Physics-Informed Neural Operators for Modeling Atrial Fibrillation

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ABSTRACT

Atrial Fibrillation (Afib) is a cardiac arrhythmia characterized by irregular electrical activity in the heart's atria, which causes irregular heartbeats. This can lead to health complications such as stroke, heart faliure, or blood clots. Current approaches focus on inverse parameter estimation and utilize Physics-Informed Neural Networks (PINNs) on the Aliev-Panfilov model, which is a simplified mathematical model that simulates electrical cardiac activity. This project explores the application of Physics-Informed Neural Operators (PINO) for inverse parameter estimation. Unlike PINNs, which require isolated training for each patient's cardiac data, PINO learns an operator mapping that can be trained once on multiple instances of patient cardiac data. This allows for a more computationally efficient and generalizable approach that approximates the underlying partial differential equation of the AP model. Using the AP model for synthetic data generation, this research demonstrates the viability of using PINO in inverse parameter estimation. The current state of the art, EP-PINNs, have average relative L_2 error of 8% ± 3%, while our experiments show that PINO can achieve $3\% \pm 0.11\%$. Additionally, this study examines the feasibility of training the models on clinical ECG data instead of the synthetically generated AP data, highlighting the challenges and limitations of utilizing raw patient data in clinical settings. These findings highlight the potential of PINOs to enhance cardiac electrophysiology modeling and support future advancements in personalized Afib diagnosis and treatment. Further research directions are proposed to expand these methods to 3D geometries and alternative cardiac models while addressing the intergration of clinical data with these models.

KEYWORDS

Physics-Informed Neural Network, Physics-Informed Neural Operator, Aliev-Panfilov Model, Atrial Fibrillation, Cardiac Electrophysiology, Action Potential Duration

1 INTRODUCTION

Atrial Fibrillation (Afib) is an arrhythmia, or irregular heart rhythm, that begins in your heart's upper chambers, known as the atria. Risk factors include high blood pressure, coronary artery disease and obesity, and symptoms include fatigue, heart palpitations, trouble breathing, and dizziness. Afib affects 0.5% of the world's population; therefore, diagnosing Afib is very important in clinical practice. The specialty which focuses on the electrical activity of the heart and is responsible for diagnosing and treating heart arrhythmias is known as Cardiac Electrophysiology (EP). To detect Afib, in an EP study known as cardiac mapping, electrocardiogram (ECG) sensors are placed on the surface of the atria, directly on the myocardium, to record the electrical activity of the heart's tissue and construct an arrival time (AT) map, which can show whether certain regions of the heart have abnormal EP properties which cause arrhythmia. These regions, characterized by EP changes such as low conduction velocity, heightened excitability, or shortened action potential duration (APD), can have many different causes, such as changes in membrane proteins, varying fiber composition or orientation, or organ shape. Once the region has been identified, a procedure known as catheter ablation is performed, in which a catheter is inserted into a blood vessel (usually in the groin), guided to the heart, and various medications are administered to increase or decrease the electrical activity on the surface of the atria.
However, identifying the problematic regions from the entire surface of the atria with sparse measurements of transmembrane potential is a difficult task for experts, so having a model that can accurately construct AT maps from these sparse measurements would be useful in aiding diagnosis.

2 PROBLEM DEFINITION

2.1 Our Task

Since this task requires learning the PDE for wave propagation, previous work has been done to develop models which can recreate wave propagation or estimate PDE parameters. Most recently, Physics-Informed Neural Networks (PINNs) have been shown to be successful for these tasks and useful in a clinical setting, but as we have studied in class, PINNs can have some weaknesses. Most notably, they only find the solution for a single instance of a PDE. In clinical settings, since each patient corresponds to a unique instance of a PDE, retraining a PINN for each patient is impractical. [See Figure 1 below, from [8], which illustrates this difference.]

To address some of this weaknesses, Physics-Informed Neural Operators (PINOs) were developed. As studied in class, PINOS learn a function mapping and directly approximate the solution operator, making them much more practical for use in clinical settings where the difference between patients is the parametrization of the same PDE. Therefore, we aim to use PINOs for the task of inverse parameter estimation of the wave propagation PDE. If successful, these models could be useful to aid clinicians in the process of diagnosing and treating Afib.

In addition to utilizing PINOs, we hope to use ECG on both the original EP-PINNs model and PINO. The EP-PINNS paper used synthetic data generation methods to generate data for the model. This brings up a new question on if EP-PINNs can be functional on

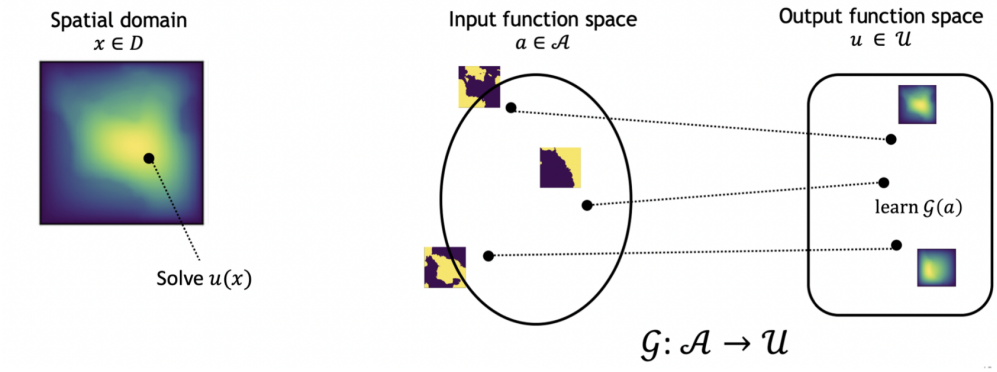

Figure 3: solve for one specific instance verse learn the entire solution operator
Left: numerical solvers and PINNs focus on solving one specific instance. Right: neural operators learn the solution operator for a family of equations.

Figure 1: [8, Figure 3]

real world data. ECG data is often collected from patients in various settings where equipment and specific procedures may vary from hospital to hospital. Testing if EP-PINNS is extendable to real-world data makes it more feasible to show that deep learning models, such as EP-PINNs, can be effectively integrated into health care systems.

2.2 Aliev-Panfilov (AP) Model

As described later on, our work revolves around the AP model, which we will describe here for reference. Full details and the original source can be found in [1].

The Aliev-Panfilov model is a simplified mathematical framework for simulating the electrical activity in cardiac tissue, especially in the context of studying arrhythmias like atrial fibrillation. This model is often chosen for its balance between computational efficiency and biological relevance, making it possible to simulate large areas of heart tissue while capturing essential dynamics like wave propagation and reentrant circuits. It consists of 2 PDEs that describe the voltage changes across the cardiac cell membrane.

$$\frac{\partial V}{\partial t} = D\nabla^2 V - kV(V-a)(V-1) - VW$$

$$\frac{\partial W}{\partial t} = \epsilon(V,W)(-W - kV(V-b-1))$$

- $V(x,t)$ is the membrane potential which represents the electrical state of cardiac cells at position x and time t. This variable models the voltage across the cell membrane, where changes in voltage represent the wave of excitation traveling through the tissue. Cardiac cells undergo rapid depolarization when excited, followed by a slower repolarization, which is captured in this model by the cubic nonlinear term $V(V-a)(V-1)$.
- $W(x,t)$ is the recovery variable which models the ionic recovery process that controls the refractory period of cardiac cells, meaning the time it takes for cells to reset after firing. This prevents immediate re-excitation, a property crucial for maintaining ordered wave propagation.
- D is the diffusion coefficient which dictates the rate of spread of the voltage across the tissue. $D\nabla^2 V$ represents the spread of electrical signals across cardiac tissue. Higher values of D indicate faster signal propagation, which is important for ensuring the timely spread of excitation through the heart muscle.
- k is the parameter controlling the strength of nonlinearity in the system.
- a is the threshold parameter for excitation which influences when cells are activated.
- b is the parameter which controls the behavior of the recovery variable and the duration of the refractory period.
- $\epsilon(V,W)$ is a dynamic recovery rate, often defined as $\epsilon(V,W) = (\epsilon_0 + \frac{\mu_1 W}{V+\mu_2})$ where ϵ_0, μ_1, and μ_2 are constants that adjust the refractory period based on the tissue's current state.

This model's simplicity allows for efficient computation of the spatiotemporal patterns of electrical activity, such as wave propagation and reentrant spirals, making it useful for training PINNs to detect Afib.

To generate our training data, we used similar values for each parameter as were used in [9], which were obtained by working with clinicians about what would be most accurate. Since we are interested specifically in Afib, which is usually characterized by a high refractory period, each run used a different randomly initialized value for b from a range specified in [9], where the low end of the range represented normal sinus rhythm and the high end represents highly irregular arrhythmia.

3 RELATED WORK

Since this is an expensive, invasive procedure, researchers have been working to develop models of electrical wave propagation through the surface of the heart. Initial ML research in this field

used knowledge-guided ML (KGML) [3] and showed that it is possible to learn the eikonal equation, a first order PDE that describes wave propagation, which can be used to efficiently construct an AT map. [5] used KGML to learn the atrial fiber orientation, which can also help identify key portions of the AT map.

The first attempts to use more advanced ML techniques like PINNs [10] attempted to learn the Aliev-Panfilov (AP) model [1], which gives a PDE for pulse propagation through cardiac tissue, and showed the efficacy of PINNs for modeling 1D and 2D single waves and spiral waves in homogeneous and heterogeneous conditions, and for modeling the effects of administering channel blockers at different regions of the heart [9]. More recent research has focused on learning the Fenton-Karma (FK) model [4], which gives PDEs for wave behavior in cardiac action potential for 3 different ions [2]. This paper showed the efficacy of PINNs for learning the AP model on 3D spherical geometries including centrifugal waves (most similar sinus rhythm, i.e. normal heart rate) and spiral waves (most similar to tachycardia, i.e. elevated heart rate), and 2D rectangular geometries for spiral wave breakup (most similar to fibrillation). It also showed the efficacy of PINNs for learning the FK model for 2D rectangular geometries for planar waves (sinus rhythm) and spiral waves (tachycardia rhythm). Lastly, it showed the efficacy of PINNs for predicting the effect of administering medication via catheter ablation in different regions by predicting a variable in the FK model with 2D rectangular geometries with planar waves and spiral waves.

As mentioned earlier, these methods can be improved upon by using PINOs [8] instead of PINNs. The main advantage is that they learn the entire operator, making them much less expensive than numerical methods for solving PDEs. They also have the advantage of being discretization-agnostic. In clinical settings, where the primary goal is inverse parameter estimation for diagnosing the cause of Afib and informing the dosage required to treat Afib, these properties are useful because they eliminate the need to retrain a model for each patient, who each present a unique instance of the wave propagation PDE. The discretization-agnostic property also allows for flexibility when recording ECG data, as different circumstances may make it difficult to collect data in the exact same manner for each patient. [7] also outlined other weaknesses of PINNs, as discussed in class. Namely, they neglect to incorporate temporal dependencies, struggle with high-frequency/multi-scale features, and have highly complex loss landscapes, which makes optimization difficult. Though we discussed various optimization techniques like the ones presented in the paper (curriculum regularization or seq2seq learning) and in class (using models with better temporal encoding such as PINNsFormers), PINOs are better suited for the problem at hand because of the clinical advantages discussed earlier.

4 METHOD

4.1 PINO

As briefly mentioned earlier, our goal is to train a PINO for the task of inverse parameter estimation to help clinicians detect Atrial Fibrillation. We will do this by using the AP model to generate many instances of wave propagation where each run has a different randomly chosen value of b indicating varying levels of Afib severity. Then, we will train a PINO by providing the model with the initial conditions, boundary conditions, solution space (which includes the values of V and W over the spatiotemporal domain), and a mesh of the spatiotemporal domain. The model will use this to generate a predicted value of b, and we will measure loss by a linear combination of data loss and PDE loss, where data loss measures how close the predicted b value was to the true value, and PDE loss uses a custom AP loss function to measure how far off the predicted value of $\partial W/\partial t$ (based on the predicted b and observed V) is based on the observed value of $\partial W/\partial t$ (computed using automatic differentiation). Then, we measure the accuracy of the model using the same data loss and AP loss functions to calculate relative L_2 error and PDE error (to match the way results are reported in [8] and [9].

Our methods for building the PINO are the same as [8], namely, using Fourier Neural Operators with physics-informed loss functions. Intuitively, we believe this should outperform EP-PINNs because they are trained on several instances of the PDE solution, so the input data is significantly more robust. Moreover, each instance having varying values of b helps the model learn more specifically how b affects the solution, rather than having to isolate this effect based on only the PDE.

4.2 ECG Data

Another primary objective of this project was to expand the models data usage to be also function on ECG data as an alternative to the synthetic data generation. A critical variable for the AP model is the transmembrane potential represented by V, which plays a crucial role in capturing electrical activity across cell membranes. Our initial approach began with utilizing raw ECG data to directly estimate TMP. Though this faced several challenges as ECG data lacks the necessary spatial and temporal resolution to extract TMP values.

We then shifted to exploring the inverse problem approach and deep-learning methods to estimate TMP. Current inverse problem approaches are designed to be used at the data collection level where extra equipment is needed to record the TMP [6]. We explored for additional datasets, though most methods did not have data that was collected on humans or would work with the AP model.

Deep Learning methods also faced similar roadblocks as no pre-existing models or pipelines were tailored to our specific use case. Implementing this solution would require designing a separate DL framework to extract the TMP features, which was not possible due to time constraints.

4.3 Data Generation

[2] and [9] use numerical forward solvers of the AP model and FK model (described further below) over various geometries to generate training data for their experiments, since collection of real data is invasive and expensive. These various scripts generate data along various factors, including geometry, stimulation points, presence of heterogeneities, and model used. Different combinations of these factors represent different heart conditions. The geometries include 1D cable, 2D square, and 3D sphere. The stimulation points are used to create either planar wave diffusion (regular sinus rhythm,

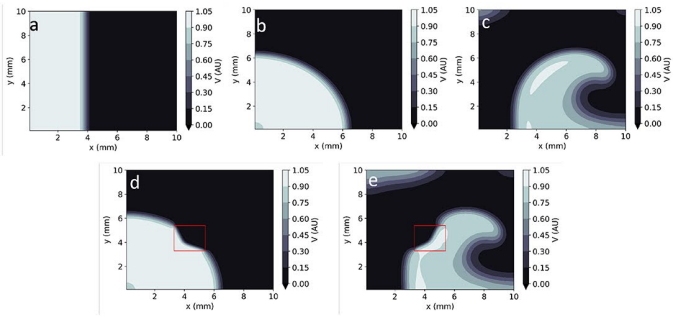

Figure 2: (a) Planar wave. (b) Centrifugal wave. (c) Spiral wave. (d) Centrifugal wave in the presence of a square heterogeneity in D. (e) Spiral wave in the presence of a square heterogeneity in D. [9, Figure 1]

in which is heart is beating normally), spiral wave diffusion (tachycardia, where the heart rate is elevated), or spiral wave breakup (atrial fibrillation, where the heart rate is irregular). The heterogeneities represent the presence of fibrosis, scar tissue, etc., which can affect different EP properties (e.g. a buildup of scar tissue on a portion of the heart's surface caused by a heart attack causes that area to diffuse the wave more slowly, creating atrial fibrillation). The models used are the AP model and the FK model, which each have their own strengths for modelling different heart conditions. Since we are primarily interested in Afib, we used the solver for the AP model for 2D spiral wave breakup to generate 50 simulations of wave propagation over the surface of the heart, each with different randomly chosen values of b, so that the PINO can be trained to estimate b in various conditions, i.e. aiding clinicians in diagnosing Afib by approximating the extent to which atrial recovery is damaged by extended refractory period and how much medication may be required to reduce the refractory period to normal. In the image below, which shows one timestep in various data generation configurations, a corresponds to planar wave, b corresponds to centrifugal wave (an in-between step in the AP model between planar wave and spiral wave breakup, not useful for our purposes), c corresponds to spiral wave breakup, d corresponds to a centrifugal wave in the presence of a heterogeneity, and e corresponds to a spiral wave breakup in the presence of a heterogeneity.

5 EXPERIMENTS

The PINO we used was similar to the one used in [8] for Darcy Flow since our problem also works on a similar spatiotemporal domain (2D spatial grid and 1 Time dimension). The model consists of 1 Linear projection layer, 4 Fourier Spectral Convolution Layers, 1 Linear projection layer, and 1 final linear layer. See Figure 3, from [8] for a reference visualization of the model architecture. The data was generated over a 100x100 spatial grid with $h = 0.1$mm and $T = 145$ timesteps, with $dt = 0.005$. For the homogeneous zone, $D = 0.1$ was used to represent no change in diffusion properties while for the heterogeneous zone, $D = 0.02$ was used to represent an 80% reduction in diffusion. The following AP model parameters were also used: $a = 0.01, k = 8, \mu_1 = 0.2, \mu_2 = 0.3, \epsilon = 0.002$. For the inverse parameter estimation task, b was sampled uniformly at random 50 times from the range $[0.075, 0.15]$ for each instance. This configuration was used for our experiments to match the experimental setting in [9]. 80% of the data was used for training and 20% was used for testing using the scikit-learn train_test_split function.

Results: Our mean relative L_2 error for the task of estimating b in a homogeneous 2D environment with spiral wave breakup was 0.03 with a standard deviation of 0.0011. In [9], when using PINNs for the same task, Martin, et. al reported mean relative L_2 error of 0.08 with a standard deviation of 0.03. Therefore, our experiments show that PINOs do indeed outperform PINNs for this task. Our mean relative L_2 error for the task of estimating b in a heterogeneous (a small subsection of the region having a different D value) 2D environment with spiral wave breakup was 0.04 with a standard deviation of 0.007. In [9], when using PINNs for the same task, Martin, et. al reported mean relative L_2 error of 0.10 with a standard deviation of 0.02. Therefore, our experiments show that PINOs outperform PINNs for this task as well. These results are promising since they imply that PINOs can achieve performance as least as well as PINNs while being significantly less computationally expensive since they can generalize to multiple instances of the PDE rather than needing to retrain a PINN for each instance. We were not able to verify the exact factor of computational time saved since we were not able to train and test PINNs for this task, but according to [8], PINOs are 3000x more computationally efficient than PINNs for some complex problems.

Method	Homogeneous	Heterogeneous
EP-PINN	8% ± 3%	10% ± 2%
PINO	3% ± 0.11%	4% ± 0.7%

6 CONCLUSION AND DISCUSSION

Our results show that PINOs are a promising replacement for PINNs in the field of EP property estimation. In the future, we aim to test their efficacy on some of the other common uses of PINNs in EP studies, such as detecting the factor by which diffusion may be reduced in certain regions of the heart, which essentially boils down to inverse estimation of D. PINNs were shown to be somewhat successful for this task (no worse than 30% error) by [9], so we would like to test PINOs and find similar results. Similarly, we would also like to test their efficacy with 3D geometries and the Fenton-Karma model, which can also generalize to different heart conditions based on the geometry and is more useful for identifying changes in specific ion channels and can therefore help decide the dosage of different medications that target these specific ion channels. PINNs were shown to be successful in these situations by [2], detecting tachycardia as well as Afib, so we would like to test the efficacy of PINOs for these tasks. Outside of inverse parameter estimation, because PINOs approximate the solution, they can be deployed in forward more as replacements for numerical solvers, so we would like to measure exactly how much faster they run and how accurate they are at producing solutions for 2D and 3D geometries modeling various heart conditions.

As of now, our results are only useful in research settings to due to limitations of generalizing the AP model domain to real-world data, so our future goals in this work would be to create a pipeline

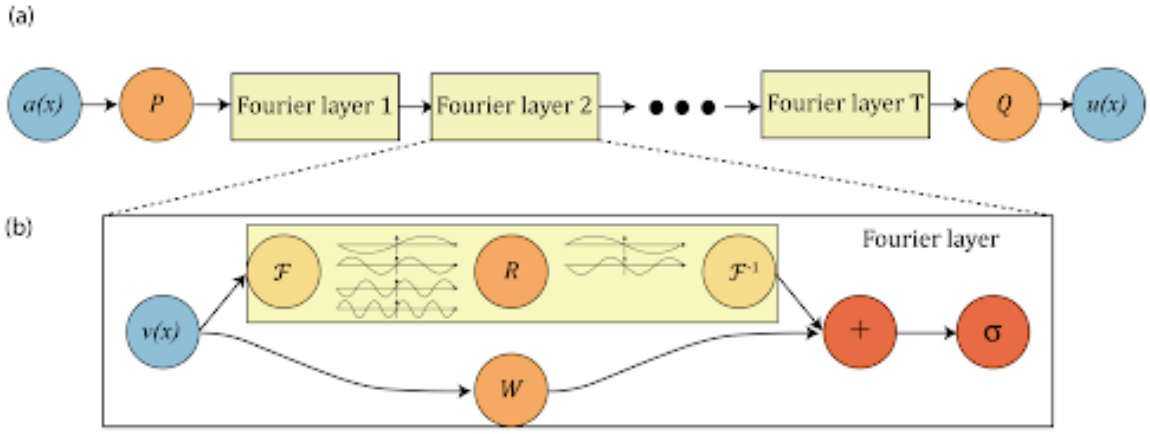

Figure 5: **top:** The architecture of the neural operators; **bottom:** Fourier layer. **(a) The full architecture of neural operator**: start from input a. 1. Lift to a higher dimension channel space by a neural network $\mathcal{P}$. 2. Apply T (typically $T = 4$) layers of integral operators and activation functions. 3. Project back to the target dimension by a neural network Q. Output u. **(b) Fourier layers**: Start from input v. On top: apply the Fourier transform $\mathcal{F}$; a linear transform R on the lower Fourier modes which also filters out the higher modes; then apply the inverse Fourier transform $\mathcal{F}^{-1}$. On the bottom: apply a local linear transform W.

Figure 3: [8, Figure 5]

to pre-process ECG data that can then be directly passed into either EP-PINNs or PINO. A recent paper utilized a Geometry-Informed Deep Learning Framework to estimate TMP directly. This method could be a better approach as other data collection methods like Optical Mapping are intravenous collection methods that are not feasible. By using an external framework, more focus can be shifted to using the standard data collected in health care checkups.

Our research also raises interesting questions on if EP models are at a state to be implemented in health care systems. ECG, which is a commonly used test and widely available proved to be unusable for current models in this domain. With current models not being able to use ECG, and no non-trivial way to process the data into a desired manner, it seems that these models will not be directly valuable to patients. More research should be focused on using routine health data, since intravenous measurement methods and high computational processing methods can be unfeasible for immediate patient benefit.

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