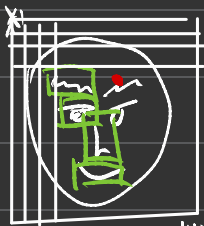Convolutional Neural Network:

motivation: $X_1, X_2, \dots X_p$: P is dimension of data.
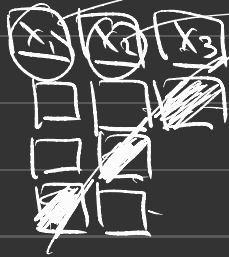
may be sparse and do not represent REAL information
of the data

$\longleftarrow$ Dad

$100 \times 100 = 10000$

2 Ways: (i) Pixels as X

Flatten
to construct
input layer

$\longrightarrow$ NN

(ii). A transformation function: Convolution.

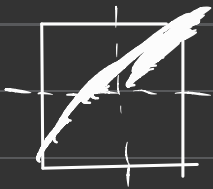→ an element-wise matrix multiplication : ⊗

$A: \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

$B: \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Note: B is artificially designed.

Def'n $A \otimes B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$.

$B: \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A \otimes B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$.
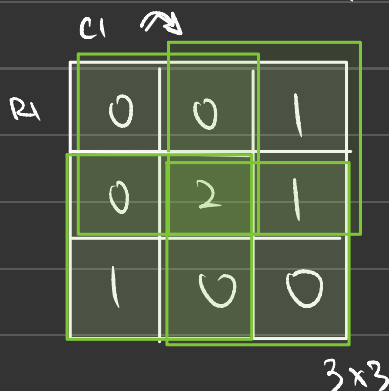
Suppose in Picture

Pixel data.
↓

in matrix form
→

$\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$.

To extract $\{1, 3\}$ out of $\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$,

we create a filter (kernel) which is the matrix B.

You already know Convolution, how to extract feature.
Now you want do it for every location of the picture.

C1

R1

0	0	1
0	2	1
1	0	0

3×3

filter: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\longrightarrow$

0	3/4
3/4	1/4

2×2

✓ max pooling
✗ average pooling

$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\frac{1}{4}(0+0+0+0)=0$ ✓

$\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ $\frac{1}{4}(0+1+2+0)=\frac{3}{4}$ ✓

$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ $\frac{3}{4}$ ✓

$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\frac{1}{4}$ ✓