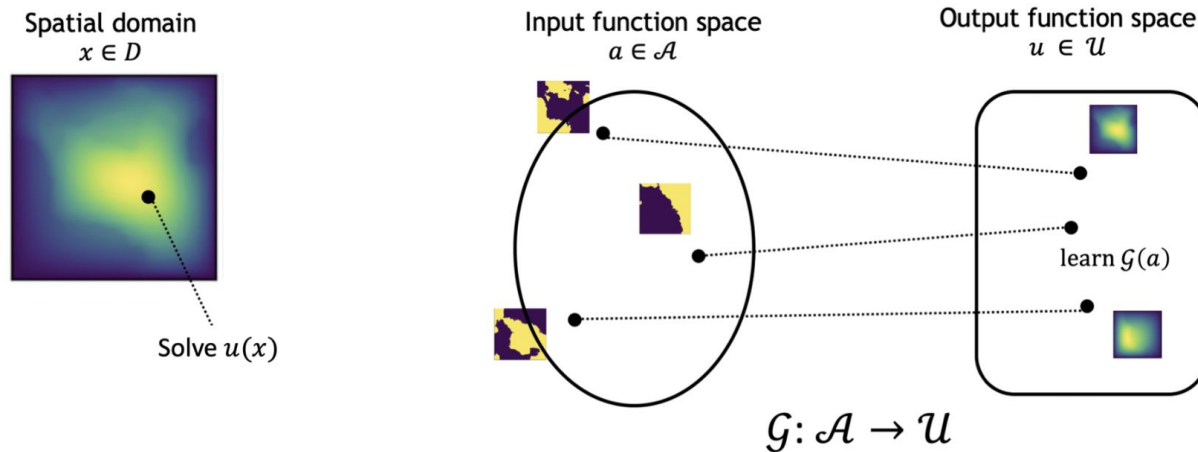


Figure 3: solve for one specific instance verse learn the entire solution operator  
Left: numerical solvers and PINNs focus on solving one specific instance. Right: neural operators learn the solution operator for a family of equations.

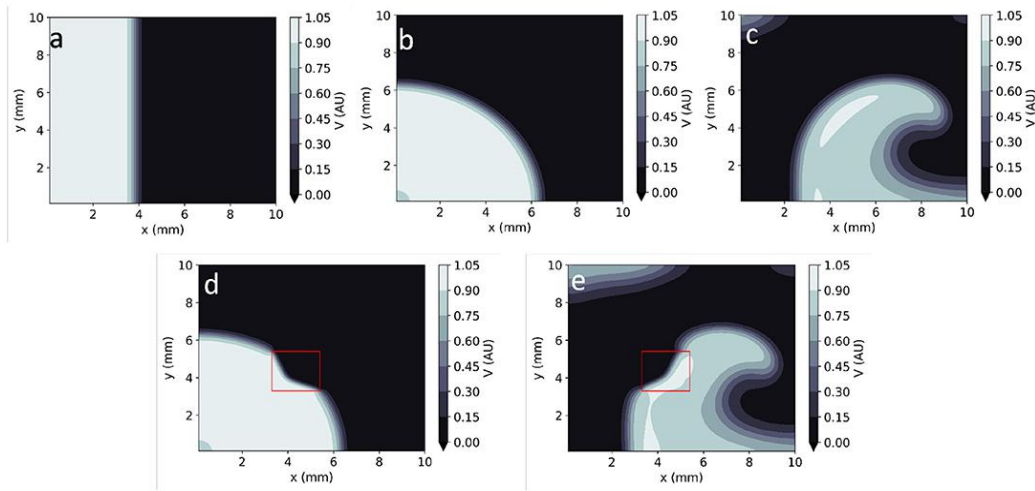
# Background: Aliev-Panfilov (AP) Model



- simplified mathematical framework for simulating the electrical activity in cardiac tissue
- $V(x, t)$  = membrane potential, represents electrical state of cardiac cells at position  $x$  and time  $t$
- $W(x, t)$  = recovery variable, models the ionic recovery process that controls the refractory period of cardiac cells
- $D$  = diffusion coefficient, dictates rate of spread of voltage across the tissue
- $k$  controls strength of nonlinearity in the system
- $a$  = threshold parameter for excitation
- $b$  = parameter which controls the behavior of the recovery variable and the duration of the refractory period
- $\epsilon(V, W)$  is a dynamic recovery rate

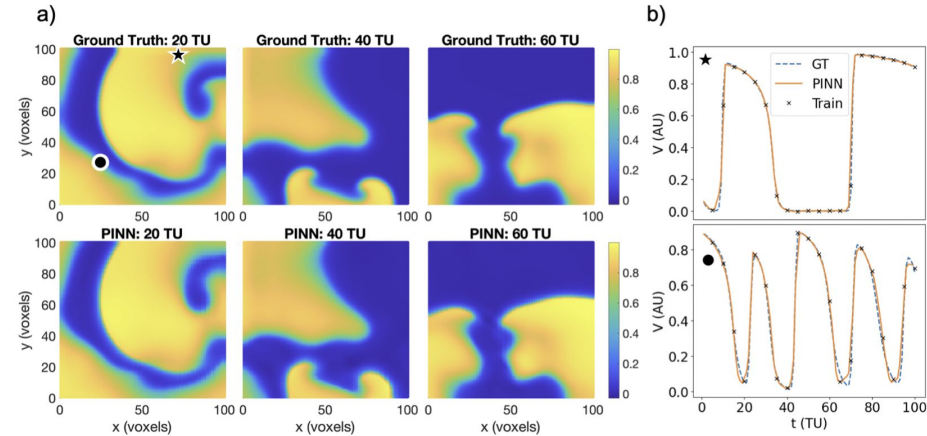
$$\frac{\partial V}{\partial t} = D \nabla^2 V - kV(V - a)(V - 1) - VW$$

$$\frac{\partial W}{\partial t} = \epsilon(V, W)(-W - kV(V - b - 1))$$



## Related Works

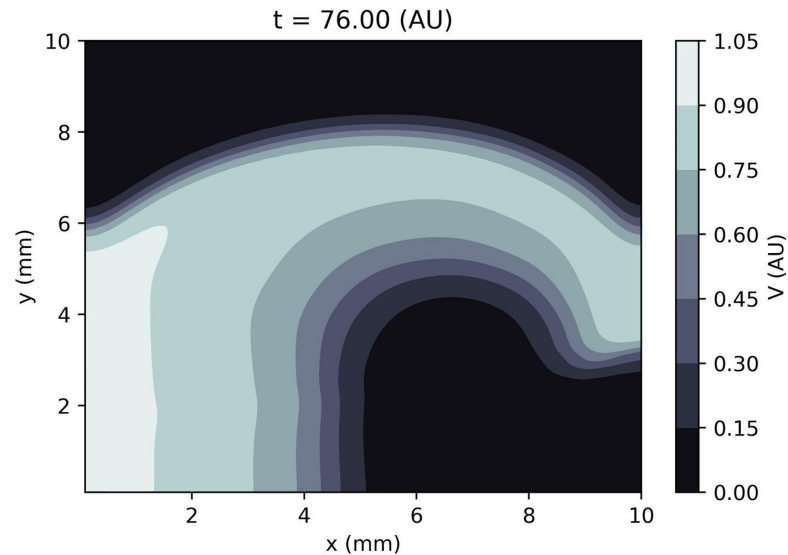
- Martin et. al. 2022 - showed PINNs can accurately approximate solution to AP model (forward problem) and somewhat accurate for inverse parameter estimation
- Chiu et. al. 2024 - showed PINNs can accurately model the effects of antiarrhythmic drug interventions using AP Model and Fenton-Karma Model (similar to AP but breaks down different ion channels)
- Chiu et. al. 2024 - showed PINNs can accurately approximate solution to FK model with 3D geometries



**Fig. 5.** Spiral wave break-up dynamics on a 2D rectangle with the AP model. (a) Comparison between the ground truth data generated by finite differences method and PINN's predictions for  $V$  at various time points (Supplementary video 3). (b)  $V(t)$  sampled at two different locations.

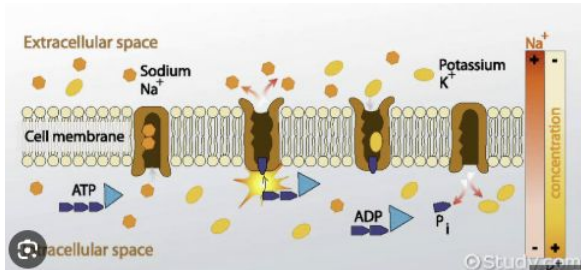
## Data Generation

- Real world data difficult and expensive to obtain
- Martin et. al. and Chiu et. al. used forward solver to generate data for the AP model, so we do the same by modifying their matlab scripts
- Generated 50 instances of AP model with varying  $b$  values, all parameters set based on clinical knowledge
- 100 x 100 mm spatial grid with 145 time steps with  $V$  and  $W$  values sampled at each point



# Applying EP-PINNS to ECG

- One of our initial goals for this project was to expand the model to utilize ecg data instead of the FD solver used to generate data
- An important value needed for the models is the Transmembrane Potential ( $V$ )
  - $W$  is a non-observable, non-diffusible variable recovery variable



$$\frac{\partial V}{\partial t} = \vec{\nabla} \cdot (D \vec{\nabla} V) - kV(V - a)(V - 1) - VW \quad (1)$$

$$\frac{dW}{dt} = \left( \epsilon + \frac{\mu_1 W}{V + \mu_2} \right) (-W - kV(V - b - 1)) \quad (2)$$



## Our Approach cont. and Roadblocks

- Initial approach was to use raw ECG data, but was ineffective
  - ECG data does not have the resolution
  - Optical Imaging datasets are limited as most are done in a wet lab environment, so data is not ML ready
- Two possible methods to extract *TMP*
  1. Inverse Problem Approach
    - a. Starts at Utilizes methods like ECG imaging to inverse solve for TMP
  2. Deep Learning Methods
    - a. Currently no out-of-box methods for our use case
    - b. Would need to design a separate model and create a pipeline







## Our Approach Pt.2

- Clinical data is sparse, so AP model is used in research to study EP properties in high resolution
  - Numerical methods for forward solvers are computationally expensive
  - Traditional inverse parameter estimation methods not as accurate
- PINNs are a somewhat better solution to this problem, but they can only learn one instance at a time
- Retraining PINNs for each instance is prohibitively expensive
- PINO can learn the function once and be deployed at relatively no cost for each patient
- Previous work more focused on forward solution, but inverse parameter estimation more clinically significant since arrhythmias easy to detect but hard to treat
- **We aim to test whether PINOs can accurately perform inverse parameter estimation for the AP model**
- Goals: Train a PINO in inverse mode to predict  $b$

# PINO Experiments

- Model: 3D Fourier Neural Operator
  - 1 Fully Connected Linear Layer - Lifting
  - 4 Spectral Convolution Layers with GeLu activation
  - 1 Fully Connected Linear Layer - Projection
  - 1 Fully Connected Linear Layer - Output
- Custom AP PDE loss function to compare  $\partial W / \partial t$  produced by estimated  $b$  with true  $\partial W / \partial t$  computed from simulation data
- Loss = data\_loss + pde\_loss
- Test accuracy: 97%

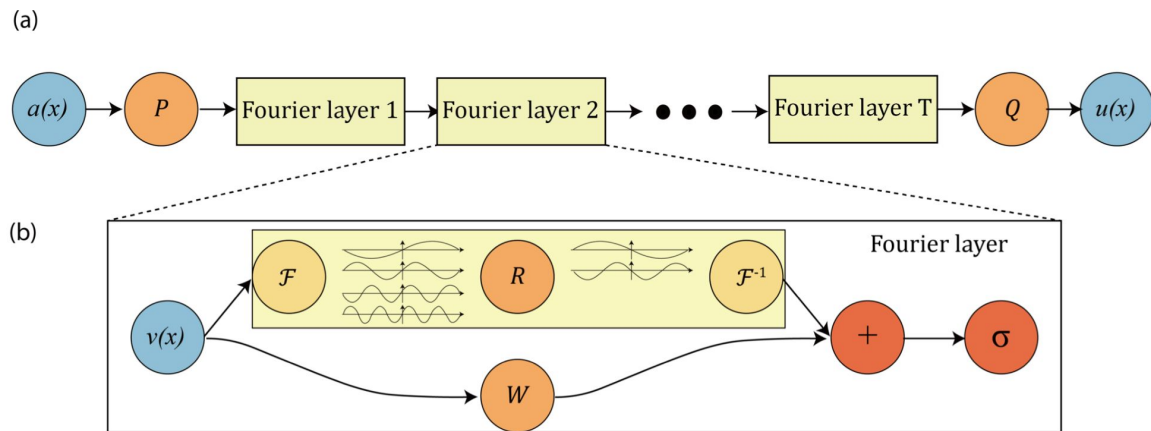


Figure 5: **top:** The architecture of the neural operators; **bottom:** Fourier layer.  
**(a) The full architecture of neural operator:** start from input  $a$ . 1. Lift to a higher dimension channel space by a neural network  $\mathcal{P}$ . 2. Apply  $T$  (typically  $T = 4$ ) layers of integral operators and activation functions. 3. Project back to the target dimension by a neural network  $\mathcal{Q}$ . Output  $u$ . **(b) Fourier layers:** Start from input  $v$ . On top: apply the Fourier transform  $\mathcal{F}$ ; a linear transform  $R$  on the lower Fourier modes which also filters out the higher modes; then apply the inverse Fourier transform  $\mathcal{F}^{-1}$ . On the bottom: apply a local linear transform  $W$ .



# Conclusion

- PINOs are indeed a viable substitute for PINNs for the task of inverse parameter estimation of  $b$  in the AP model
- Future goals:
  - Test efficacy of PINOs for other common EP tasks such as estimating  $D$  for the AP model and various parameters in the FK model
  - Preprocess other forms of cardiac data to needed format
  - Create a pipeline with other deep learning methods to utilize raw ECG data