HW 5

→ Recurrance Relations

– Master's theorem

(1) $T(N) = 2T(N-1) + 1$ $a > 1$; $k=0, a=2, b=1$

- $T(N) = O(n^k \cdot a^{\frac{n}{b}})$
- $T(N) = O(n^0 \cdot 2^n)$
- $\boxed{T(N) = O(2^n)}$

(2) $T(N) = 3T(N-1) + n^1$ $a > 1$; $k=1; a=3; b=1$

→ $f(n) = O(n^1)$

- $T(n) = O(n^k \cdot a^{\frac{n}{b}})$
- $T(n) = O(n^1 \cdot 3^{\frac{n}{1}})$

→ $\boxed{T(n) = O(n3^n)}$

(3) $T(N) = 9T(\frac{N}{2}) + N^2$ $a = 9$, $b = 2$, $d = 2$

– $f(n) = O(N^2)$

– $\log_2 9 > 2$

case 3:

→ $T(n) = \theta(n^{\log_b a})$

$\boxed{T(n) = \theta(n^{\log_2 9})}$

(5) $T(N) = 4T(\frac{N}{2}) + n^2 \log n$

$f(n) = \theta(n^k \log^p n)$ $k \geq 0$

$a = 4$ $k = 2$

$b = 2$ $p = 1$

$\log_b a = \log_2 4 = 2$

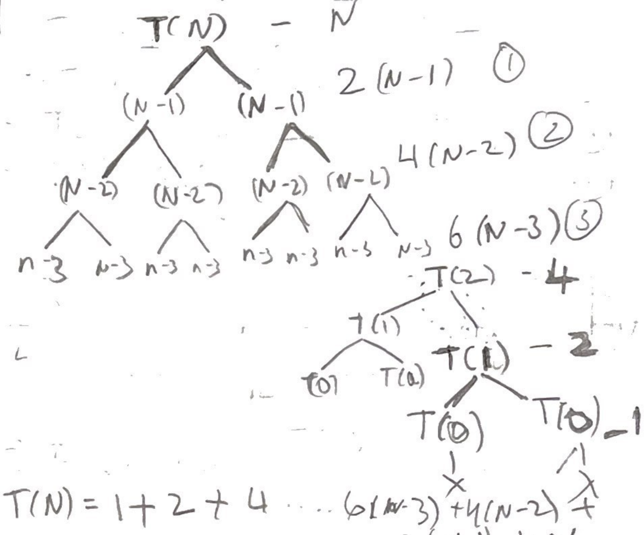
since $2 = 2$ & $p > -1$ → case 2

$\boxed{T(n) = \theta(n^2 \cdot \log^2(n))}$

Masters theorem.

– Algebric or Tree Method

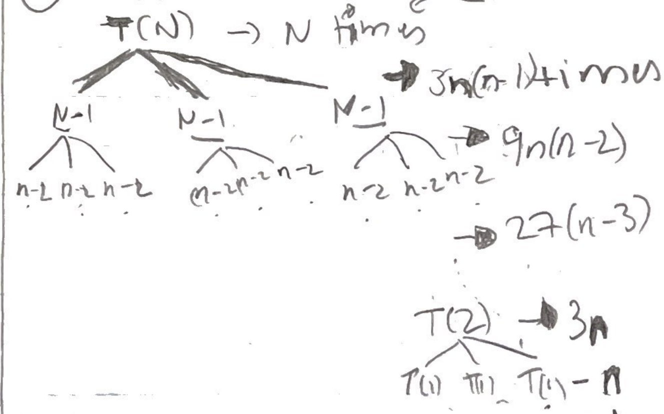
(1) $T(N) = 2T(N-1) + 1$



$T(N) = 1 + 2 + 4 \ldots 6(N-3) + 4(N-2) + 2(N-1) + N$

→ Follows 2^n → $\boxed{O(2^n)}$

(2) $T(N) = 3T(N-1) + n$



$T(N) = 1 + n + 3n + 9n \ldots\ldots 9n(n-2) + 3n(n-1) + N =$

$\boxed{T(N) = O(n3^n)}$

Masters Theorem

$T(N) = 100T\left(\frac{N}{2}\right) + n^{\log_2(N+1)}$

$f(n) = \theta(n^k . \log^p n)$ = 1

Since $\log^p n = 1 \rightarrow \boxed{p=0}$

- $a = 100$
- $b = 2$
- $\log_b a = \log_2 100$
- $k = \log_2 c.N + 1$
- $p = 0$

$\rightarrow \log_2 100 < \log_2 c.N + 1$ [k]

$\rightarrow$ Case 3: and $p \geq 0 \rightarrow T(N) = \theta(n^k \log^p n)$

$\rightarrow T(N) = \theta(n^{\log_2 c.N + 1} . \log^0(n))$

$\rightarrow \boxed{T(N) = \theta(n^{\log_2 c.N+1})}$

⑥ $T(N) = 5T\left(\frac{N}{2}\right) + \frac{n^2}{\log(n)}$

rewrite: $T(N) = 5T\left(\frac{N}{2}\right) + n^2 . \log^{-1} n$

$f(n) = \theta(n^k \log^p n)$

- $a = 5$
- $b = 2$
- $k = 2$
- $p = -1$
- $\log_2 5 = 2.32$

$\rightarrow$ Since $\log_b a$ [2.32] > k [2]

$\rightarrow$ Case 1

$\rightarrow T(N) = \theta(n^{\log_b a})$

$\rightarrow \boxed{T(N) = \theta(n^{\log_2 5})}$

Alg or tree

③ $T(N) = 9T\left(\frac{N}{2}\right) + n^2$

$\therefore T\left(\frac{N}{2}\right) = 9T\left(\frac{N}{2^2}\right) + \frac{n^2}{2}$ Substitute

- $T(N) = 9\left(9T\left(\frac{N}{2^2} + \frac{n^2}{2}\right)\right) + n^2$
- $T(N) = 9^2T\left(\frac{N}{2^2} + n^2\right) + n^2$
- $T(k) = 9^k T\left(\frac{N}{2^k}\right) + \frac{n^2}{2^{k-1}} + \ldots + n^2$

Assume $\frac{N}{2^k} = 1$

$T(N) = T(1) + 9n + 9\frac{n^2}{2^{k-1}} \ldots$

$\boxed{T(N) = \theta(n^{\log_2 9})}$

④ $T(N) = 100T\left(\frac{N}{2}\right) + n^{\log_2(N+1)}$

$\rightarrow$ same logic as above, only different changes $\rightarrow a = 100$, n

$\therefore T(N) = \theta(n^{\log_2 CN + 1})$

⑤ $T(N) = 4T\left(\frac{N}{2}\right) + n^2\log(n)$

$T(N)$

$\frac{N}{2}$ $\frac{N}{2}$ $\frac{N}{2}$ $\frac{N}{2}$ $\quad 4\left(\frac{n}{2}\right)^2 \log\left(\frac{n}{2}\right)$

$4(1)$

$T(N) = 1 + 4 + \ldots 4\left(\frac{n}{2}\right)^2 \log\left(\frac{n}{2}\right) + n$

$\boxed{T(N) = \theta(n^2 \log^2(n))}$

Problem 2:

yet Another (n): $\rightarrow T(N)$

if $n > 1$ $\rightarrow$ c - ignore

for (int i = 0; i < 10n; i++) $\rightarrow$ need to find

do sum

yet Another (n/2) $\rightarrow T(N/2)$

yet Another (n/2) $\rightarrow T(N/2)$

$T(N) = 2T\left(\frac{N}{2}\right)$ + need to find

→ **Problem 2**: continue: find time complexity of the for loop.

- for (i = 0 ; i < 10n ; i++)
  - do sum

iterations	i
1	0
1	1
1	2
1	3
⋮	⋮
10k	10k − 1

→ Let $10k > n$

$k > n/10$

→ follows $\Theta(N)$ → $f(n) = N$

continue: $T(N) = 2T\left(\frac{N}{2}\right) + N^1$

- $a = 2$ · $k = 1$ · $\log_b^a = 1$
- $B = 2$ · $p = 0$

→ Apply Masters theorem

→ $f(n) = \Theta(n^k \log^p n)$

→ $f(n) = \Theta(N^1 \cdot \log^0(n))$

→ $\log_b^a[1] = k[1]$ AND $p > -1$ ⇒ case 2, a

→ $T(N) = \Theta(n^k \cdot \log^{k+1}(n))$

→ $\boxed{T(N) = \Theta(n \cdot \log^2(n))}$

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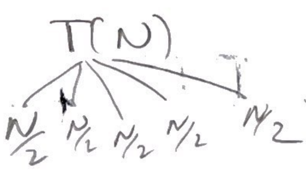
(6) Tree Method → Part 1 Assume $n - k = 0$

→ $T(N) = 5T\left(\frac{N}{2}\right) + \frac{n^2}{\log(n)}$



$5\left(\frac{\frac{n^2}{2}}{\log\left(\frac{n}{2}\right)}\right)$ (1)

$5^2\left(\frac{\frac{n}{4}^2}{\log \frac{n}{4}}\right)$ (2)

$5^k\left(\frac{\frac{n^2}{2^k}}{\log \frac{n}{k}}\right)$ (k)

$T(N) = 1 + \frac{\frac{n^2}{2^k}}{\log \frac{n}{k}} + \frac{\frac{n^2}{2^{k-1}}}{\log \frac{n}{k-1}} \ldots \frac{\frac{n^2}{2}}{\log(n)} + n$

$\boxed{T(N) = O(n^{\log_2 5})}$